Answers

For full worked solutions, visit:
www.scholastic.co.uk/gcse

Number

Factors, multiples and primes

1 **a** 5 **b** 1, 12 **c** 1, 5, 45
2 HCF = 10, LCM = 1050
3 $2 \times 3^2 \times 5$
4 **a** 10 **b** 840
5 12 and 18

Ordering integers and decimals

1 **a** false **c** true **e** true
b true **d** true
2 −0.3, −1.5, −2.5, −4.2, −7.2
3 0.049, 0.124, 0.412, 0.442, 1.002
4 **a** < **b** < **c** >

Calculating with negative numbers

Stretch it! negative, yes

1 **a** −11 **c** −6 **e** 0
b 99 **d** 18 **f** 25
2 −8 and 9
3 32°C

Multiplication and division

Stretch it! 148 419

1 **a** 2115 **b** 56 364
2 **a** 47 **c** 126 remainder 4 or $126\frac{4}{17}$
b 516
3 **a** 33 boxes **b** 1 pencil
4 £91.25
5 £288
6 $307\frac{2}{3}$
7 28 805
8 37 boxes
9 He has not placed a zero in the ones column before multiplying through by 5.

Calculating with decimals

Stretch it! 18.2

1 **a** 2.33 **c** 0.035 **e** 1.563
b 24.391 **d** 6.099
2 £4.64
3 Erica: £54.92; Freya: £27.46

Rounding and estimation

Stretch it! **a** 1.0 **b** 1.00 **c** 1.000 – they are all 1

Stretch it! 55.25 m² – an overestimate.

1 **a** 0.35 **c** 32.6
b 10 **d** 33 100
2 **a** $150 \leq x < 250$ **c** $3.15 \leq x < 3.25$
b $5.5 \leq x < 6.5$ **d** $5.055 \leq x < 5.065$
3 $\frac{30}{0.5 \times 6} = 10$
4 b is false since $18 \times 1 = 18$ so 18×0.9 cannot be 1.62
c is false since if you divide by a number smaller than 1 the answer will be larger.
5 Tarik should choose One tariff.

Converting between fractions, decimals and percentages

Stretch it! $0.\dot{1}$, $0.\dot{2}$, $0.\dot{3}$, … $0.\dot{4}$, $0.\dot{5}$

1 **a** $\frac{32}{100} = \frac{8}{25}$ **c** $\frac{33}{100}$
b $1\frac{24}{100} = 1\frac{6}{25}$ **d** $\frac{95}{100} = \frac{19}{20}$
2 **a** $0.41\dot{6}$ **c** 0.49 **e** $0.\dot{4}28571\dot{4}$
b 0.375 **d** 0.185
3 **a** 91% **c** 80%
b 30% **d** 60%
4 37.5%
5 30%, 0.35, $\frac{2}{5}$
6 $\frac{15}{20} = \frac{75}{100} = 75\%$ – Amy
Rudi was highest

Ordering fractions, decimals and percentages

1 $\frac{7}{12}, \frac{3}{8}, \frac{1}{3}$
2 $-2.2, -\frac{1}{10}$, 1%, 0.1, 15%, $\frac{1}{5}$, 7 (so the middle is 0.1)
3 Yes, if the numerator of a fraction is $\frac{1}{2}$ the denominator the fraction is equivalent to $\frac{1}{2}$. If the numerator is smaller than this the fraction must be smaller than $\frac{1}{2}$.

Calculating with fractions

Stretch it! No, you could add the whole number parts then the fraction parts, giving:
$1 + 2 = 3$
$\frac{3}{5} + \frac{1}{4} = \frac{17}{20}$
$= 3\frac{17}{20}$

1 **a** $1\frac{5}{8}$ **c** $\frac{10}{21}$ **e** $\frac{2}{25}$
b $\frac{6}{17}$ **d** $8\frac{3}{20}$
2 **a** 12 **b** £35 **c** 808 mm
3 20
4 35

Percentages

1 **a** 1.8 cm **b** £0.30 **c** 4 ml
2 **a** 33 **b** 540 **c** £101.92
3 **a** 480 **b** 133 **c** £14.58
4 3052
5 £14 300

Order of operations

1 **a** 7
b −1.9
c −13
2 30
3 (8 − 3 + 5) × 4

Exact solutions

1 **a** π
b 36π
c $2\frac{1}{2}\pi$
2 **a** 7π
b $\frac{5}{8}\pi$
3 Area $= \frac{6}{28} = \frac{3}{14}\text{cm}^2$. Perimeter $= 2\frac{1}{14}\text{cm}$
4 **a** 18π cm
b $144\pi\,\text{cm}^2$
5 $\frac{1}{2}\pi\,\text{cm}$

Indices and roots

1 **a** $\frac{1}{3}$
b $2\frac{1}{2}$
c $1\frac{1}{9}$
2 $1^3, \sqrt[3]{8}, \sqrt[3]{27}, 3^2$
3 **a** −8 **c** 81
b 1 **d** 1
4 **a** $\frac{1}{4}$ **c** 1
b $\frac{1}{49}$ **d** $\frac{1}{3}$
5 5^4

Standard form

1 **a** 45 000 000 **b** 0.091
2 **a** 6.45×10^8 **b** 7.9×10^{-8}
3 345 800
4 3.1×10^{-2} 3.09×10 $3 + (2.1 \times 10^2)$ 3.2×10^2
5 3×10^8
6 $0.022 = 2.2 \times 10^{-2}\text{m} = 2.2\,\text{cm}$

Listing strategies

Stretch it!
red + small, red + medium, red + large,
green + small, green + medium, green + large,
blue + small, blue + medium, blue + large.

1 111
112, 121, 211, 113, 131, 311

222
221, 212, 122, 223, 232, 322

333
331 313 133 332 323 233

123 132 213 231 312 321

2 444 446 449
464 466 469
494 496 499
3 Small A, Small B, Small C, Small D
Medium A, Medium B, Medium C, Medium D
Large A, Large B, Large C, Large D.

Review it!

1 7 and 6 (or 11 and 2, where both are prime and 2 is also a factor of 12)
2 $630 = 2 \times 3 \times 3 \times 5 \times 7 = 2 \times 3^2 \times 5 \times 7$
3 $18 = 2 \times 3 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
$40 = 2 \times 2 \times 2 \times 5$
HCF = 2
4 −11.5, −8.3, −3.5, −3.2, 1.4
5 **a** £51.73
b £18.33
6 **a** 3.22
b 4023
7 27
8 $31\frac{4}{11}$
9 **a** 0.375
b 70%
10 **a** $\frac{7}{10}$
b $\frac{4}{5}$
11 All of them
12 **a** $\frac{26}{35}$
b $1\frac{1}{2}$
c $1\frac{1}{2}$
13 0.25 − 0.07
14 $\frac{3}{4}$
15 $\frac{9}{200}$
16 £2.15
17 **a** 9 **b** 5
18 **a** 3.4×10^9 **b** 3.04×10^{-7}
19 $37.55 \leq x < 37.65$
20 **a** 51
b 12, 15, 21, 51, 25, 52
21 **a** $200 \times 9 \times 10 = 18\,000 = £180.00$
b Underestimate since all numbers were rounded down.
22 240
23 35%
24 no since 2 is a prime number and odd + odd + even = even
25 £279.20
26 **a** 3.1 **b** 3.05
27 **a** 325 000 **b** 320 000
28 729
29 £16.62
30 3420
31 **a** 2010 and 2011 **b** $1.1 \times 102.3 = 112.53$

Algebra

Understanding expressions, equations, formulae and identities

1 **a** $3a + 6 = 10$ **c** $3(a + 2)$
b $C = \pi D$ **d** $3ab + 2ab = 5ab$

2 James is correct.

$4x - 2 = 2x$ can be solved to find the value of x so it is an equation.

Or, the two sides of $4x - 2 = 2x$ are not equal for all values of x so it cannot be an identity.

Simplifying expressions

Stretch it! $12t \times t \times t$, $2t \times 6t \times t$, $2t \times 3t \times 2t$, $3t \times 4t \times t$

1 **a** p^3 **c** $12ab$ **e** $-8g^2$
b $28bc$ **d** $20x^2$ **f** $6pqr$

2 **a** $5x$ **c** 6 **e** $4x$
b $-7w$ **d** $4n$ **f** $9a$

Collecting like terms

1 **a** $5f$ **g** $2x^2$
b $7b$ **h** t^3
c $5mn$ **i** $7\sqrt{x}$
d $4d - 2e$ **j** $3\sqrt{x}$
e $5x + 3y - 2$ **k** $7\sqrt{x}$
f $-3a - b - 3$

Using indices

1 **a** x^9 **c** $6m^8$ **e** u^3
b p^5 **d** $15m^6n^4$ **f** t

2 **a** x^2 **d** $2x^3$ **g** $\frac{x}{3}$
b y^4 **e** $\frac{1}{m^2}$
c p **f** $\frac{x^4}{3}$

3 **a** x^6 **c** p^{10} **e** $\frac{1}{x^6}$
b y^{16} **d** $16m^{10}$ **f** n^8

4 **a** $6x^3$ **c** $\frac{x}{4}$ **e** $\frac{1}{y^4}$
b x^7 **d** $27y^9$ **f** a^5b^3

Expanding brackets

Stretch it!

1 $(x + 2)(x + 4) = x^2 + 6x + 8$

2 **a** $2x^2 + 8x + 6$ **c** $6x^2 + 7x - 3$
b $3x^2 + 10x - 8$

1 **a** $3a + 6$ **e** $4x + 4y + 8$
b $4b - 16$ **f** $-2y - 4$
c $10c + 25$ **g** $x^2 - 2x$
d $6 - 2e$ **h** $2a^2 + 10a$

2 **a** $8x + 26$ **b** $17y - 3$ **c** $2m + 31$

3 **a** $x^2 + 5x + 6$ **c** $a^2 - 4a - 21$
b $y^2 + y - 12$ **d** $m^2 - 7m + 6$

4 **a** $x^2 + 2x + 1$ **c** $m^2 - 4m + 4$
b $x^2 - 2x + 1$ **d** $y^2 + 6y + 9$

Factorising

Stretch it! $(x + 1)$

1 **a** $3(a + 3)$ **c** $7(1 + 2c)$
b $5(b - 2)$ **d** $d(d - 2)$

2 **a** $4(2a + 5)$ **c** $9(2 + c)$
b $4(b - 3)$ **d** $d(2d - 3)$

3 **a** $2(2x - 3y)$ **d** $n(2 - 9n)$
b $m(a + b)$ **e** $5x(1 + 2y)$
c $x(4x + 3y)$ **f** $4p(q - 3)$

4 **a** $(x + 1)(x + 7)$ **e** $(x - 3)(x - 3)$
b $(x - 1)(x + 5)$ **f** $(x + 3)(x + 4)$
c $(x + 2)(x - 4)$ **g** $(x - 2)(x + 5)$
d $(x - 2)(x - 3)$ **h** $(x + 4)(x - 5)$

5 **a** $(x + 4)(x - 4)$ **c** $(x + 9)(x - 9)$
b $(x + 6)(x - 6)$ **d** $(y + 10)(y - 10)$

Substituting into expressions

1 11

2 **a** 10 **c** -22 **e** 28
b -24 **d** 20 **f** -1

3 False.

When $a = 3$: $3a^2 = 3 \times 3^2 = 3 \times 9 = 27$

4 **a** 20 **d** 0.012
b 2 **e** 2400
c 200 000 $(= 2 \times 10^5)$ **f** 10

Writing expressions

1 **a** $4 - q$ **d** xy (or yx)
b $n + m$ (or $m + n$) **e** p^2
c $8t$ **f** a^3

2 $x + y$

3 $100n + 75b$

4 $9a + 2$

5 $4a + 10$

Solving linear equations

1 **a** 7 **c** 16
b 17 **d** -2

2 **a** 5 **c** -4 **e** 11
b 5 **d** 33 **f** 19

3 **a** 2 **c** $\frac{5}{2}$ (Or 2.5, or $2\frac{1}{2}$)
b 3 **d** -2

4 Hannah has not subtracted 4 from *both* sides.

5 **a** 3 **d** -4
b 3 **e** $-\frac{4}{8} = -\frac{1}{2}$ (Or -0.5)
c 3

6 **a** 3 **d** 3
b 3 **e** -6
c $\frac{5}{2}$ (Or 2.5 or $2\frac{1}{2}$)

7 **a** -4 **c** 1
b 2 **d** $\frac{9}{6} = \frac{3}{2}$ (Or 1.5 or $1\frac{1}{2}$)

Writing linear equations

1 **a** $8s + 12$ **b** 9 cm

2 **a** 50 **b** 115°

3 30 years old

4 4

5 8 cm

6 $a = 18$, $b = 28$

Linear inequalities

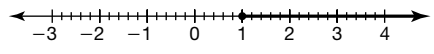
1 **a** $x = 3, 4, 5$ **c** $x = 0, 1, 2, 3$

b $x = 2, 3, 4, 5$ **d** $x = -3, -2, -1, 0, 1$

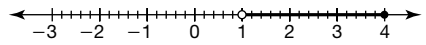
2 **a** $x < 3$ **c** $-1 \leq x \leq 5$

b $x \geq -2$

3 **a**



b



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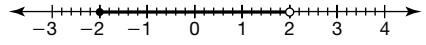
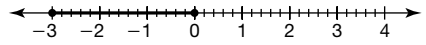
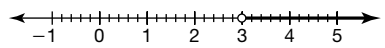


d

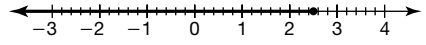


4 **a** $x > 3$

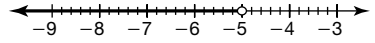


b $x \leq \frac{10}{4} = \frac{5}{2}$

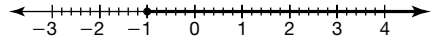
$\left(\text{Or } x \leq 2.5 \text{ or } x \leq 2\frac{1}{2}\right)$



c $x < -5$



d $x \geq -1$



5 Olivia has not multiplied all the terms in the bracket by the term outside.

6 **a** $-\frac{5}{2} < x$ $\left(\text{Or } x > -\frac{5}{2}\right)$ **b** -2

7 No, Lee is not correct.

$6 \leq 2x + 4 < 16$ (-4 from each term in the inequality)

$-2 \leq 2x < 12$ ($\div 2$)

$-1 \leq x < 6$

Inequality is not true when $x = 6$ so $x = 6$ is not a possible solution.

8 **a** $3 \leq x$ (Or $x \geq 3$) **c** $\frac{1}{2} \geq x$ $\left(\text{Or } x \leq \frac{1}{2}\right)$

b $-1 > x$ (Or $x < -1$) **d** $2 > x \geq -3$

Formulae

1 £305

2 27

3 -90

4 $C = 25d + 50$

5 $A = l^2$

6 **a** $4a + 6$ **b** 30 cm

7 -65

8 **a** $\frac{v - u}{t} = a$ **d** $\frac{v^2 - u^2}{2a} = s$

b $\frac{3V}{A} = h$ **e** $g = \frac{2s}{T^2}$

c $\frac{y + 9}{3} = x$

Or: $x = \frac{y}{3} + 3$

Linear sequences

1 **a** **i** 14, 17 **ii** 7, 3 **iii** 27, 33 **iv** 24, 29

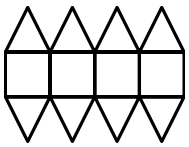
b **i** 29 **ii** -13 **iii** 57 **iv** 49

2 **a** 1st term = 2, 2nd term = 6, 3rd term = 10, 4th term = 14

b 78

3 3, 10

4 **a**



b 16

c No. The number of triangles forms an even number sequence and 35 is odd.

5 **a** $4n - 1$

b $4n - 1 = 99$ ($+ 1$)

$4n = 100$ ($\div 4$)

$n = 25$

Yes, 99 is a term in the sequence because 25 is an integer.

Non-linear sequences

1 1, 3, 5, 7, 9, … Arithmetic sequence

1, 2, 4, 8, 16, … Geometric sequence

1, 4, 5, 9, 14, … Fibonacci-type sequence

1, 4, 9, 16, 25, … Square-number sequence

2 **a** $\frac{1}{2}, \frac{1}{4}$ **d** 3, 9

b 0.005, 0.0005 **e** $-0.8, -1.6$

c $\frac{1}{16}, \frac{1}{32}$ **f** $-24, 48$

3 71

4 **a** 4th term $= b + a + b = a + 2b$

5th term $= a + b + a + 2b = 2a + 3b$

b 4

Show that...

1 **a** LHS $= 4(x - 3) + 2(x + 5) = 4x - 12 + 2x + 10$

$= 6x - 2$

RHS $= 3(2x - 1) + 1 = 6x - 3 + 1 = 6x - 2$

LHS = RHS

So $4(x - 3) + 2(x + 5) \equiv 3(2x - 1) + 1$

b LHS $= (x + 2)(x - 2) = x^2 - 2x + 2x - 4 = x^2 - 4$

LHS = RHS

So $(x + 2)(x - 2) \equiv x^2 - 4$

2 Rod A $= n$

Rod B $= n + 1$

Rod C $= n + 2$

Rod A + Rod C $= n + n + 2$

$= 2n + 2$ (Factorise)

$= 2(n + 1)$

Rod A + Rod C is 2 times the length of rod B.

Functions

1 **a** 27

b 4

c If $x = y$,

$3x - 3 = x\ (-x)$

$2x - 3 = 0\ (+3)$

$2x = 3\ (\div 2)$

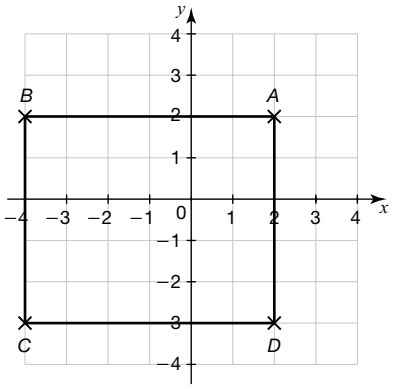
$x = \frac{3}{2}$

(Or $x = 1\frac{1}{2}$)

Coordinates and midpoints

1 **a** $A(2, 2)$

b and **c**

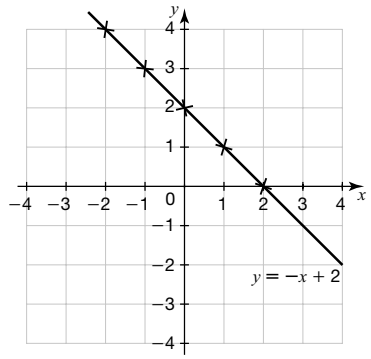


d $(-1, -0.5)$

Straight-line graphs

Stretch it! $x = 0.5$

1



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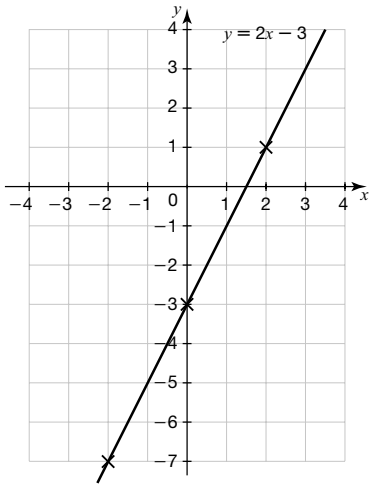
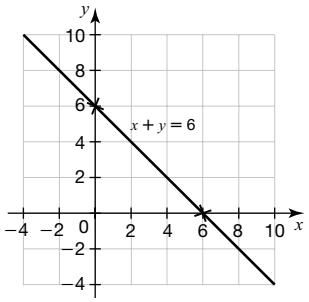


3 $y = x + 1$

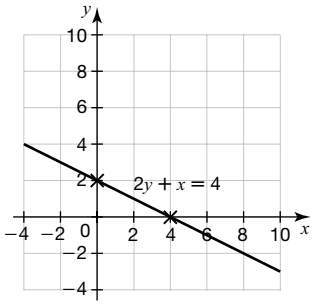
4 $y = 2x - 4$

5 $y = -\frac{1}{2}x + 4$

6 **a**



b



7 Lines A and D are parallel

8 $(0, -2)$

9 **a** $y = 3.4$ **b** $x = 0.5$ **c** $x \approx 2.5$ **d** $x \approx -0.5$

Solving simultaneous equations

1 **a** $x = 1, y = 2$ **b** $x = 3, y = -2$ **c** $x = 6, y = -4$ **d** $x = 3, y = 2$ **e** $x = 4, y = -1$ **f** $x = 2, y = 1$

2 7 and 14

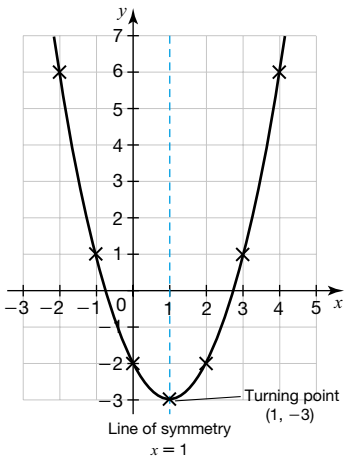
3 A burger costs 95 p. A cola costs £1.10.

4 **a** $x = 2, y = 8$ **b** $x = 8, y = 1$

Quadratic graphs

Stretch it! $x \approx 3.6$ and $x \approx -0.6$

1 **a**



b **i** $x = 1$ **ii** $(1, -3)$

2 **a** $x = 0$ and $x = 2$ **b** $x \approx -1.2$ and $x \approx 3.2$ **c** $x = 1$

Solving quadratic equations

Stretch it!

$x = 4$ or $x = -4$

$x = 5$ or $x = -5$

1 **a** $x = 0$ or $x = 4$

b $x = 0$ or $x = -7$

c $x = -4$ or $x = 4$

d $x = -1$ or $x = -9$

e $x = 3$ or $x = -4$

f $x = -2$ or $x = 8$

2 **a** $x = -7$ or $x = 7$

b $x = 0$ or $x = 3$

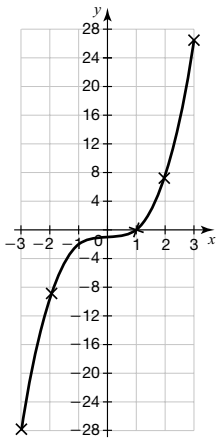
c $x = -1$ or $x = -6$

Cubic and reciprocal graphs

1 **a**

x	−3	−2	−1	0	1	2	3
y	−28	−9	−2	−1	0	7	26

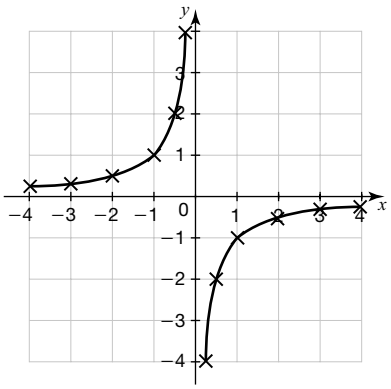
b



2 **a**

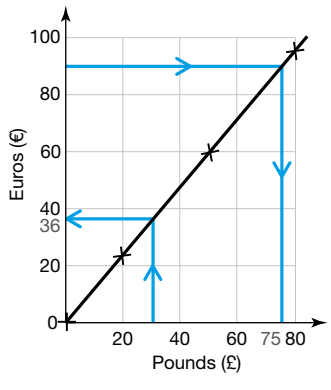
x	−4	−3	−2	−1	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4
y	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	4	−4	−2	−1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$

b



Drawing and interpreting real-life graphs

1 **a**



b **i** €36 **ii** £75

c From the graph: £30 = €36

So £90 = €36 × 3 = €108

Ring is cheaper in France.

2 **a** £10

b 13p

3 A: The temperature is steadily increasing.

B: The temperature remains constant.

C: The temperature rises steadily for a period and then remains constant.

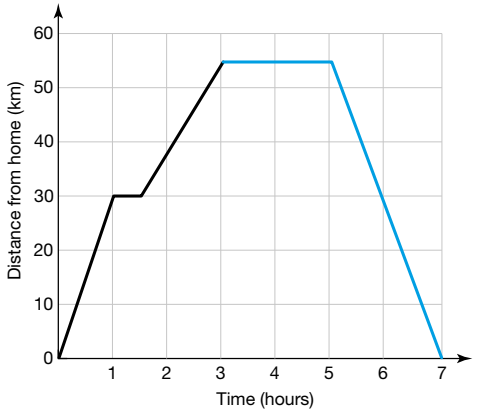
4 **a** 30 minutes

b 55 km

c Speed before break = 30 km/hr

Speed after break = 16.7 km/hr

d



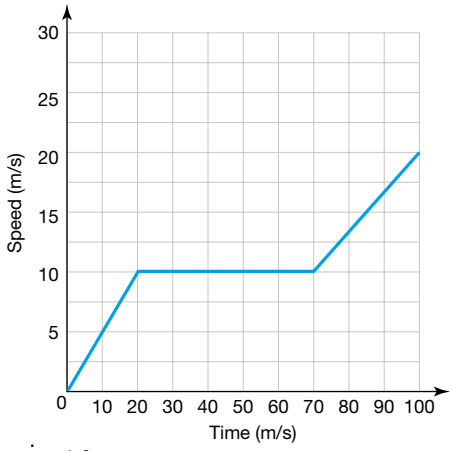
5 **a** 6 m/s

b 4 seconds

c 6 seconds

d 1.5 m/s²

6 **a**



b $0.\dot{3}$ m/s²

7 **a** 35 cm

b The person stays in the bath.

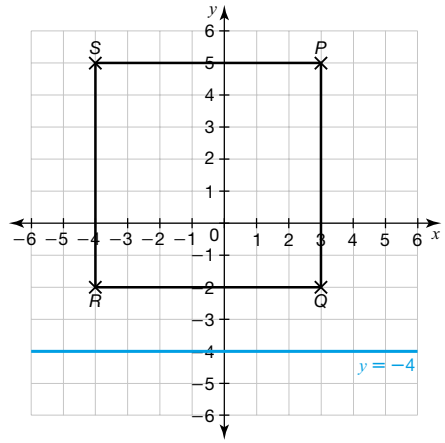
c The person got out of the bath.

d Running water in the bath was quicker. The slope of the line between *O* and *A* (filling the bath) is steeper than the slope of the line between *E* and *F* (emptying the bath).

Review it!

1 **a** $P(3, 5)$

b, c and **e**



d $(-0.5, 1.5)$

2 **a** -2

b 7

c Millie is correct.

When $x = 4$, $3x^2 = 3 \times 4^2 = 3 \times 16 = 48$

3 **a** $4(2x + 3)$

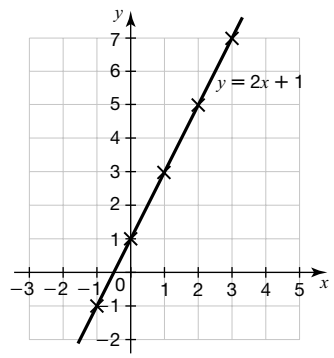
b m^5

c x^5

4 **a**

x	−1	0	1	2	3
y	−1	1	3	5	7

b

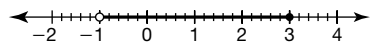


c 2

5 $x = 3$

6 **a** $x \leq 2$

b



c $x = 1, 2, 3$

d 1

7 **a** 33

b No. This is a sequence of odd numbers and 44 is even.

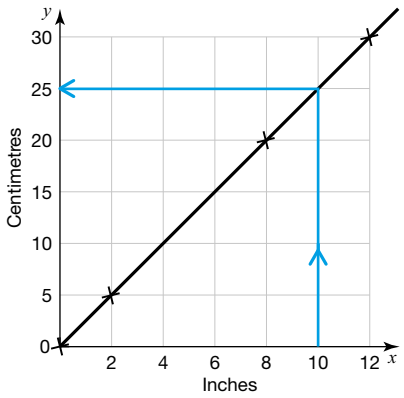
c 47

8 $x^2 + 7x + 12$

9 $x = 4, y = 2$

10 **a** $12m$ **b** $12p^2$ **c** $6x$

11 **a**



b **i** 25 cm **ii** 20 inches

c £3.00

12 **a** $T = 12.50x + 10$

b 5 days

13 **a** $x \leq \frac{6}{4} = \frac{3}{2}$ (Or $x \leq 1\frac{1}{2}$) **b** 5 and 6

14 Ollie has squared each term inside the brackets rather than squaring the whole bracket.

15 $4(P - R) = Q$

16 **a** $m(m + 8)$

b $(x + 3)(x + 4)$

17 **a** $3n - 1$

b $2n - 3 = 112$ $(+ 3)$

$2n = 115$ $(\div 2)$

$n = 57.5$

No, Kadeena is incorrect.

112 cannot be a term in the sequence because 57.5 is not an integer.

18 **a** $-2x + 23$ **b** $20a^5b^3$

19 $x = 4.5$

20 **a** 4th term: $b + a + b = a + 2b$

5th term: $a + b + a + 2b = 2a + 3b$

6th term: $a + 2b + 2a + 3b = 3a + 5b$

7th term: $2a + 3b + 3a + 5b = 5a + 8b$

b $a = 2, b = 3$

Ratio, proportion and rates of change

Units of measure

1 **a** 3000 m **c** 13 000 cm^2 **e** 7200 seconds
b 75 mins **d** 3.52 litres **f** 14 kg

2 4.175 kg or 4175 g

3 $2.\dot{2}\dot{7}$ kg

Ratio

Stretch it! $\frac{31}{56}$

1 **a** 1 : 4 **b** 1 : 3 : 4 **c** 4 : 5

2 7 : 1

3 **a** 7 : 1 **b** 100 tickets

4 **a** 3 : 2 **b** 120

5 0.6 kg

6 12 cm and 15 cm

7 $s = 20t$

8 40 g

Scale diagrams and maps

Stretch it! 50 miles on ground = $\frac{50}{x}$ × 161 000cm on map

1 A, B, F

2 **a** 36 km **b** 1.25 cm

3 120 m

4 **a** 1 km **b** 250°

Fractions, percentages and proportion

1 $\frac{1}{175}$

2 $\frac{11}{24}$

3 **a** $\frac{3}{4}$ **b** 25%

4 10%

5 School A: 125 : 145 = 25 : 29
School B: 100 : 120 = 5 : 6
No since the ratios are not equivalent.

6 4.125 g

Direct proportion

Stretch it! Since for two values to be in direct proportion when one is 0 the other must be 0.

1 A and E

2 **a** **i** 30 meringues **b** 7 eggs
ii 100 meringues

3 2 hours 30 mins

4 A, D

Inverse proportion

1 D

2 22.5 mins

3 $1\frac{2}{3}$ of a day.

4 **a** 2

b The age of the chicken and the number of eggs it lays are in inverse proportion. This means that as the age of the chicken increases so the number of legs it lays decreases.

Working with percentages

Stretch it! £128
Stretch it! 2% (1.02^5 × 100 = £110.41)

1 **a** £51.50 **b** 992 **c** 12.48

2 12.5% **3** 20.9°C **4** 25 920 **5** £150

Compound units

Stretch it! $\frac{100}{x}$ mph

1 164 units **3** 0.24 g/cm^3

2 40 minutes **4** 6 N/m^2

5 10.8 km/hour

6 475 gallons per hour (to the nearest whole number)

7 Bolt: 100 m in 9.58 seconds = 10.4 m/s
Cheetah: 120 km/hr = 120 000 m/hour = 120 000 ÷ 60 m/min = 2000 m/min = 2000 ÷ 60 m/sec = 33.3 m/s The cheetah is fastest.

Review it!

1 **a** 3200 m **b** 540 seconds **c** 400 ml

2 4.6 km

3 150 minutes

4 3520 cm^2 or 0.352 m^2

5 30 000 cm^2

6 $\frac{5}{12}$

7 13 : 9

8 5 minutes

9 2300 kg/m^3

10 25%

11 $\frac{12}{25}$ or 48%

12 $\frac{23}{90}$

13 25 hours = 1 day and 1 hour

14 **a** 30 more **b** $C = 4J$

15 28%

16 £864

17 6 cm

18 £1591.81

19 8000 people

20 3 hours 9 minutes

21 £15.75

22 20%

23 £3675

24 18 : 13

25 1750 men

26 No – for two things to be in direct proportion when one is zero the other must be zero, the graph does not go through the origin so this is not the case.

27 Neither, since the time taken to cook increases as the weight increases it is not in indirect proportion. It is not in direct proportion since a graph to illustrate the relationship would not go through the origin.

28 11 seconds

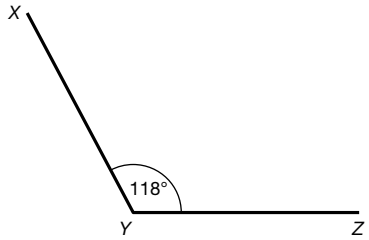
29 She is incorrect since the ratio of females to males must be the same for them to have equivalent proportions.

Geometry and measures

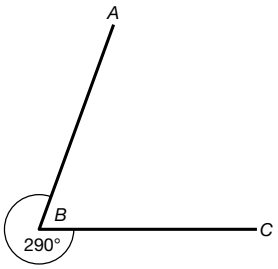
Measuring and drawing angles

1 **a** 43° **b** Acute

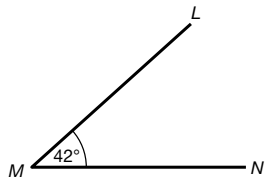
2 **a**



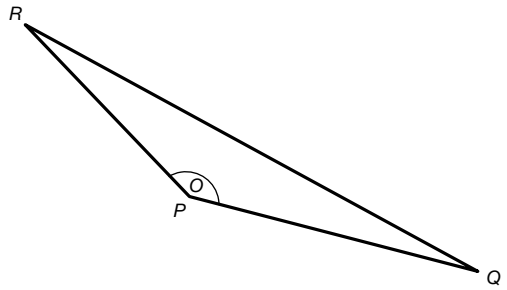
b



c



3 **a**



b 18°

Using the properties of angles

1 54°

2 **a** **i** 70°

ii Base angles of an isosceles triangle are equal.

b 110°

3 18°

4 **a** **i** 54°

ii Angles on a straight line add up to 180°.

b 83°

5 **a** 96°

b **i** $y = 96°$

ii Use the fact that corresponding angles are equal, then vertically opposite angles are equal.

Or, use the fact that alternate angles are equal, then use angles on a straight line add up to 180°.

6 **a** 58° **b** 64°

c 58° (Alternate angles are equal; or, opposite angles of a parallelogram are equal)

7 Angle $BAD = 62°$ (Opposite angles of a parallelogram are equal)

Angle $ADE = 62°$ (Alternate angles are equal)

$x = 180 - 62 - 62$ (Base angles of an isosceles triangle are equal)

$x = 56°$

8 Angle $ACB = 36°$ (Base angles of an isosceles triangle are equal)

Angle $ABC = 180 - 36 - 36$ (Angles in a triangle add up to 180°)

Angle $ABC = 108°$

$x = 108°$ (Alternate angles are equal)

Using the properties of polygons

Stretch it!

1 The angle sum of a triangle is 180.

Sum of interior angles of a hexagon $= 4 \times 180° = 720°$.

2

Polygon	Number of sides (n)	Number of triangles formed	Sum of interior angles
Triangle	3	1	180°
Quadrilateral	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°
Heptagon	7	5	900°
Octagon	8	6	1080°
Decagon	10	8	1440°

3 $n - 2$

4 $180 \times (n - 2)$

Stretch it!

Angles around a point add up to 360°.

No, 360° is not divisible by 108° (interior angle of a regular pentagon).

1 36°

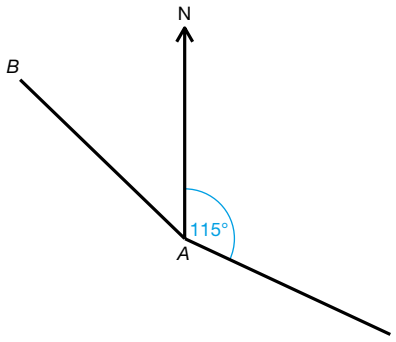
2 **a** 24 **b** 3960°

3 135°

Using bearings

1 **a** 315°

b



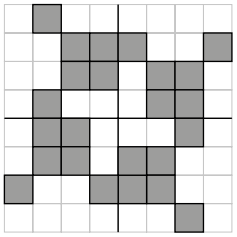
2 344°

3 Kirsty is correct.

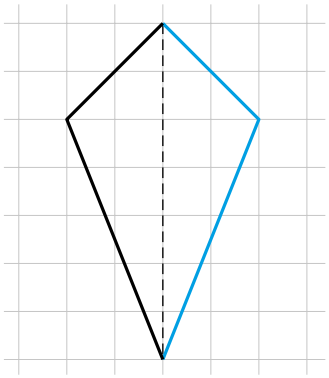
The bearing is 314° (360° − 46°) as it must be measured clockwise from North.

Properties of 2D shapes

Stretch it!

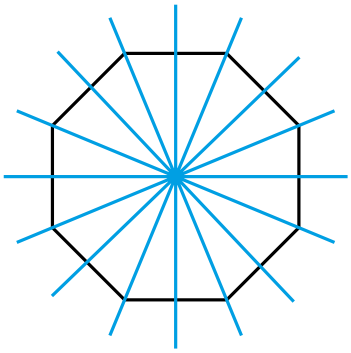


1 a



b kite

2 a 8 possible lines of symmetry:



b 8

3 a 2 **c** 1, no

b rhombus **d** square, rhombus

Congruent shapes

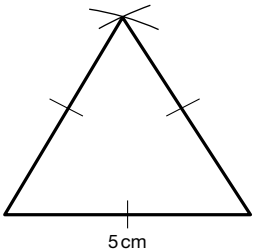
1 Any accurate copy of shape A.

2 a 120° **b** 12 cm

3 a SSS **b** ASA

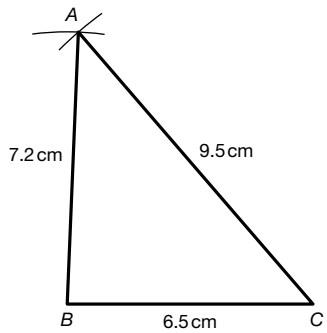
Constructions

Stretch It!



60°

1



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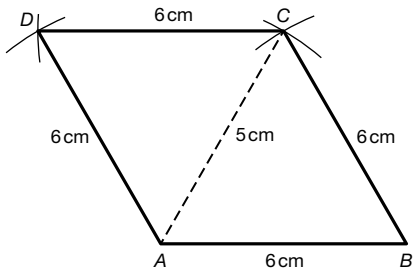

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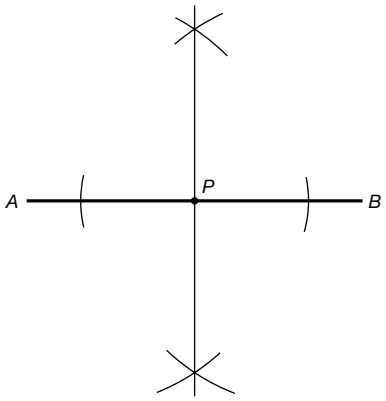

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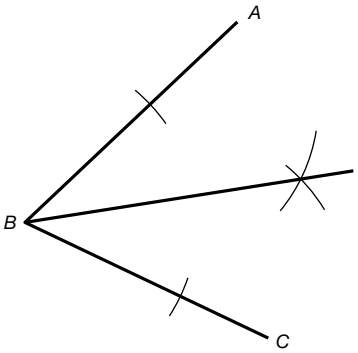

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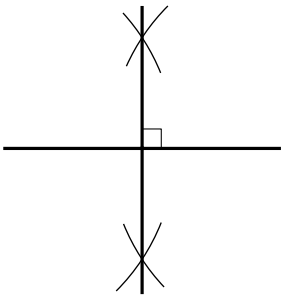


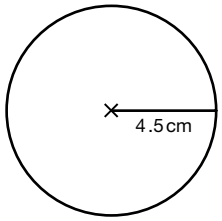
Drawing circles and parts of circles

1 a chord **d** arc

b tangent **e** radius

c diameter **f** segment

2



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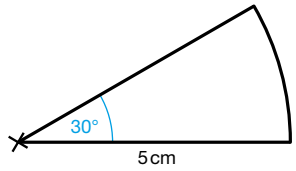
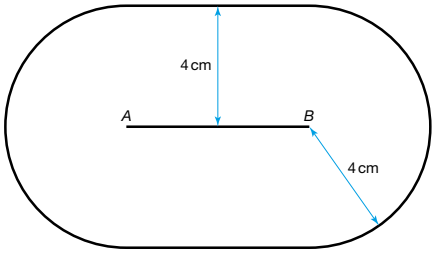
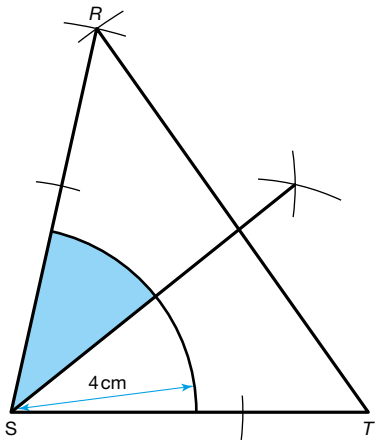


Loci

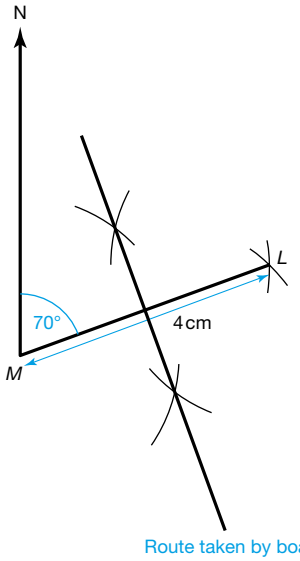
1



2



3 **a** and **b**



Route taken by boat

Perimeter

1 28.8 cm

2 42 cm

3 $k = 4$, $b = 8$

4 $200 + 30\pi$ m

5 £1.80

Area

Stretch it! Area of a semicircle $= \frac{\pi r^2}{2}$,

Area of quarter circle $= \frac{\pi r^2}{4}$

1 **a** 9.0 cm^2 **c** 28.0 cm^2 **e** 63.6 cm^2

b 4.5 cm^2 **d** 5.0 cm^2

2 9 cm^2

3 $\frac{1}{4}$

4 18 cm^2

5 454.1 cm^2

Sectors

1 Area $= 39.3$ cm^2

Perimeter $= 25.7$ cm

2 Area $= 12\pi$ cm^2

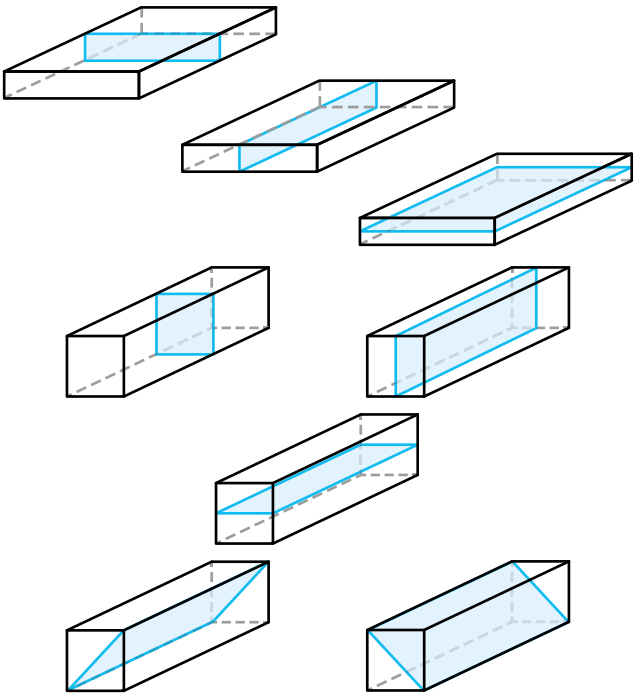
3 £119.92

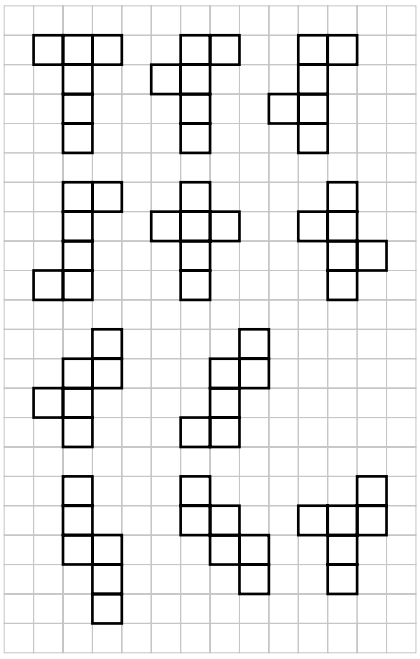
3D shapes

Stretch it!

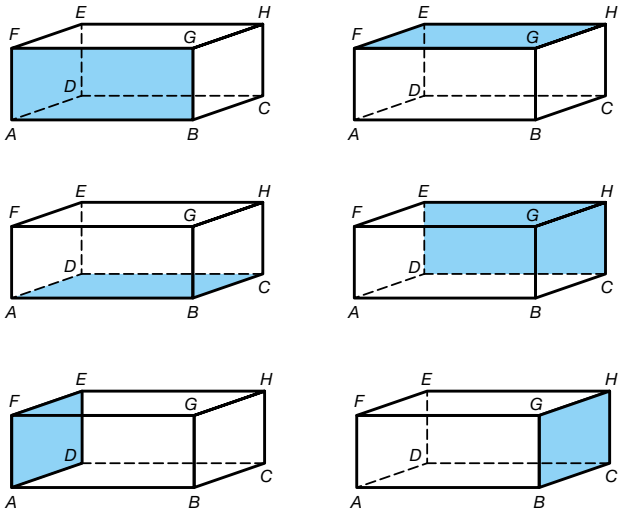
3D shape	Faces	Edges	Vertices
Cube	6	12	8
Cuboid	6	12	8
Square-based pyramid	5	8	5
Tetrahedron	4	6	4
Triangular prism	5	9	6
Hexagonal prism	8	18	12

Stretch it!

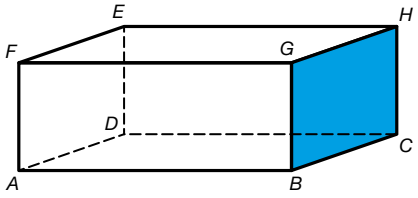




1 a 6 possible rectangular faces:

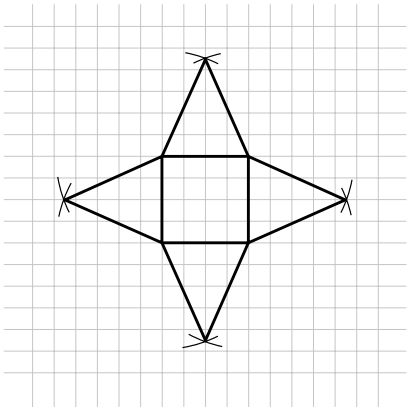


b

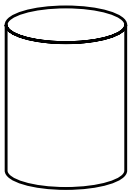


c Kelli has not counted the hidden edges.

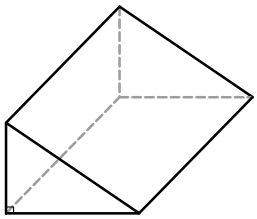
2



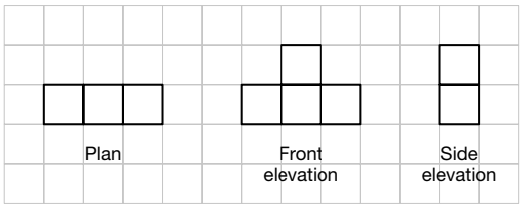
3 a



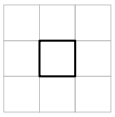
b



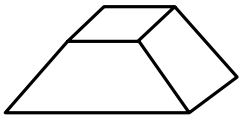
4



5 a



b



Volume

1 382 cm^3
2 1.96 m^3
3 $k = 264$
4 6.3 cm

Surface area

1 150 cm^2
2 36π cm^2
3 14 cm^2
4 76.3%

Using Pythagoras' theorem

1 10.2 cm
2 4.8 m
3 54 cm^2
4 If the triangle is right-angled, $PQ^2 = PR^2 + RQ^2$
$PQ^2 = 13^2 = 169$
$PR^2 + RQ^2 = 8^2 + 5^2 = 89$
$PQ^2 \neq PR^2 + RQ^2$
Claudia is not correct.
5 10.63
6 $\sqrt{13}$
7 £910

Trigonometry

Stretch it! Any lengths for the opposite and hypotenuse in the ratio 1:2.

1 a 0.4 **b** 0.6 **c** 1.0 **d** 26.6 **e** 48.6 **f** 54.7
2 4.6 cm
3 40.6°
4 $\text{Sin } 15° = \frac{\text{opposite}}{10}$
Opposite = 2.6 m

Exact trigonometric values

1 **a** 0.5 **c** 0 **e** $\sqrt{3}$
b 0 **d** $\frac{1}{\sqrt{2}}$

2 $AC = 4$ cm
$BC = 4\sqrt{2}$ cm

3 30° and 60°

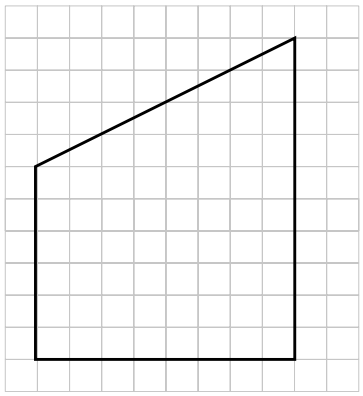
4 $\sin 30° = \frac{1}{2}$ therefore, $ABC = 30°$

5 0.5, $\frac{3}{4}$, cos 30°, tan 45°

Transformations

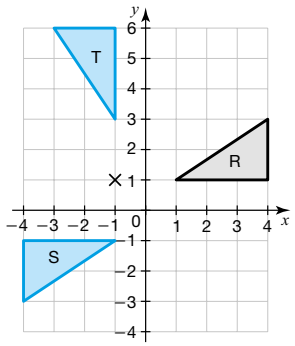
Stretch it! Yes

1



2 Translation by vector $\begin{pmatrix}-4\\-2\end{pmatrix}$

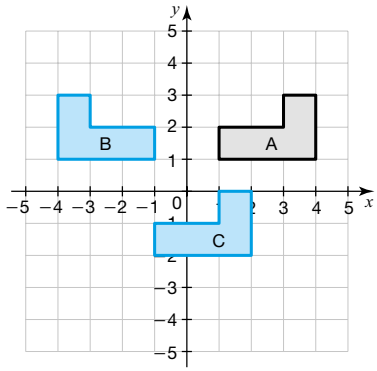
3 **a** and **b**



4 Reflection in the y-axis

5 Enlargement by scale factor $\frac{1}{2}$, centre (3, 3)

6 **a** and **b**



c Rotation of 90° clockwise about (0, 0)

Similar shapes

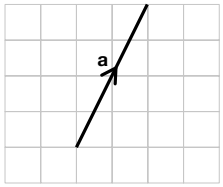
Stretch it! Perimeter of triangle $ABC = 14$ cm; perimeter of triangle $DEF = 28$ cm. The perimeter of a shape enlarged by scale factor 2 will also be enlarged by scale factor 2. When a shape is enlarged by any scale factor, the perimeter of the shape is enlarged by the same scale factor.

1 **a** 30° **b** 16 cm **c** 2 cm

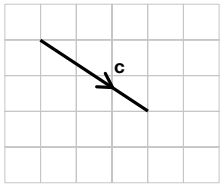
2 **a** 80° **b** 13.2 cm **c** 4 cm

Vectors

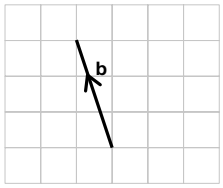
1 **a**



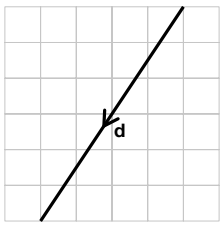
c



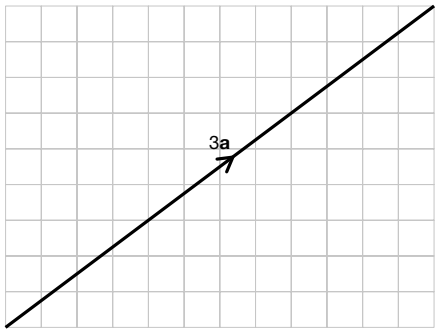
b



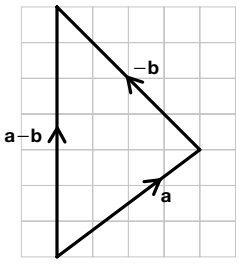
d



2 **a**



b



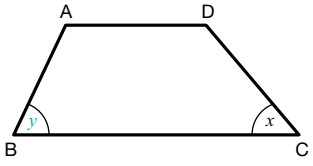
c $\begin{pmatrix}-8\\-6\end{pmatrix}$

3 **a** $\begin{pmatrix}-3\\9\end{pmatrix}$ **c** $\begin{pmatrix}7\\3\end{pmatrix}$
b $\begin{pmatrix}-8\\12\end{pmatrix}$ **d** $\begin{pmatrix}-9\\-9\end{pmatrix}$

4 No, only Jordan is correct.
Vectors **a** and **d** are parallel as $10\mathbf{a} = \begin{pmatrix}20\\30\end{pmatrix}$
Vectors **a** and **b** are not parallel as $-2\mathbf{a} = \begin{pmatrix}-4\\-6\end{pmatrix}$

Review it!

1 **a**



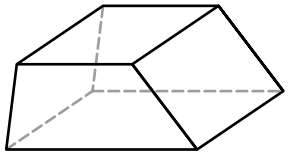
b 3.8 cm

c 50°

2 **a** 5

b 6

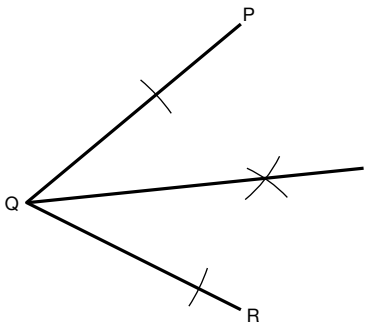
3



4 72 cm^2

5 24 cm

6



7 Rotation of 180° about (1, 0)

8 Angle *CFE* = 112° (Corresponding angles are equal)

Angle *CFG* = 180 − 112 = 68° (Angles on a straight line add up to 180°)

Angle *GCF* = angle *CFG* (Base angles of an isosceles triangle are equal)

x = (180 − 68 − 68) = 44° (Angles in a triangle add up to 180°)

9 $\frac{9}{20}$ or 45%

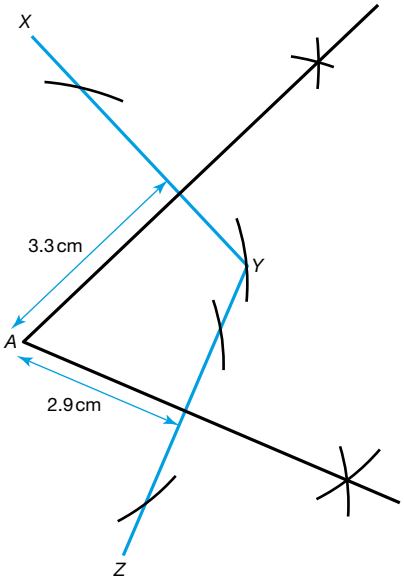
10 If triangle *ABC* is right-angled, $c^2 = a^2 + b^2$

$c^2 = 8^2 = 64$

$a^2 + b^2 = 6^2 + 4^2 = 36 + 16 = 52$

$c^2 \neq a^2 + b^2$ so triangle *ABC* is not right-angled.

11 **a**

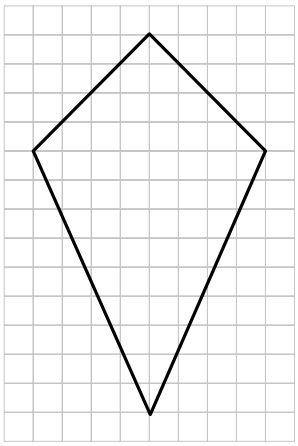


b 0.8 m

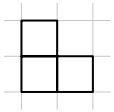
12 **a** $\frac{1}{\sqrt{2}}$ **b** *AB* = 3 cm

13 **a** $\begin{pmatrix}8\\1\end{pmatrix}$ **b** $\begin{pmatrix}-6\\13\end{pmatrix}$

14



15



16 **a** **i** 35°

ii Base angles of an isosceles triangle are equal.

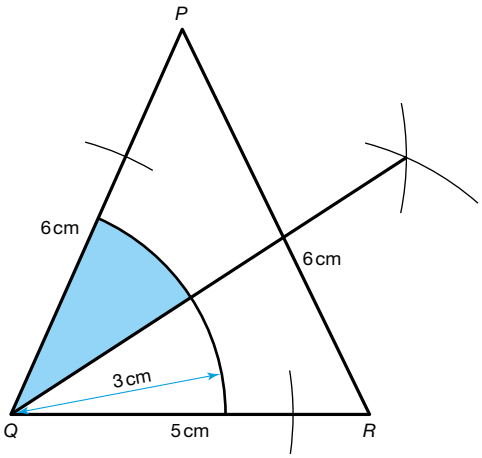
b 110°

c Triangle *XYZ* is isosceles so angle *XYZ* = angle *YXZ*. Angle *c* = 55°

17 12.6 cm

18 135°

19



20 29.06 cm

21 32.3 cm

22 7.7 cm^2

23 4 times

24 3.87 cm

25 **a** 300 cm^2

b 300 cm^3

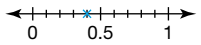
26 53.1°

27 Translation by vector $\begin{pmatrix}-7\\-6\end{pmatrix}$

Probability

Basic probability

Stretch it! No – each time the probability of getting an even number is $\frac{1}{2}$. You would expect to get even numbers approximately 50 times but cannot guarantee it.

1 

2 **a** $\frac{3}{25}$ **b** $\frac{13}{25}$

3 a

4 b

5 0.43

6 0.4

Two-way tables and sample space diagrams

1

	Chicken	Beef	Vegetarian
Fruit	12	6	4
Cake	5	3	8
Total	17	9	12

a 12 **b** as above

2 **a**

		Dice 1					
		1	2	3	4	5	6
Dice 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

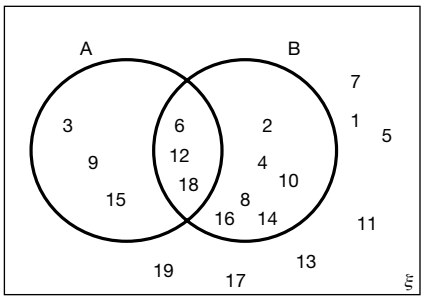
b **i** $\frac{1}{18}$ **ii** $\frac{1}{12}$ **iii** 0

3 1, 1, 3, 3

Sets and Venn diagrams

Stretch it! None

1 **a**



b Multiples of 6

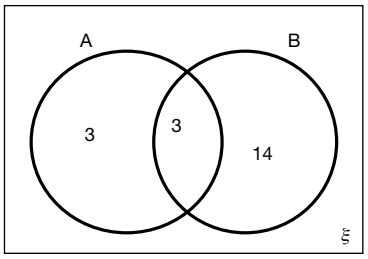
2 **a** C ∩ T is the set of students who travel by car AND train.

C′ ∩ B is the set of students who do NOT travel by car AND travel by bus.

b **i** $P(C) = \frac{19}{50}$

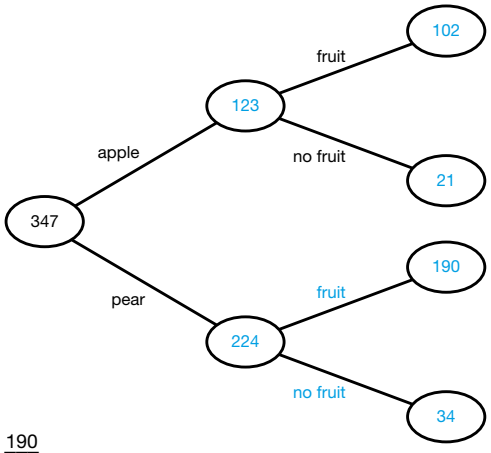
ii $P(B \cup T) = \frac{3}{5}$ **iii** $P(B' \cap T) = \frac{7}{25}$

3



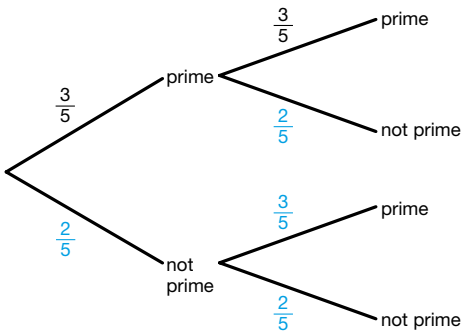
Frequency trees and tree diagrams

1 **a**



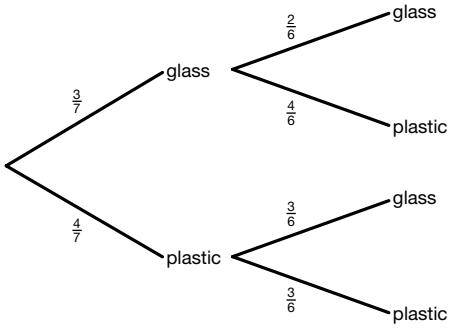
b $\frac{190}{347}$

2 **a**



b $\frac{9}{25}$ **c** $\frac{21}{25}$

3 **a**



b $\frac{5}{7}$

Expected outcomes and experimental probability

Stretch it! The dice has not been rolled enough times – carry out further tests.

1 135

2 20 red sweets

3 50 primes

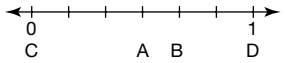
4 **a** Charlie – he has carried out the most tests.

b 6

Review it!

1 30

2



3 0.7

4 B, C

5 **a** $\frac{3}{5}$

b 5

6

	Pizza	Pasta	Risotto	Total
Cake	12	6	1	19
Ice cream	10	11	10	31
Total	22	17	11	50

7 0.7

8 **a** No, he has not tested his dice enough times.

b 18

9 0.55

10 **a**

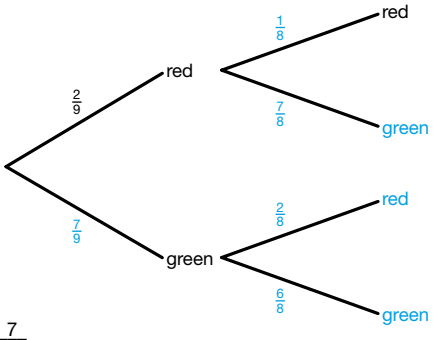
		Dice					
		1	2	3	4	5	6
Coin	Head	2	4	6	8	10	12
	Tail	3	4	5	6	7	8

b **i** $\frac{1}{6}$ **ii** $\frac{1}{6}$

11 **a** **i** 6 **ii** 1 **iii** 5

b $\frac{4}{8} = \frac{1}{2}$

12 **a**



b $\frac{7}{12}$

13 **a** 10

b $\frac{1}{3}$ is only a theoretical probability and therefore will not necessarily be accurate in real life.

14 0.74 or $\frac{37}{50}$

15 £50

16 **a** Milo, he has surveyed a larger sample.

b 263

Statistics

Data and sampling

Stretch it! A random sample could be taken; you could allocate a number to each student and randomly generate the numbers to survey. Any method is acceptable as long as each person in the school has an equally likely chance of being chosen. Alternatively a stratified sample could be taken.

1 Primary Source; Recording the data by measuring it yourself.

Secondary source: Any sensible source, e.g. the Met Office, local paper etc.

2 Qualitative data.

3 It is cheaper and quicker than surveying the whole population.

4 **a** The people working for an animal charity are more likely to be opposed to wearing real fur. Every member of the population does not have an equal chance of being chosen.

b Surveying people in the street, a random telephone survey. Any sensible method that ensure that each member of the population does has an equal chance of being chosen.

5 **a** $\frac{1}{5}$ **b** 10 bottles

6 **a** 12 000

b The sample is relatively small. The sample is not a random sample as it is taken on one day in a year.

Frequency tables

1

Number of people on the bus	Frequency
0–9	4
10–19	12
20–29	3
30–39	1

2 **a**

Number of courgettes	Frequency
0	1
1	0
2	1
3	1
4	9
5	3
6	0

b 56

3 There are gaps between his groups – where would he record someone who spent 15.5 hours training?

His groups do not have the same width.

Bar charts and pictograms

1 **a** 20 **b** 25%

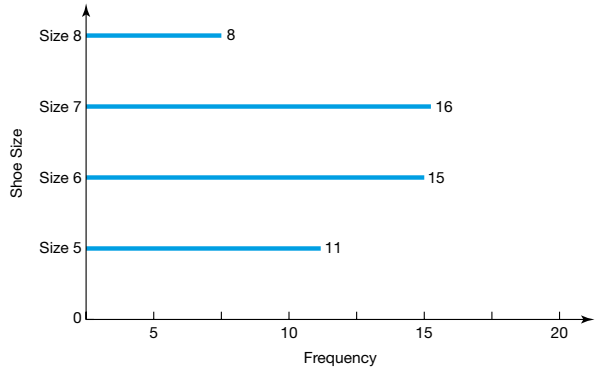
2 **a** 4 **b** 39.2%

c Proportion of boys who play two sports $= \frac{6}{18} = \frac{1}{3}$

Proportion of boys who play three sports $= \frac{3}{12} = \frac{1}{4}$

$\frac{1}{3} > \frac{1}{4}$ so Jasmine is incorrect and you can't prove that what she says is true.

3

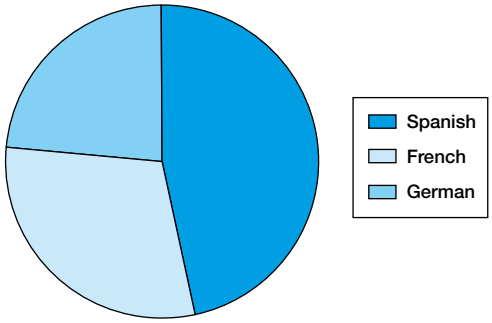


4 **a** 70 **b** ~~51~~

Pie charts

Stretch it! Round appropriately – but check the angles sum to 360°.

1 Spanish = 168°
French = 108°
German = 84°



2 **a** 1 **b** 97.5% or $\frac{39}{40}$

3 **a** 28
b The bar chart, since the frequency is easy to read from the bar chart.

Stem and leaf diagrams

1 **a** 7 **b** 0.2 kg/200 g

2 1 Age of people using a dentist

2	0 0 0 0 1 1 1
3	2 5 5 7
4	1 2 2 6

The leaves were not in ascending order. The spaces between leaves were not regular.

3 **a** Stem and leaf diagram – you can see the smallest number of passengers was 3, however on the bar chart you only know it is between 0 and 9.
b Both since the shape of the data is preserved in both.

Measures of central tendency: mode

1 The other three must be 12.2.

2 $1 < t \leq 2$

3 17

Measures of central tendency: median

1 5.4

2 $2 < b \leq 4$

3 **a** Group A = 83.5
Group B = 77
b Group A

Measures of central tendency: mean

Stretch it! **a** Mode **b** Mean/Median **c** Mean/Median

1 **a** Mean = 4.6
b You are using the midpoint of the groups as an estimate of the actual value for each group.

2 8.5

3 No – they could be any pair of numbers which sum to 10.

Range

1 8.8

2 **a** Girls = 3
b Boys = 2

3 Athlete A

4 35% or 75%

Comparing data using measures of central tendency and range

1 **a** **i** Mean = 83.5 minutes
ii Median = 30.5 minutes
b The extreme values affects the mean but not the median.

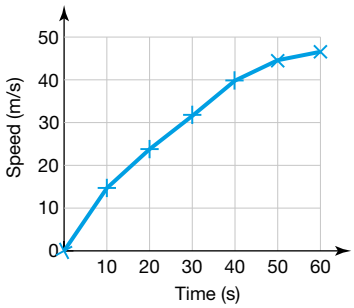
2 All the data is used to find the mean.

3 Either as long as suitably justified:
Car A – although the mean time is higher, it is more consistent in performance since the range is smaller.
Car B – the acceleration is quicker on average.

4 **a** and **b** The mode or median since the mean will not be a whole number and therefore not meaningful.

Time series graphs

1

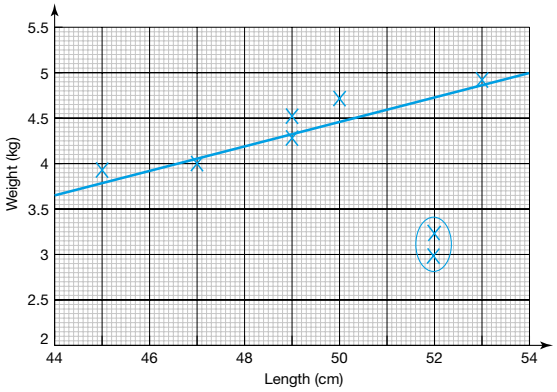


2 **a** 67°C
b Approx. 27°C
c No, since it is extrapolation (beyond the limits of the data).

3 **a** 17 000
b **i** April **ii** August
c The number of tourists peaks in April and again in December. The low seasons are February/March and July/August/September.

Scatter graphs

1 **a** and **b**

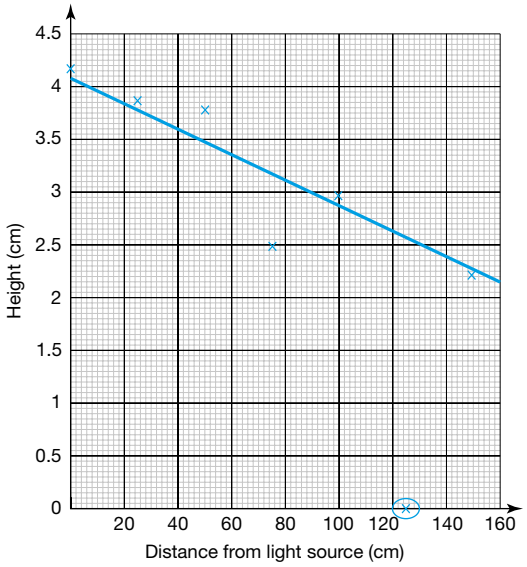


c Positive

d This will vary according to the line of best fit: approximately 4.8 kg

e This is beyond the limits of the data and therefore extrapolation.

2 **a**, **b** and **c**



c The seeds failed to germinate or the seedling died.

d The further the seedling is from the light source the shorter its height.

3 No, although the two things correlate one does not cause another. There may be many reasons why the crime rate is high in the area, perhaps there is poverty and inequality causing social tension.

Review it!

1 The sample is too small and he only asked his friends, therefore not representative of the population of TV viewers.

2 **a** Margherita **b** 5% **c** 18°

3 **a** $\frac{1}{4}$

b 480 cars

c $\left(\frac{105}{360}\right) \times 480 = 140$ cars

4 **a** The number of people doing their grocery shopping online is increasing.

b Any sensible answer: approximately 75%

c No – it is outside the limits of the data therefore extrapolation.

5 **a** Outside: **i** Mode = 21 and 31 **ii** Median = 28.5 **iii** Range = 21

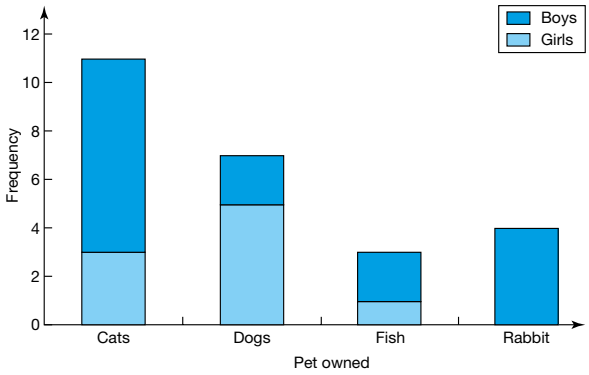
Greenhouse: **i** Mode = 47 **ii** Median = 47 **iii** Range = 14

b The seedlings are taller in the greenhouse since both mode and median is larger, the range of data is smaller in the greenhouse so the height the seedlings reach is more consistent.

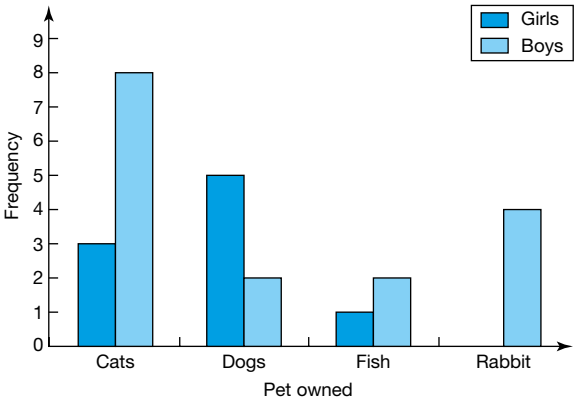
c Range = 31

6 Comparative bar chart or compound bar chart:

a



or:



b $\frac{11}{25}$

7 **a** 40–59

b The youngest person is between 0 and 19, the youngest may be any age in this range and the oldest is between 80 and 99 therefore any age in this range.

8 **a** 7

b Size 5

c Mean = $5.\dot{3}$

d Mode – the mean is not an actual shoe size.

9 **a** Time for 800 m (seconds)

11	2 2 5 8 9
12	0 1 9
13	1 2

Key: 11| 2 = 112 seconds

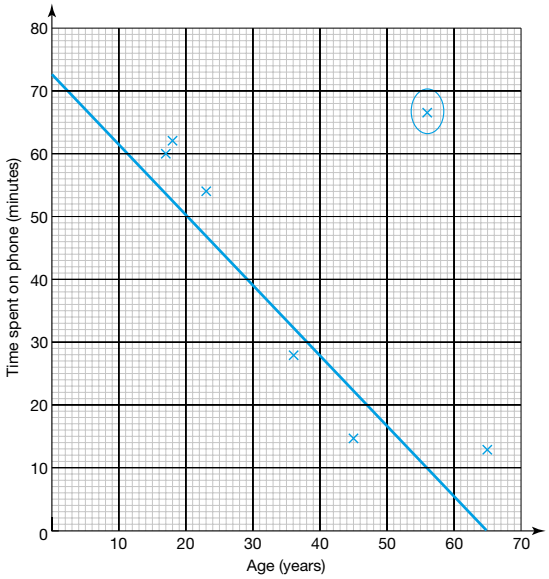
b $\frac{2}{5}$

10 **a** $\frac{1}{3}$

b 20

c Biology

11 **a** and **c**



b Negative

d Approximately 40 minutes: depends on line of best fit.

e This is outside the limits of the data and therefore extrapolation.

f As the age of the customer increases the time spent on the phone decreases.

12 **a** 74

b The midpoint of the class is used as the age of each of the patients rather than the actual age.

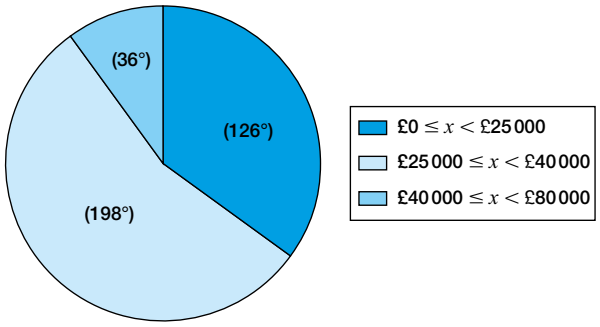
13 Male $< 50 = 40$

Female $< 50 = 35$

Male $\geq 50 = 19$

Female $\geq 50 = 26$

14 Annual income for surveyed population



15 Mean = 10.2 (1 decimal place)

16 2, 3, 3, 5, 6