Higher Mathematics Revision Guide for All Exam Boards

Full worked solutions

Number

Integers, decimals and symbols

1 a $23 \times 8.7 = 200.1$

   b $2.3 \times 0.87 = 2.001$

   c $\frac{2.001}{0.87} = 2.3$

   d $\frac{2001}{23} = 87$

2 a $4.86 \times 29 = 140.94$

   b $0.486 \times 2.9 = 1.4094$

   c $14094 \div 48.6 = 290$

   d $140.94 \div 29 = 4.86$

3 Ascending order means going up in size.

   $-0.5 \qquad 0 \qquad 0.012 \qquad 0.12 \qquad 12$

4 a $\frac{5}{0.5} = 10$

   b $1\frac{5}{9} > \frac{4}{3}$

   c $-3 < -1$

5 a $-2 + 7 = 5$

   b $-3 - 5 = -8$

   c $3 \times -5 = -15$

   d $-12 \div -2 = 6$

   e $-1 + 7 - 10 = -4$

Addition, subtraction, multiplication and division

1 a $1083 + 478 = 1561$

   b $2445 + 89 + 513 = 3047$

   c $66.55 + 3.38 = 69.93$

   d $7.08 + 4.5 + 12.343 = 23.923$

2 a $4556 - 1737 = 2819$

   b $674 - 387 = 287$

   c $12.935 - 4.75 = 8.185$

   d $5.77 - 0.369 = 5.401$

3 a $634 \times 47 = 29798$

   b $7.7 \times 3.8 = 29.26$

   c $8.32 \times 4.9 = 40.768$

4 a $1058 \div 23 = 46$

   b $617.4 \div 1.8 = 343$

   c $88.5 \div 2.5 = 35.4$

Using fractions

1 a $\frac{16}{5} = 3\frac{1}{5}$

   b $1\frac{1}{5} = \frac{6}{5}$

   c In ascending order: $\frac{5}{8} \quad \frac{3}{4} \quad \frac{9}{10} \quad 1\frac{1}{5} \quad \frac{16}{5}$

   d $1\frac{1}{5} + \frac{16}{5} = 1\frac{1}{5} + 3\frac{1}{5} = 4\frac{2}{5}$

   e $\frac{16}{5} - \frac{5}{8} = \frac{128}{40} - \frac{25}{40} = \frac{103}{40} = 2\frac{23}{40}$

   $45 \div 15 = 3$

   $12 \div 4 = 3$

   $48 \div 16 = 3$

2 $\frac{15}{45}, \frac{4}{12}, \frac{16}{48}$

3 a $3\frac{1}{3} \times 2\frac{1}{5} = \frac{10}{3} \times \frac{11}{5} = \frac{22}{3} = 7\frac{1}{3}$

   b $1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1} = \frac{7}{2} = 3\frac{1}{2}$

4 Fraction of weekly wage spent $= \frac{1}{3} + \frac{1}{5} + \frac{1}{4}$

   $= \frac{20 + 12 + 15}{60} = \frac{47}{60}$

   Ravi has $1 - \frac{47}{60} = \frac{13}{60}$ of his wage left.

Different types of number

1 a **16** is one more than 15, which is a multiple of 5.

   b **5** is one less than 6, which is a factor of 12.

   c **16** is not a prime number as it has factors 1, 2, 4, 8 and 16.

2 $300 = 30 \times 10 = 15 \times 2 \times 5 \times 2 = 5 \times 3 \times 2 \times 5 \times 2$

         $= 2 \times 2 \times 3 \times 5 \times 5$ ◄———

> It is best to write the product of prime factors in ascending order (i.e. smallest number first).

3 Find the multiples of each number.

   12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156 ...

   16, 32, 48, 64, 80, 96, 112, 128, 144, 160 ...

   18, 36, 54, 72, 90, 108, 126, 144, 162 ...

   Now look for the lowest number that appears in all three lists: 144.

   This means that all three grandchildren will call on the same day every **144 days**. (The lowest number to appear in only two lists is 36, so the earliest that only two grandchildren will call on the same day is after 36 days.)

4 a $756 = 252 \times 3 = 126 \times 2 \times 3 = 63 \times 2 \times 2 \times 3$
$= 7 \times 9 \times 2 \times 2 \times 3$
$= 7 \times 3 \times 3 \times 2 \times 2 \times 3 = 2^2 \times 3^3 \times 7$

b Comparing the two products and looking for the factors that are common we see that $2^2 \times 3^2 = 36$ Hence **36** is the highest common factor.

Listing strategies

1 There are 2 possible outcomes to tossing a coin.

There are 6 possible outcomes to rolling a six-sided dice.

So $2 \times 6 = 12$ different possible outcomes.

2 As the first number cannot be zero, it could be any number from 1 to 9 (i.e. any of 9 numbers). The second number could be any number from 0 to 9 (i.e. any of 10 numbers). For the whole 3-digit number to be divisible by 5 the last number must be 0 or 5 (i.e. 2 numbers).

So total number of 3-digit numbers that could be picked $= 9 \times 10 \times 2 = 180$.

The order of operations in calculations

1 a $10 + 4 \times 2 = 10 + 8 = 18$

b $5 \times 3 - 4 \div 2 = 15 - 2 = 13$

c $(7 - 4)^2 + (8 \div 2)^2 = 3^2 + 4^2 = 9 + 16 = 25$

2 a $2 + 3 \div \frac{1}{3} - 1 = 2 + 9 - 1 = 10$ ◀

> Note that the division must be performed first and that $3 \div \frac{1}{3} = 3 \times \frac{3}{1} = 9$

b $15 - (4 - 6)^3 = 15 - (-2)^3 = 15 - (-8) = 15 + 8 = 23$

> Brackets are worked out first.

c $\sqrt{4 - 3 \times (-7)} = \sqrt{4 + 21} = \sqrt{25} = 5$ or -5

> Note that the whole calculation inside the square root sign should be completed first.

Indices

1 a $7^7 \times 7^3 = 7^{7+3} = 7^{10}$

b $3^{-2} \div 3^4 = 3^{(-2-4)} = 3^{-6}$

c $(5^4)^5 = 5^{4 \times 5} = 5^{20}$

2 a $\frac{5^4 \times 5^6}{5^3} = 5^{4+6-3} = 5^7$

b $6^{(-2) \times 3} \div 6^{-3} = 6^{-6} \div 6^{-3} = 6^{-6-(-3)} = 6^{-3}$

c $2^8 \times 2^2 \times 5^4 \times 5^{-7} = 2^{10} \times 5^{-3}$

d $\frac{7^4 \times 7^6 \times 11^3}{11^4} = 7^{4+6} \times 11^{3-4} = 7^{10} \times 11^{-1}$

3 a $16^0 = 1$

b $100^{\frac{1}{2}} = \sqrt{100} = 10$ or -10

c $64^{\frac{2}{3}} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$

d $25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$ (or 0.2) or $-\frac{1}{5}$ (or -0.2)

e $27^{\frac{1}{3}} \times 36^{\frac{1}{2}} = \sqrt[3]{27} \times \sqrt{36} = 3 \times 6 = 18$ or 3×-6
$= -18$

4 $(3^{2x})^2 = 81$
$3^{4x} = 3^4$

The powers on each side must be the same, so
$4x = 4$
$x = 1$

Surds

1 a $\sqrt{3} \times \sqrt{2} = \sqrt{6}$

b $(\sqrt{5})^2 = 5$

c $2 \times 3 \times \sqrt{3} \times \sqrt{3} = 18$

d $(2\sqrt{5})^2 = 2 \times 2 \times \sqrt{5} \times \sqrt{5} = 20$

2 $\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$, hence $a = 2$

3 a $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

b $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$

4 a $\frac{16}{3\sqrt{2}} = \frac{16 \times \sqrt{2}}{3 \times \sqrt{2} \times \sqrt{2}} = \frac{16\sqrt{2}}{3 \times 2} = \frac{8\sqrt{2}}{3}$

b $\frac{18}{4 + \sqrt{7}} \times \frac{4 - \sqrt{7}}{4 - \sqrt{7}} = \frac{72 - 18\sqrt{7}}{16 - 7} = 8 - 2\sqrt{7}$

5 a $(1 + \sqrt{5})(1 - \sqrt{5}) = 1 - \sqrt{5} + \sqrt{5} - 5 = -4$

b $(2 + \sqrt{3})^2 = (2 + \sqrt{3})(2 + \sqrt{3}) = 4 + 2\sqrt{3} + 2\sqrt{3} + 3$
$= 7 + 4\sqrt{3}$

c $(1 + \sqrt{3})(2 + \sqrt{3}) = 2 + \sqrt{3} + 2\sqrt{3} + 3 = 5 + 3\sqrt{3}$

Standard form

1 a $5 \times 10^{-3} = 0.005$

b $5.65 \times 10^5 = 565000$

2 a $25000 = 2.5 \times 10^4$

b $0.00125 = 1.25 \times 10^{-3}$

c $0.05 \times 10^4 = 5 \times 10^2$

d $14 \times 10^{-3} = 1.4 \times 10^{-2}$

3 a $(3 \times 10^{-2})^2 = 9 \times 10^{-4}$

b $8 \times 10^4 \times 3 \times 10^{-2} = 24 \times 10^2 = 2.4 \times 10^3$

c $8 \times 10^4 \div 4 \times 10^2 = 2 \times 10^2$

d $8 \times 10^4 + 4 \times 10^2 = 10^2 (8 \times 10^2 + 4) = 10^2 \times 804$
$= 8.04 \times 10^4$

4 a 13.3 billion $= 1.33 \times 10^{10}$ pounds

b Number of people $= \frac{1.33 \times 10^{10}}{500} = 1.33 \times 2 \times 10^7$
$= 26600000$ people

5 a $15500 = 1.55 \times 10^4$

b $6.55 \times 10^5 = 655000$

c $\frac{1.25 \times 10^5}{2.5 \times 10^2} = \frac{12.5 \times 10^4}{2.5 \times 10^2} = 5 \times 10^2$

d $\sqrt{1.6 \times 10^7} = (1.6 \times 10^7)^{\frac{1}{2}} = 4 \times 10^3$

Converting between fractions and decimals

1 a $\frac{43}{100} = 0.43$

b $\frac{3}{8} = 0.375$

c $\frac{11}{20} = 0.55$

2 a $0.8 = \frac{8}{10} = \frac{4}{5}$

b $0.45 = \frac{45}{100} = \frac{9}{20}$

c $0.584 = \frac{584}{1000} = \frac{73}{125}$

3 a Let $x = 0.\dot{7} = 0.777777...$ and $10x = 7.777777...$

To eliminate the block of recurring digits after the decimal point we take x from $10x$.

$10x - x = 7.777777... - 0.777777...$

$9x = 7$

$x = \frac{7}{9}$

b Let $x = 0.0\dot{4} = 0.0444444...$

So $10x = 0.444444$ and $100x = 4.444444$

Subtract: $100x - 10x = 4.444444... - 0.444444...$

$90x = 4$

$x = \frac{4}{90} = \frac{2}{45}$

c Let $x = 0.9\dot{5}\dot{4} = 0.954545454...$

So $10x = 9.54545454...$ and $1000x = 954.545454...$

Subtract: $1000x - 10x = 954.545454...$
$- 9.54545454...$

$990x = 945$

$x = \frac{945}{990}$

Dividing both parts of fraction by 5 and then by 9

gives $= \frac{21}{22}$

4 a Let $x = 0.\dot{5}1\dot{8} = 0.518518...$

$1000x = 518.518518...$

$1000x - x = 518.518518... - 0.518518...$

$999x = 518$

$x = \frac{518}{999}$

Dividing both parts of fraction by 37 gives $x = \frac{14}{27}$

b $0.76 = \frac{76}{100} = \frac{19}{25}$

5 $0.\dot{7} = \frac{7}{9}$

So $0.\dot{7} + \frac{2}{9} = \frac{7}{9} + \frac{2}{9}$

$= \frac{9}{9}$

$= 1$

6 $\frac{7}{8} = 0.875$

$\frac{4}{5} = 0.8$

$\frac{7}{10} = 0.7$

Putting the decimals in ascending order:

$-0.9, 0.7, 0.8, 0.85, 0.875$

Rewriting using the numbers in their original form:

$-0.9, \frac{7}{10}, \frac{4}{5}, 0.85, \frac{7}{8}$

Converting between fractions and percentages

1 a $25\% = \frac{1}{4}$

b $85\% = \frac{17}{20}$

c $68\% = \frac{17}{25}$

2 Maths: $\frac{65}{80} \times 100 = 81.25\%$

$81.25 > 80$

So Charlie did better at maths.

3 a $\frac{3}{10} = 30\%$

b $\frac{4}{25} = 16\%$

c $\frac{3}{7} = 42.9\%$ (to 3 s.f.)

Fractions and percentages as operators

1 a $\frac{3}{4}$ of £640 $= \frac{3}{4} \times 640 = 3 \times 160 = £480$

b 15% of £30 $= \frac{15}{100} \times 30 = £4.50$

c 195% of 80 kg $= \frac{195}{100} \times 80$

$= 156$ kg

2 Number of boys in School A = 56% of 600

$= \frac{56}{100} \times 600 = 336$

Percentage of boys in School B $= 100 - 35 = 65\%$

Number of boys in School B = 65% of 700

$= \frac{65}{100} \times 700 = 455$

Standard measurement units

1 a 9.7 kg $= 9.7 \times 1000 = 9700$ g

b 850 cm³ $= 850$ ml $= 850 \div 1000 = 0.85$ litres

c 2.05 km $= 2.05 \times 1000$ m $= 2050$ m

$= 2050 \times 100$ cm

$= 205\,000$ cm

2 1 day = 24 hours $= 24 \times 60$ minutes

$= 24 \times 60 \times 60$ seconds = 86400 seconds

$= 8.64 \times 10^4$ seconds

3 $0.34 \times 20 \times 12 = £81.60$

Rounding numbers

1 a 1259 = 1260 (correct to 3 s.f.)

b 14.919 = 14.9 (correct to 3 s.f.)

c 0.0003079 = 0.000308 (correct to 3 s.f.) ◄

> Remember, you don't start counting the digits for significant figures until the first non-zero digit.

d 9 084 097 = 9 080 000 (correct to 3 s.f.)

e $1.8099 \times 10^{-4} = 1.81 \times 10^{-4}$ (correct to 3 s.f.)

2 a 10.565 = 10.6 (correct to 1 decimal place)

b 123.9765 = 123.977 (correct to 3 decimal places)

c 0.02557 = 0.03 (correct to 2 decimal places)

d 3.9707 = 3.971 (correct to 3 decimal places)

e 0.00195 = 0.002 (correct to 3 decimal places)

f 4.098 = 4.10 (correct to 2 decimal places)

3 a 1989 = 2000 (correct to 1 significant figure)

b 1989 = 2000 (correct to 2 significant figures)

c 1989 = 1990 (correct to 3 significant figures)

4 $3.755 \times 10^{-4} = 0.0003755 = 0.0004$ (correct to 4 decimal places)

Estimation

1 a $\frac{5.9 \times 189}{0.54} \approx \frac{6 \times 200}{0.5} \approx \frac{1200}{0.5} \approx 2400$

> Write each number to 1 significant figure.

> There are twice as many halves as units in 1200.

b $\sqrt{4.65 + 28.9 \div 6} \approx \sqrt{5 + 30 \div 6} \approx \sqrt{5 + 5} \approx \sqrt{10} \approx 3$

2

	Question	Estimation	Answer
a	$3.45 \times 2.78 \times 0.09$	$\approx 3 \times 3 \times 0.09 \approx 0.8$	A
b	$12.56 \times 1.87 \times 0.45$	$\approx 10 \times 2 \times 0.5 \approx 10$	C
c	$120 \div 0.45$	$\approx 100 \div 0.5 \approx 200$	B
d	$0.01 \times 0.15 \times 109$	$\approx 0.01 \times 0.2 \times 100 \approx 0.2$	B
e	$0.12 \times 300 \times 0.53$	$\approx 0.1 \times 300 \times 0.5 \approx 15$	A
f	$6.07 \times 3.67 \times 0.1$	$\approx 6 \times 4 \times 0.1 \approx 2$	B
g	$20.75 \div 6.98$	$\approx 20 \div 7 \approx 3$	C
h	$0.01 \times 145 \times 35$	$\approx 0.01 \times 100 \times 40 \approx 40$	A
i	$6.5 \times 0.3 \times 0.01$	$\approx 7 \times 0.3 \times 0.01 \approx 0.02$	B
j	$65 \div 1050$	$\approx 70 \div 1000 \approx 0.07$	A

3 a $\sqrt{36} < \sqrt{45} < \sqrt{49}$ so $\sqrt{45}$ is between 6 and 7.
$\sqrt{45} \approx 6.7$ (accept 6.5 to 6.9)

   b $\sqrt{100} < \sqrt{104} < \sqrt{121}$ so $\sqrt{104}$ is between 10 and 11.
$\sqrt{104} \approx 10.2$ (accept 10.1 to 10.3)

   c $(2)^3 < (2.3)^3 < (3)^3$ so $(2.3)^3$ is between 8 and 27.
$(2.3)^3 \approx 12$ (accept 10 to 14)

   d $3^1 < 3^{1.4} < 3^2$ so $3^{1.4}$ is between 3 and 9.
$3^{1.4} \approx 5$ (accept 4 to 6)

Upper and lower bounds

1 a   Half of the smallest unit is 0.5 cm
Lower bound = least length = 144.5 cm

   b   Upper bound = greatest length = 145.5 cm

   c $144.5 \leq l < 145.5$ cm

Review it!

1 a   24 647.515

   b $21.5443469 = 21.5$ (1 d.p.)

2 a $8 - 1.5 = 6.5$

   b $-1 + 4 + 1 = 4$

   c $27 \times 3 = 81$

   d $1.65 \times 3.6 = 4 \times 1.65 \times 9 \div 10 = 6.6 \times 9 \div 10 = (66 - 6.6) \div 10 = 59.4 \div 10 = 5.94$

3 a $\frac{64}{12} = \frac{16}{3} = 5\frac{1}{3}$

   b $\frac{124}{13} = 9\frac{7}{13}$

4 $4\frac{3}{4} + \frac{1}{2}$ ◀

Use the largest integer as the whole number and the next largest as the numerator.

5 a   Half the smallest unit is 0.005 cm.
Least width = 4.545 cm and least length
= 2.225 cm
Least area = $4.545 \times 2.225 = 10.112625$ cm²
= 10.113 cm² (3 d.p.)
Greatest width = 4.555 cm and greatest length
= 2.235 cm
Greatest area = $4.555 \times 2.235 = 10.180425$ cm²
= 10.180 cm² (3 d.p.)

   b   The upper and lower bounds for area are the same when written to the nearest cm² (5 and 2).
Area = $5 \times 2 = 10$ cm²

6 a $6.02 \times 10^{23} \div 18 \times 10 = 3.34 \times 10^{23}$ molecules (3 s.f.)

   b   18 g = 0.018 kg
$0.018 \div 6.02 \times 10^{23} = 2.99 \times 10^{-26}$ kg (3 s.f.)

7 $(0.45 \times 0.78)^2 \approx (0.5 \times 0.8)^2 \approx 0.4^2 \approx 0.16$

8 a $7^0 = 1$

   b $9^{\frac{1}{2}} = \sqrt{9} = 3$ or -3

   c $8^{-2} = \frac{1}{8^2} = \frac{1}{64}$

   d $64^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$ or $-\frac{1}{4}$

9   Half the smallest unit = $0.1 \div 2 = 0.05$
Upper bound = $5.6 + 0.05 = 5.65$
Lower bound = $5.6 - 0.05 = 5.55$
Error interval is $5.55 \leq y < 5.65$

10 $\frac{1 - \sqrt{2}}{1 + \sqrt{2}} = \frac{1 - \sqrt{2}}{1 + \sqrt{2}} \times \frac{1 - \sqrt{2}}{1 - \sqrt{2}} = \frac{1 - 2\sqrt{2} + 2}{1 - 2} = \frac{3 - 2\sqrt{2}}{-1} = 2\sqrt{2} - 3$

11   Upper bound = 112.5 cm and lower bound = 111.5 cm
Error interval is $111.5 \leq a < 112.5$

12   Let $x = 0.\dot{7}\dot{2} = 0.727272...$
So $100x = 72.727272...$
$100x - x = 72.727272... - 0.727272...$
$99x = 72$
$x = \frac{72}{99} = \frac{8}{11}$

13 a a: upper bound = 0.67545, lower bound = 0.67535
b: upper bound = 2.345, lower bound = 2.335
Greatest value of $c = \frac{\text{Upper bound of } a}{\text{Lower bound of } b} = \frac{0.67545}{2.335}$
= 0.289272
Least value of $c = \frac{\text{Lower bound of } a}{\text{Upper bound of } b} = \frac{0.67535}{2.345}$
= 0.287996
Error interval is $0.287996 \leq c < 0.289272$

   b   The value of c is the same when rounded to 2 significant figures.
So $c = 0.29$ (to 2 s.f.)

14 a $\frac{3}{25} \div \frac{9}{50} = \frac{3}{25} \times \frac{50}{9} = \frac{1}{1} \times \frac{2}{3} = \frac{2}{3}$

   b $25 \div \frac{5}{16} = \frac{25}{1} \times \frac{16}{5} = \frac{5}{1} \times \frac{16}{1} = 80$

15 a $0.00000045 = 4.5 \times 10^{-7}$

   b   12 million = 12 000 000 = 1.2×10^7

   c $5640 = 5.64 \times 10^3$

16 $(8 \times 10^{-5}) \times (4 \times 10^9) = 8 \times 4 \times 10^{-5} \times 10^3$
= $32 \times 10^{-2} = 3.2 \times 10^{-1}$

17 a   Factors of 64: 1, 2, 4, 8, 16, 32, 64

   b   Factors of 100: 1, 2, 4, 5, 10, 20, 25, 50, 100
Highest common factor = 4

Algebra

Simple algebraic techniques

1 a   Formula

   b   Identity

   c   Expression

   d   Identity

   e   Equation

2 a $15x^2 - 4x + x^2 + 9x - x - 6x^2 = 10x^2 + 4x$

  b $7a + 5b - b - 4a - 5b = 3a - b$

  c $8yx + 5x^2 + 2xy - 8x^2 = -3x^2 + 10xy$ (or $10xy - 3x^2$)

  d $x^3 + 3x - 5 + 2x^3 - 4x = 3x^3 - x - 5$

3 $P = I^2R = \left(\frac{2}{3}\right)^2 \times 36 = \frac{4}{9} \times 36 = 16$

4 $v = u + at = 20 + (-8)(2) = 20 - 16 = 4$

Removing brackets

1 a $2x + 8$

  b $63x + 21$

  c $-1 + x$ or $x - 1$

  d $3x^2 - x$

  e $3x^2 + 3x$

  f $20x^2 - 8x$

2 a $2(x + 3) + 3(x + 2) = 2x + 6 + 3x + 6 = 5x + 12$

  b $6(x + 4) - 3(x - 7) = 6x + 24 - 3x + 21 = 3x + 45$

  c $3x^2 + x + x^2 + x = 4x^2 + 2x$

  d $3x^2 - 4x - 6x + 8 = 3x^2 - 10x + 8$

3 a $p^{-\frac{4}{3}} \times p^{\frac{8}{3}} = p^{-\frac{4}{3}+\frac{8}{3}} = p^{\frac{4}{3}}$

  b $\dfrac{r^5 \times r^{-2}}{r^4} = \dfrac{r^{5-2}}{r^4}$

      $= \dfrac{r^3}{r^2}$

      $= r^{3-2}$

      $= r^{-1}$

  c $\left(b^{\frac{3}{2}}\right)^{-4} = b^{\frac{3}{2} \times (-4)}$

      $= b^{-6}$

4 a $(t + 3)(t + 5) = t^2 + 5t + 3t + 15 = t^2 + 8t + 15$

  b $(x - 3)(x + 3) = x^2 + 3x - 3x - 9 = x^2 - 9$

  c $(2y + 9)(3y + 7) = 6y^2 + 14y + 27y + 63$

      $= 6y^2 + 41y + 63$

  d $(2x - 1)^2 = (2x - 1)(2x - 1) = 4x^2 - 2x - 2x + 1$

      $= 4x^2 - 4x + 1$

5 a $(x + 7)(x + 2)(2x + 3) = (x^2 + 9x + 14)(2x + 3)$

                   $= 2x^3 + 21x^2 + 55x + 42$

  b $(2x - 1)(3x - 2)(4x - 3) = (6x^2 - 7x + 2)(4x - 3)$

                   $= 24x^3 - 28x^2 + 8x - 18x^2$

                         $+ 21x - 6$

                   $= 24x^3 - 46x^2 + 29x - 6$

Factorising

1 a $24t + 18 = 6(4t + 3)$

  b $9a - 2ab = a(9 - 2b)$

  c $5xy + 15yz = 5y(x + 3z)$

  d $24x^3y^2 + 6xy^2 = 6xy^2(4x^2 + 1)$

2 a $(x + 7)(x + 3)$

  b $(x + 5)(x - 3)$

  c To get 6, use factors 2 and 3, and to get 10 use factors 2 and 5. This gives $2x \times 2 = 4x$ and $3x \times 5 = 15x$, total $19x$; so solution is $(2x + 5)(3x + 2)$

  d Difference of two squares. Factorises to $(2x + 7)(2x - 7)$

  e $25x^2 - 7 = (5x + \sqrt{7})(5x - \sqrt{7})$

3 $\dfrac{2x^2 + 7x - 4}{x^2 + 7x + 12} = \dfrac{(2x - 1)(x + 4)}{(x + 3)(x + 4)}$

              $= \dfrac{2x - 1}{x + 3}$

4 $\dfrac{1}{x - 7} - \dfrac{x + 10}{2x^2 - 11x - 21} = \dfrac{1}{x - 7} - \dfrac{x + 10}{(2x + 3)(x - 7)}$

> Factorise the denominator of the 2nd fraction.

> Make both denominators the same.

  $= \dfrac{2x + 3}{(x - 7)(2x + 3)} - \dfrac{x + 10}{(2x + 3)(x - 7)}$

> Combine into one fraction and simplify

  $= \dfrac{2x + 3 - x - 10}{(x - 7)(2x + 3)}$

  $= \dfrac{x - 7}{(x - 7)(2x + 3)}$

  $= \dfrac{1}{2x + 3}$

Changing the subject of a formula

1 a $A = \pi r^2$ $\dfrac{A}{\pi} = r^2$

    $r = \sqrt{\dfrac{A}{\pi}}$

  b $A = 4\pi r^2$

    $\dfrac{A}{4\pi} = r^2$

    $r = \sqrt{\dfrac{A}{4\pi}}$

  c $V = \dfrac{4}{3}\pi r^3$

    $3V = 4\pi r^3$

    $\dfrac{3V}{4\pi} = r^3$

    $r = \sqrt[3]{\dfrac{3V}{4\pi}}$

2 a $y = mx + c$ (c)

> Subtract mx from both sides.

    $c = y - mx$

  b $v = u + at$ (a)

> Subtract u from both sides.

    $v - u = at$

    $a = \dfrac{v - u}{t}$

> Divide both sides by t.

  c $v^2 = 2as$ (s)

> Divide both sides by $2a$.

    $s = \dfrac{v^2}{2a}$

  d $v^2 = u^2 + 2as$ (u)

> Subtract $2as$ from both sides.

    $v^2 - 2as = u^2$

    $u = \sqrt{v^2 - 2as}$

> Square root both sides.

  e $s = \dfrac{1}{2}(u + v)t$ (t)

> Multiply both sides by 2.

    $2s = (u + v)t$

    $t = \dfrac{2s}{u + v}$

> Divide both sides by $(u + v)$.

  f $\dfrac{1}{x} = \dfrac{1}{2p} + \dfrac{1}{q}$

    $2pq = xq + 2px$

  $x(2p + q) = 2pq$

        $x = \dfrac{2pq}{2p + q}$

Solving linear equations

1 a $x - 7 = -4$

    $x = -4 + 7 = 3$

  b $9x = 27$

    $x = 27 \div 9 = 3$

  c $\dfrac{x}{5} = 4$

    $x = 4 \times 5 = 20$

2 a $3x + 1 = 16$

$\qquad 3x = 15$

$\qquad\quad x = 5$

b $\frac{2x}{3} = 12$

$\qquad 2x = 36$

$\qquad\ \ x = 18$

c $\frac{3x}{5} + 4 = 16$

$\qquad\ \ \frac{3x}{5} = 12$

$\qquad 3x = 60$

$\qquad\ \ x = 20$

3 a $5(1 - x) = 15$

$\qquad 5 - 5x = 15$

$\qquad\ \ -5x = 10$

$\qquad\quad\ \ x = -2$

b $2m - 4 = m - 3$

$\qquad m - 4 = -3$

$\qquad\quad\ m = 1$

c $9(4x - 3) = 3(2x + 3)$

$\qquad 36x - 27 = 6x + 9$

$\qquad 30x - 27 = 9$

$\qquad\quad\ 30x = 36$

$\qquad\quad\ \ x = \frac{36}{30} = \frac{6}{5},\ 1\frac{1}{5}$ or 1.2

> Always cancel fractions so that they are in their lowest terms. Here both top and bottom can be divided by 6.

Solving quadratic equations using factorisation

1 a $(x + 3)(x + 2) = 0$ giving $x = -2$ or -3

b $(x + 3)(x - 4) = 0$ giving $x = -3$ or 4

c $(2x + 7)(x + 5) = 0$ giving $x = -\frac{7}{2}$ or -5

2 a   Area of triangle $= \frac{1}{2} \times$ base $\times$ height

$$\frac{1}{2}(2x + 3)(x + 4) = 9$$

$$2x^2 + 11x + 12 = 18$$

$$2x^2 + 11x - 6 = 0$$

b $2x^2 + 11x - 6 = 0$

$\qquad (2x - 1)(x + 6) = 0$

So $x = \frac{1}{2}$ or $x = -6$

Since x represents a height, only the positive value is valid.

$$x = \frac{1}{2}$$

c $x = 0.5$, so base is $2 \times 0.5 + 3 = 4\,\text{cm}$ and height is $0.5 + 4 = 4.5\,\text{cm}$

3   By Pythagoras' theorem

$$(x + 1)^2 + (x + 8)^2 = 13^2$$

$$x^2 + 2x + 1 + x^2 + 16x + 64 = 169$$

$$2x^2 + 18x - 104 = 0$$

Dividing by 2 gives

$$x^2 + 9x - 52 = 0$$

$$(x - 4)(x + 13) = 0$$

$$\text{so } x = 4 \text{ or } x = -13$$

(Disregard $x = -13$ as x is a length.)

Hence, $x = 4$ cm

(This also means the sides of the triangle are 5, 12 and 13 cm.)

Solving quadratic equations using the formula

1   Comparing the equation given, with $ax^2 + bx + c$ gives $a = 2$, $b = -1$ and $c = -7$.

Substituting these values into the quadratic equation formula gives:

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-7)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{57}}{4}$$

$$= \frac{1 + 7.550}{4} \text{ or } \frac{1 - 7.550}{4} \text{ (to 4 s.f.)}$$

$$x = 2.14 \text{ or } -1.64 \text{ (to 3 s.f.)}$$

2 a $\frac{2x + 3}{x + 2} = 3x + 1$

$\qquad 2x + 3 = (x + 2)(3x + 1)$

$\qquad 2x + 3 = 3x^2 + x + 6x + 2$

$\qquad\qquad 0 = 3x^2 + 5x - 1$

or $3x^2 + 5x - 1 = 0$

b   Comparing the equation given, with $ax^2 + bx + c = 0$ gives $a = 3$, $b = 5$ and $c = -1$

Substituting these values into the quadratic equation formula gives:

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-5 \pm \sqrt{25 + 12}}{6} = \frac{-5 \pm \sqrt{37}}{6} = \frac{-5 + \sqrt{37}}{6} \text{ or } \frac{-5 - \sqrt{37}}{6}$$

Hence $x = 0.18$ or -1.85 (2 d.p.)

Solving simultaneous equations

1 a   Firstly write the second equation so that in both equations the x value and the numerical value are aligned.

$$y = 3x - 7 \qquad\qquad (1)$$

$$y = -2x + 3 \qquad\qquad (2)$$

Notice that the coefficient of y (the number multiplying y, i.e. 1) is the same for both equations. We can eliminate y by subtracting equation (2) from equation (1).

Subtracting $(1) - (2)$ we obtain

$$0 = 5x - 10$$

$$5x = 10$$

$$x = 2$$

Substituting $x = 2$ into equation (1) we obtain

$$y = 3(2) - 7$$

$$= 6 - 7$$

$$= -1$$

Checking by substituting $x = 2$ into equation (2) we obtain

$$y = -2x + 3$$

$$= -2(2) + 3$$

$$= -1$$

Hence solutions are $x = 2$ and $y = -1$.

b $y = 2x - 6$ (1)

$y = -3x + 14$ (2)

Subtracting (1) − (2) we obtain

$0 = 5x - 20$

$x = 4$

$y = 2 \times 4 - 6$ (1)

$y = 2.$

2 Equating expressions for y gives

$10x^2 - 5x - 2 = 2x - 3$

$10x^2 - 7x + 1 = 0$

Factorising this quadratic gives

$(5x - 1)(2x - 1) = 0$

Hence $x = \frac{1}{5}$ or $x = \frac{1}{2}$

Substituting $x = \frac{1}{5}$ into $y = 2x - 3$ gives

$y = -2\frac{3}{5}$

Substituting $x = \frac{1}{2}$ into $y = 2x - 3$ gives

$y = -2$

Hence $x = \frac{1}{5}$ and $y = -2\frac{3}{5}$ or $x = \frac{1}{2}$ and $y = -2$

3 Equating the y values gives

$x^2 + 5x - 4 = 6x + 2$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or -2

When $x = 3$, $y = 6 \times 3 + 2 = 20$

When $x = -2$, $y = 6 \times (-2) + 2 = -10$

Points are (3, 20) and (−2, −10)

Solving inequalities

1 a $1 - 2x < -11$

> Subtract 1 from both sides.
> Divide both sides by –2 and reverse sign.

$-2x < -12$

$x > 6$

$\{x: x > 6\}$

b $2x - 7 \geq 15$

> Add 7 to both sides.
> Divide both sides by 2.

$2x \geq 22$

$x \geq 11$
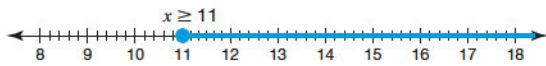
$\{x: x \geq 11\}$

c $\frac{x - 5}{3} < 7$

> Multiply both sides by 3.
> Add 5 to both sides.

$x - 5 < 21$

$x < 26$
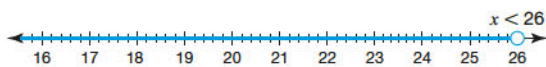
$\{x: x < 26\}$

2 a $2x - 4 > x + 6$

$x - 4 > 6$

$x > 10$

b $4 + x < 6 - 4x$

$4 < 6 - 5x$

$-2 < -5x$

$\frac{-2}{-5} > x$

$x < 0.4$ or $\frac{2}{5}$

c $2x + 9 \geq 5(x - 3)$

$2x + 9 \geq 5x - 15$

$9 \geq 3x - 15$

$24 \geq 3x$

$8 \geq x$

or $x \leq 8$

3 a
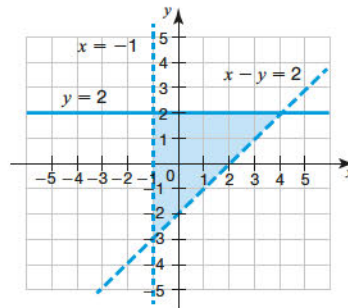
b (0, 2), (0,1), (0, 0), (0, −1), (1, 2), (1, 1), (1, 0), (2, 2), (2, 1), (3, 2)

4 $x^2 > 3x + 10$

$x^2 - 3x - 10 > 0$

$(x - 5)(x + 2) > 0$

$x < -2$ and $x > 5$

Problem solving using algebra

1 Let the larger number $= x$ and the smaller number $= y$.

$x + y = 77$

$x - y = 25$

Adding gives $2x = 102$ which gives $x = 51$ so y must be 26.

2 Let the number added $= x$

$\frac{15 + x}{31 + x} = \frac{5}{6}$

$6(15 + x) = 5(31 + x)$

$90 + 6x = 155 + 5x$

$x = 65$

Check the answer $\frac{15 + 65}{31 + 65} = \frac{80}{96} = \frac{5}{6}$

3 Perimeter: $2x + 2y = 24$ so $x + y = 12$ (1)

Area: $xy = 27$ (2)

From equation (1) $y = 12 - x$

Substitute into equation (2):

$x(12 - x) = 27$

So, $12x - x^2 = 27$

Hence, $x^2 - 12x + 27 = 0$

Factorising gives $(x - 3)(x - 9) = 0$

So $x = 3$ or $x = 9$

Substituting each of these values into equation (1) we have

$3 + y = 12$ or $9 + y = 12$, giving $y = 9$ or $y = 3$.

Hence, length = 9 cm and width = 3 cm.

Use of functions

1 a Divide by 5, then subtract 4: $\frac{x}{5} - 4$

b Add 4, then multiply by 5: $5(x + 4)$

2 a i $3 - 1 = 2$; reciprocal of 2 $= \frac{1}{2}$

ii $1\frac{1}{4} - 1 = \frac{1}{4}$; reciprocal of $\frac{1}{4} = 4$

b reciprocal of $-3 = \frac{-1}{3}$; $\frac{-1}{3} + 1 = \frac{2}{3}$

c Subtract 1, then find the reciprocal: $\frac{1}{x-1}$

3 a $f(0) = \frac{1}{0-1} = -1$

b $f(-\frac{1}{2}) = \frac{1}{-\frac{1}{2}-1} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$

c Let $y = \frac{1}{x-1}$

$y(x - 1) = 1$

$xy - y = 1$

$xy = y + 1$

$x = \frac{y+1}{y}$

$f^{-1}(x) = \frac{x+1}{x}$

4 a $fg(x) = \sqrt{((x+4)^2 - 9)}$

$= \sqrt{x^2 + 8x + 16 - 9}$

$= \sqrt{x^2 + 8x + 7}$

b $gf(x) = \sqrt{(x^2 - 9)} + 4$

c $gf(3) = \sqrt{(3^2 - 9)} + 4$

$= 4$

Iterative methods

1

x	$3x^3 - 2x^2 - 3$
0	−3
0.1	−2.977
0.2	−2.896
0.3	−2.739
0.4	−2.488
0.5	−2.125
0.6	−1.632
0.7	−0.991
0.8	−0.184
0.9	0.807
0.81	−0.093
0.82	−0.001
0.83	0.093

$x = 0.8$

or

x	$3x^3 - 2x^2 - 3$
0	−3
1	2
0.5	−2.125
0.75	−0.609
0.85	0.287

$x = 0.8$

2 $x_0 = 1.5$

$x_1 = 1.5182945$

$x_2 = 1.5209353$

$x_3 = 1.5213157$

$x_4 = 1.5213705 \approx 1.521$ (correct to three decimal places)

Check value of $x^3 - x - 2$ for $x = 1.5205, 1.5215$

x	$f(x)$
1.5205	−0.005225
1.5215	0.0007151

Since there is a change of sign, $a = 1.521$ is correct to three decimal places.

Equation of a straight line

1 a $2y = 4x - 5$

$y = 2x - \frac{5}{2}$

Comparing this to $y = mx + c$ we have gradient $m = 2$

b Gradient $= -\frac{1}{m} = -\frac{1}{2}$

c $y = -\frac{1}{2}x + 5$ (or $2y = -x + 10$)

2 $y - y_1 = m(x - x_1)$ where $m = 3$ and $(x_1, y_1) = (2, 3)$.

$y - 3 = 3(x - 2)$

$y - 3 = 3x - 6$

$y = 3x - 3$

3 $y - y_1 = m(x - x_1)$ where $m = 2$ and $(x_1, y_1) = (-1, 0)$

$y - 0 = 2(x - (-1))$

$y = 2(x + 1)$

$y = 2x + 2$

$-y + 2x + 2 = 0$ (or $2x - y + 2 = 0$)

4 a Gradient $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{6 - (-2)} = \frac{4}{8} = \frac{1}{2}$

b $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 6}{2}, \frac{0 + 4}{2}\right) = (2, 2)$

c i Gradient $= -2$ (i.e invert $\frac{1}{2}$ and change the sign)

ii $y - y_1 = m(x - x_1)$

$y - 2 = -2(x - 2)$

$y - 2 = -2x + 4$

$y = -2x + 6$

Quadratic graphs

1 a $2x^2 - 12x + 1 = 2\left[x^2 - 6x + \frac{1}{2}\right]$

$= 2\left[(x - 3)^2 - 9 + \frac{1}{2}\right]$

$= 2\left[(x - 3)^2 - \frac{17}{2}\right]$

$= 2(x - 3)^2 - 17$

b i Turning point is at $(3, -17)$

ii At the roots,

$2(x - 3)^2 - 17 = 0$

$2(x - 3)^2 = 17$

$(x - 3)^2 = \frac{17}{2}$

$x - 3 = \sqrt{\frac{17}{2}}$

$x = \sqrt{\frac{17}{2}} + 3$

Roots are $x = 0.1$ and $x = 5.9$ (1 d.p.)

c

(3, −17)
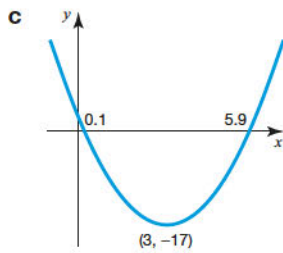
2 a $y = (x + 1)(x - 5)$ or $y = x^2 - 4x - 5$

b $y = -(x - 2)(x - 7)$ or $y = -x^2 + 9x - 14$

3 a $x^2 + 12x - 16 = (x + 6)^2 - 36 - 16$

$\qquad = (x + 6)^2 - 52$

b Turning point is at $(-6, -52)$

Recognising and sketching graphs of functions

1 a B

b F

c E

d A

e D

f C

2 a
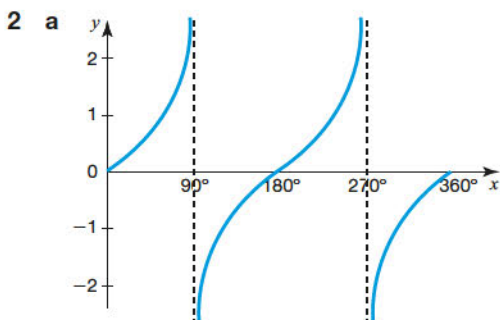
b Read up from 60° to the graph, then read across until you hit the graph again.

$x = 240°$

3 a A

b G

c F

d E

Translations and reflections of functions

1 a (3, 5) (i.e. a movement of one unit to the right)

b (−1, 5) (i.e. a movement of three units to the left)

c (2, −5) (i.e. a reflection in the x-axis)

d (−2, 5) (i.e. a reflection in the y-axis)

2
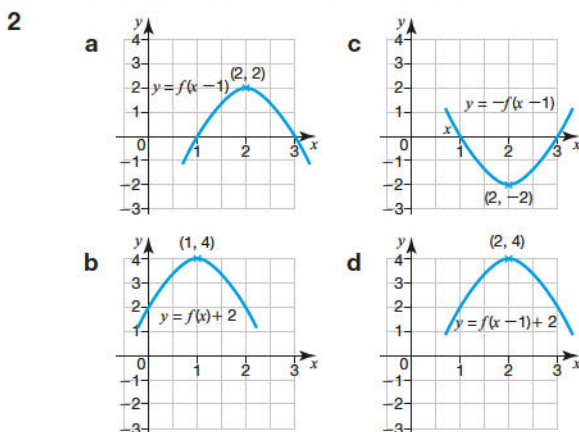
Equation of a circle and tangent to a circle

1 a Centre is (0, 0)

b radius $= \sqrt{49} = 7$

2 a $x^2 + y^2 = 100$

b Gradient of radius to (8, 6) $= \frac{6}{8} = \frac{3}{4}$

Gradient of tangent $= -\frac{4}{3}$

c $y - y_1 = m(x - x_1)$

$y - 6 = -\frac{4}{3}(x - 8)$

$y = -\frac{4}{3}x + 16\frac{2}{3}$ (or $3y = -4x + 50$)

Real-life graphs

1 a 08:00 to 09:00 is 1 hour (h), which is 1 unit on x axis.

Average speed $=$ gradient $= \frac{2.5}{0.5} = 5$ km/h

b 15 mins $= 0.25$ hours

c Average speed $=$ gradient between 09:30 and 09:45

$\qquad = \frac{6}{0.25} = 24$ km/h

2

NAILIT!

When drawing a velocity-time graph, ensure that the axes are labelled with quantities and units.

Any values and letters for quantities that need to be found should be labelled on the graph.

a
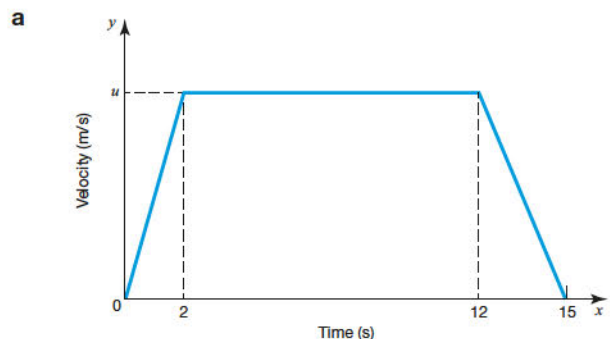
b Total distance travelled $=$ Area under the velocity-time graph

Use the formula for the area of a trapezium:

Distance $= \frac{1}{2}(15 + 10) \times u$

$\qquad = 12.5u$

Use the formula for the area of a trapezium.

The total distance travelled $= 50$ m

Hence $50 = 12.5u$

$\qquad u = 4$ m/s

c Velocity $= 4$ m/s and time for deceleration $= 3$ s

Deceleration $=$ gradient $= \frac{4}{3} = 1.33$ m/s²

Since deceleration is negative acceleration, a positive answer is appropriate.

9

Generating sequences

1 **a** 17: sequence goes up by 3

   b 3.0: sequence goes up by 0.2

   c −12: sequence goes down by 3

   d 432: last term is multiplied by 6

   e $\frac{1}{48}$: last term is multiplied by $\frac{1}{2}$

   f $-\frac{1}{16}$: last term is multiplied by $-\frac{1}{2}$

2 Second term is $(-4)^2 + 1 = 17$ and third term is $17^2 + 1 = 290$

   Second term is 17, third term is 290.

3 Reverse the process: to find the preceding term, subtract 1 and halve.

   Second term is $(12 - 1) \div 2 = \frac{11}{2} = 5.5$

   First term is $(5.5 - 1) \div 2 = \frac{4.5}{2} = 2.25$

   First term is 2.25, second term is 5.5

The nth term

1 **a** When $n = 1$, $50 - 3(1) = 47$

   When $n = 2$, $50 - 3(2) = 44$

   When $n = 3$, $50 - 3(3) = 41$

   First three terms are 47, 44, 41

   b Use the nth term formula to find the value of n when the nth term $= 34$

   $50 - 3n = 34$

   $3n = 16$

   $n = 16 \div 3$

   The value of n is not an integer so 34 is not a number in the sequence.

   c Use the nth term formula to find the value of n when the nth term is less than zero (i.e. negative).

   $50 - 3n < 0$ (subtracting 50 from both sides)

   $-3n < -50$ (dividing both sides and reversing the inequality sign)

   $n > \frac{50}{3}$

   $n > 16\frac{2}{3}$

   As n has to be an integer, its lowest possible value is $n = 17$.

   Check that you get a negative term when $n = 17$ is put back into the nth term formula.

   17th term $= 50 - 3 \times 17 = 50 - 51 = -1$

2 **a** The first four terms are: $2 \times 3^1, 2 \times 3^2, 2 \times 3^3, 2 \times 3^4$

   $= 6, 18, 54, 162$

   b As the nth term formula is 2×3^n both 2 and 3 are factors, so 6 must also be a factor.

3 **a** By inspection: nth term $= \frac{1}{n}$

   b By inspection: nth term $= \frac{1}{(\sqrt{2})^n}$ or $\left(\frac{1}{\sqrt{2}}\right)^n$

4 **a** Common difference between terms $= 2$ so formula will start with $2n$.

   When $n = 1$, you need to subtract 3 from $2n$ to get an answer of -1.

   Therefore nth term $= 2n - 3$

   b $59 = 2x - 3$

   $2x = 62$

   $x = 31$

5

	4,	17,	38,	67
First differences		13	21	29
Second differences			8	8

As a second difference is needed before a constant difference is found, there is an n^2 term in the nth term. The number in front of this n^2 will be $\frac{8}{2} = 4$.

So first part of the nth term will be $4n^2$.

n	1	2	3	4
Term	4	17	38	67
$4n^2$	4	16	36	64
Term − $4n^2$	0	1	2	3

Use this set of information to work out the linear part of the sequence (the part with an n term and a number).

Difference	1	1	1

This means that the linear sequence will start with n.

When $n = 2$, 'Term − $4n^2$' is 1, not 2, so if n is in the term you also need to subtract 1.

This makes the linear part of the sequence $n - 1$.

Check it with a different value of n. When $n = 3$, $n - 1$ equals 2. This is the correct value for 'Term − $4n^2$'.

Combining the terms gives nth term $= 4n^2 + n - 1$

Arguments and proofs

1 **a** $2n$ is always even as it has 2 as a factor. Adding 1 to an even number always gives an odd number. The statement is true.

   b $x^2 - 9 = 0$ so $x^2 = 9$ and $x = \sqrt{9} = \pm3$

   The statement is false.

   c n could be a decimal such as 4.25 so squaring it would not give an integer.

   The statement is false.

   d If n was 1, or a fraction smaller than 1, this would not be true.

   The statement is false.

2 Let the consecutive integers be n, $n + 1$, $n + 2$ and $n + 3$, where n is an integer that can be either odd or even.

   Sum of the integers $= n + n + 1 + n + 2 + n + 3$

   $= 4n + 6 = 2(2n + 3)$

   As 2 is a factor of this expression, the sum of four consecutive integers must be a multiple of 2, and therefore even.

3 Let the consecutive integers be x, $x + 1$ and $x + 2$, where x is an integer that can be either odd or even.

   Sum of the integers $= x + x + 1 + x + 2 = 3x + 3$

   $= 3(x + 1)$

   As 3 is a factor of this expression, the sum of three consecutive integers must be a multiple of 3.

4 **a** The numerator is larger than the denominator so the fraction will always be greater than 1. The statement is false.

   b As a is larger than b, squaring a will result in a larger number than squaring b. Hence $a^2 > b^2$ so the statement is false.

c The square root of a number can have two values, one positive and the other negative so, this statement is false.

Review it!

1 a $-3(3x - 4) = -9x + 12$

b $4x + 3(x + 2) - (x + 2) = 4x + 3x + 6 - x - 2$
$$= 6x + 4$$

c $(x + 3)(2x - 1)(3x + 5) = (2x^2 + 5x - 3)(3x + 5)$
$$= 6x^3 + 25x^2 + 16x - 15$$

2 a $2x^2 + 7x - 4 = (2x - 1)(x + 4)$

b $2x^2 + 7x - 4 = 0$
$x = \frac{1}{2}$ or $x = -4$

3 a $(2x^2y)^3 = 8x^6y^3$

b $2x^{-3} \times 3x^4 = 6x$

c $\frac{15a^3b}{3a^3b^2} = \frac{5}{b}$

4
$$3x + 2y = 8 \quad (1)$$
$$5x + y = 11 \quad (2)$$
$$(2) \times 2: 10x + 2y = 22 \quad (3)$$
$$(3) - (1): 7x = 14$$
$$x = 2$$
Substitute into (2) to find y
$$5 \times 2 + y = 11$$
$$y = 11 - 10$$
$$y = 1$$

5 a $\frac{3}{x + 7} = \frac{2 - x}{x + 1}$
$$3(x + 1) = (2 - x)(x + 7)$$
$$3x + 3 = 2x + 14 - x^2 - 7x$$
$$3x + 3 = -x^2 - 5x + 14$$
$$x^2 + 8x - 11 = 0$$

b $x^2 + 8x - 11 = 0$
$$x = \frac{-8 \pm \sqrt{(8)^2 - 4 \times 1 \times (-11)}}{2 \times 1}$$
$$x = \frac{-8 \pm \sqrt{108}}{2}$$
$$x = \frac{-8 \pm 6\sqrt{3}}{2}$$
$$x = -4 + 3\sqrt{3} \text{ or } x = -4 - 3\sqrt{3}$$
So $x = 1.20$ or $x = -9.20$ (to 2 d.p.)

6 $\frac{3y - x}{z} = ax + 2 \quad (x)$
$$3y - x = z(ax + 2)$$
$$3y - x = axz + 2z$$
$$3y - 2z = axz + x$$
$$3y - 2z = x(az + 1)$$
$$x = \frac{3y - 2z}{az + 1}$$

7 a $y = \frac{x}{3} + 5$
$$3y = x + 15$$
$$x = 3y - 15$$
Now replace x with $f^{-1}(x)$ and y with x.
$$f^{-1}(x) = 3x - 15 \text{ or } f^{-1}(x) = 3(x - 5)$$

b $fg(x) = \frac{(2(x)^2 + k)}{3} + 5$
So $fg(2) = \frac{(8 + k)}{3} + 5$
We know that $fg(2) = 10$
So $\frac{(8 + k)}{3} + 5 = 10$
$$(8 + k) + 15 = 30$$
$$8 + k = 15$$
$$k = 7$$

8 a Let $n = 1: 30 - 4 \times 1 = 26$
Let $n = 2: 30 - 4 \times 2 = 22$
Let $n = 3: 30 - 4 \times 3 = 18$
First three terms are 26, 22, 18.

b $30 - 4n < 0$
$$-4n < -30$$
$$n > \frac{-30}{-4}$$
$$n > 7.5$$
n must be an integer, so the lowest possible value of n is $n = 8$
Therefore the first negative term of the sequence is:
$30 - 4 \times 8 = -2$

9 $x = 4, y = 3$
$(4)^2 + (3)^2 = 16 + 9 = 25$
So $x^2 + y^2 > 21$
Hence the point (4, 3) lies outside the circle.

10 $(\sqrt{x} + \sqrt{9y})(\sqrt{x} - 3\sqrt{y})$
Simplify terms inside the brackets if possible
$(\sqrt{x} + 3\sqrt{y})(\sqrt{x} - 3\sqrt{y}) = x + 3\sqrt{xy} - 3\sqrt{xy} - 9y$
$$= x - 9y$$

11 a $2x^2 + 8x + 1 = 2(x^2 + 4x) + 1$
$$= 2(x + 2)^2 - 8 + 1$$
$$= 2(x + 2)^2 - 7$$

b **i** Turning point is $(-2, -7)$.
ii For $2x^2 + 8x + 1 = 0$
$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 2 \times 1}}{2 \times 2}$$
$$x = \frac{-8 \pm \sqrt{56}}{4}$$
$$x = \frac{-8 \pm 2\sqrt{14}}{4}$$
$$x = \frac{-4 \pm \sqrt{14}}{2}$$
So roots are at $x = -3.9$ and $x = -0.1$ (1 d.p.)

c
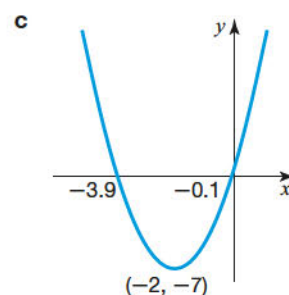
12 Perimeter of $ABCD = 2 \times (4x + (2x - 3)) = 12x - 6$
Perimeter of $EFG = 2x - 1 + x + 9 + 5x - 2 = 8x + 6$
Equate the perimeters to find x

$12x - 6 = 8x + 6$

$4x = 12$

$x = 3$

The height of the triangle, $EF = 2 \times 3 - 1 = 5$ cm

The base of the triangle, $EG = 3 + 9 = 12$ cm

Area of the triangle $= \frac{1}{2} \times 12 \times 5 = 30\,cm^2$

13 Side AB is parallel to side CD, so $k = 5$.

Gradient of $BD = \frac{5-(-2)}{-1-(-2)} = \frac{7}{1} = 7$

Using point B $(-2, -2)$

$y - (-2) = 7(x - (-2))$

$y + 2 = 7x + 14$

Equation of BD is $y = 7x + 12$

14 $x^2 + y^2 = 4$ (1)

$2y - x = 2$ (2)

Rearrange (2) for y: $y = \frac{1}{2}x + 1$ (3)

Substitute (3) into (1):

$x^2 + \left(\frac{1}{2}x + 1\right)^2 = 4$

$x^2 + \frac{x^2}{4} + x + 1 = 4$

$5x^2 + 4x + 4 = 16$

$5x^2 + 4x - 12 = 0$

$(5x - 6)(x + 2) = 0$

$x = \frac{6}{5} = 1.2$ or $x = -2$

Substitute into (2) to find y

So $x = \frac{6}{5}, y = \frac{8}{5}$ or $x = -2, y = 0$

Ratio, proportion and rates of change

Introduction to ratios

1 a $2 : 6 = 1 : 3$ (divide both sides by 2)

b $25 : 60 = 5 : 12$ (divide both sides by 5)

c $1.6 : 3.6 = 4 : 9$ (divide both sides by 0.4, or multiply by ten then divide by four)

2 a $250\,g : 2\,kg = 250\,g : 2000\,g = 250 : 2000 = 1 : 8$

b $25\,m : 250\,mm = 25\,000\,mm : 250\,mm$
$= 25\,000 : 250 = 100 : 1$

c $2\,cl : 1\,l = 2\,cl : 100\,cl = 2 : 100 = 1 : 50$

3 Ratio $= 3.5 : 2.1 = 35 : 21 = 5 : 3$

Total shares $= 5 + 3 = 8$

1 share $= £\frac{400}{8} = £50$

5 shares $= 5 \times £50 = £250$

3 shares $= 3 \times £50 = £150$

4 4 parts $= 180$

1 part $= 45$ (Dividing both sides by 4)

3 parts $= 3 \times 45$

$= 135$

Hence there are $180 + 135 = 315$ members of the gym.

5 The ratio is $21 : 25 : 29$

Total shares $= 21 + 25 + 29 = 75$

One share $= £\frac{150\,000}{75} = £2000$

Youngest daughter receives $21 \times 2000 = £42\,000$

6 Total number of parts in the ratio $= 5 + 2 = 7$

Now pick a number that is divisible by 7. We will choose 70.

Dividing this into the ratio $5 : 2$ gives 50 male guests and 20 female guests.

60% of male guests are under 40 and 70% of female guests are under 40.

60% of 50 = 30 (males under 40)

70% of 20 = 14 (females under 40)

So if there were 70 guests, 44 of them would be under 40 years old. Use this information to work out the correct percentage, whatever the number of guests:

$\frac{44}{70} \times 100 = 62.9\%$

7 There are 2 more parts of the ratio for 20p coins, and 6 more 20p coins.

So 2 parts = 6 coins

1 part = 3 coins.

Hence there are $5 \times 3 = 15$ 10p coins, and $7 \times 3 = 21$ 20p coins

Total amount (£) in the money box $= 15 \times 0.1 + 21 \times 0.2$
$= 1.5 + 4.2 = £5.70$

8 Let x be the number of yellow marbles.

So $5x$ = number of red marbles, and $2 \times 5x = 10x$ = number of blue marbles.

Hence the ratio of blue to red to yellow marbles
$= 10x : 5x : x = 10 : 5 : 1$

Scale diagrams and maps

1 First convert 150 km to cm.

150 km = 150 000 m = 15 000 000 cm

500 000 cm is equivalent to 1 cm on the map.

Distance in cm on the map $= \frac{15\,000\,000}{500\,000} = 30$ cm

2 a Distance between the ship and the port = 2 cm.

> This is measured using a ruler on the map.

We now need to get the units the same.

10 km = 10 000 m = 1 000 000 cm

The scale is $2 : 1\,000\,000$

Dividing both sides of the ratio by 2 gives

$1 : 500\,000$

b Measuring the actual distance between the two ships gives 1.2 cm

Actual distance $= 1.2 \times 500\,000 = 600\,000$ cm

Divide this by 100 and then 1000 to give the actual distance in km.

$600\,000\,cm = 6$ km

Percentage problems

1 $\frac{8}{300} \times 100 = 2.\dot{6} = 2.67\%$ to 2 d.p.

2 Increase = Final earnings − Initial earnings
$= 1\,100\,000 - 600\,000 = £500\,000$

% increase $= \frac{Increase}{original\ value} \times 100$

$= \frac{500\,000}{600\,000} \times 100$

$= 83.3\%$ to 1 d.p.*

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

3 $3.5\% = \frac{3.5}{100} = 0.035$

Add 1 to create a multiplier for the original number: 1.035

New salary = $1.035 \times 38\,000$ = £39 330

4 $18\% = \frac{18}{100} = 0.18$

$1 - 0.18 = 0.82$

82% of original price = £291.92

1% of original price = $\frac{291.92}{82} = 3.56$

100% of original price = $3.56 \times 100 = 356$

Original price = £356

5 Amount of interest in one year = 3.5% of £12 000

$= \frac{3.5}{100} \times 12000 = £420$

Total interest paid over 6 years = $6 \times £420 = £2520$

Direct and inverse proportion

1 Inverse proportion means that if one quantity doubles the other quantity halves.

2 a $y = kx$

b $8 = k \times 3$ giving $k = \frac{8}{3}$

$y = \frac{8}{3}x$

When $x = 4$, $\frac{8}{3} \times 4 = \frac{32}{3} = 10.7$ (1 d.p.)

3 a Find the equivalent price in £

$€\frac{120}{1.27} = £94.49$

The sunglasses are cheaper in the UK.

b £94.49 − £89 = £5.49 cheaper

4 $V \propto r^3$ so $V = kr^3$

When $V = 33.5$, $r = 2$ so $33.5 = k \times 2^3$

$k = \frac{33.5}{8} = 4.1875$

Substituting this value of k back into the equation gives

$V = 4.1875r^3$

When $r = 4$, $V = 4.1875 \times 4^3 = 268$ cm³

5 $P \propto \frac{1}{V}$ so $P = \frac{k}{V}$

Hence $100\,000 = \frac{k}{1}$, giving $k = 100\,000$

Formula is $P = \frac{100\,000}{V}$

When $V = 3$,

$P = \frac{100\,000}{3}$

$= 33\,333$ (to the nearest whole number)

6 $a = kb^2$

When $a = 96$, $b = 4$ so $96 = k \times 4^2$ giving $k = 6$

Hence $a = 6b^2$ and when $b = 5$, $a = 6 \times 5^2 = 150$

7 a A square of side x cm has an area of x^2.

$A = kx^2$

As there are 6 faces to a cube, surface area = $6x^2$

Hence $A = 6x^2$

So constant of proportionality $k = 6$

b $A = 6x^2$

When $x = 4$ cm

$A = 6 \times 4^2 = 96$ cm²

Graphs of direct and inverse proportion and rates of change

1 Graph C

2 Graph B

3 As P and V are inversely proportional, $P = \frac{k}{V}$

At point A, when $P = 12$, $V = 3$ so $12 = \frac{k}{3}$ hence $k = 36$

Substituting this value of k back into the equation we have $P = \frac{36}{V}$

When $V = 6$, $P = \frac{36}{6} = 6$

Hence $a = 6$

4 As x and y are inversely proportional, $y \propto \frac{1}{x}$, so $y = \frac{k}{x}$

When $x = 1$, $y = 4$ so $4 = \frac{k}{1}$ so $k = 4$

The equation of the curve is now $y = \frac{4}{x}$

When $x = 4$, $y = \frac{4}{4} = 1$ so $a = 1$

When $y = 0.8$, $0.8 = \frac{4}{x}$ giving $x = 5$ so $b = 5$.

Hence $a = 1$ and $b = 5$.

5 $y = kx^2$

When $x = 2$, $y = 16$ so $16 = k \times 2^2$ giving $k = 4$.

$y = 4x^2$

Hence when $y = 36$,

$36 = 4x^2$ so $x = 3$ or -3

Since a is positive,

$a = 3$

Growth and decay

1 For a, b and c, multiplier is $\frac{100\% + \text{percentage given}}{100}$

a 1.05

b 1.25

c 1.0375

d Multiplier is $\frac{100\% - \text{percentage given}}{100} = 0.79$

2 Multiplier = $1 - \frac{18}{100} = 0.82$

Value at the end of n years = $A_0 \times (\text{multiplier})^n$ where A_0 is the initial value.

Value at the end of 3 years = $9000 \times (0.82)^3 = £4962.312$

$= £4962$ (nearest whole number)

3 multiplier for years 1 and 2 = 1.04

multiplier for years 3 and 4 = 1.03

amount after 4 years = $650 \times 1.04^2 \times 1.03^2 = £745.86$ (to nearest penny)

4 Number of restaurants after n years, $R_n = R_0 \times (\text{multiplier})^n$

Multiplier = 1.25, $n = 3$, $R_n = 4000$

So $R_0 = \frac{4000}{(1.25)^3} = 2048$

There were 2048 restaurants 3 years ago.

Ratios of lengths, areas and volumes

1 Scale factor = $\frac{6}{9} = \frac{2}{3}$

$x = \frac{2}{3} \times 10 = \frac{20}{3} = 6\frac{2}{3}$ cm

2 As the shapes are similar $\frac{V_A}{V_B} = \left(\frac{r_A}{r_B}\right)^3$

Cube rooting both sides gives $\frac{r_A}{r_B} = \sqrt[3]{\frac{V_A}{V_B}}$

$$\frac{r_A}{r_B} = \sqrt[3]{\frac{27}{64}}$$

$$= \frac{3}{4}$$

$$\frac{\text{radius of cylinder A}}{\text{radius of cylinder B}} = \frac{3}{4}$$

$$\frac{\text{surface area of cylinder A}}{\text{surface area of cylinder B}} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

So surface area of cylinder A = surface area of cylinder

$$B \times \frac{9}{16}$$

Surface area of cylinder A = $96 \times \frac{9}{16} = 54\,\text{cm}^2$

3 a As *BE* is parallel to *CD* all the corresponding angles in both triangles are the same so triangles *ABE* and *ACD* are similar.

$\frac{BE}{8} = \frac{5}{10}$ giving *BE* = 4 cm

b *BE* = 4 cm and *AB* = 5 cm. Angle *ABE* is a right angle because angle *ACD* is a right angle and the triangles are similar.

Hence area $ABE = \frac{1}{2} \times 4 \times 5 = 10\,\text{cm}^2$

Gradient of a curve and rate of change

1 a The gradient represents the acceleration.

b Find the gradient between when time = 2 s and time = 6 s.
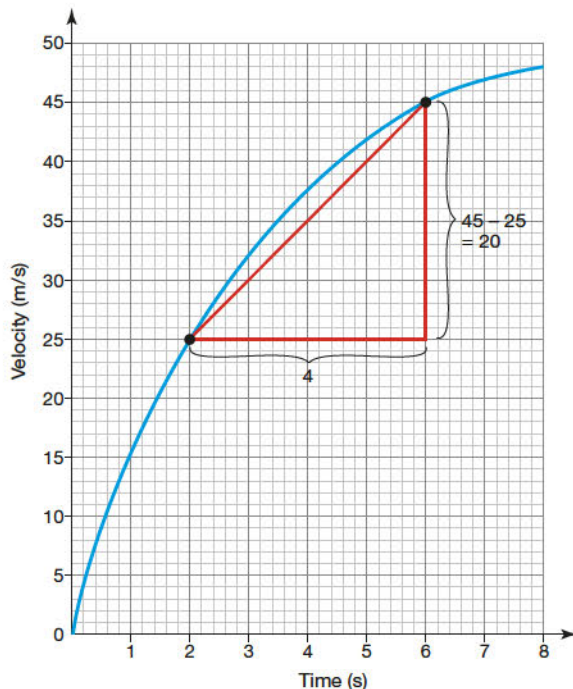
From the graph, acceleration between time = 2 s and time = 6 s

$$= \frac{45 - 25}{6 - 2}$$

$$= \frac{20}{4}$$

$$= 5\,\text{m/s}^2.$$

c We need to find a point on the graph where the gradient equals that of the tangent already drawn (5 m/s²).
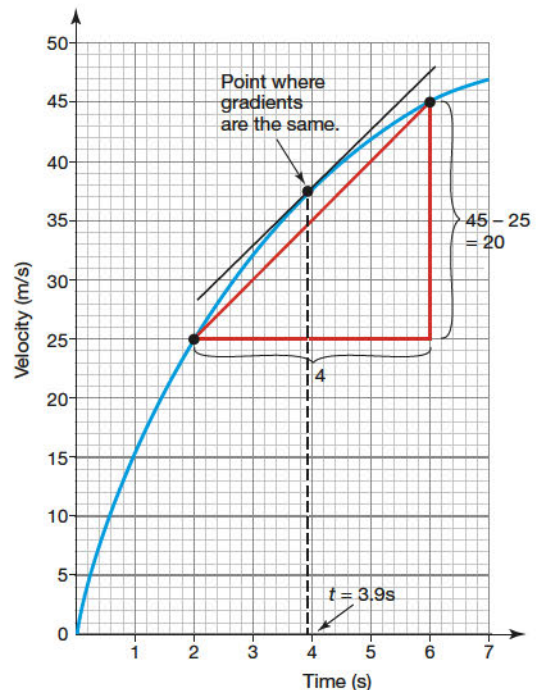
Time when the instantaneous acceleration is the same as the average acceleration = 3.9 s

Converting units of areas and volumes, and compound units

1 a Surface area = $2 \times 4 \times 6 + 2 \times 4 \times 5 + 2 \times 6 \times 5$
= 148 cm²

i 1 cm² = 10 × 10 mm² = 100 mm²

148 cm² = 148 × 100 mm² = 14 800 mm²

ii 1 m² = 100 × 100 cm² = 10 000 cm²

148 cm² = 148 ÷ 10 000 m² = 0.0148 m²

b Volume = 6 × 5 × 4 = 120 cm³

1 m³ = 100 × 100 × 100 cm³ = 1 000 000 cm³

120 cm³ = 120 ÷ 1 000 000 = 0.00012 cm³

2 Density = $\frac{\text{mass}}{\text{volume}} = \frac{1159}{600} = 1.932$ g/cm³ (3 d.p.)

3 First get the units the same.

1 m³ = 100 × 100 × 100 cm³ = 1 000 000 cm³

Number of ball bearings = $\frac{1\,000\,000}{0.5} = 2\,000\,000$
(i.e. 2 million)

4 50 km/h = 50 000 m/h

1 h = 60 × 60 s = 3600 s

Now as fewer m/s will be covered compared to m/h we divide by 3600 to convert the speed to m/s.

Hence 50 000 m/h = $\frac{50\,000}{3600} = 13.89$ m/s (2 d.p.)

5 a Distance = speed × time = 70 × 4 = 280 km

Mary's speed = 280/5 = 56 km/h

b If Mary's route was longer, her average speed would increase. If her route was shorter, her average speed would decrease.

Review it!

1 80% of the original price = £76.80

10% of the original price = $\frac{76.80}{8}$ = £9.60

100% of the original price = £9.60 × 10 = £96

2 a $A_1 = 1.02 \times A_0$

$\quad\quad = 1.02 \times 5$

$\quad\quad = 5.10\text{ m}^2$ (2 d.p.)

b $A_2 = 1.02 \times A_1 = 1.02 \times 5.1 = 5.202\text{ m}^2$

$\quad A_3 = 1.02A_2 = 1.02 \times 5.202 = 5.31\text{ m}^2$ (2 d.p.)

3 $y = \frac{k}{x}$

$4 = \frac{k}{2.5}$

$k = 10$

Hence $y = \frac{10}{x}$

When $x = 5$, $y = \frac{10}{5} = 2$

4 2 parts of the ratio = 54 students, so 1 part = 27 students.

Students in the whole year = 2 + 7 parts = 9 parts.

In Year 11, there are 9 × 27 = 243 students.

5 The price of the house has more than doubled.

Multiplier = 1 + 1.2 = 2.2

So £220 000 × 2.2 = £484 000

The value of the house = £485 000 to the nearest £5000

6 a Simple interest: One year = 2000 × 0.025 = £50

$\quad$ 50 × 5 = 250

$\quad$ After 5 years, there will be £2250 in the account.

b Compound interest: 2000 × 1.025^5 = £2262.82

7 a 400 × 8.55 = 3420 yuan

b Travel agent: 800 ÷ 8.6 = £93.02

$\quad$ Commission = 93.02 × 0.025 = £2.33

$\quad$ So Tom would get £93.02 − £2.33 = £90.69

$\quad$ From the post office, Tom would get 800 ÷ 8.9 = £89.89

$\quad$ Tom will get a better deal from the travel agent.

8 The difference between Luke's and Amy's payouts was £4000. The difference in parts is 7 − 5 = 2.

So 2 parts = £4000 and 1 part = £2000

The profits are split into 3 + 5 + 7 = 15 parts.

Total profits = 15 × 2000 = £30 000

9 $\frac{\text{Surface area A}}{\text{Surface area B}} = \left(\frac{\text{Length A}}{\text{Length B}}\right)^2$

So $\frac{25}{4} = \frac{5^2}{2^2}$

Similarly, $\frac{\text{Volume A}}{\text{Volume B}} = \left(\frac{\text{Length A}}{\text{Length B}}\right)^3$

$\frac{10}{\text{Volume B}} = \frac{5^3}{2^3}$

So Volume B = $\frac{10 \times 8}{125} = \frac{80}{125} = 0.64\text{ cm}^3$

10 7 + 4 = 11 parts in the ratio.

Let this be 1100 members.

There would then be 700 male club members: 25% of this is 0.25 × 700 = 175 junior members.

There would be 400 female club members: 10% of this is 0.1 × 400 = 40 junior members.

In total, there would be 175 + 40 = 215 junior members.

As a percentage, this is $\frac{215}{1100} \times 100 = 19.54$

= 20% to the nearest integer.

11 Each cm³ of the alloy contains 9 parts (= 0.9 cm³) copper and 1 part (= 0.1 cm³) tin.

So the mass of copper in 1 cm³ = 8.9 × 0.9 = 8.01 g

The mass of tin in 1 cm³ = 7.3 × 0.1 = 0.73 g

Mass of 1 cm³ of alloy = 8.01 + 0.73 = 8.74 g

So the density of the alloy = 8.7 g/cm³ (1 d.p.)

Geometry and measures

2D shapes

1 a True

b False

c True

d False

e True

f True

Constructions and loci

1 The line which bisects the angle will be the locus of point X.
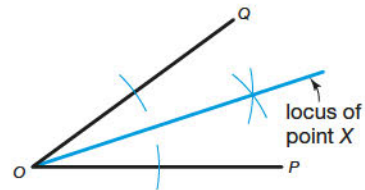
2 Draw two circles: one with a radius of 3 cm from A, another with a radius of 6 cm from B. Shade the area outside the two circles. The area outside the two circles shows all possible locations for the wind farm.
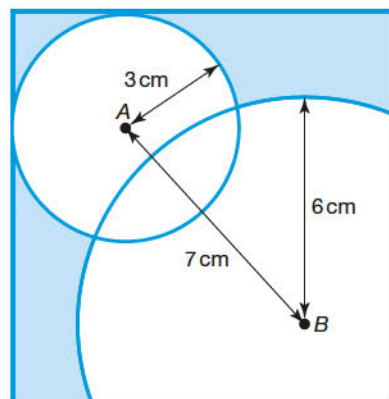
Properties of angles

1 Exterior angle = $\frac{360°}{\text{number of sides}} = \frac{360}{10} = 36°$

Interior angle + exterior angle = 180°, so interior angle = 144°

> The interior and exterior angle always add up to 180°.

2 a Number of sides $= \frac{360°}{\text{exterior angle}} = \frac{360}{40} = 9$

b Interior angle $= 180° - $ exterior angle
$= 180 - 40 = 140°$

As there are 9 sides, total of interior angles
$= 140 \times 9 = 1260°$

3 Angle $BAC = 50°$ (corresponding angles)

Angle $ABC = 180 - (50 + 60) = 70°$ (angles in a triangle add up to 180°)

$x = 180 - (90 + 70) = 20°$ (angles in triangle ABD add up to 180°)

4 There are several different ways of working the answers out and the method shown is only one of them.

a Angle x is an alternate angle to angle EBC.
Hence $x = 55°$.

b Angle $EHI = 180 - (85 + 55) = 40°$ as angles in a triangle add up to 180°.

Angle DEH is an alternate angle to angle EHI so it is 40°.

5 Opposite angles in a rhombus are equal.

So $2(3a + 5) + 2(5a - 17) = 360$
$6a + 10 + 10a - 34 = 360$
$16a - 24 = 360$
$16a = 384$
$a = \frac{384}{16}$
$a = 24°$

There are two angles of $3 \times 24 + 5 = 77°$, and two angles of $180 - 77 = 103°$

Congruent triangles

1
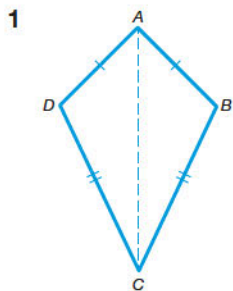
There are two triangles with a common side AC.

$AB = AD$ (given in the question)

$BC = CD$ (given in the question)

Triangles ACB and ACD are congruent (SSS).

Hence, angle $ABC = $ angle ADC.

2 $AB = AC$ (given in the question)

$BM = MC$ (as M is the midpoint of BC)

$AM = AM$ (common to both triangles)
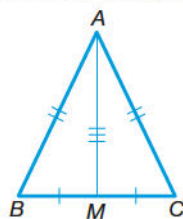
There are three pairs of equal corresponding sides so the triangles ABM and ACM are congruent (SSS).

Hence angle $AMB = $ angle AMC.

Angle $AMB + $ angle $AMC = 180°$, (angles on a straight line)

So angle $AMB = \frac{180}{2} = 90°$

Transformations

1 a and b
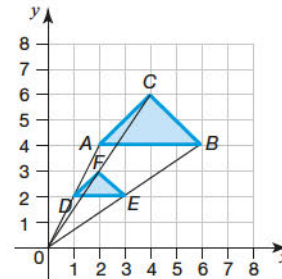
c Enlargement, scale factor 2, centre (0, 0)

2
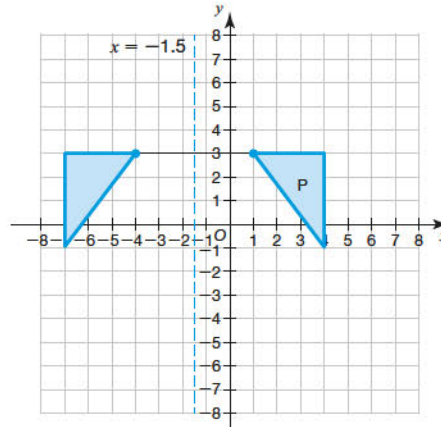
3
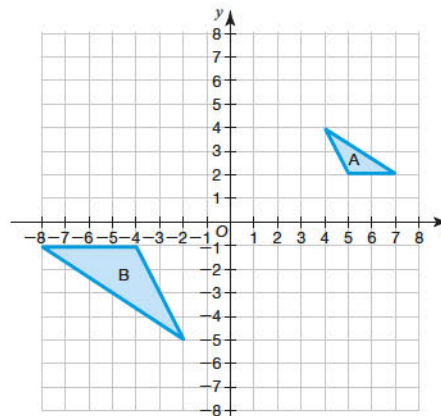
4 A translation by $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$

Invariance and combined transformations

1 a, b, c
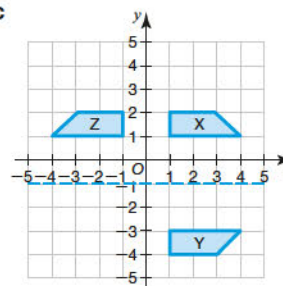
d Reflection in the y-axis, or reflection in the line $x = 0$

2 a, b
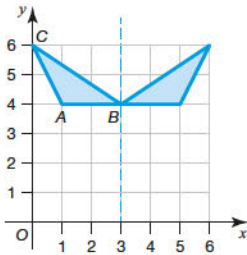
c Invariant point is (3, 4)

3 a, b
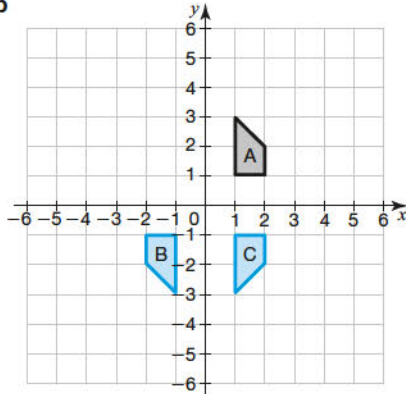
c A reflection in the *x*-axis.

3D shapes

1 a

Shape	Number of vertices, V	Number of faces, F	Number of edges, E
Triangular-based pyramid	4	4	6
Cone	1	2	1
Cuboid	8	6	12
Hexagonal prism	12	8	18

b $V = 16, F = 10, E = 24; V + F - E = 16 + 10 - 24 = 2$

Parts of a circle

1 a radius

b chord

c minor arc

d minor segment

e minor sector

Circle theorems

1 a Angle $ACB = 30°$ as it is equal to angle YAB (alternate segment theorem)

b Angle $ABC = 90°$ (angle in a semicircle is always a right angle)

c Angle $ADC = 90°$ (angle in a semicircle is always a right angle)

2 a Angle $OAX = 90°$ (angle formed by a tangent to a radius or diameter is always a right angle)

b Angle $AOX = 180 - (90 + 30) = 60°$ (angles in a triangle add up to 180°)

c Angle $ACB = 60 ÷ 2 = 30°$ (angle at the centre is twice the angle at the circumference standing on the same arc)

3 a Angle $ADB = 40°$ as it is equal to angle ACB (angles at the circumference on the same arc are equal)

b Angle $ABD =$ angle $ADE = 50°$ (alternate segment theorem)

Angle $EDB = 90°$ (i.e. $40° + 50°$) which means BD must be a diameter, as a tangent and a diameter are at right angles to each other.

As BD is a diameter, angle BAD is 90° as it is the angle in a semicircle.

4 Angle $ACB = 46°$ (angles bounded by the same chord in the same segment are equal)

Angle $ABC = 90°$ (angle in a semicircle is a right-angle)

Angle $BAC = 180 - (90 + 46) = 44°$ (angles in a triangle add up to 180°)

5 a Angle $ABC = 60°$ (angle in an equilateral triangle)

Angle $ADC = 180 - 60 = 120°$ (opposite angles in a cyclic quadrilateral add up to 180°)

b Angle $BAC = 60°$ (angle in an equilateral triangle)

Angle $CBQ =$ angle BAC (alternate segment theorem)

$= 60°$

Projections

1 a
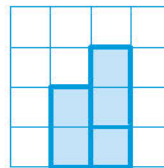
c
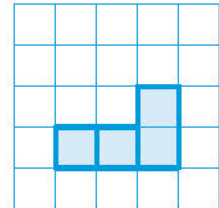
b
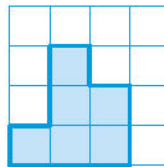
2

Bearings

1

Bearing $= 180 + 65 = 245°$
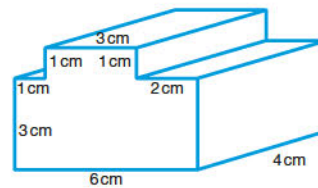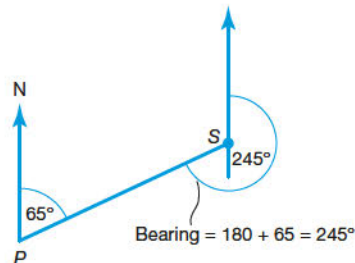
Bearing $= 245°$

2 a Bearing of P from Q $= 180 + 45 = 225°$

b Bearing of Q from R $= 180 + 140 = 320°$

Pythagoras' theorem

1 a By Pythagoras' theorem
$$x^2 = 5^2 + 9^2 = 106$$
$$x = \sqrt{106} = 10.3 \text{ cm (1 d.p.)}$$

b By Pythagoras' theorem
$$13.5^2 = 10.2^2 + x^2$$
$$x^2 = 182.25 - 104.04 = 78.21$$
$$x = \sqrt{78.21} = 8.8 \text{ cm (1 d.p.)}$$

2
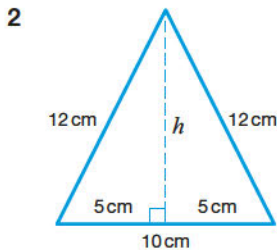
By Pythagoras' theorem
$$12^2 = 5^2 + h^2$$
$$144 = 25 + h^2$$
$$h^2 = 119$$
$$h = \sqrt{119}$$
$$= 10.9087 \text{ cm (4 d.p.)}$$

Area $= \frac{1}{2} \times$ base $\times$ height
$$= \frac{1}{2} \times 10 \times 10.9087$$
$$= 54.54 \text{ cm}^2 \text{ (2 d.p.)}$$

3 $AC^2 = 3^2 + 11^2 = 130$
$$x^2 = 14^2 - AC^2 = 196 - 130 = 66$$
$$x = \sqrt{66} = 8.12 \text{ cm (2 d.p.)}$$

4 Let the perpendicular height of the triangle $= h$

Area of triangle $= \frac{1}{2} \times 14 \times h$

Hence $\frac{1}{2} \times 14 \times h = 90$
$$h = \frac{90 \times 2}{14} = \frac{180}{14} = \frac{90}{7}$$
$$h = 12.8571 \text{ cm (4 d.p.)}$$

By Pythagoras' theorem, $AC^2 = 7^2 + 12.8571^2$

$AC = \sqrt{214.305}$ (3 d.p.)

$AC = 14.6$ cm (3 s.f.)

Area of 2D shapes

1 Right angle and parallel lines show that the office is a trapezium.

Area of a trapezium $= \frac{1}{2} \times (a + b) \times h = \frac{1}{2} \times (9 + 12) \times 4$

Office area $= 42 \text{ m}^2$

Cost of new floor $= £38 \times 42 = £1596$

2 Length of side of square $= \frac{\text{perimeter}}{4} = \frac{20}{4} = 5 \text{ cm}$

Area of square $= 5 \times 5 = 25 \text{ cm}^2$

Area of small triangle $= \frac{1}{2} \times$ base $\times$ height
$$= \frac{1}{2} \times 2.5 \times 2.5 = 3.125 \text{ cm}^2$$

Area of large triangle $= \frac{1}{2} \times$ base $\times$ height
$$= \frac{1}{2} \times 2.5 \times 5 = 6.25 \text{ cm}^2$$

Shaded area $= 25 - (3.125 + 6.25) = 15.625$

Proportion shaded $= \frac{15.625}{25}$

Multiply by 8 to get rid of the decimal:
$$= \frac{125}{200}$$

Cancel fraction (divide by 25):
$$= \frac{5}{8} \text{ of the square.}$$

3 a Area of semicircle $= \frac{1}{2} \times \pi \times x^2 = \frac{\pi x^2}{2}$

Area of rectangle $= 4x \times 2x = 8x^2$

Area of shape $= \frac{\pi x^2}{2} + 8x^2 = x^2(8 + \frac{\pi}{2})$

b Perimeter $= 2x + 4x + 4x + \frac{1}{2} \times 2\pi x = 10x + \pi x = x(10 + \pi)$.

An exact answer is required, so you should leave π in your answer.

Volume and surface area of 3D shapes

1
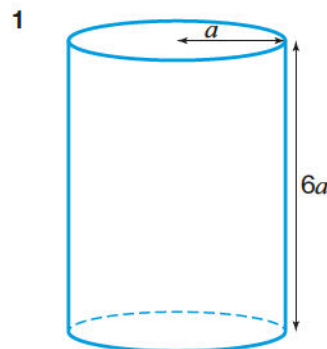
Volume of cylinder = cross-sectional area $\times$ length
$$= \pi a^2 \times 6a$$
$$= 6\pi a^3$$
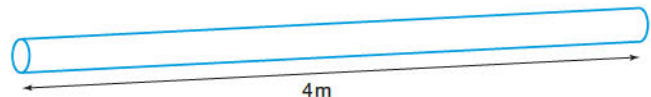
Volume of rod = cross-sectional area $\times$ length
$$= \pi r^2 \times 4$$
$$= 4\pi r^2$$

The two volumes are the same so we can write
$$4\pi r^2 = 6\pi a^3 \text{ (Dividing both sides by } \pi)$$
$$4r^2 = 6a^3 \text{ (Dividing both sides by 2)}$$
$$2r^2 = 3a^3 \text{ (Dividing both sides by 2)}$$
$$r^2 = \frac{3}{2} a^3 \text{ (Square rooting both sides)}$$
$$r = \sqrt{\frac{3}{2} a^3}$$

2 a Area of trapezium $= \frac{1}{2}(a + b)h = \frac{1}{2}(0.75 + 1.75) \times 10 = 12.5 \text{ m}^2$

Cross-sectional area $= 12.5 \text{ m}^2$

b Volume $=$ Cross-sectional area $\times$ length
$$= 12.5 \times 5 = 62.5 \text{ m}^3$$

c Total volume per minute of water entering $= 2 \times 0.05 = 0.1 \text{ m}^3$ per minute

Number of minutes it takes to fill $= \frac{62.5}{0.1}$
$$= 625 \text{ mins}$$

Number of hours it takes to fill $= \frac{625}{60}$
$$= 10.42 \text{ hours (2 d.p.)}$$
$$= 10 \text{ hours (nearest hour)}$$

3 Need to first find the slant height, l, as this is needed for the formula.

By Pythagoras' theorem $\quad l^2 = 4^2 + 15^2 = 241$

$$l = 15.524 \text{ m (3 d.p.)}$$

Curved surface area of a cone $= \pi r l = \pi \times 15 \times 15.524$
$$= 731.55 \text{ m}^2 \text{ (2 d.p.)}$$

Curved surface area of a cylinder $= 2\pi r h$
$$= 2 \times \pi \times 15 \times 5$$
$$= 471.24 \text{ m}^2 \text{ (2 d.p.)}$$

Total surface area of the tent $= 731.55 + 471.24$
$$= 1202.79 \text{ m}^2$$

The tent uses 1200 m² of fabric (to 3 s.f.)

4 Volume = volume of large cone – volume of small cone
$$= \tfrac{1}{3} \times \pi \times 5^2 \times 20 - \tfrac{1}{3} \times \pi \times 3.5^2 \times 14$$
$$= \tfrac{500}{3}\pi - \tfrac{171.5}{3}\pi$$
$$= \tfrac{328.5}{3}\pi$$
$$= 109.5\pi \text{ cm}^3$$

Trigonometric ratios

1 a $\cos 30° = \frac{15}{x}$

> x is the hypotenuse and the 15cm side is the adjacent so we use $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.

$$x = \frac{15}{\cos 30°} = 17.32 \text{ cm (2 d.p.)}$$

b $\cos 40° = \frac{x}{12}$

> x is the adjacent and the 12 cm side is the hypotenuse so we use $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.

Hence, $x = 12 \cos 40°$
$$= 9.19 \text{ cm (2 d.p.)}$$

2 a $\tan \theta = \frac{10}{3}$
$$\theta = \tan^{-1}\left(\frac{10}{3}\right)$$
$$= 73.3° \text{ (nearest } 0.1°)$$

b $\sin \theta = \frac{10}{13}$

> The 13cm side is the hypotenuse and the 10cm side is the opposite, so we use $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$.

$$\theta = \sin^{-1}\left(\frac{10}{13}\right)$$
$$= 50.3° \text{ (nearest } 0.1°)$$

3 $\sin \theta = \frac{b}{c}$ (i.e. $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$)

and $\cos \theta = \frac{a}{c}$ (i.e. $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$)

> Remember that when you divide by fractions you turn the bottom fraction upside down and replace the division by a multiplication.

Hence $\frac{\sin \theta}{\cos \theta} = \frac{\frac{b}{c}}{\frac{a}{c}}$
$$= \frac{b}{c} \times \frac{c}{a}$$
$$= \frac{b}{a}$$

Now, $\frac{b}{a} = \frac{\text{opposite}}{\text{adjacent}} = \tan \theta$

Hence, $\frac{\sin \theta}{\cos \theta} = \tan \theta$

4 a By Pythagoras' theorem, $BD^2 = AB^2 + AD^2$
$$= 10^2 + 7^2$$
$$= 149$$
$$BD = \sqrt{149}$$
$$= 12.207 \text{ cm (3 d.p.)}$$

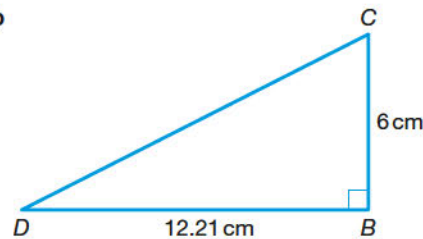Applying Pythagoras' theorem to triangle BCD
$$CD^2 = BD^2 + BC^2$$
$$= 12.207^2 + 6^2$$
$$= 185.011 \text{ (3 d.p.)}$$
$$CD = \sqrt{185.011}$$
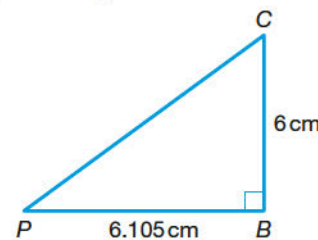$$= 13.60 \text{ cm (2 d.p.)}$$

b

Let angle $BCD = x$
$$\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{12.21}{6}$$
$$x = \tan^{-1}\left(\frac{12.21}{6}\right)$$

So $x = 63.8°$ (1 d.p.)

c $BP = \tfrac{1}{2} \times BD = \tfrac{1}{2} \times 12.21 = 6.105 \text{ cm}$
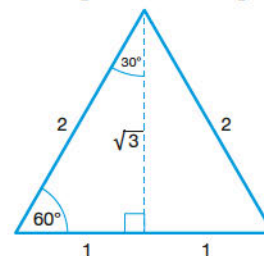

By Pythagoras' theorem, $PC^2 = 6.105^2 + 6^2$
$$PC = 8.56 \text{ cm (2 d.p.)}$$

Exact values of sin, cos and tan

1 First draw an equilateral triangle with sides 2 cm and mark on the following sides and angles.

From the diagram $\tan 30° = \frac{1}{\sqrt{3}}$ and $\cos 60° = \frac{1}{2}$

$$\sqrt{3} \tan 30° + \cos 60° = \sqrt{3} \times \frac{1}{\sqrt{3}} + \frac{1}{2} = 1 + \frac{1}{2} = \frac{3}{2}$$

where $a = 3$ and $b = 2$.

2 $\sin 30 = \frac{1}{2}$ and $\cos 30 = \frac{\sqrt{3}}{2}$

$\sin^2 30 = \frac{1}{4}$ and $\cos^2 30 = \frac{3}{4}$

$\sin^2 30 + \cos^2 30 = \frac{1}{4} + \frac{3}{4} = 1$

Sectors of circles

1 $l = \frac{\theta}{360} \times 2\pi r$

$10 = \frac{\theta}{360} \times 2\pi \times 12$

$\theta = \frac{10 \times 360}{2\pi \times 12} = \frac{5 \times 30}{\pi}$

$= 47.7°$ (1 d.p.)

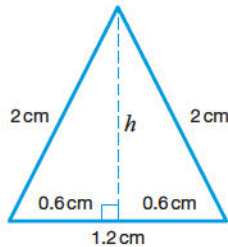**2 a**   Perimeter of logo = 2 × arc length of sector + 2 × radius of sector + base of triangle

Arc length = $\frac{\theta}{360} \times 2\pi r = \frac{40}{360} \times 2 \times \pi \times 2 = 1.396$ cm

So perimeter = 2 × 1.396 + 2 × 2 + 1.2
$= 7.99$ cm (3 s.f.)

b   Area of logo = 2 × area of sector + area of triangle

Area of sector = $\frac{\theta}{360} \times \pi r^2 = \frac{40}{360} \times \pi \times 2^2$
$= 1.396$ cm² (3 d.p.)

Find the height of the isosceles triangle to find its area.

Use Pythagoras' theorem

$2^2 = 0.6^2 + h^2$

So $h = 1.9079$ cm (4 d.p.)

Area of triangle = $\frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 1.2 \times 1.9079 = 1.1447$ cm²

Area of logo = 2 × 1.396 + 1.1447 = 3.9367
$= 3.94$ cm² (3 s.f.)

3   Area of sector = $\frac{85}{360} \times \pi \times r^2$

Area = 25π so $\frac{85}{360} \times \pi \times r^2 = 25\pi$

$r^2 = \frac{25\pi \times 360}{85\pi} = 105.88$ (2 d.p.)

$r = OA = \sqrt{105.88} = 10.3$ cm (1 d.p.)

4   First find the radius of the circle using the length of arc.

$\frac{85}{360} \times \pi \times d = 10$

$d = \frac{3600}{85\pi}$

$r = \frac{3600}{85\pi} \div 2$

$= 6.7407$ (4 d.p.)

Area of sector = $\frac{275}{360} \times \pi \times 6.7407^2 = 109$ cm² to the nearest whole number

Sine and cosine rules

1 a   Area of triangle $ABC = \frac{1}{2} bc \sin A$

$= \frac{1}{2} \times 10 \times 6 \sin 150°$

$= 15$ cm²

b   Using the cosine rule

$BC^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \cos 150°$

$BC^2 = 36 + 100 - 120 \cos 150°$

$BC^2 = 136 + 103.92$

$BC = \sqrt{239.92}$

$BC = 15.5$ cm (to 3 s.f.)

2   The smallest angle of any triangle is always opposite the smallest side.

Draw a sketch of the triangle and let the smallest angle be θ.
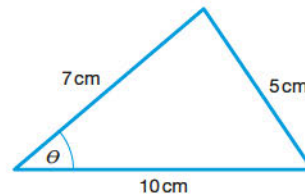
Use the cosine rule

$5^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \cos \theta$
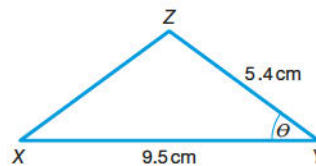
$25 = 49 + 100 - 140 \cos \theta$

$140 \cos \theta = 124$

$\cos \theta = 0.8857$

$\theta = \cos^{-1}(0.8857)$

$= 27.7°$ (nearest 0.1°)

3 a

Let angle $XYZ = \theta$

Area $= \frac{1}{2} \times XY \times YZ \times \sin \theta$

From the question, area = 16 cm²

So $16 = \frac{1}{2} \times 9.5 \times 5.4 \times \sin \theta$

$\sin \theta = \frac{16 \times 2}{9.5 \times 5.4}$

$\theta = \sin^{-1}\left(\frac{32}{51.3}\right)$

So $\theta = 38.6°$ (1 d.p.)

b   If angle XZY is not obtuse, then angle XYZ can be obtuse.

So an alternative answer for $\theta = 180 - 38.6$
$= 141.4°$ (1 d.p.).

Vectors

1 a $\mathbf{b} - \mathbf{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$

b $3\mathbf{a} + 5\mathbf{b} = 3\begin{pmatrix} 4 \\ -5 \end{pmatrix} + 5\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 12 \\ -15 \end{pmatrix} + \begin{pmatrix} -10 \\ 15 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

2 a $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= -\mathbf{a} + \mathbf{b}$
$= \mathbf{b} - \mathbf{a}$

b $\overrightarrow{AP} = \frac{3}{5} \overrightarrow{AB}$
$= \frac{3}{5}(\mathbf{b} - \mathbf{a})$

c　$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

> Multiply out the bracket and then simplify the terms.

$= \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$

$= \mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{3}{5}\mathbf{a}$

$= \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

$= \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$

3　a　$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$\overrightarrow{BA} = -\overrightarrow{AB} = -\mathbf{a}$

$\overrightarrow{AC} = \overrightarrow{AP} + \overrightarrow{PC} = -2\mathbf{b} - \mathbf{b} = -3\mathbf{b}$

So $\overrightarrow{BC} = -\mathbf{a} - 3\mathbf{b}$

b　If M is the midpoint of BC, $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BC}$

$= \frac{1}{2}(-\mathbf{a} - 3\mathbf{b}) = -\frac{1}{2}(\mathbf{a} + 3\mathbf{b})$

If B is the midpoint of AD, $AB = BD = \mathbf{a}$

So $\overrightarrow{PM} = \overrightarrow{PA} + \overrightarrow{AB} + \overrightarrow{BM}$

$= 2\mathbf{b} + \mathbf{a} - \frac{1}{2}(\mathbf{a} + 3\mathbf{b})$

$= 2\mathbf{b} + \mathbf{a} - \frac{1}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}$

$= \frac{1}{2}(\mathbf{a} + \mathbf{b})$

Similarly, $\overrightarrow{MD} = \overrightarrow{MB} + \overrightarrow{BD}$

$= \frac{1}{2}(\mathbf{a} + 3\mathbf{b}) + \mathbf{a}$

$= \frac{1}{2}\mathbf{a} + \frac{3}{2}\mathbf{b} + \mathbf{a}$

$= \frac{3}{2}(\mathbf{a} + \mathbf{b})$

$\overrightarrow{PM}$ and $\overrightarrow{MD}$ have the same vector part, $(\mathbf{a} + \mathbf{b})$, therefore they are parallel. Both lines pass through point M and parallel lines cannot pass through the same point unless they are the same line. Hence PMD is a straight line.

Review it!

1　Angle $ADE = y°$ (alternate segment theorem)

Angle $ADC = (180 - x)°$ (opposite angles in cyclic quadrilateral add up to 180°)

EDF is a tangent, so it is a straight line.

So angle $CDF = 180 -$ angle $ADE -$ angle ADC
$= 180 - y - (180 - x) = (x - y)°$

2　Draw the following triangles and mark on the angles.
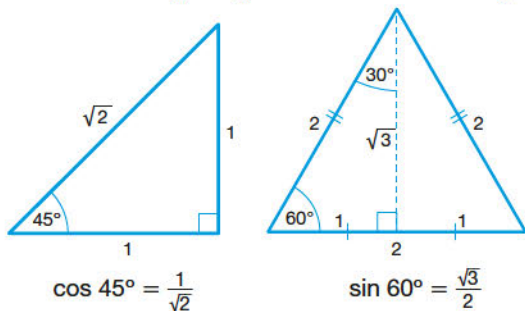
$\cos 45° = \frac{1}{\sqrt{2}}$　　　$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 45° + \sin 60° = \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}$

$= \frac{1}{2}(\sqrt{2} + \sqrt{3})$

3　Using Pythagoras' theorem we obtain

$(4x - 3)^2 = (x + 1)^2 + (3x)^2$

$16x^2 - 24x + 9 = x^2 + 2x + 1 + 9x^2$

So $6x^2 - 26x + 8 = 0$

Dividing both sides by 2 we obtain
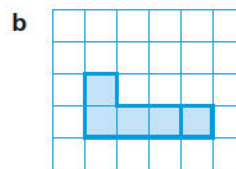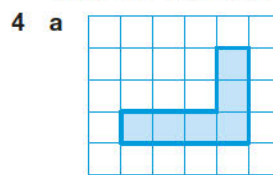
$3x^2 - 13x + 4 = 0$

$(3x - 1)(x - 4) = 0$

$3x - 1 = 0$　or $x - 4 = 0$
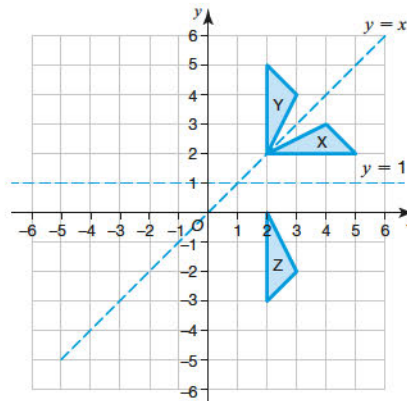
Hence $x = \frac{1}{3}$ or $x = 4$.

We need to put both values into each of the expressions for the sides to see if any of the sides end up negative. When $x = \frac{1}{3}$ is substituted into $4x - 3$ the result is negative. As you cannot have a negative length for a side, $x = \frac{1}{3}$ is disregarded. Hence $x = 4$.

When x is 4 the sides are 5, 12 and 13.

4　a

b

5　a, b i and c

b　**ii** Invariant point (vertex) is (2, 2)

d　Rotation 90° clockwise about (1, 1)

6　a　Work out angle ABC in the triangle.

The angle between AB and due south at B is 70° (alternate with 70° marked at A).

The angle between due south at B and BC is 30° (angles on a straight line add up to 180°).

So angle $ABC = 100°$

Using the Cosine Rule:

$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos B$

$AC^2 = 15^2 + 10^2 - 2 \times 15 \times 10 \times \cos 100°$

$AC^2 = 325 - (-52.094)$

So $AC = \sqrt{377.094} = 19.4$ km (3 s.f.)

b　Let angle $= \theta$

Using the Sine Rule, $\frac{19.4}{\sin 100} = \frac{10}{\sin \theta}$

So $\sin \theta = \frac{10 \sin 100}{19.4} = \frac{9.84}{19.4}$

$\theta = \sin^{-1}\left(\frac{9.84}{19.4}\right)$

$\theta = 30.48°$

Bearing of C from $A = 70 + 30.48 = 100.48°$

So bearing of A from $C = 100.48 + 180 = 280°$ to the nearest degree.

Probability

The basics of probability

1 There are two possible even numbers (i.e. 2, 4)

Probability of landing on an even number

$= \frac{\text{Number of ways something can happen}}{\text{Total of number of possible outcomes}} = \frac{2}{5}$

Probability of getting an even number on each of three

spins $= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$

2 a As there are no white counters, the probability of picking one $= 0$

  b Probability of picking a black counter $= \frac{4}{20} = \frac{1}{5}$

  c Probability of picking a green counter $= \frac{9}{20}$

  Prob of picking a green counter + prob of not picking a green counter $= 1$

  Prob of not picking a green counter

  $= 1 - \frac{9}{20} = \frac{11}{20}$

Probability experiments

1 a Relative frequency for a score of 3 $= \frac{25}{120}$

    $= 0.21$ (2 d.p.)

  b Relative frequency for a score of 6 $= \frac{17}{120}$

    $= 0.14$ (2 d.p.)

  c Sean is wrong. 120 spins is a small number of spins and it is only over a very large number of spins that the relative frequencies may start to be nearly the same.

2 a Estimated probability = relative frequency

    $= \frac{\text{Frequency}}{\text{Total Frequency}} = \frac{20}{500} = 0.04$

  b Number of cans containing less than 330 ml $= 0.04 \times 15\,000 = 600$

  Another way to do this is to see how many times 500 divides into 15 000. This is 30. So there will be $30 \times 20 = \mathbf{600\ cans}$ containing less than 30 ml.

3 Expected frequency = Probability of the event
                                 $\times$ number of events

    $= \frac{3}{40} \times 600$

    $= 45$ apples

The AND and OR rules

1 a Independent events are events where the probability of one event does not influence the probability of another event occurring. Here it means that the probability of the first set of traffic lights being red does not affect the probability of the second set being red.

  b P(A AND B) = P(A) $\times$ P(B)

  P(stopped at first AND stopped at second)
  = P(stopped first) $\times$ P(stopped second)

  $= 0.2 \times 0.3$

  $= 0.06$

  c P $= 0.8 \times 0.7 = 0.56$

You can work out the probability of lights not being on red by subtracting the probability of being red from 1. So the probability of them not being red at the first set is $1 - 0.2 = 0.8$ and for the second set it is $1 - 0.3 = 0.7$
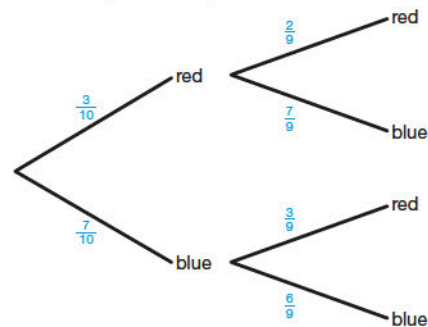
2 a Probability of all events taking place $= \frac{1}{4} \times \frac{2}{3} \times \frac{7}{8}$

    $= \frac{2 \times 7}{12 \times 8} = \frac{7}{48}$

  b Probability of none of the events taking place

    $= \frac{3}{4} \times \frac{1}{3} \times \frac{1}{8} = \frac{3}{3 \times 32} = \frac{1}{32}$

Tree diagrams

1 a The following tree diagram is drawn.

P(red AND red) $= \frac{3}{10} \times \frac{2}{9} = \frac{6}{90} = \frac{1}{15}$

  b P(red AND blue) = P(RB) + P(BR)

Note that red and blue does not specify an order. There are two paths that need to be considered on the tree diagram.

Remember to fully cancel fractions. Use your calculator to help you.

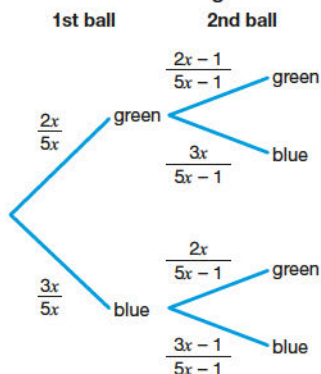$= \frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{3}{9}$

$= \frac{7}{30} + \frac{7}{30}$

$= \frac{14}{30}$

$= \frac{7}{15}$

2 Let the number of green balls in the bag be $2x$. Let the number of blue balls be $3x$. So the total number of balls in the bag is $5x$.

Put these values into a tree diagram:

1st ball   **2nd ball**

$\frac{2x}{5x}$ green

$\frac{2x-1}{5x-1}$ green

$\frac{3x}{5x-1}$ blue

$\frac{3x}{5x}$ blue

$\frac{2x}{5x-1}$ green

$\frac{3x-1}{5x-1}$ blue

$P(\text{blue AND blue}) = \frac{3x}{5x} \times \frac{3x-1}{5x-1} = \frac{33}{95}$

Divide both sides by $\frac{3}{5}$ and cancel the x top and bottom on the left.

So $\frac{3x-1}{5x-1} = \frac{11}{19}$

$19(3x-1) = 11(5x-1)$

$57x - 19 = 55x - 11$

$2x = 8$

$x = 4$

Hannah put $5x = 5 \times 4 = 20$ balls into the bag.

3 a   **Check in**   **Security check**

0.7 long queue

0.5 long queue

0.5 short queue

0.3 short queue

0.5 long queue
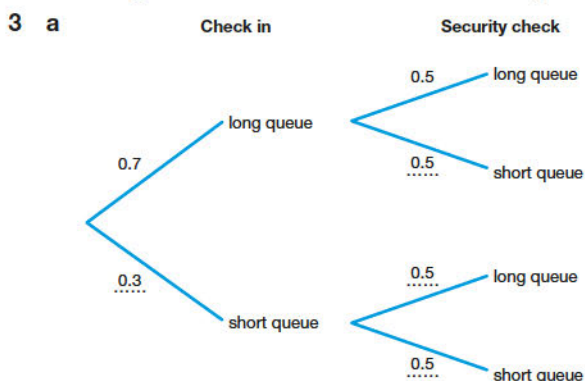
0.5 short queue

b Probability $= 0.3 \times 0.5 = 0.15$

c Probability $= 1 -$ probability of short queue at both
$= 1 - 0.15 = 0.85$

4 a Total number of students in school $= 450 + 500$
$= 950$

Number of male students in upper school
$= 0.6 \times 450 = 270$

Number of female students in upper school
$= 450 - 270 = 180$

Number of male students in lower school
$= 0.55 \times 500 = 275$

Number of female students in lower school
$= 500 - 275 = 225$

This information is added to the frequency tree.
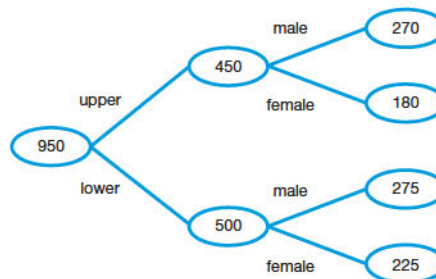
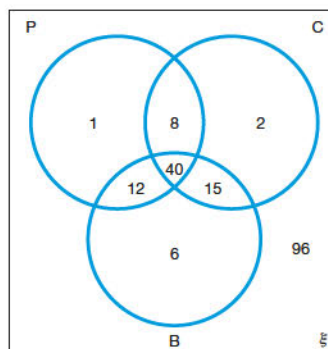**b**   Total males $= 270 + 275$

$= 545$

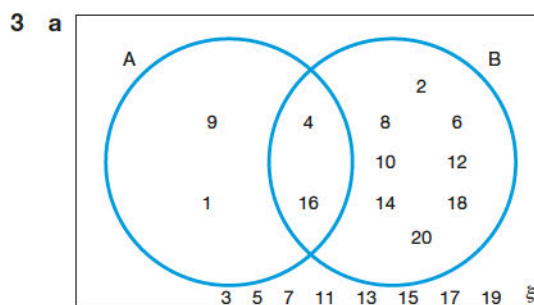$P(\text{student is male}) = \frac{545}{950}$

$= \frac{109}{190}$

$= 0.57$

Venn diagrams and probability

1 a   **i** $A \cup B = \{1, 3, 4, 5, 8, 9, 10, 11\}$

ii $A \cap B = \{8, 9\}$

iii $A' = \{2, 5, 6, 10, 13\}$

b $P(B') = \frac{7}{11}$

2 Complete the diagram, using the information you are given to work out the unknown areas.

a $P(\text{all 3 sciences}) = \frac{40}{84} = \frac{10}{21}$

b $P(\text{only one science}) = \frac{(1+2+6)}{84} = \frac{9}{84} = \frac{3}{28}$

c $P(\text{chemistry if study physics}) = \frac{(8+40)}{(8+40+12+1)} = \frac{48}{61}$

3 a

b $P(A \cup B)' = \frac{8}{20} = \frac{2}{5}$

4 a $P(A \cap B) = P(A \mid B) \times P(B)$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{48}}{\frac{1}{4}}$$

$$= \frac{4}{48}$$

$$= \frac{1}{12}$$

b $P(A \cap B) = P(B \mid A) \times P(A)$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{1}{48}}{\frac{1}{10}}$$

$$= \frac{10}{48}$$

$$= \frac{5}{24}$$

5 a $165 + 74 + 235 + 236 = 710$

b $A = 165 + 235$ claims

$B = 165 + 74$ claims

$165 = A \cap B$

There is a value for $A \cap B$: the two groups intersect.
So A and B are not independent.

Or:

$$P(B) = \frac{165 + 74}{710}$$

$$= \frac{239}{710}$$

$$= 0.34$$

$$P(B \text{ given } A) = P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{165}{710}}{\frac{165 + 235}{710}}$$

$$= \frac{165}{400}$$

$$= \frac{33}{80}$$

$$= 0.41$$

As P(B given A) ≠ P(B) the two events are not independent.

6 a $P(P \cap Q) = P(P \mid Q) \times P(Q)$

$$P(P \mid Q) = \frac{P(P \cap Q)}{P(Q)}$$

$$= \frac{0.3}{0.3 + 0.4} = \frac{0.3}{0.7} = \frac{3}{7} \text{ (or } 0.4285\dot{7}\dot{1})$$

b $P(Q \mid P) = \frac{P(P \cap Q)}{P(P)}$

$$= \frac{0.3}{0.2 + 0.3} = \frac{0.3}{0.5} = \frac{3}{5} \text{ (or } 0.6)$$

Review it!

1 a
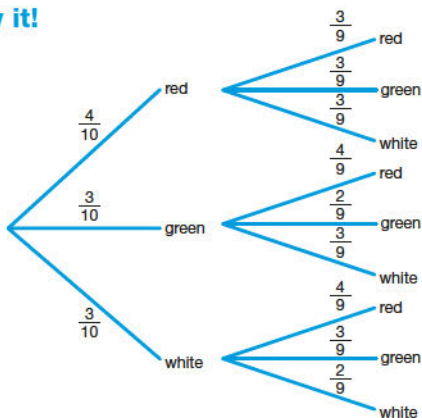
b Probability the same colour
= P (2 red) + P(2 green) + P(2 white)

$$= \left(\frac{4}{10} \times \frac{3}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right)$$

$$= \frac{12}{90} + \frac{6}{90} + \frac{6}{90}$$
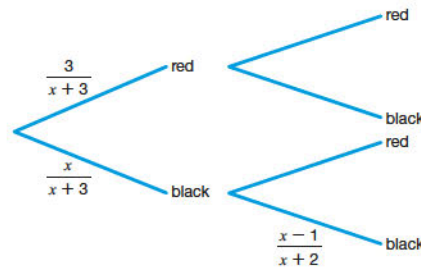
$$= \frac{24}{90}$$

$$= \frac{4}{15}$$

c Probability different colours = 1 − probability of the same colour

$$= 1 - \frac{4}{15}$$

$$= \frac{11}{15}$$

2 a

Probability two black balls chosen $= \left(\frac{x}{x+3}\right) \times \left(\frac{x-1}{x+2}\right)$

Also, Probability two black balls chosen $= \frac{7}{15}$

$$\left(\frac{x}{x+3}\right) \times \left(\frac{x-1}{x+2}\right) = \frac{7}{15}$$

$$15x(x-1) = 7(x+3)(x+2)$$

$$15x^2 - 15x = 7x^2 + 35x + 42$$

$$8x^2 - 50x - 42 = 0$$

$$4x^2 - 25x - 21 = 0$$

b Solving the quadratic equation
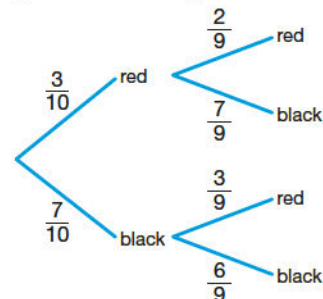
$$4x^2 - 25x - 21 = 0$$

$$(4x + 3)(x - 7) = 0$$

$x = -\frac{3}{4}$ (which is impossible as x has to be a positive integer) or $x = 7$.

Hence $x = 7$

Total number of balls in the bag $= 3 + 7 = 10$

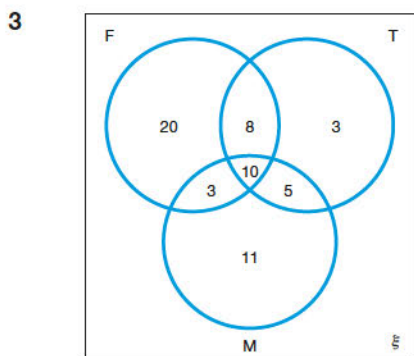**c** Producing a new tree diagram now that x is known:
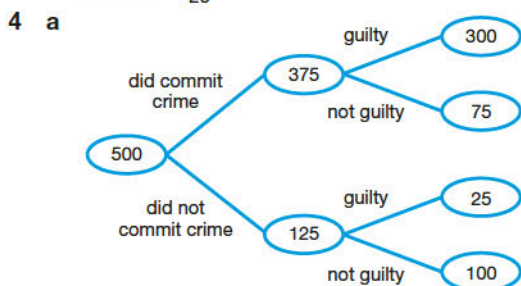

Probability of two different colours

$$= \left(\frac{3}{10} \times \frac{7}{9}\right) + \left(\frac{7}{10} \times \frac{3}{9}\right)$$

$$= \frac{21}{90} + \frac{21}{90}$$

$$= \frac{7}{15}$$

3

a P(only liked motor racing) $= \frac{11}{60}$

b P(student who liked motor racing also liked tennis) $= \frac{15}{29}$

4 **a**

b P(random defendant found guilty) $= \frac{300 + 25}{500}$
$$= \frac{13}{20}$$

c P(defendant who did not commit crime found guilty) $= \frac{25}{125} = \frac{1}{5}$

Statistics

Sampling

1 Find the % of males in the youth club

$$= \frac{\text{Number of males}}{\text{Total number of members}} \times 100$$

The total number of members is 185 + 165 = 350

$$= \frac{185}{350} \times 100$$
$$= 52.86\%$$

Now find this same % of the sample = 52.86% of 50
= 0.5286 × 50 = 26 (nearest whole number)

A quicker way to do this would be to notice that 50
= 350 ÷ 7, so:

Males chosen $= \frac{185}{7} = 26$ to nearest whole number.

Two-way tables and pie charts

1 **a** Total frequency = 13 + 7 + 10 = 30

30 newts = 360° so 1 newt represents $\frac{360}{30} = 12°$

Smooth newt angle = 13 × 12 = 156°

Great crested newt angle = 7 × 12 = 84°

Palmate newt angle = 10 × 12 = 120°
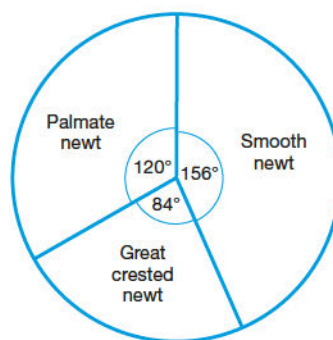
b No. The other pond might have had more newts in total. The proportion of smooth newts in the second pond is lower, but there may be more newts.

Line graphs for time series data

1 As these values are decreasing with time the trend is decreasing sales.

2 **a, b** and **c**
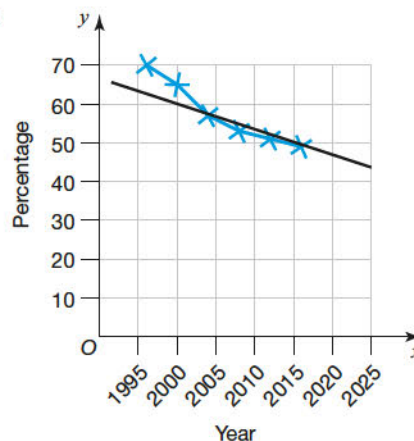
b The percentage of people visiting the local shop is decreasing.

c Drawing a trend line gives 44%

d There are no points, so you would be trying to predict the future. There may be a change of ownership or a refurbishment, making it more popular. It could even close down before then.

Averages and spread

1 Mean $= \frac{\text{Total age of members}}{\text{Number of members}}$

Rearranging, we have:

Total age of members = Mean × Number of members

Total age for boys = 13 × 10 = 130

Total age for girls = 14 × 12 = 168

Total age for boys and girls = 130 + 168 = 298

Total number of boys and girls in club = 10 + 12 = 22

Mean $= \frac{\text{Total age of members}}{\text{Number of members}} = \frac{298}{22} = 13.5$ years

2 Mean $= \frac{\text{Total number of marks}}{\text{Number of students}}$

Total mark for boys = 50 × 10 = 500

Total mark for girls = 62 × 15 = 930

Total mark for the whole class of 25 = 500 + 930
= 1430

Mean for whole class $= \frac{\text{Total number of marks}}{\text{Number of students}} = \frac{1430}{25}$
$$= 57.2\%$$

Joshua is wrong, because he didn't take account of the fact that there were different numbers of boys and girls.

3 a

Cost (£C)	Frequency	Mid-interval value	Frequency × mid-interval value
$0 < C \le 4$	12	2	24
$4 < C \le 8$	8	6	48
$8 < C \le 12$	10	10	100
$12 < C \le 16$	5	14	70
$16 < C \le 20$	2	18	36

b Estimated mean $= \dfrac{24 + 48 + 100 + 70 + 36}{37} = £7.51$ (to nearest penny)

Histograms

1 Notice there is no scale for frequency density on the histogram. It is necessary to use a bar on the histogram where there is a known frequency so the frequency density can be found which will then enable the scale to be found.

Take the class $2.5 < m \le 3.0$ which has a high frequency of 38.

frequency density $= \dfrac{\text{frequency}}{\text{class width}} = \dfrac{38}{0.5} = 76$

This bar has a height of 76 and is also the highest bar so the scale can be worked out.
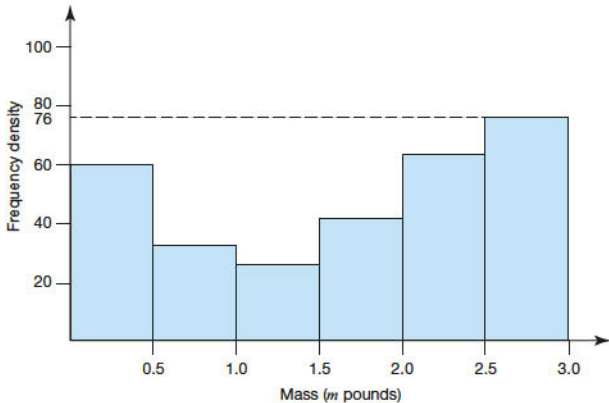
For the class $0.5 < m \le 1.0$, the frequency density can be obtained from the graph (i.e. 32).

frequency = frequency density × class width

$\qquad = 32 \times 0.5 = 16$

For the class $2.0 < m \le 2.5$, the frequency density from the graph is 64.

frequency = frequency density × class width

$\qquad = 64 \times 0.5 = 32$

The table can now be completed with the new frequencies.

Mass (m pounds)	Frequency
$0.0 < m \le 0.5$	30
$0.5 < m \le 1.0$	16
$1.0 < m \le 1.5$	13
$1.5 < m \le 2.0$	21
$2.0 < m \le 2.5$	32
$2.5 < m \le 3.0$	38

Cumulative frequency graphs

1 a
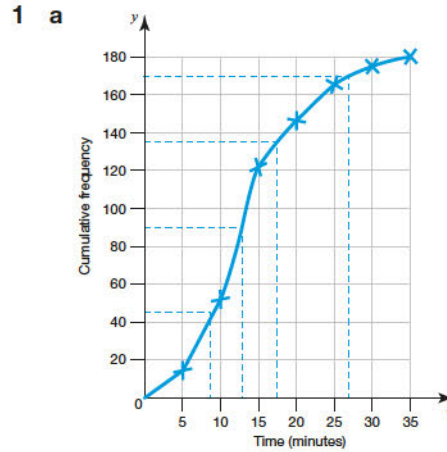
b To read off the median find half-way through the cumulative frequency and then draw a horizontal line and where it meets the curve draw a vertical line down. The median is the value where this line cuts the horizontal axis.

Hence median = 13 minutes

c **i** To find the upper quartile read off $\frac{3}{4}$ of the way through the cumulative frequency to the curve and read off the value on the horizontal axis. Upper quartile = 17.5 minutes

ii To find the lower quartile read off $\frac{1}{4}$ of the way through the cumulative frequency to the curve and read off the value on the horizontal axis. Lower quartile = 9 minutes

iii Interquartile range = upper quartile − lower quartile = 17.5 − 9 = 8.5 minutes

d Read vertically up from 27 minutes to the curve and then across and read off the cumulative frequency. This gives 170 which means 180 − 170 = 10 waited more than 27 minutes.

$\dfrac{\text{Number of people waiting 27 minutes or more}}{\text{Total number of people}} \times 100 = \dfrac{10}{180} \times 100 = 5.6\%$ (1 d.p.)

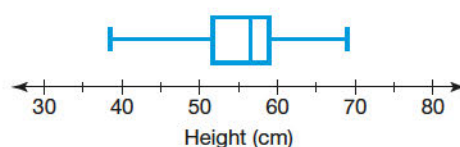So 5.6% of passengers waited for 27 minutes or more at security.

Comparing sets of data

1

	Height (cm)
Lowest height	38
Lower quartile	52
Median	57
Upper quartile	59
Highest height	69

Measurements from the scale on the box plot are transferred to the table.

The lowest height and the highest height are added to complete the box plot.

2 a   i Range = 120 − 0 = 120 marks

ii Median = 65 marks

iii Upper quartile = 75 marks

iv Lower quartile = 51 marks

v Interquartile range = 75 − 51 = 24 marks

b For the girls:

> You need to find the median and upper and lower quartiles from the graph for the girls.

Median mark = 74 marks

Upper quartile = 89 marks

Lower quartile = 58 marks

Interquartile range = 89 − 58 = 31 marks

Comparison:

The median mark for the girls is higher.

The interquartile range is lower for the boys showing that their marks are less spread out for the middle half of the marks.

Scatter graphs

1   a, b
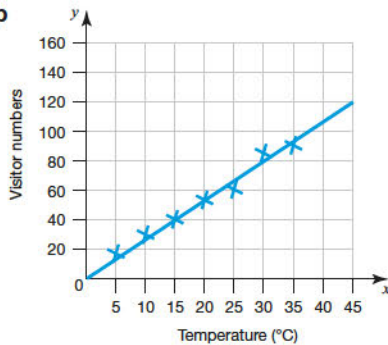
c 71 visitors

d   i 120 visitors

ii There are no points near this temperature so you cannot assume the trend continues. As the temperature gets extremely high people might choose to stay away from the beach.

Review it!

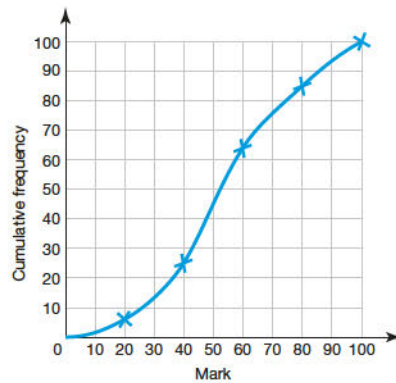**1   a** Number of students in class = 6 + 15 + 9 + 6 = 36

b

Amount of pocket money (£a per week)	Frequency	Mid-interval value	Frequency × mid-interval value
$2 < a \le 4$	6	3	$6 \times 3 = 18$
$4 < a \le 6$	15	5	$15 \times 5 = 75$
$6 < a \le 8$	9	7	$9 \times 7 = 63$
$8 < a \le 10$	6	9	$6 \times 9 = 54$

Estimate for mean = $\frac{18 + 75 + 63 + 54}{36}$ = £5.83 (to nearest penny)

c Modal class interval is $4 < a \le 6$

2   a

b Median = 54 (i.e. the mark corresponding to half-way through the cumulative frequency)

c The top 75% would pass so the bottom 25% would fail. This 25% corresponds to a cumulative frequency of 25 which gives a pass mark of 40.

3   a D (the median mark is furthest to the right)

b C (the largest gap between the quartiles)

c D (the median mark is the highest and the interquartile range is small, which means 50% of pupils got near to the median mark).

4   a The sample needs to be representative of the people living on the street, so they should take a stratified sample that includes different groups such as males and females, adults and children, in the right proportions.

b $\frac{15}{40} \times 320 = 120$

They should buy lemonade for 120 people.

All Boards Higher Mathematics Exam Practice Book
Full worked solutions

Number
Integers, decimals and symbols

1 $(-1)^3 = -1$

 $(0.1)^2 = 0.01$

 $\frac{1}{1000} = 0.001$

 0.1

 $\frac{1}{0.01} = 100$

 Descending order: $\frac{1}{0.01}$ 0.1 $(0.1)^2$ $\frac{1}{1000}$ $(-1)^3$

2 a $0.035 \times 1000 = 35$

 b $12.85 \div 1000 = 0.01285$

 c $(-3) \times 0.09 \times 1000 = -0.27 \times 1000 = -270$

 d $(-1) \times (-0.4) \times 100 = 0.4 \times 100 = 40$

3 a $86 \times 54 = (0.86 \times 100) \times 54 = 46.44 \times 100 = 4644$

 b $8.6 \times 540 = (0.86 \times 10) \times (54 \times 10) = 46.44 \times 100 = 4644$

 c $\frac{4644}{54} = \frac{46.44 \times 100}{54} = 0.86 \times 100 = 86$

 d $\frac{46.44}{0.086} = \frac{46.44}{0.86 \div 10} = 54 \times 10 = 540$

4 a $12.56 \times 3.45 = 0.1256 \times 345$

 b $(-8)^2 = 64$, so $(-8)^2 > -64$

 c $6 - 12 = -6$ and $8 - 14 = -6$, so $6 - 12 = 8 - 14$

 d $(-7) \times (0) = 0$ and $(-7) \times (-3) = 21$,

   so $(-7) \times (0) < (-7) \times (-3)$

Addition, subtraction, multiplication and division

1 a $67.78 + 8.985 = 76.765$

 b $124.706 + 76.9 + 0.04 = 201.646$

 c $93.1 - 1.77 = 91.33$

 d $23.7 + 8.94 - 22.076 = 32.64 - 22.076 = 10.564$

2 a $147 \times 8 = 1176$

 b $57 \times 38 = 2166$

 c $9.7 \times 4.6 = 44.62$

 d $1.24 \times 0.53 = 0.6572$

 e $486 \div 18 = 27$

 f $94.5 \div 1.5 = 945 \div 15 = 63$

3 a $34^2 = 34 \times 34 = 1156$

 b $\frac{1.5 \times 2.5}{0.5} = 3 \times 2.5 = 7.5$

 c $2.4^2 = 2.4 \times 2.4 = 5.76$

Using fractions

1 $\frac{2}{5} = \frac{16}{40} = \frac{30}{75} = \frac{50}{125}$

2 a $\frac{64}{12} = 5\frac{4}{12} = 5\frac{1}{3}$

 b $\frac{124}{13} = 9\frac{7}{13}$

3 a $4\frac{1}{4} \times 1\frac{2}{3} = \frac{17}{4} \times \frac{5}{3} = \frac{85}{12} = 7\frac{1}{12}$

 b $1\frac{7}{8} \div \frac{1}{4} = \frac{15}{8} \times \frac{4}{1} = \frac{60}{8} = \frac{15}{2} = 7\frac{1}{2}$

 c $3\frac{1}{5} - \frac{3}{4} = 3\frac{4}{20} - \frac{15}{20} = 2\frac{24}{20} - \frac{15}{20} = 2\frac{9}{20}$

4 $1 - \left(\frac{2}{7} + \frac{3}{8} + \frac{1}{4}\right) = 1 - \frac{2 \times 8 + 3 \times 7 + 1 \times 14}{56} = 1 - \frac{51}{56} = \frac{5}{56}$

 $\frac{5}{56}$ of the students travel by car.

5 The lowest common multiple of all the denominators is 24.

 $\frac{2}{3} = \frac{16}{24}$ $\frac{3}{4} = \frac{18}{24}$ $\frac{7}{8} = \frac{21}{24}$ $\frac{1}{2} = \frac{12}{24}$ $\frac{7}{12} = \frac{14}{24}$

 Order with the smallest first:

 $\frac{1}{2}$ $\frac{7}{12}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{7}{8}$

Different types of number

1 a 7

 b 49

 c 2

 d 6

 e 6

2 a $693 = 3^2 \times 7 \times 11$

 b $945 = 3^3 \times 5 \times 7$

   3^2 and 7 are common to both lists.

   Highest common factor $= 3^2 \times 7 = 63$

 c Lowest common multiple $= 3^3 \times 5 \times 7 \times 11 = 10\,395$

3 $49 = 7^2$

 $63 = 9 \times 7 = 3 \times 3 \times 7 = 3^2 \times 7$

 Lowest common multiple $= 3^2 \times 7^2 = 441$

4 $15 = 3 \times 5$

 $20 = 4 \times 5 = 2^2 \times 5$

 $25 = 5^2$

 Lowest common multiple $= 2^2 \times 3 \times 5^2 = 300$ seconds
 $= 5$ minutes

 They will make a note together after 5 minutes.

Listing strategies

1 Multiples of 14:

 14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154, 168, 182, 196, 210, …

 Multiples of 15:

 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, 195, 210, …

 210 is common to both lists, so they will sound together after 210 seconds.

2 Multiples of 6:

 6, 12, 18, 24, 30, 36, 42, …

 Multiples of 6 add 1:

 7, 13, 19, 25, 31, 37, 43, …

 Multiples of 7:

 7, 14, 21, 28, 35, 42, 49, 56, …

 Multiples of 7 minus 4:

 3, 10, 17, 24, 31, 38, 45, 52

 31 is common to both lists. This means 6 chocolates could be given to 5 friends with one chocolate left over.

 Hence number of friends $= 5$

3 List all the ratios that are equivalent to $5 : 6$, find the difference between them and look for a difference of 100.

 Since this is a college, the numbers are likely to be large.

Male	Female	Difference
50	60	10
500	600	100

 There are a total of $500 + 600 = 1100$ students.

4   Call the students A, B, C, D, E and F.

Possible pairs beginning with A:

AB, AC, AD, AE, AF

Possible pairs beginning with B (excluding pairs in first list):

BC, BD, BE, BF

Possible pairs beginning with C (excluding pairs in previous lists):

CD, CE, CF

Possible pairs beginning with D (excluding pairs in previous lists):

DE, DF

Possible pairs beginning with E (excluding pairs in previous lists):

EF

Total number of pairs $= 5 + 4 + 3 + 2 + 1 = 15$

The order of operations in calculations

1   a   Ravi has worked out the expression from left to right, instead of using BIDMAS. He should have performed the division and multiplication before the addition.

b $24 \div 3 + 8 \times 4 = 8 + 32$

$= 40$

2   a $9 \times 7 \times 2 - 24 \div 6 = 126 - 4 = 122$

b $2 - (-27) \div (-3) + 4 = 2 - 9 + 4 = -3$

c $(4 - 16)^2 \div 4 + 32 \div 8 = (-12)^2 \div 4 + 4 = 144 \div 4 + 4$
$= 36 + 4 = 40$

3   a $1 + 4 \div \frac{1}{2} - 3 = 1 + 8 - 3 = 6$

b $15 - (1 - 2)^2 = 15 - (-1)^2 = 15 - 1 = 14$

c $\sqrt{4 \times 12 - 2 \times (-8)} = \sqrt{48 + 16} = \sqrt{64} = 8$

Indices

1   a $10^5 \times 10 = 10^{5+1} = 10^6$

b $(10^4)^2 = 10^{4 \times 2} = 10^8$

c $\frac{10^5 \times 10^3}{10^2} = 10^{5+3-2} = 10^6$

d $(10^6)^{\frac{1}{2}} = 10^{6 \times \frac{1}{2}} = 10^3$

2   a $5^0 = 1$

b $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

c $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

d $49^{\frac{1}{2}} = \sqrt{49} = 7$

3   a $\left(\frac{8}{27}\right)^{-\frac{1}{3}} = \left(\frac{27}{8}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2}$

b $\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2 = 4^2 = 16$

c $36^{-\frac{1}{2}} = \frac{1}{\sqrt{36}} = \frac{1}{6}$

d $16^{\frac{3}{2}} = \left(16^{\frac{1}{2}}\right)^3 = (\sqrt{16})^3 = 4^3 = 64$

4   $125 = 5^3$, so $5^{2x} = 5^3$

$2x = 3$ giving $x = 1.5$

Surds

1   a $\left(\sqrt{5}\right)^2 = \sqrt{5} \times \sqrt{5} = 5$

b $3\sqrt{2} \times 5\sqrt{2} = 3 \times 5 \times \sqrt{2} \times \sqrt{2} = 15 \times 2 = 30$

c $(3\sqrt{2})^2 = 3^2 \times (\sqrt{2})^2 = 9 \times 2 = 18$

2   $\frac{15}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{15 \times \sqrt{3}}{4 \times 3} = \frac{5\sqrt{3}}{4}$

3   $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 2\sqrt{3} + 2\sqrt{3} - 3 = 4 - 3 = 1$

4   $\sqrt{3}\,(3\sqrt{24} + 4\sqrt{6}) = 3\sqrt{72} + 4\sqrt{18}$
$= 3\sqrt{36 \times 2} + 4\sqrt{9 \times 2}$

$= 3 \times 6\sqrt{2} + 4 \times 3\sqrt{2}$

$= 18\sqrt{2} + 12\sqrt{2}$

$= 30\sqrt{2}$

$a = 30$

5   $(1 - \sqrt{5})(3 + 2\sqrt{5}) = 3 + 2\sqrt{5} - 3\sqrt{5} - 10 = -\sqrt{5} - 7$

6   $\frac{1}{\sqrt{2}} + \frac{1}{4} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{1}{4}$

$= \frac{\sqrt{2}}{2} + \frac{1}{4}$

$= \frac{2\sqrt{2}}{4} + \frac{1}{4}$

$= \frac{1 + 2\sqrt{2}}{4}$

7   $\frac{2}{1 - 1/\sqrt{2}} = \frac{2}{\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}}}$

$= \frac{2}{\frac{\sqrt{2} - 1}{\sqrt{2}}}$

$= \frac{2\sqrt{2}}{\sqrt{2} - 1}$

$= \frac{2\sqrt{2}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$

$= \frac{4 + 2\sqrt{2}}{2 - 1}$

$= 4 + 2\sqrt{2}$

8   $\frac{3}{\sqrt{3}} + \sqrt{75} + (\sqrt{2} \times \sqrt{6}) = \frac{3\sqrt{3}}{3} + \sqrt{3 \times 25} + \sqrt{12}$

$= \sqrt{3} + 5\sqrt{3} + \sqrt{3 + 4}$

$= \sqrt{3} + 5\sqrt{3} + 2\sqrt{3}$

$= 8\sqrt{3}$

Standard form

1   a 2.55×10^{-3}

b 1.006×10^{10}

c 8.9×10^{-8}

2   a $3 \times 10^6 \times 2 \times 10^8 = 3 \times 2 \times 10^{6+8} = 6 \times 10^{14}$

b $5.5 \times 10^{-3} \times 2 \times 10^8 = 5.5 \times 2 \times 10^{-3+8} = 11 \times 10^5$
$= 1.1 \times 10^6$

c $\frac{8 \times 10^5}{4 \times 10^3} = \frac{8}{4} \times 10^{5-3} = 2 \times 10^2$

d $\frac{5 \times 10^{-3}}{0.5} = \frac{5}{0.5} \times 10^{-3} = 10 \times 10^{-3} = 1 \times 10^{-2}$

e $\frac{4.5 \times 10^{-6}}{0.5 \times 10^{-3}} = \frac{4.5}{0.5} \times \frac{10^{-6}}{10^{-3}} = 9 \times 10^{-6 - (-3)} = 9 \times 10^{-3}$

3   $2.4 \times 10^3 + 2.8 \times 10^2 = 2400 + 280 = 2680$

4   $3.3 \times 10^4 = 33\,000$ and $3.3 \times 10^2 = 330$

$3.3 \times 10^4 + 3.3 \times 10^2 = 33\,330$

$a = 3.3$

Converting between fractions and decimals

1   a $\frac{11}{20} = \frac{55}{100} = 0.55$

b $\frac{3}{8} = 3 \div 8 = 0.375$

2   a $16 = 2 \times 2 \times 2 \times 2$

All the prime factors in the denominator are 2, so it will produce a **terminating** decimal.

b   The prime factors of the denominators are 7 and 1 so this will produce a **recurring** decimal.

c   Prime factors of 35 are 7 and 5 so this will produce a **recurring** decimal.

3   Let $x = 0.40\dot{2} = 0.402402402...$

$1000x = 402.402402...$

$1000x - x = 402.402402... - 0.402402402...$

$999x = 402$

$x = \frac{402}{999} = \frac{134}{333}$

Hence $0.40\dot{2} = \frac{134}{333}$

4   Let $x = 0.65\dot{2} = 0.652525252\ldots$

$10x = 6.52525252\ldots$

$1000x = 652.52525252\ldots$

$1000x - 10x = 652.52525252\ldots - 6.52525252\ldots$

$990x = 646$

$x = \frac{646}{990} = \frac{323}{495}$

Hence $0.65\dot{2} = \frac{323}{495}$

Converting between fractions and percentages

1   a $35\% = \frac{35}{100} = \frac{7}{20}$

b $7\% = \frac{7}{100}$

c $76\% = \frac{76}{100} = \frac{19}{25}$

d $12.5\% = \frac{12.5}{100} = \frac{125}{1000} = \frac{1}{8}$

2   a $\frac{1}{5} = \frac{1 \times 20}{5 \times 20} = \frac{20}{100} = 20\%$

b $\frac{17}{25} = \frac{17 \times 4}{25 \times 4} = \frac{68}{100} = 68\%$

c $\frac{150}{60} = \frac{50}{20} = \frac{250}{100} = 250\%$

d $\frac{7}{40} = 7 \div 40 = 0.7 \div 4 = 0.175 = 17.5\%$

3   $\frac{8}{15} = \frac{8}{15} \times 100\% = 53.33\%$ (to 2 d.p.)

4   $\frac{66}{90} = \frac{66}{90} \times 100\% = 73.3\%$ (to 1 d.p.)

Jake did better in chemistry.

Fractions and percentages as operators

1   70% of $£49.70 = 0.7 \times 49.70 = £34.79$

2   8% of $600 = \frac{8}{100} \times 600 = 48$

48 apples are bad.

3   12% of $8000 = 0.12 \times 8000 = 960$ components were rejected.

$8000 - 960 = 7040$ components were accepted.

4   a   VAT $= 20\%$ of $£12\,000 = 0.2 \times 12\,000 = £2400$

Total cost $= 12\,000 + 2400 = £14\,400$

b   Deposit $= 20\%$ of $£14\,400 = 0.2 \times 14\,400 = £2880$

Remainder $= 14\,400 - 2880 = £11\,520$

Monthly payment $= \frac{11\,520}{36} = £320$

5   $\frac{2}{3}$ of $\frac{7}{11} = \frac{2}{3} \times \frac{7}{11} = \frac{14}{33}$

Standard measurement units

1   $1.75\,\text{km} = 1.75 \times 1000\,\text{m} = 1750\,\text{m}$

$1\,\text{m} = 100\,\text{cm}$, so $1750\,\text{m} = 1750 \times 100 = 175\,000\,\text{cm}$

Hence $1.75\,\text{km} = 175\,000\,\text{cm}$

2   3 litres $= 3000\,\text{ml}$

Number of glasses $= \frac{3000}{175} = 17.14$

Number of complete glasses $= 17$

3   900 litres $= 900 \times 1000 = 900\,000\,\text{cm}^3$

Number of containers needed to fill pool $= \frac{900\,000}{700} = 1286$ (to nearest whole number)

4   a   Mass of one atom $= \frac{12}{6.02 \times 10^{23}} = 1.99 \times 10^{-23}\,\text{g}$ (to 3 s.f.)

b   Mass of one atom $= \frac{12 \times 10^{-3}}{6.02 \times 10^{23}} = 1.99 \times 10^{-26}\,\text{kg}$ (to 3 s.f.)

5   $79 \times 9.11 \times 10^{-31}\,\text{kg} = 79 \times 9.11 \times 10^{-31} \times 10^{3}\,\text{g}$
$= 7.20 \times 10^{-26}\,\text{g}$ (to 3 s.f.)

Rounding numbers

1   a   35   c   0   e   2

b   101   d   0

2   a   34.88   b   34.877

3   a   12 800

b   0.011

c 7×10^{-5}

4   a $(0.18 \times 0.046)^2 - 0.01 = -0.00993$ (to 3 s.f.)

b $\frac{1200 \times 1.865}{2.6 \times 25} = 34.4$ (to 3 s.f.)

c $\frac{36}{0.07} \times 12 \div \frac{1}{2} = 12\,300$ (to 3 s.f.)

Estimation

1   $4.6 \times 9.8 \times 3.1 \approx 5 \times 10 \times 3 = 150 \approx 200$ (to 1 s.f.)

2   a   236.2298627

b $19.87^2 - \sqrt{404} \times 7.89 \approx 20^2 - \sqrt{400} \times 8$

$= 400 - 20 \times 8 = 240$

3   $(0.52 \times 0.83)^2 \approx (0.5 \times 0.8)^2 = (0.4)^2 = 0.16$

4   $\sqrt{5.08 + 4.10 \times 5.45} \approx \sqrt{5 + 4 \times 5} = \sqrt{25} = 5$

5   $\sqrt{100} = 10$ and $\sqrt{121} = 11$

$\sqrt{112}$ is slightly over halfway between the two roots.

Hence $\sqrt{112} = 10.6$ (to 1 d.p.) is a reasonable estimate.

6   $\frac{0.89 \times 7.51 \times 19.76}{2.08 \times 5.44 \times 3.78} \approx \frac{1 \times 8 \times 20}{2 \times 5 \times 4} = \frac{160}{40} = 4$

7   a   mass $\approx 9 \times 10^{-31} \times 6 \times 100 = 5.4 \times 10^{-28} \approx 5 \times 10^{-28}\,\text{kg}$

b   This will be an underestimate, as the mass of one electron has been rounded down.

Upper and lower bounds

1   upper bound $= 2.345\,\text{kg}$

lower bound $= 2.335\,\text{kg}$

$2.335 \leq m < 2.345\,\text{kg}$

2   a   i $V = \frac{P}{I}$

The upper bound for V will be when P has its upper bound and I has its lower bound.

upper bound for $P = 3.0525$

lower bound for $I = 1.235$

upper bound for $V = \frac{3.0525}{1.235} = 2.472$

ii   The lower bound for V will be when P has its lower bound and I has its upper bound.

lower bound of $P = 3.0515$

upper bound of $I = 1.245$

lower bound for $V = \frac{3.0515}{1.245} = 2.451$

b   The upper bound and the lower bound are the same when the numbers are given to 2 significant figures.

$V = 2.5$ (to 2 s.f.)

3   lower bound for shelf length $= 1.05\,\text{m} = 105\,\text{cm}$

upper bound for book thickness $= 3.05\,\text{cm}$

number of books on shelf $= \frac{\text{length of shelf}}{\text{thickness of book}}$

number of books that will definitely fit onto shelf $=$

minimum value for $\frac{\text{length of shelf}}{\text{thickness of book}}$

minimum value for $\frac{\text{length of shelf}}{\text{thickness of book}}$ is when shelf length is at

its lower bound and book thickness is at its upper bound.

$\frac{\text{minimum length of shelf}}{\text{maximum thickness of book}} = \frac{105}{3.05} = 34.4$

The answer must be a whole number, so **34** books will definitely fit on the shelf.

Algebra

Simple algebraic techniques

1 **a** formula   **c** expression   **e** formula
   b identity   **d** identity

2 $4x + 3x \times 2x - 3x = 4x + 6x^2 - 3x = x + 6x^2$

3 $y^3 - y = (1)^3 - 1 = 0$ so $y = 1$ is correct.
   $y^3 - y = (-1)^3 - (-1) = -1 + 1 = 0$ so $y = -1$ is correct.

4 **a** $6x - (-4x) = 6x + 4x = 10x$
   b $x^2 - 2x - 4x + 3x^2 = 4x^2 - 6x$
   c $(-2x)^2 + 6x \times 3x - 4x^2 = 4x^2 + 18x^2 - 4x^2 = 18x^2$

5 **a** $s = \frac{3^2 - 1^2}{2 \times 2} = \frac{8}{4} = 2$
   b $s = \frac{(-4)^2 - 3^2}{2 \times 4} = \frac{7}{8}$
   c $s = \frac{5^2 - (-2)^2}{2 \times (-7)} = \frac{21}{-14} = -\frac{3}{2}$

Removing brackets

1 **a** $8(3x - 7) = 8 \times 3x - 8 \times 7$
      $= 24x - 56$
   b $-3(2x - 4) = -3 \times 2x - 3 \times (-4)$
      $= -6x + 12$

2 **a** $3(2x - 1) - 3(x - 4) = 6x - 3 - 3x + 12$
      $= 3x + 9$
   b $4y(2x + 1) + 6(x - y) = 8xy + 4y + 6x - 6y$
      $= 8xy + 6x - 2y$
   c $5ab(2a - b) = 10a^2b - 5ab^2$
   d $x^2y^3(2x + 3y) = 2x^3y^3 + 3x^2y^4$

3 **a** $(m - 3)(m + 8) = m^2 + 8m - 3m - 24$
      $= m^2 + 5m - 24$
   b $(4x - 1)(2x + 7) = 8x^2 + 28x - 2x - 7$
      $= 8x^2 + 26x - 7$
   c $(3x - 1)^2 = (3x - 1)(3x - 1)$
      $= 9x^2 - 3x - 3x + 1$
      $= 9x^2 - 6x + 1$
   d $(2x + y)(3x - y) = 6x^2 - 2xy + 3xy - y^2$
      $= 6x^2 + xy - y^2$

4 **a** $(x + 5)(x + 2) = x^2 + 2x + 5x + 10$
      $= x^2 + 7x + 10$
   b $(x + 4)(x - 4) = x^2 - 4x + 4x - 16$
      $= x^2 - 16$
   c $(x - 7)(x + 1) = x^2 + x - 7x - 7$
      $= x^2 - 6x - 7$
   d $(3x + 1)(5x + 3) = 15x^2 + 9x + 5x + 3$
      $= 15x^2 + 14x + 3$

5 **a** $(x + 3)(x - 1)(x + 4) = (x^2 - x + 3x - 3)(x + 4)$
      $= (x^2 + 2x - 3)(x + 4)$
      $= x^3 + 4x^2 + 2x^2 + 8x - 3x - 12$
      $= x^3 + 6x^2 + 5x - 12$

b $(3x - 4)(2x - 5)(3x + 1) = (6x^2 - 15x - 8x + 20)(3x + 1)$
   $= (6x^2 - 23x + 20)(3x + 1)$
   $= 18x^3 + 6x^2 - 69x^2 - 23x + 60x$
      $+ 20$
   $= 18x^3 - 63x^2 + 37x + 20$

Factorising

1 **a** $25x^2 - 5xy = 5x(5x - y)$
   b $4\pi r^2 + 6\pi x = 2\pi(2r^2 + 3x)$
   c $6a^3b^2 + 12ab^2 = 6ab^2(a^2 + 2)$

2 **a** $9x^2 - 1 = (3x + 1)(3x - 1)$
   b $16x^2 - 4 = (4x + 2)(4x - 2)$
      $= 4(2x + 1)(2x - 1)$

3 **a** $a^2 + 12a + 32 = (a + 4)(a + 8)$
   b $p^2 - 10p + 24 = (p - 6)(p - 4)$

4 **a** $a^2 + 12a = a(a + 12)$
   b $b^2 - 9 = (b + 3)(b - 3)$
   c $x^2 - 11x + 30 = (x - 5)(x - 6)$

5 **a** $3x^2 + 20x + 32 = (3x + 8)(x + 4)$
   b $3x^2 + 10x - 13 = (3x + 13)(x - 1)$
   c $2x^2 - x - 10 = (2x - 5)(x + 2)$

6 $\dfrac{x + 15}{2x^2 - 3x - 9} + \dfrac{3}{2x + 3} = \dfrac{x + 15}{(2x + 3)(x - 3)} + \dfrac{3}{(2x + 3)}$
   $= \dfrac{x + 15 + 3(x - 3)}{(2x + 3)(x - 3)}$
   $= \dfrac{4x + 6}{(2x + 3)(x - 3)}$
   $= \dfrac{2(2x + 3)}{(2x + 3)(x - 3)}$
   $= \dfrac{2}{(x - 3)}$

7 $\dfrac{1}{8x^2 - 2x - 1} \div \dfrac{1}{4x^2 - 4x + 1} = \dfrac{1}{8x^2 - 2x - 1} \times (4x^2 - 4x + 1)$
   $= \dfrac{1}{(4x + 1)(2x - 1)} \times (2x - 1)(2x - 1)$
   $= \dfrac{2x - 1}{4x + 1}$

Changing the subject of a formula

1 $PV = nRT$
   $T = \dfrac{PV}{nR}$

2 $2y + 4x - 1 = 0$
      $2y = 1 - 4x$
      $y = \dfrac{1 - 4x}{2}$

3 $v = u + at$
   $at = v - u$
   $a = \dfrac{v - u}{t}$

4 $y = \dfrac{x}{5} - m$
   $\dfrac{x}{5} = y + m$
   $x = 5(y + m)$

5 $E = \frac{1}{2}mv^2$
   $v^2 = \dfrac{2E}{m}$
   $v = \sqrt{\dfrac{2E}{m}}$

6 **a** $V = \frac{1}{3}\pi r^2 h$
      $r^2 = \dfrac{3V}{\pi h}$
      $r = \sqrt{\dfrac{3V}{\pi h}}$
   b $r = \sqrt{\dfrac{3 \times 100}{\pi \times 8}}$
      $= 3.45\,\text{cm}$ (to 2 d.p.)

7 a $y = 3x - 9$

$3x = y + 9$

$x = \frac{y + 9}{3}$

b $x = \frac{3 + 9}{4}$

$= 4$

8 $3y - x = ax + 2$

$3y - x - 2 = ax$

$3y - 2 = ax + x$

$3y - 2 = x(a + 1)$

$x = \frac{3y - 2}{a + 1}$

9 a $c^2 = \frac{(16a^2b^4c^2)^{\frac{1}{2}}}{4a^2b}$

$c^2 = \frac{4ab^2c}{4a^2b}$

$c^2 = \frac{bc}{a}$

$c = \frac{b}{a}$

b upper bound of $a = 2.85$ lower bound of $a = 2.75$

upper bound of $b = 3.25$ lower bound of $b = 3.15$

upper bound for $c = \frac{\text{upper bound for } b}{\text{lower bound for } a} = \frac{3.25}{2.75} = 1.18$ (to 3 s.f.)

lower bound for $c = \frac{\text{lower bound for } b}{\text{upper bound for } a} = \frac{3.15}{2.85} = 1.11$ (to 3 s.f.)

Solving linear equations

1 a $2x + 11 = 25$

$2x = 14$

$x = 7$

b $3x - 5 = 10$

$3x = 15$

$x = 5$

c $15x = 60$

$x = 4$

d $\frac{x}{4} = 8$

$x = 32$

e $\frac{4x}{5} = 20$

$4x = 100$

$x = 25$

f $\frac{2x}{3} = -6$

$2x = -18$

$x = -9$

g $5 - x = 7$

$5 = 7 + x$

$-2 = x$

$x = -2$

h $\frac{x}{7} - 9 = 3$

$\frac{x}{7} = 12$

$x = 84$

2 $5x - 1 = 2x + 1$

$5x = 2x + 2$

$3x = 2$

$x = \frac{2}{3}$

3 a $\frac{1}{4}(2x - 1) = 3(2x - 1)$

$2x - 1 = 12(2x - 1)$

$2x - 1 = 24x - 12$

$11 = 22x$

$x = \frac{11}{22}$

$x = \frac{1}{2}$

b $5(3x + 1) = 2(5x - 3) + 3$

$15x + 5 = 10x - 6 + 3$

$15x + 5 = 10x - 3$

$5x = -8$

$x = -\frac{8}{5}$

Solving quadratic equations using factorisation

1 a $x^2 - 7x + 12 = (x - 3)(x - 4)$

b $x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

So $x - 3 = 0$ or $x - 4 = 0$, giving $x = 3$ or $x = 4$

2 a $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

b $2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

So $2x - 1 = 0$ or $x + 3 = 0$, giving $x = \frac{1}{2}$ or $x = -3$

3 $x^2 - 3x - 20 = x - 8$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

So $x + 2 = 0$ or $x - 6 = 0$, giving $x = -2$ or $x = 6$

4 a $x(x - 8) - 7 = x(5 - x)$

$x^2 - 8x - 7 = 5x - x^2$

$2x^2 - 13x - 7 = 0$

b $2x^2 - 13x - 7 = 0$

$(2x + 1)(x - 7) = 0$

So $2x + 1 = 0$ or $x - 7 = 0$, giving $x = -\frac{1}{2}$ or $x = 7$

5 area of trapezium $= \frac{1}{2}(a + b)h$

$= \frac{1}{2}(x + 4 + x + 8)x$

$= \frac{1}{2}(2x + 12)x$

$= (x + 6)x$

$= x^2 + 6x$

area $= 16\,\text{cm}^2$ so $x^2 + 6x = 16$

$x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$

Solving gives $x = -8$ or $x = 2$

$x = -8$ is impossible as x is the height and so cannot be negative.

Hence $x = 2\,\text{cm}$

Solving quadratic equations using the formula

1 a $\frac{3}{x + 7} = \frac{2 - x}{x + 1}$

$3(x + 1) = (2 - x)(x + 7)$

$3x + 3 = 2x + 14 - x^2 - 7x$

$3x + 3 = -x^2 - 5x + 14$

$x^2 + 8x - 11 = 0$

b $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times (-11)}}{2 \times 1}$

$= \frac{-8 \pm \sqrt{64 + 44}}{2}$

$= \frac{-8 \pm \sqrt{108}}{2}$

$= \frac{-8 + \sqrt{108}}{2}$ or $\frac{-8 - \sqrt{108}}{2}$

$= 1.1962$ or -9.1962

$x = 1.20$ or -9.20 (to 2 d.p.)

2 area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2}(3x + 1)(2x + 3)$

$= \frac{1}{2}(6x^2 + 9x + 2x + 3)$

$= \frac{1}{2}(6x^2 + 11x + 3)$

$= 3x^2 + 5.5x + 1.5$

area $= 40$, so $3x^2 + 5.5x + 1.5 = 40$

Hence, $3x^2 + 5.5x - 38.5 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-5.5 \pm \sqrt{5.5^2 - 4 \times 3 \times (-38.5)}}{2 \times 3}$

$= \dfrac{-5.5 \pm \sqrt{492.25}}{6}$

$= \dfrac{-5.5 + \sqrt{492.25}}{6} = $ or $\dfrac{-5.5 - \sqrt{492.25}}{6}$

$= 2.78$ or -4.61 (to 2 d.p.)

$x = -4.61$ would give negative lengths, which are impossible.

$x = 2.78$ cm (to 2 d.p.)

3   $x^2 - 2x - 9 = x - 8$

$x^2 - 3x - 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$

$= \dfrac{3 \pm \sqrt{9 + 4}}{2}$

$= \dfrac{3 \pm \sqrt{13}}{2}$

$= \dfrac{3 + \sqrt{13}}{2} = $ or $\dfrac{3 - \sqrt{13}}{2}$

$x = 3.30$ or -0.30 (to 2 d.p.)

Solving simultaneous equations

1   $2x - 3y = -5$ (1)

$5x + 2y = 16$ (2)

Equation (1) $\times$ 2 and equation (2) $\times$ 3 gives:

$4x - 6y = -10$ (3)

$15x + 6y = 48$ (4)

Equation (3) + equation (4) gives:

$19x = 38$

$x = 2$

Substituting $x = 2$ into equation (1):

$2 \times 2 - 3y = -5$

$4 - 3y = -5$

$3y = 9$

$y = 3$

Checking by substituting $x = 2$ and $y = 3$ into equation (2) gives:

$5 \times 2 + 2 \times 3 = 10 + 6 = 16$

$x = 2$ and $y = 3$

2   a   Table of values for plotting graph of $y = 3x - 2$:

x	1	2	3	4
y	1	4	7	10
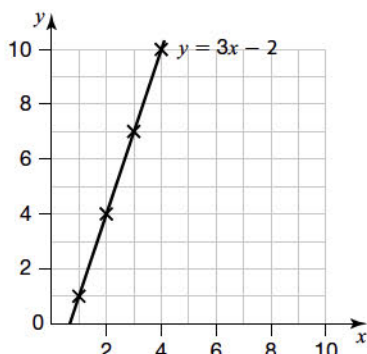
b   Table of values for plotting graph of $y = 10 - x$:

x	0	5	10
y	10	5	0
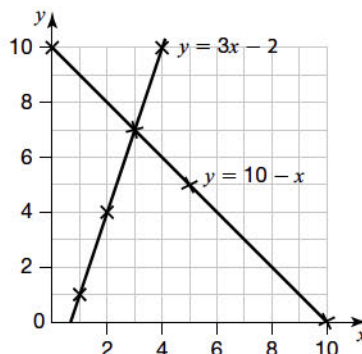
The graphs intersect at (3, 7).

$x = 3, y = 7$

3   $x - y = 3$ (1)

$x^2 + y^2 = 9$ (2)

Rearrange equation (1) as $y = x - 3$.

Substitute in equation (2):

$x^2 + (x - 3)^2 = 9$

$x^2 + x^2 - 6x + 9 = 9$

$2x^2 - 6x = 0$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0$ or $x = 3$

Substituting into equation (1) gives:

$x = 0, y = -3$ or $x = 3, y = 0$

Solving inequalities

1   a $\dfrac{x + 5}{4} \geq -1$

$x + 5 \geq -4$

$x \geq -9$

b $3x - 4 > 4x + 8$

$-x - 4 > 8$

$-x > 12$

$x < -12$

2

3   a
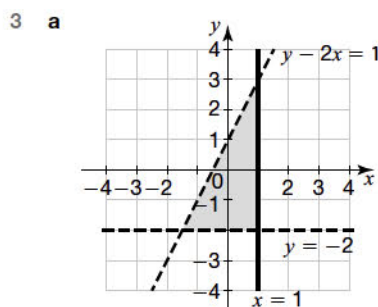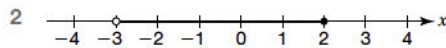
b   Coordinates of points that lie in the shaded region or on the solid line:

(1, 2), (1, 1), (0, 0), (1, 0), (0, −1), (1, −1)

4 $x^2 + 2x \leq 3$

$x^2 + 2x - 3 \leq 0$

Solving $x^2 + 2x - 3 = 0$:

$(x - 1)(x + 3) = 0$, giving $x = 1$ and $x = -3$

Sketch of the curve $y = x^2 + 2x - 3$:
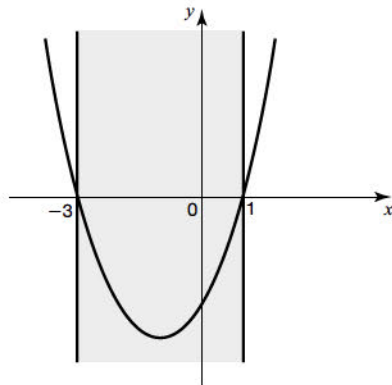
From graph, $x^2 + 2x - 3 \leq 0$ when:

$-3 \leq x \leq 1$

5 Solving $x^2 - 2x - 15 = 0$:

$(x - 5)(x + 3) = 0$, giving $x = 5$ and $x = -3$

Sketch of the curve $y = x^2 - 2x - 15$:
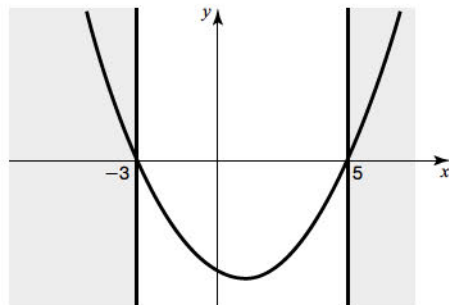
From graph, $x^2 - 2x - 15 > 0$ when:

$x < -3$ and $x > 5$

Problem solving using algebra

1 Let width $= x$

length $= x + 1$

perimeter $= x + x + 1 + x + x + 1 = 4x + 2$

perimeter $= 26$ so

$4x + 2 = 26$

$4x = 24$

$x = 6$

width $= 6\,m$ and length $= 7\,m$

area $= 6 \times 7 = 42\,m^2$

2 Let cost of adult ticket $= x$ and cost of child ticket $= y$

$2x + 5y = 35$ (1)

$3x + 4y = 38.5$ (2)

Equation (1) × 3 and equation (2) × 2 gives:

$6x + 15y = 105$ (3)

$6x + 8y = 77$ (4)

Equation (3) − equation (4) gives:

$7y = 28$

$y = 4$

Substituting $y = 4$ into equation (1) gives:

$2x + 20 = 35$

$2x = 15$

$x = 7.5$

cost of adult ticket $= £7.50$

cost of child ticket $= £4$

3 **a** Let Rachel be x years and Hannah be y years.

$xy = 63$

$(x + 2)(y + 2) = 99$

$xy + 2x + 2y + 4 = 99$

Substitute for xy:

$63 + 2x + 2y + 4 = 99$

$2x + 2y = 32$

$x + y = 16$

The sum of their ages is 16 years.

b $y = x - 2$

$x + x - 2 = x + y$

$2x - 2 = 16$

$2x = 18$

$x = 9$

Rachel is 9 years old.

Use of functions

1 **a** $f(3) = 5 \times 3 + 4 = 19$

b Set $f(x) = -1$

$5x + 4 = -1$

$5x = -5$

$x = -1$

2 **a** $fg(x) = f(g(x)) = f(x - 6) = (x - 6)^2$

b $gf(x) = g(f(x)) = g(x^2) = x^2 - 6$

3 **a** $f(5) = \sqrt{5 + 4} = \sqrt{9} = 3$ or -3

b $gf(x) = 2(\sqrt{x + 4})^2 - 3$

$= 2(x + 4) - 3$

$= 2x + 5$

4 Let $y = 5x^2 + 3$

$x = \sqrt{\dfrac{y - 3}{5}}$

$f^{-1}(x) = \sqrt{\dfrac{x - 3}{5}}$

Iterative methods

1 Let $f(x) = 2x^3 - 2x + 1$

$f(-1) = 2(-1)^3 - 2(-1) + 1 = 1$

$f(-1.5) = 2(-1.5)^3 - 2(-1.5) + 1 = -2.75$

There is a sign change of $f(x)$, so there is a solution between $x = -1$ and $x = -1.5$.

2 $x_1 = (x_0)^3 + \dfrac{1}{9} = (0.1)^3 + \dfrac{1}{9} = 0.1121111111$

$x_2 = (x_1)^3 + \dfrac{1}{9} = (0.1121111111)^3 + \dfrac{1}{9} = 0.1125202246$

$x_3 = (x_2)^3 + \dfrac{1}{9} = (0.1125202246)^3 + \dfrac{1}{9} = 0.1125357073$

3 **a** $x_1 = 1.5182945$

$x_2 = 1.5209353$

$x_3 = 1.5213157$

$x_4 = 1.5213705 \approx 1.521$ (to 3 d.p.)

b Checking value of $x^3 - x - 2$ for $x = 1.5205, 1.5215$:

When $x = 1.5205$ $\quad$ f(1.5205)$= -0.0052$

$\quad\quad x = 1.5215$ $\quad$ f(1.5215) $= 0.0007$

Since there is a change of sign, the root is 1.521 correct to 3 decimal places.

4 a i
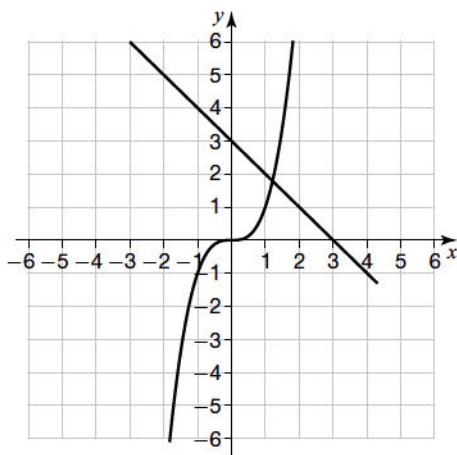
ii There is a root of $x^3 + x - 3 = 0$ where the graphs of $y = x^3$ and $y = 3 - x$ intersect. The graphs intersect once so there is one real root of the equation $x^3 + x - 3 = 0$.

b $x_1 = 1.216440399$

$\quad x_2 = 1.212725591$

$\quad x_3 = 1.213566964$

$\quad x_4 = 1.213376503$

$\quad x_5 = 1.213419623$

$\quad x_6 = 1.213409861 = 1.2134$ (to 4 d.p.)

Equation of a straight line

1 Comparing the equation with the equation of a straight line, $y = mx + c$:

$y = -2x + 3$

gradient of line, $m = -2$ (line has a negative gradient).

intercept on the y-axis, $c = 3$

Correct line is A.

2 a gradient, $m = \dfrac{\text{change in } y-\text{values}}{\text{change in } x-\text{values}} = -\dfrac{4}{3}$

b gradient of CD, $m = \dfrac{1-5}{5-(-3)} = \dfrac{-4}{8} = -\dfrac{1}{2}$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 1 = -\dfrac{1}{2}(x - 5)$

$y = -\dfrac{1}{2}x + \dfrac{7}{2}$ (or $x + 2y = 7$)

c M is at $\left(\dfrac{-3+5}{2}, \dfrac{5+1}{2}\right) = (1, 3)$

gradient of perpendicular to $CD = \dfrac{-1}{-\frac{1}{2}} = 2$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 3 = 2(x - 1)$

$y = 2x + 1$

3 P is the point (x, y).

gradient of line $OP = \dfrac{y}{x} = 3$, so $y = 3x$

Using Pythagoras' theorem:

$OP^2 = x^2 + y^2$

$12^2 = x^2 + y^2$

$12^2 = x^2 + (3x)^2$

$144 = x^2 + 9x^2$

$144 = 10x^2$

$14.4 = x^2$

$\quad x = 3.8$ (to 1 d.p.)

$\quad y = 3x$

$\quad\quad = 3 \times 3.8$

$\quad\quad = 11.4$

P is the point (3.8, 11.4) (to 1 d.p.).

Quadratic graphs

1 a $\quad x^2 + 4x + 1 = 0$

$\quad (x + 2)^2 - 4 + 1 = 0$

$\quad\quad (x + 2)^2 = 3$

$\quad\quad\quad x + 2 = \pm\sqrt{3}$

$\quad x = \sqrt{3} - 2 \text{ or } -\sqrt{3} - 2$

$\quad x = -0.3 \text{ or } -3.7$ (to 1 d.p.)

b $x^2 + 4x + 1 = (x + 2)^2 - 3$, so turning point is at $(-2, -3)$.
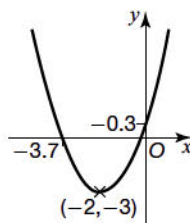
2 $5x^2 - 20x + 10 = 5[x^2 - 4x + 2]$

$\quad\quad\quad\quad\quad = 5[(x - 2)^2 - 4 + 2]$

$\quad\quad\quad\quad\quad = 5(x - 2)^2 - 10$

$a = 5, b = -2$ and $c = -10$

3 $2x^2 + 12x + 3 = 2[x^2 + 6x + \frac{3}{2}]$

$\quad\quad\quad\quad\quad = 2[(x + 3)^2 - 9 + \frac{3}{2}]$

$\quad\quad\quad\quad\quad = 2[(x + 3)^2 - \frac{15}{2}]$

$\quad\quad\quad\quad\quad = 2(x + 3)^2 - 15$

$a = 2, b = 3$ and $c = -15$

Recognising and sketching graphs of functions

1

Equation	Graph
$y = x^2$	B
$y = 2^x$	D
$y = \sin x°$	E
$y = x^3$	C
$y = x^2 - 6x + 8$	A
$y = \cos x°$	F

2 a

b
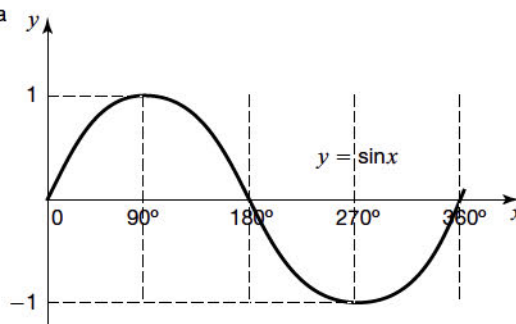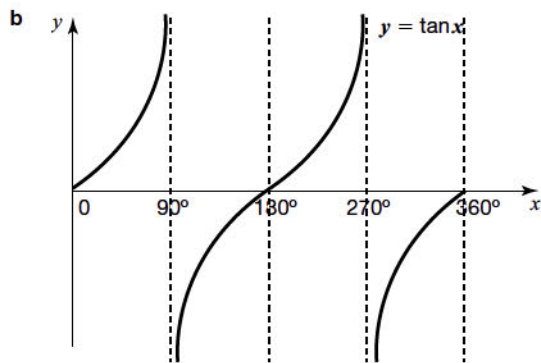
3 $3\cos\theta = 1$

$\cos\theta = \frac{1}{3}$
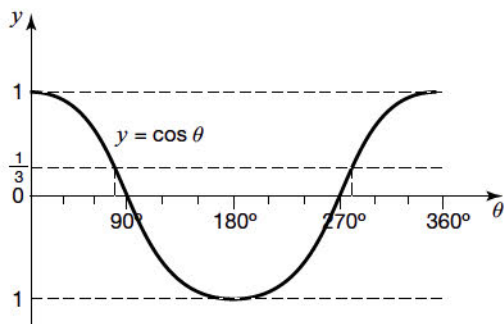
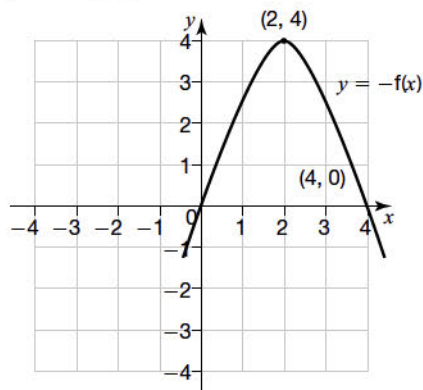$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.5°$ (to 1 d.p.)

From the graph, $\cos\theta$ is also $\frac{1}{3}$ when $\theta = 360 - 70.5 = 289.5°$

$\theta = 70.5°$ or $289.5°$ (to 1 d.p.)
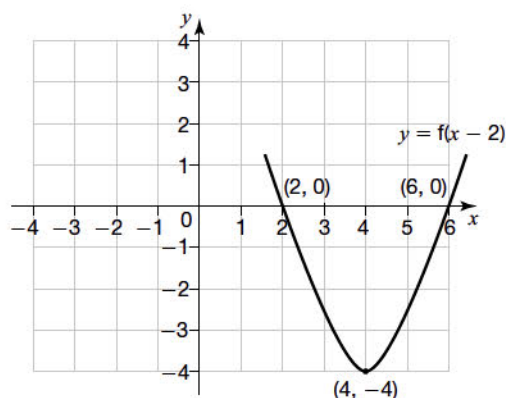
Translations and reflections of functions

1 a $y = -f(x)$ is a reflection in the x-axis of the graph $y = f(x)$

The points on the x-axis stay in the same place and the turning point at $(2, -4)$ is reflected to become a turning point at $(2, 4)$.

b $y = f(x - 2)$ is a translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ of the graph $y = f(x)$.

The y-coordinates stay the same but the x-coordinates are shifted to the right by 2 units.

2 a $y = -f(x)$: reflection in the x-axis.

b $y = f(x) + 2$: translation of 2 units vertically upwards.

c $y = f(-x)$: reflection in the y-axis.
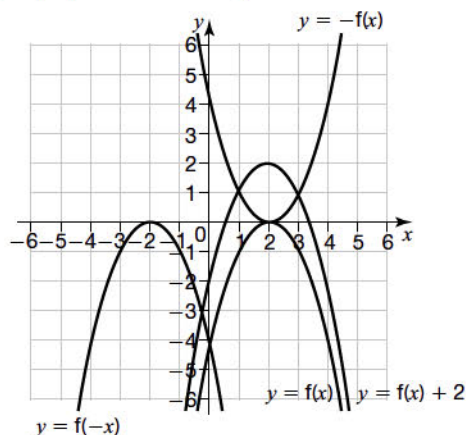
3 The cosine graph is shifted two units in the vertical direction, i.e. a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.
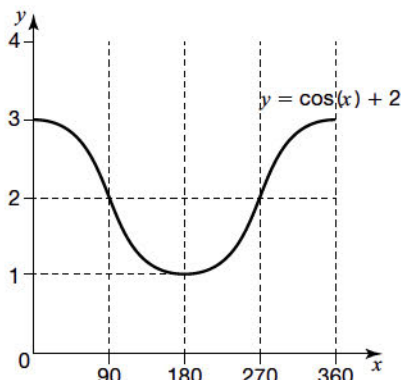
Equation of a circle and tangent to a circle

1 a $x^2 + y^2 = 25$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{25} = 5$

b $x^2 + y^2 - 49 = 0$

$x^2 + y^2 = 49$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{49} = 7$

c $4x^2 + 4y^2 = 16$

$x^2 + y^2 = 4$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{4} = 2$

2 Radius of the circle $= \sqrt{21} = 4.58$

Distance of the point $(4, 3)$ from the centre of the circle $(0, 0)$

$= \sqrt{16 + 9} = \sqrt{25} = 5$

This distance is greater than the radius of the circle, so the point lies outside the circle.

3 a

Using Pythagoras' theorem:

$OP^2 = 5^2 + 7^2$

$= 25 + 49$

$= 74$
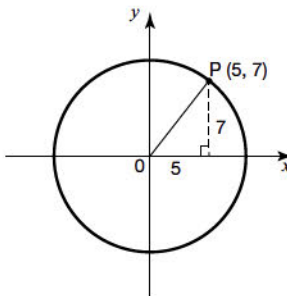
OP = radius of the circle = $\sqrt{74}$

b equation of a circle, radius r, centre the origin: $x^2 + y^2 = r^2$

$x^2 + y^2 = 74$

c gradient of line $OP = \frac{7}{5}$

gradient of the tangent at $P = -\frac{5}{7}$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 7 = -\frac{5}{7}(x - 5)$

$7y - 49 = -5x + 25$

$7y = -5x + 74$

$y = -\frac{5}{7}x + \frac{74}{7}$

Real-life graphs

1 a acceleration = gradient of line = $\frac{10}{10}$ = 1 m/s^2

b total distance travelled = area under the velocity−time graph

$= \frac{1}{2}(30 + 15) \times 10 = 225$ m

2 a The graph is a straight line starting at the origin, so this represents constant acceleration from rest of $\frac{15}{6}$ = 2.5 m/s^2.

b The gradient decreases to zero, so the acceleration decreases to zero.

c area = area of 3 trapeziums
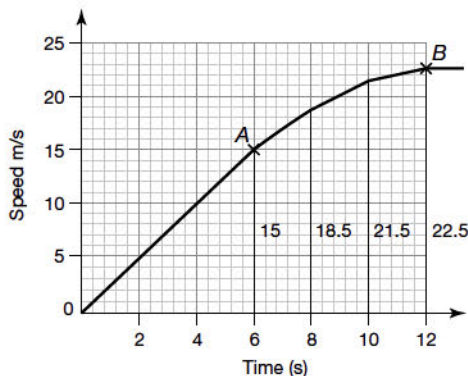
area $= \frac{1}{2}(15 + 18.5) \times 2 + \frac{1}{2}(18.5+21.5) \times 2 +$

$\frac{1}{2}(21.5 + 22.5) \times 2$

$= 117.5$

distance travelled between A and B = 118 m (to nearest integer)

d It will be a slight underestimate, as the curve is always above the straight lines forming the tops of the trapeziums.

Generating sequences

1 a **i** $\frac{1}{2}$ (term-to-term rule is divide by 2)

ii 243 (term-to-term rule is multiply by 3)

iii 21 (term-to-term rule is add 4)

b 4th term − 1st term = −12 − 27 = −39

common difference = −39 ÷ 3 = −13

missing terms are 14, 1

2 3rd term = 2 × 1 − 5 = −3

4th term = 2 × −3 − 5 = −11

3 a 25, 36 (square numbers)

b 15, 21 (triangular numbers)

c 8, 13 (Fibonacci numbers)

The nth term

1 a

n	1	2	3	4
Term	2	6	10	14
Difference		4	4	4

nth term starts with $4n$

$4n$	4	8	12	16
Term − $4n$	−2	−2	−2	−2

nth term = $4n - 2$

b nth term = $4n - 2 = 2(2n - 1)$

2 is a factor, so the nth term is divisible by 2 and therefore is even.

c Let nth term = 236

$4n - 2 = 236$

$4n = 238$

$n = 59.5$

n is not an integer, so 236 is not a term in the sequence.

2 a 2nd term = $9 - 2^2 = 5$

b 20th term = $9 - 20^2 = 9 - 400 = -391$

c n^2 is always positive, so the largest value value $9 - n^2$ can take is 8 when $n = 1$. All values of n above 1 will make $9 - n^2$ smaller than 8. So 10 cannot be a term.

3

Term	1		1	3	7	13
First difference		0		2	4	6
Second difference			2	2	2	

The formula starts n^2.

n	1	2	3	4	
Term	1	1	3	7	13
n^2	1	4	9	16	25
Term − n^2	0	−3	−6	−9	−12
Difference		−3	−3	−3	−3

The linear part of the sequence starts with $-3n$.

$-3n$		−3	−6	−9	−12	−15
Linear term − $(-3n)$	3	3	3	3	3	

nth term = $n^2 - 3n + 3$

Checking:

When $n = 1$, term is $1^2 - 3 \times 1 + 3 = 1$

$n = 2$, term is $2^2 - 3 \times 2 + 3 = 1$

$n = 3$, term is $3^2 - 3 \times 3 + 3 = 3$

Arguments and proofs

1 The only even prime number is 2.

Hence, statement is false because 2 is a prime number that is not odd.

2 a true: $n = 1$ is the smallest positive integer and this would give the smallest value of $2n + 1$, which is 3.

b true: 3 is a factor of $3(n + 1)$ so $3(n + 1)$ must be a multiple of 3.

c false: $2n$ is always even and subtracting 3 will give an odd number.

3 Let first number $= x$ so next number $= x + 1$

Sum of consecutive integers $= x + x + 1 = 2x + 1$

Regardless of whether x is odd or even, $2x$ will always be even as it is divisible by 2.

Hence $2x + 1$ will always be odd.

4 $(2x - 1)^2 - (x - 2)^2 = 4x^2 - 4x + 1 - (x^2 - 4x + 4)$

$$= 4x^2 - 4x + 1 - x^2 + 4x - 4$$

$$= 3x^2 - 3$$

$$= 3(x^2 - 1)$$

The 3 outside the brackets shows that the result is a multiple of 3 for all integer values of x.

5 Let two consecutive odd numbers be $2n - 1$ and $2n + 1$.

$(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1)$

$$= 8n$$

Since 8 is a factor of $8n$, the difference between the squares of two consecutive odd numbers is always a multiple of 8.

(If you used $2n + 1$ and $2n + 3$ for the two consecutive odd numbers, difference of squares $= 8n + 8 = 8(n + 1)$.)

Ratio, proportion and rates of change

Introduction to ratios

1 3 parts $= 18$ black balls so 1 part $= 6$ black balls

There are $3 + 2 = 5$ parts in total, so the total number of balls

$$= 5 \times 6 = 30$$

2 total shares $= 15 + 17 + 18 = 50$

50 shares $= £25\,000$

1 share $= \frac{£25\,000}{50} = £500$

15-year-old will get $15 \times £500 = £7500$.

17-year-old will get $17 \times £500 = £8500$.

18-year-old will get $18 \times £500 = £9000$.

3 40% of 800 $= = 320$ acres

remainder of land area $= 800 - 320 = 480$ acres

total shares $= 9 + 7 = 16$

1 share $= \frac{480}{16} = 30$

area devoted to sheep $= 7$ shares $= 7 \times 30 = 210$ acres

4 $\frac{3x + 1}{x + 4} = \frac{2}{3}$

$3(3x + 1) = 2(x + 4)$

$9x + 3 = 2x + 8$

$\quad 7x = 5$

$\quad\quad x = \frac{5}{7}$

5 For 2 oak trees there are 3 ash trees, so if there are 8 oak trees there would be 12 ash trees.

pine : oak: ash $= 5 : 8 : 12$

There are a total of $5 + 8 + 12 = 25$ parts.

1 part $= \frac{300}{25} = 12$

number of ash trees $= 12 \times 12 = 144$

Scale diagrams and maps

1 10 cm on map $= 10 \times 50\,000$ cm $= 500\,000$ cm actual distance

$500\,000$ cm $= 5000$ m $= 5$ km

Towns are 5 km apart.

2 a length of road $= 2.3 \times 40\,000$ cm

$$= 92\,000\,\text{cm}$$

$$= 920\,\text{m}$$

$$= 0.92\,\text{km}$$

b length of road $= 3 \times 40\,000$ mm

$$= 120\,000\,\text{mm}$$

$$= 12\,000\,\text{cm}$$

$$= 120\,\text{m}$$

$$= 0.12\,\text{km}$$

3 5 cm : 40 m $= 5$ cm : 4000 cm

$$= 5 : 4000$$

$$= 1 : 800$$

scale of drawing $= 1 : 800$

4 length measured on map $= 6$ cm

Scale is:

6 cm : 12 km $= 6 : 12 \times 1000 \times 100$

$$= 6 : 1\,200\,000$$

$$= 1 : 200\,000$$

Percentage problems

1 increase $= £35$

percentage increase $= \frac{\text{increase}}{\text{original price}} \times 100$

$$= \frac{35}{350} \times 100$$

$$= 10\%$$

2 increase $=$ final earnings $-$ initial earnings

$$= 1\,100\,000 - 600\,000$$

$$= £500\,000$$

% increase $= \frac{\text{increase}}{\text{original value}} \times 100$

$$= \frac{500\,000}{600\,000} \times 100$$

$$= 83.3\%$$

3 multiplier $= 1 - 0.28 = 0.72$

value of car $= 0.72 \times 25\,000$

$$= £18\,000$$

4 88% of the original price $= £14\,300$

1% of the original price $= \frac{14\,300}{88}$

100% of the original price $= \frac{14\,300}{88} \times 100 = 16\,250$

original price $= £16\,250$

5 interest earned $= \frac{2.8}{100} \times 8000 \times 4$

$$= £896$$

Direct and inverse proportion

1 a $P = kT$

b $P = kT$ so $k = \frac{P}{T} = \frac{200\,000}{540} = 370.370$

$P = 370.370T = 370.370 \times 200 = 74\,074$ Pascals

(to nearest whole number)

2 $C \propto r^2$

$C = kr^2$

$480 = k \times 3^2$

$k = 53.33$

$C = 53.33r^2$

cost $= 53.33 \times 4^2 = £853.28 = £853$ (to nearest whole number)

3 **a** $c \propto \frac{1}{h}$ so $c = \frac{k}{h}$

   $3 = \frac{k}{12}$, so $k = 36$

   $c = \frac{36}{h}$

   b $c = \frac{36}{h}$

   $= \frac{36}{15}$

   $= 2.4$

4 **a** $350 \times 1.15 = $€$402.50$

   b $80 \div 1.11 = $£$72.07$ (to nearest penny)

   c The €80 she had left cost her $80 \div 1.15 = $£$69.57$ before her holiday.

   If she hadn't changed this amount, she would have saved $72.07 - 69.57 = $£$2.50$

Graphs of direct and inverse proportion and rates of change

1 straight line through the origin: B

2 curve that gets close to but does not cross either axis: B

3 **a**
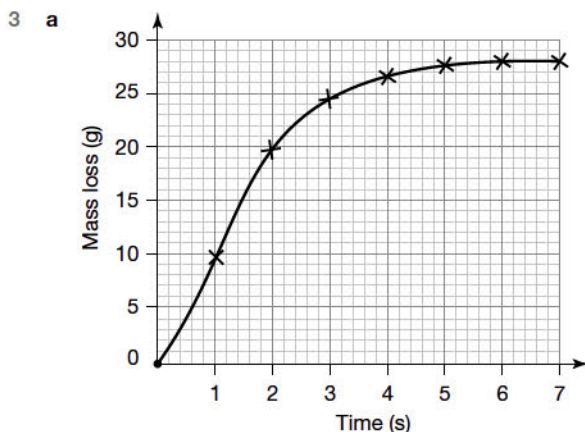
   b **i** initial rate of change = gradient over first 2 minutes

   $= \frac{19.6}{2}$

   $= 9.8$ g/minute

   ii initial rate of change $= \frac{9.8}{60} = 0.16$ g/second (to 2 d.p.)

Growth and decay

1 **a** multiplier $= 1 + \frac{\% \text{ increase}}{100} = 1 + \frac{6}{100} = 1.06$

   population after n years $= A_0 \times (\text{multiplier})^n$,

   where A_0 = initial population and n = number of years

   population after 3 years $= 150\,000 \times 1.06^3 = 178\,652$

   b Try $n = 5$ years: population $= 150\,000 \times 1.06^5 = 200\,733$

   Try $n = 4$ years: population $= 150\,000 \times 1.06^4 = 189\,372$

   After **5 years**, the population will have risen to over $200\,000$.

2 multiplier $= 1 - \frac{\% \text{ decrease}}{100} = 1 - \frac{12}{100} = 0.88$

   value of car after 4 years $= 21\,000 \times 0.88^4 = $£$12\,594$

3 multiplier $= 1 - \frac{\% \text{ decrease for each time unit}}{100} = 1 - \frac{50}{100} = 0.5$

   2 minutes = 120 seconds

   number of time intervals, $n = \frac{120}{12} = 10$

   amount at the end of n time intervals $= A_0 \times (\text{multiplier})^n$

   $= 100 \times 0.5^{10}$

   $= 0.1$ (to 1 s.f.)

Ratios of lengths, areas and volumes

1 **a** scale factor for enlargement $\left(\frac{big}{small}\right)^3 = \left(\frac{12}{8}\right)^3 = 3.375$ or $\frac{27}{8}$

   b $\frac{\text{area of triangle of large prism}}{\text{area of triangle of small prism}} = \left(\frac{12}{8}\right)^2$

   area of triangle of large prism $= \left(\frac{12}{8}\right)^2 \times 10 = 22.5$ cm²

   c $\frac{\text{volume of small prism}}{\text{volume of large prism}} = \left(\frac{8}{12}\right)^3$

   volume of small prism $= \left(\frac{8}{12}\right)^3 \times 450$

   $= 133.33$

   $= 133$ cm³ (to nearest whole number)

2 $\frac{\text{volume of larger cuboid}}{\text{volume of smaller cuboid}} = \left(\frac{h}{12}\right)^3$

   $\frac{\text{volume of larger cuboid}}{\text{volume of smaller cuboid}} = 1.953$

   $\left(\frac{h}{12}\right)^3 = 1.953$

   $h^3 = 3374.784$

   $h = 15$ cm (to nearest cm)

3 **a** **i** angle XYZ = angle TUZ (corresponding angles)

   angle ZXY = angle ZTU (corresponding angles)

   angle XZY = angle TZU (same angle)

   Hence triangles XYZ and TUZ are similar.

   $\frac{XY}{TU} = \frac{YZ}{UZ}$

   $\frac{XY}{3} = \frac{15}{5}$

   $XY = 9$ cm

   ii angle YUT = angle YZW (corresponding angles)

   angle YTU = angle YWZ (corresponding angles)

   angle TYU = angle WYZ (same angle)

   Hence triangles YUT and YZW are similar.

   $\frac{WZ}{TU} = \frac{YZ}{YU}$

   $\frac{WZ}{3} = \frac{15}{10}$

   $WZ = 4.5$ cm

   b angle YTX = angle ZTW (vertically opposite angles)

   angle YXT = TZW (alternate angles)

   Two angles of both triangles are equal so the third angles must also be equal.

   Hence triangles XYT and ZWT are similar.

   ratio of the areas of TYX and $TWZ = \left(\frac{XY}{WZ}\right)^2 = \left(\frac{9}{4.5}\right)^2 = 4$

   ratio of areas = 4 : 1

Gradient of a curve and rate of change

1 a acceleration = gradient of straight line = $\frac{10-0}{15-0} = \frac{2}{3}$ m/s²

b Instantaneous acceleration = gradient of the tangent to the curve at 45 s.
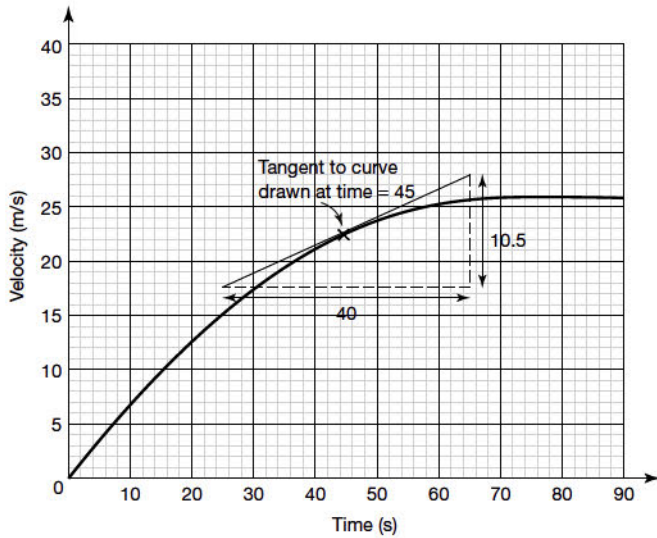
Gradient = $\frac{10.5}{40}$ = 0.2625

Instantaneous acceleration at 45 s = 0.26 m/s² (to 2 d.p.)

c average acceleration over the first 70 s = gradient of the line from (0, 0) to (70, 26)

gradient = $\frac{26-0}{70-0}$ = 0.3714...

acceleration = 0.37 m/s² (to 2 d.p.)

d
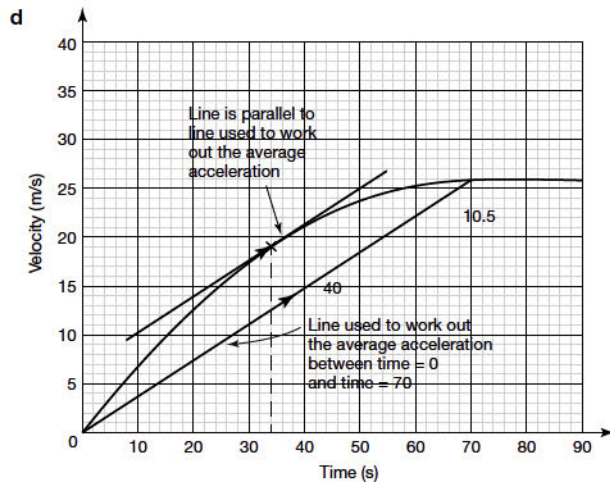
gradients of the two lines are parallel when time = 34 s

Converting units of areas and volumes, and compound units

1 pressure = $\frac{force}{area} = \frac{200}{0.4}$ = 500 N/m²

2 1 m² = 100 × 100 cm² = 10 000 cm²

1 cm² = $\frac{1}{10000}$ m²

200 cm² = $\frac{200}{(10\,000)}$ m² = 0.02 m²

pressure = $\frac{force}{area} = \frac{500}{0.02}$ = 25 000 N/m²

3 density = $\frac{mass}{volume}$

mass = volume × density

= 12 × 8.92

= 107.04 g

mass of wire = 107 g (to nearest g)

4 He has worked out the area in m² by dividing the area in cm² by 100, which is incorrect.

There are 100 × 100 = 10 000 cm² in 1 m², so the area should have been divided by 10 000.

Correct answer:

area in m² = $\frac{9018}{10\,000}$

= 0.9018

= 0.90 m² (to 2 d.p.)

5 distance in first two hours = time × speed

= 2 × 60

= 120 km

distance in next three hours = time × speed

= 3 × 80

= 240 km

total distance = 120 + 240

= 360 km

average speed = $\frac{distance}{time}$

= $\frac{360}{5}$

= 72 km/h

Geometry and measures

2D shapes

1 a true   **c** true   **e** true

b false   **d** true   **f** false (this would be true only for a regular pentagon)

2 a rhombus   **c** equilateral triangle

b parallelogram   **d** kite

Constructions and loci

1 The locus of the point is the line that bisects and is perpendicular to *AB*.
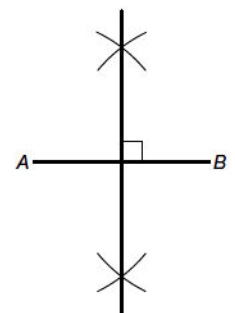
2 The line AB is 6 cm long, with semicircles of radius 1.5 cm at either end, joined by two lines parallel to and 1.5 cm from the line.

Not drawn to scale
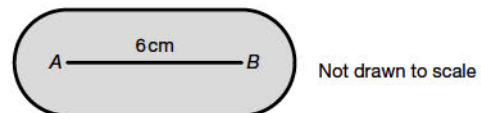
3 a bisector of angle *YXZ*

b path of fly: arc with centre *X* of radius 6 cm between the two walls
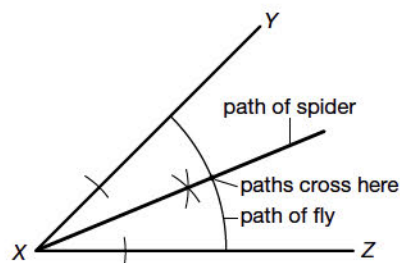
Properties of angles

1 a angle ACB = angle BAC = 30° (base angles of isosceles triangle ABC, since AB and BC are equal sides of a rhombus)

   b angle AOB = 90° (diagonals of a rhombus intersect at right angles)

   c angle ABO = 180 − (90 + 30) = 60° (angle sum of a triangle)

   angle BDC = angle ABO = 60° (alternate angles between parallel lines AB and DC)

2 angle $BAC = \frac{(180 - 36)}{2} = 72°$ (angle sum of a triangle and base angles of an isosceles triangle)

   angle BDC = 180 − 90 = 90° (angle sum on a straight line)

   angle ABD = 180 − (90 + 72) = 18° (angle sum of a triangle)

3 a $2x + 4x = 180°$ (angle sum on a straight line)

   $$6x = 180°$$
   $$x = 30°$$

   b If lines AB and CD are parallel, the angles $4x$ and $3x + 30$ would be corresponding angles, and so equal.

   $4x = 4 \times 30 = 120°$

   $3x + 30 = 3 \times 30 + 30 = 120°$

   These two angles are equal so lines AB and CD are parallel.

4 For the regular pentagon:

   exterior angle $= \frac{360°}{\text{number of sides}} = \frac{360}{5} = 72°$

   exterior angle of a square = interior angle of a square = 90°

   angle x = 90 + 72 = 162°

Congruent triangles

1 BD common to triangles ABD and CDB

   angle ADB = angle CBD (alternate angles)

   angle ABD = angle CDB (alternate angles)

   Therefore triangles ABD and CDB are congruent (ASA).

   Hence angle BAD = angle BCD

2 Draw the triangle and the perpendicular from A to BC.

   $AX = AX$ (common)

   $AB = AC$ (triangle ABC is isosceles)

   angle AXB = AXC = 90° (given)

   Therefore triangles ABX and ACX are congruent (RHS).

   Hence $BX = XC$, so X bisects BC.

3 OQC = 90° (corresponding angles), so $PB = OQ$ (perpendicular distance between 2 parallel lines).

   $AP = PB$ (given), so $AP = OQ$

   $PO = QC$ (Q is the midpoint of BC)

   angle ABC = angle APO = angle OQC = 90° (OQ is parallel to AB and OP parallel to BC)

   Therefore triangles AOP and OCQ are congruent (SAS).
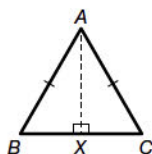
Transformations

1 translation of $\begin{pmatrix} -7 \\ -5 \end{pmatrix}$

2

3 a translation of $\begin{pmatrix} -7 \\ 0 \end{pmatrix}$

   b reflection in the line $y = 3$

   c rotation of 90° clockwise about (0, 1)

Invariance and combined transformations

1 a 1

   b i invariant point (3, 3)
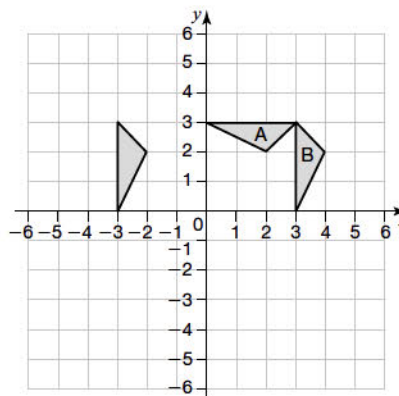
   ii rotation 90° anticlockwise about the point (3, 3)

2 a The shaded triangle is the image after the two transformations.
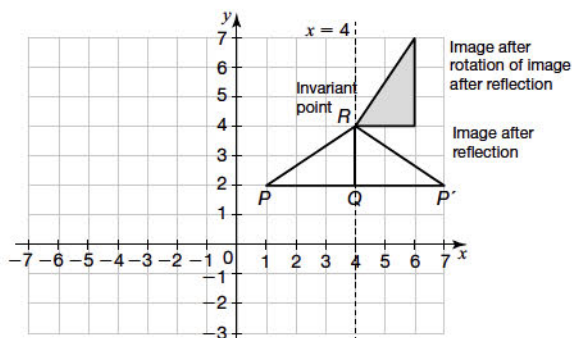
   b invariant point is R (4, 4)

3D shapes

1 a G   c A, H   e C
 b B   d B   f A, H

Parts of a circle

1 a radius   c chord
  b diameter   d arc

2 a minor sector   c major sector
  b major segment   d minor segment

## Circle theorems

1 angle $OTB = 90°$ (angle between tangent and radius)

angle $BOT = 180 - (90 + 28) = 62°$ (angle sum in a triangle)

angle $AOT = 180 - 62 = 118°$ (angle sum on a straight line)

$AO = OT$ (radii), so triangle AOT is isosceles

angle $OAT = \frac{(180 - 118)}{2} = 31°$ (angle sum in a triangle)

2 a angle $ACB = 30°$ (angle at centre twice angle at circumference)

b angle BAC = angle $CBX = 70°$ (alternate segment theorem)

c $OA = OB$ (radii), so triangle AOB is isosceles

angle $AOB = 60°$, so triangle AOB is equilateral

angle $OAB = 60°$ (angle of equilateral triangle)

angle $CAO = 70 - 60 = 10°$

Projections

1

2 a

b

3

Bearings

1
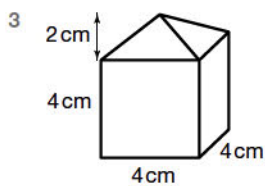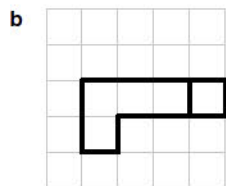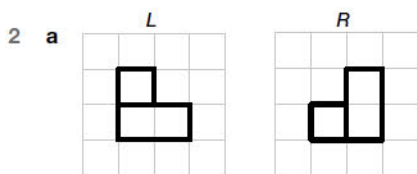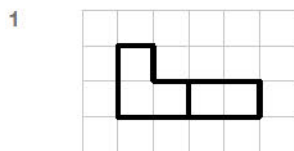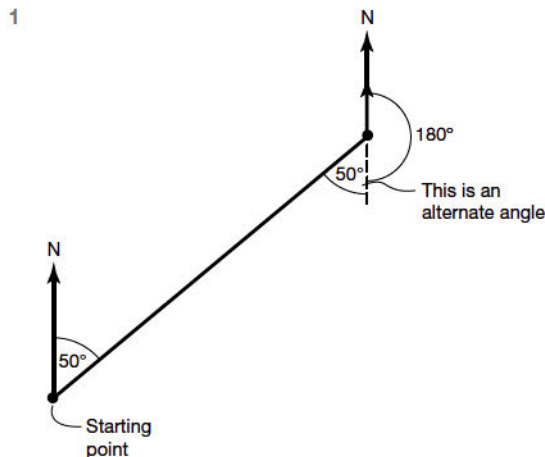
bearing to return to starting point $= 180 + 50 = 230°$

2
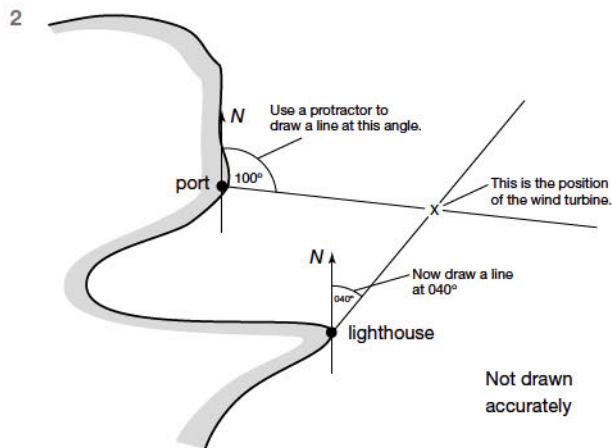
Pythagoras' theorem

1 $AC^2 = AB^2 + BC^2 = 24^2 + 16^2 = 832$

$AD^2 = AC^2 - CD^2 = 832 - 9^2 = 751$

$AD = 27\,m$ (to nearest m)

perimeter $= 24 + 16 + 9 + 27 = 76\,m$ (to nearest m)

2 In triangle ABC, $AC^2 = AB^2 + BC^2 = 16^2 + 7^2 = 305$

In triangle ACG, $AG^2 = AC^2 + CG^2 = 305 + 5^2 = 330$

$AG = 18.2\,cm$ (to 1 d.p.)

extra distance $= 16 + 7 + 5 - 18.2 = 9.8\,cm$ (to 1 d.p.)

3 a angle $ADC = 90°$ (angle in a semicircle)

Using Pythagoras' theorem:

$10.8^2 = 5.8^2 + AD^2$

$AD = 9.11\,cm =$ (to 2 d.p.)*

b area of triangle $ACD = 1/2 \times 5.8 \times 9.11 = 26.419\,cm^2$

angle $ABC = 90°$ (angle in a semicircle)

$\frac{AB}{10.8} = \cos 65°$

$AB = 4.564\,cm$

$\frac{BC}{10.8} = \sin 65°$

$BC = 9.788\,cm$

area of triangle ABC $= \frac{1}{2} \times 4.564 \times 9.788 = 22.336\,cm^2$

area of quadrilateral $ABCD = 26.419 + 22.336$
$= 48.76\,cm^2$ (to 2 d.p.)

Area of 2D shapes

1 a i Using Pythagoras' theorem:

$7^2 + AE^2 = 9^2$

$49 + AE^2 = 81$

$AE^2 = 32$

$AE = \sqrt{32} = 5.6569 = 5.66\,cm$ (to 2 d.p.)

ii area of triangle $ADE = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 5.6569 \times 7 = 19.80\,cm^2$
(to 2 d.p.)

b Using Pythagoras' theorem:

$7^2 + BF^2 = 8^2$

$49 + BF^2 = 64$

$BF^2 = 15$

$BF = \sqrt{15} = 3.87\,cm$ (to 2 d.p.)

area of triangle $BCF = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 3.87 \times 7$

$= 13.56\,cm^2$

*This answer differs from that in the Exam Practice Book due to an error in our first edition. It has now been re-checked and corrected.

area of trapezium $CDEF$ = area of rectangle − area of ADE
− area of BCF

$$= 84 - 19.81 - 13.56$$
$$= 50.65\,\text{cm}^2\text{ (to 2 d.p.)}$$

2 a perimeter of triangle $= x + 1 + 2x + 1 + x + 2 = 4x + 4$

perimeter of rectangle $= x + 1 + 3 + x + 1 + 3 = 2x + 8$

perimeters are the same:

$$4x + 4 = 2x + 8$$
$$2x = 4$$
$$x = 2\,\text{cm}$$

area of $DEFG = 3 \times (x + 1) = 9\,\text{cm}^2$

b area of triangle $= \frac{1}{2} \times 4 \times 3 = 6\,\text{cm}^2$

3 3 circles in stencil

a area $= 3 \times \pi r^2 = 3 \times \pi \times 3^2 = 27\pi\,\text{cm}^2$

b perimeter = 3 × circumference + 2 × radius

$$= 3 \times 2\pi r + 2r$$
$$= 3 \times 2 \times \pi \times 3 + 2 \times 3$$
$$= 18\pi + 6\,\text{cm}$$

Volume and surface area of 3D shapes

1 a area of trapezium $= \frac{1}{2}(a + b)h$

$$= \frac{1}{2}(3.5 + 1.8) \times 1.5$$
$$= 3.975\,\text{m}^2$$

b volume of prism = area of cross−section × length

$$= 3.975 \times 1.6$$
$$= 6.36\,\text{m}^3$$

2 volume of sphere $= \frac{4}{3}\pi r^3$

volume of hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3}\pi \times 4^3$$
$$= 134.04\,\text{cm}^3$$

volume of drink in bottle $= 750\,\text{ml} = 750\,\text{cm}^3$

number of glasses $= \frac{750}{134.04} = 5.6$

A bottle will fill 5 glasses.

3 a Using Pythagoras' theorem:

$$l^2 = h^2 + r^2$$
$$= 5.6^2 + 4.8^2$$
$$= 54.4$$
$$l = 7.3756\ldots = 7.4\,\text{cm}\text{ (to 1 d.p.)}$$

b surface area of cone $= \pi r l + \pi r^2$

$$= \pi \times 4.8 \times 7.3756 + \pi \times 4.8^2$$
$$= 183.60\,\text{cm}^2$$

surface area of sphere $= 4\pi r^2 = 183.60$

$r = \sqrt{\frac{183.60}{4\pi}} = 3.8\,\text{cm}\text{ (to 1 d.p.)}$

4 volume of water in cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi \times 2^2 \times 12$$
$$= 50.27\,\text{cm}^3$$

volume of water in cylinder $= \pi r^2 h = 50.27\,\text{cm}^3$
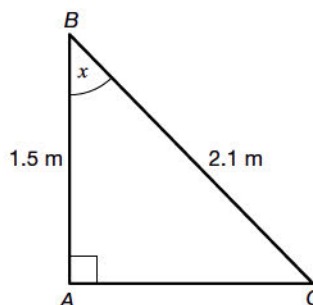
$$\pi \times 5^2 \times h = 50.27$$
$$h = 0.64\,\text{cm}$$

depth of water in the cylinder $= 0.64\,\text{cm}$

Trigonometric ratios

1 a $\cos 48° = \frac{x}{9}$

$x = 9\cos 48° = 6.0\,\text{cm}\text{ (to 1 d.p.)}$

b $\tan y = \frac{9}{12}$

$y = \tan^{-1}\left(\frac{9}{12}\right) = 36.9°\text{ (to 1 d.p.)}$

2

Let angle $ABC = x$

$\cos x = \frac{1.5}{2.1}$

$x = \cos^{-1}\left(\frac{1.5}{2.1}\right) = 44.4°\text{ (to 2 d.p.)}$

angle $ABC = 44.4°\text{ (to 1 d.p.)}$

3 $\sin 70° = \frac{h}{10}$

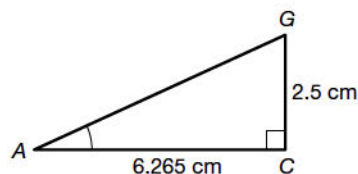$h = 10\sin 70° = 9.40\,\text{cm}\text{ (to 2 d.p.)}$*

4 In triangle ABC:

$$AC^2 = AB^2 + BC^2$$
$$= 5.5^2 + 3^2$$
$$= 39.25$$
$$AC = \sqrt{39.25} = 6.265\,\text{cm}$$

In triangle ACG, you know the lengths AC and CG, and you want to find the angle CAG, so use tan (CG is 'opposite' and AC is 'adjacent').
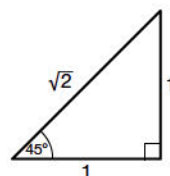

$\tan(\text{angle } CAG) = \frac{CG}{AC} = \frac{2.5}{6.265} = 0.3990$

angle $CAG = \tan^{-1}(0.3990) = 21.8°\text{ (to 1 d.p.)}$

Exact values of sin, cos and tan

1

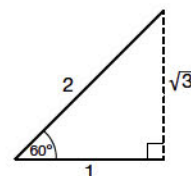$\tan 45° = \frac{1}{1} = 1$ $\cos 60° = \frac{1}{2}$

Hence, $\tan 45° + \cos 60° = 1 + \frac{1}{2} = \frac{3}{2}$

2 a i $\sin 45° = \frac{1}{\sqrt{2}}$ b $\frac{\sin 45°}{\cos 45°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

ii $\cos 45° = \frac{1}{\sqrt{2}}$ $\tan 45° = 1$

Hence $\frac{\sin 45°}{\cos 45°} = \tan 45°$

3 a Using Pythagoras' theorem:

$$AC^2 = 1^2 + 2^2$$
$$AC = \sqrt{5}$$

b i $\sin x = \frac{1}{\sqrt{5}}$

ii $\cos x = \frac{2}{\sqrt{5}}$

*This answer differs from that in the Exam Practice Book due to an error in our first edition. It has now been re-checked and corrected.

c $(\sin x)^2 = \frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{1}{5}$

$(\cos x)^2 = \frac{2}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{4}{5}$

$(\sin x)^2 + (\cos x)^2 = \frac{1}{5} + \frac{4}{5}$

$$= 1$$

4 $\tan 30° + \tan 60° + \cos 30° = \frac{1}{\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$

$$= \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{3} + \sqrt{3} + \frac{\sqrt{3}}{2}$$

$$= \frac{2\sqrt{3} + 6\sqrt{3} + 3\sqrt{3}}{6}$$

$$= \frac{11\sqrt{3}}{6}$$

Sectors of circles

1 $l = \frac{\theta}{360} \times 2\pi r$

$3 = \frac{\theta}{360} \times 2\pi \times 5$

$\theta = 34°$ (to nearest degree)

2 area of sector $= \frac{\theta}{360} \times \pi r^2$

$$= \frac{290}{360} \times \pi \times 12^2$$

$$= 364.4\,\text{cm}^2$$

3 a area of sector $= \frac{\theta}{360} \times \pi r^2$

area of sector A $= \frac{74}{360} \times \pi \times 3^2$

area of sector B $= \frac{360 - 122}{360}\,\pi r^2 = \frac{238}{360}\,\pi r^2$

$\frac{238}{360}\,\pi r^2 = \frac{74}{360} \times \pi \times 3^2$

$r^2 = \frac{74}{238} \times 3^2 = 2.798$

$r = 1.67\,\text{cm}$ (to 3 s.f.)

b perimeter of sector $= \frac{\theta}{360} \times 2\pi r + 2r$

perimeter of sector A $= \frac{74}{360} \times 2 \times \pi \times 3 + 2 \times 3$

$$= 9.875\,\text{cm}^2$$

perimeter of sector B $= \frac{238}{360} \times 2 \times \pi \times 1.67 + 2 \times 1.67$

$$= 10.277\,\text{cm}^2$$

perimeter A : perimeter B $= 9.875 : 10.277 = 1 : 1.04$

4 a Length of arc $= \frac{\theta}{360} \times 2\pi r$

$5.4 = \frac{\theta}{360} \times 2 \times \pi \times 6$

$\theta = 51.5662°$

area of sector $AOB = \frac{\theta}{360} \times \pi r^2$

$$= \frac{51.5662}{360} \times \pi \times 6^2$$

$$= 16.2\,\text{cm}^2 \text{ (to 1 d.p.)}$$

b area of triangle $= \frac{1}{2} \times 6 \times 6 \times \sin 51.5662° = 14.09988\,\text{cm}^2$

area of segment $=$ area of sector $-$ area of triangle

$$= 16.2 - 14.1$$

$$= 2.1\,\text{cm}^2 \text{ (to 1 d.p.)}$$

Sine and cosine rules

1 a area $= \frac{1}{2} \times 25 \times 30 \times \frac{3}{5}$

$$= 225\,\text{cm}^2$$

b Because of the value of $\sin\theta$, this must be a right-angled triangle with sides in the ratio $3 : 4 : 5$. Draw the simplest form of this triangle.
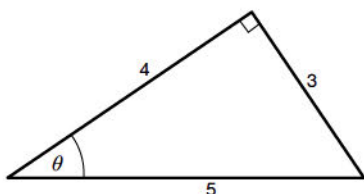
adjacent $= 4$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$

c Using the cosine rule:

$AC^2 = 25^2 + 30^2 - 2 \times 25 \times 30 \times \frac{4}{5}$

$$= 325$$

$AC = 18.0\,\text{cm}$ (to 3 s.f.)

2 a Using the sine rule:

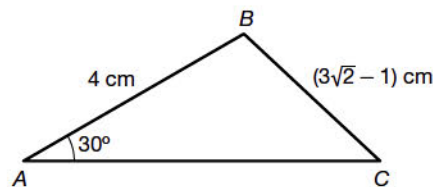$\frac{x}{\sin 84°} = \frac{12}{\sin 40°}$

$x = \frac{12 \sin 84°}{\sin 40°}$

$$= 18.5664$$

$$= 18.6\,\text{cm} \text{ (to 3 s.f.)}$$

b angle $BAC = 180 - (84 + 40) = 56°$

area $= \frac{1}{2} \times 18.5664 \times 12 \times \sin 56°$

$$= 92.4\,\text{cm}^2 \text{ (to 3 s.f.)}$$

If you use your answer from part a (to 3 s.f) you get 92.5, but taking x to 4 d.p. gives the answer of 92.4.

3

Using the sine rule:

$\frac{3\sqrt{2} - 1}{\sin 30°} = \frac{4}{\sin ACB}$

$\sin ACB = \frac{4 \sin 30°}{3\sqrt{2} - 1}$

$$= \frac{2}{3\sqrt{2} - 1}$$

$$= \frac{2(3\sqrt{2} + 1)}{(3\sqrt{2} - 1)(3\sqrt{2} + 1)}$$

$$= \frac{6\sqrt{2} + 2}{18 - 1}$$

$$= \frac{2 + 6\sqrt{2}}{17}$$

Vectors

1 a $\frac{1}{2}(\mathbf{p} + \mathbf{q}) = \frac{1}{2}\left(\binom{-2}{3} + \binom{6}{-1}\right) = \frac{1}{2}\binom{4}{2} = \binom{2}{1}$

b $2\mathbf{p} - 3\mathbf{q} = 2\binom{-2}{3} - 3\binom{6}{-1} = \binom{2 \times -2 - 3 \times 6}{2 \times 3 - 3 \times -1}$

$$= \binom{-22}{9}$$

2 a $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$

b $\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AB} = \frac{3}{5}(\mathbf{b} - \mathbf{a})$

c $\overrightarrow{OQ} = \frac{2}{5}\overrightarrow{OA} = \frac{2}{5}\mathbf{a}$

$\overrightarrow{QP} = \overrightarrow{QA} + \overrightarrow{AP}$

$$= \frac{3}{5}\mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$$

$$= \frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{3}{5}\mathbf{a}$$

$$= \frac{3}{5}\mathbf{b}$$

As $\overrightarrow{QP} = \frac{3}{5}\mathbf{b}$ and $\overrightarrow{OB} = \mathbf{b}$ they both have the same vector part and so are parallel.

3 a $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$$= -3\mathbf{b} + \mathbf{a}$$

$$= \mathbf{a} - 3\mathbf{b}$$

b $\overrightarrow{PB} = \frac{1}{3}\overrightarrow{AB} = \mathbf{b}$

$\overrightarrow{PM} = \overrightarrow{PB} + \overrightarrow{BM}$

$$= \overrightarrow{PB} + \frac{1}{2}\overrightarrow{BC}$$

$$= \mathbf{b} + \frac{1}{2}(\mathbf{a} - 3\mathbf{b})$$

$$= \tfrac{1}{2}\mathbf{a} - \tfrac{1}{2}\mathbf{b}$$
$$= \tfrac{1}{2}(\mathbf{a} - \mathbf{b})$$
$$\overrightarrow{MD} = \overrightarrow{MC} + \overrightarrow{CD}$$
$$= \tfrac{1}{2}\overrightarrow{BC} + \overrightarrow{CD}$$
$$= \tfrac{1}{2}(\mathbf{a} - 3\mathbf{b}) + \mathbf{a}$$
$$= \tfrac{3}{2}\mathbf{a} - \tfrac{3}{2}\mathbf{b}$$
$$= \tfrac{3}{2}(\mathbf{a} - \mathbf{b})$$

Both $\overrightarrow{PM}$ and $\overrightarrow{MD}$ have the same vector part $(\mathbf{a} - \mathbf{b})$ so they are parallel. Since they both pass through M, they are parts of the same line, so PMD is a straight line.

Probability

The basics of probability

1 a

Dice 1

Dice 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b P(prime number) $= \frac{15}{36} = \frac{5}{12}$

c 7, as there are six 7s which is more than all the other scores.

2 a total number of chocolates $= 2x + 1 + x + 2x = 5x + 1$

P(mint) $= \frac{x}{5x + 1} = \frac{4}{21}$

$$21x = 20x + 4$$
$$x = 4$$

total number of chocolates $= 5x + 1 = 5 \times 4 + 1 = 21$

b number of truffles $= 2 \times 4 + 1 = 9$

P(truffle) $= \frac{9}{21} = \frac{3}{7}$

3

Beth

Amy	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

a number of possible outcomes $= 36$

number of pairs the same $= 6$

P(scores are equal) $= \frac{6}{36} = \frac{1}{6}$

b The possible scores where Amy's score is higher are:

(2, 1)

(3, 1), (3, 2)

(4, 1), (4, 2), (4, 3)

(5, 1), (5, 2), (5, 3), (5, 4)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5)

P(Amy's score is higher) $= \frac{15}{36} = \frac{5}{12}$

Probability experiments

1 a He is wrong because 100 spins is a very small number of trials. To approach the theoretical probability you would have to spin many more times. Only when the number of spins is extremely large will the frequencies start to become similar.

b relative frequency $= \frac{\text{frequency of event}}{\text{total frequency}} = \frac{22}{100} = \frac{11}{50}$

c Spinning 100 times gives a frequency of 19.

Spinning 500 times gives an estimate for the frequency $= 5 \times 19 = 95$.

2 a $3x + 0.05 + 2x + 0.25 + 0.20 + 0.1 = 5x + 0.6$

The relative frequencies have to add up to 1.

So $5x + 0.6 = 1$

$$5x = 0.4$$
$$x = 0.08$$

b relative frequency for a score of $1 = 3 \times 0.08 = 0.24$

c number of times $= 0.20 \times 80 = 16$

The AND and OR rules

1 a P(picture card) $= \frac{12}{52} = \frac{3}{13}$

P(2 picture cards) $= \frac{3}{13} \times \frac{3}{13} = \frac{9}{169}$

b P(ace) $= \frac{4}{52} = \frac{1}{13}$

P(ace and picture card) $= \frac{1}{13} \times \frac{3}{13} = \frac{3}{169}$

c P(queen of hearts and queen of diamonds) $= \frac{1}{52} \times \frac{1}{52}$
$$= \frac{1}{2704}$$

2 a When an event has no effect on another event, they are said to be independent events. Here the colour of the first marble has no effect on the colour of the second marble.

b P(red) $= \frac{3}{10}$

P(2 reds) $= \frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$

c P(red then blue) $= \frac{3}{10} \times \frac{5}{10} = \frac{15}{100}$

P(blue then red) $= \frac{5}{10} \times \frac{3}{10} = \frac{15}{100}$

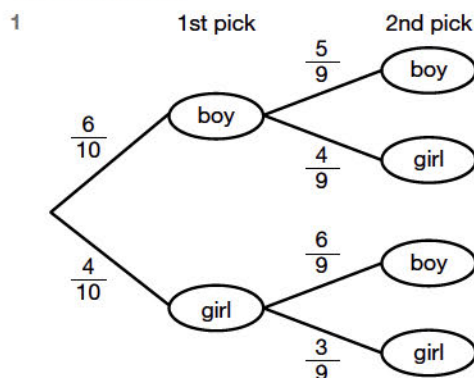P(red and blue) $= \frac{15}{100} + \frac{15}{100} = \frac{30}{100} = \frac{3}{10}$

3 a P(homework in all 3 subjects) $= \frac{3}{5} \times \frac{3}{7} \times \frac{1}{4} = \frac{9}{140}$

b P(homework not given in any of the subjects)

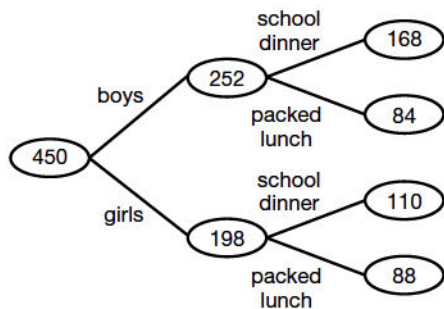$$= \frac{2}{5} \times \frac{4}{7} \times \frac{3}{4} = \frac{24}{140} = \frac{6}{35}$$

Tree diagrams

1

1st pick	2nd pick

a P(2 girls) $= \frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$

b P(boy and girl) = P(BG) + P(GB)

$$= \left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9}\right)$$

$$= \frac{8}{15}$$

2 **a** number of boys = $0.56 \times 450 = 252$

number of girls = $450 - 252 = 198$

number of boys who have a packed lunch = $\frac{1}{3} \times 252 = 84$

number of boys who have a school dinner = $252 - 84 = 168$

number of girls who have a school dinner = $\frac{5}{9} \times 198 = 110$

number of girls who have a packed lunch = $198 - 110 = 88$

b P(girl who has a school dinner) = $\frac{110}{450} = \frac{11}{45}$

c P(boy who has a school dinner) = $\frac{168}{450}$

P(school dinner) = $\frac{110}{450} + \frac{168}{450} = \frac{278}{450} = \frac{139}{225}$

3 **a** Possible ways of one marble of each colour:

RGY   RYG   GYR   GRY   YGR   YRG

P(RGY) = $\frac{5}{9} \times \frac{3}{8} \times \frac{1}{7} = \frac{15}{504}$

P(RYG) = $\frac{5}{9} \times \frac{1}{8} \times \frac{3}{7} = \frac{15}{504}$

P(one of each colour) = $6 \times \frac{5}{9} \times \frac{3}{8} \times \frac{1}{7} = 6 \times \frac{15}{504} = \frac{90}{504}$

$= \frac{5}{28} = 0.179$ (to 3 d.p.)

b P(no green) = $\frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{120}{504} = \frac{5}{21} = 0.238$ (to 3 d.p.)

c P(all red) = $\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{5}{42}$

P(all green) = $\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} = \frac{1}{84}$

P(same colour) = $\frac{5}{42} + \frac{1}{84} = \frac{11}{84} = 0.131$ (to 3 d.p.)

Venn diagrams and probability

1 **a** 9, 8

b 1, 2, 3, 5, 7, 8, 9, 12, 15

c 1, 3, 4, 10, 12, 15

d 4, 10

2 **a**
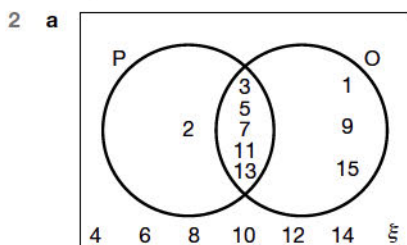
b P(number in P ∩ O) = $\frac{5}{15} = \frac{1}{3}$

3 **a** P(team sports) = P(not only individual) = $1 - \frac{15}{100} = \frac{85}{100}$

$= \frac{17}{20}$

Alternatively, you could find the total of all those who play team sports.

b P(student playing large team also plays small team sports)

$= \frac{18}{73}$

Statistics

Sampling

1 **a** Ling's, as he has a larger sample so it is more likely to represent the whole population (i.e. students at the school).

b % of boys in school = $\frac{400}{900} \times 100 = 44.4\%$

number of boys in sample = $\frac{44.4}{100} \times 50 = 22.2 = 22$ (as number has to be an integer)

2 The sample should be taken randomly, with each member of the population having an equal chance of being chosen.

The sample size should be large enough to represent the population, since the larger the sample size, the more accurate the results.

3 **a** 52% males and 48% females.

12% males who smoke and 88% males who do not smoke

10% females who smoke and 90% females who do not smoke

number of females in sample = 48% of 400 = 192

number of females who do not smoke in sample = $0.9 \times 192 = 172.8 = 173$

173 questionnaires needed

b number of males in sample = 52% of 400 = 208

number of males who smoke in sample = $0.12 \times 208 = 24.96 = 25$
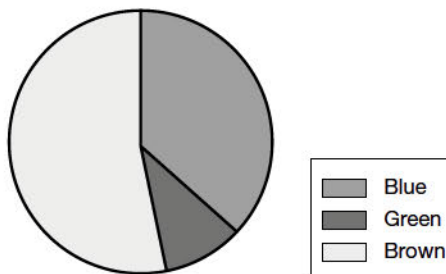
25 questionnaires needed

Two-way tables and pie charts

1 **a**

	Coronation Street	EastEnders	Emmerdale	Total
Boys	12	31	20	63
Girls	18	12	7	37
Total	30	43	27	100

b P(student's favourite is Emmerdale) = $\frac{27}{100}$

c P(girl's favourite is Eastenders) = $\frac{12}{37}$

2 total frequency = $11 + 3 + 16 = 30$

angle for one person = $\frac{360}{30} = 12°$

| Colour	Frequency	Angle
Blue	11	$11 \times 12 = 132°$
Green	3	$3 \times 12 = 36°$
Brown	16	$16 \times 12 = 192°$

3 **a** The median is the 11th student, working from the smallest value.

median = 11 students

b range = highest value − lowest value = $26 - 3 = 23$ students

Line graphs for time series data

1 a An upward trend, rising slowly at first, then rising quickly, then continuing to rise more slowly.

b Mode, because it shows that June is the month when the environment offers the largest sample of insects to study. The median would give May, when there are also a lot of insects.

2 a, b and c
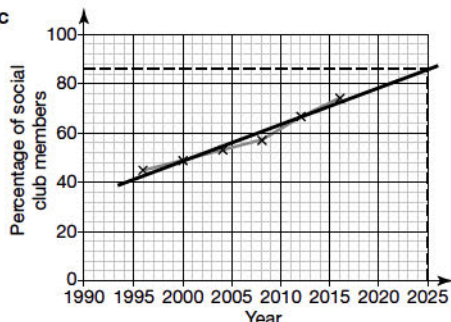
b An upward trend – people are increasingly likely to assume it should be an option.

c Reading from graph: 86% (or a close value).

d Future data may change so that the line of best fit is no longer accurate. Also, the relationship may not be best represented by a straight line, but by a curve.

Averages and spread

1 a $5 + 3 + 1 + 4 + 3 + 5 + 0 + 1 + 4 + 1 + 2 + 3 = 32$

b mean $= \frac{32}{12} = 2.7$ (to 1 d.p.)

c Ordering the data gives:

0 1 1 1 2 3 3 3 4 4 5 5

median $= 3$

2 mean mark $= \frac{\text{total marks}}{\text{total number of students}}$

total marks = mean mark × total number of students

For the group of 25 students:

total marks $= 60 \times 25 = 1500$

For the 30 students:

total marks $= 72 \times 30 = 2160$

total marks for both groups $= 1500 + 2160 = 3660$

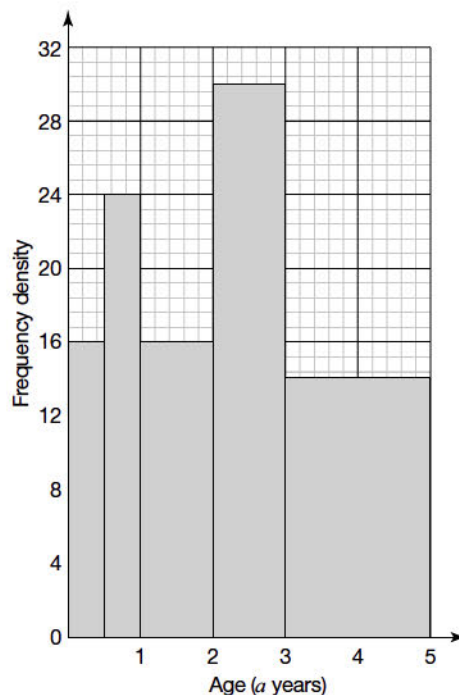mean mark for both groups $= \frac{3660}{55} = 66.5$ (to 1 d.p.)

3 a

Age (t years)	Frequency	Mid-interval value	Frequency × mid-interval value
$0 < t \le 4$	8	2	16
$4 < t \le 8$	10	6	60
$8 < t \le 12$	16	10	160
$12 < t \le 14$	1	13	13

b 35

c estimated mean $= \frac{16 + 60 + 160 + 13}{35} = 7.1$ years (to 2 s.f.)

Histograms

1

| Age (a years)	Frequency	Frequency density
$0 < a \le 0.5$	8	$\frac{8}{0.5} = 1$
$0.5 < a \le 1$	12	$\frac{12}{0.5} = 24$
$1 < a \le 2$	16	$\frac{16}{1} = 16$
$2 < a \le 3$	30	$\frac{30}{1} = 30$
$3 < a \le 5$	28	$\frac{28}{2} = 14$

2 frequency = frequency density × class width

class $0 < w \le 5$: frequency $= (5 - 0) \times 0.4 = 2$

class $25 < w \le 40$: frequency $= (40 - 25) \times 0.6 = 9$

The first bar on the chart given in the book is inaccurate: it should go up to 0.4 on the vertical scale but it actually goes up to 0.5.*

Wingspan (w cm)	Frequency
$0 < w \le 5$	2
$5 < w \le 10$	6
$10 < w \le 15$	24
$15 < w \le 25$	30
$25 < w \le 40$	9

Cumulative frequency graphs

1 a

| Number of items of junk mail per day (n)	Frequency	Cumulative frequency
$0 < n \le 2$	21	21
$2 < n \le 4$	18	39
$4 < n \le 6$	10	49
$6 < n \le 8$	8	57
$8 < n \le 10$	3	60
$10 < n \le 12$	1	61

*This is an error in the first edition of our Exam Practice Book. It has been re-checked and will be corrected in future editions.

b
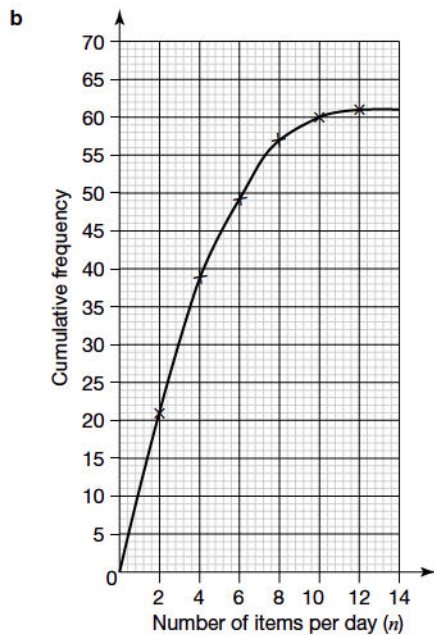
c median = value at frequency of 30.5 = 3 (acceptable values: 2.8 to 3.0)

2 a interquartile range = 17.5 − 11 = 6.5 kg

b number of penguins with mass below 10 kg = 8 (from graph)

number of penguins with mass above 10 kg = 40 − 8 = 32

Comparing sets of data

1 a Chloe's scores in ascending order:

10 12 12 14 14 15 15 16 17 17 17 17 18 18 19

median = 16

range = 19 − 10 = 9

lower quartile = 14 (halfway between the median and the lowest data value)

upper quartile = 17 (halfway between the median and the highest data value)

interquartile range (IQR) = 17 − 14 = 3
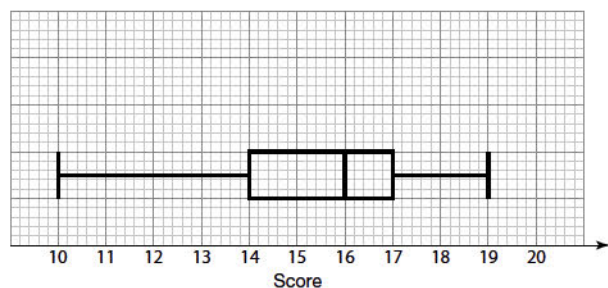
b Sasha's median score is higher (17 compared to 16). The IQR for Sasha is 2 compared to Chloe's 3. The range for Sasha is 5 compared to Chloe's 9. Both these are measures of spread, which means that Sasha's scores are less spread out (i.e. more consistent).

2 a i Draw a horizontal line, from halfway up the vertical axis across to the curve, then draw a line down to the horizontal axis and read off your result:
median = 138 minutes

ii lower quartile = value at frequency of 25 = 128 minutes

upper quartile = value at frequency of 75 = 150 minutes

interquartile range = 150 − 128 = 22 minutes. (21 or 20 minutes are also acceptable answers, depending on your values for the upper and lower quartiles from the graph.)

b From the box plot for the female runners:

median = 145 minutes
range = 180 − 106 = 74 minutes
lower quartile = 134 minutes
upper quartile = 156 minutes
interquartile range = 156 − 134 = 22 minutes

On average the men were faster as the median is lower.

The variation in times were greater for the men as their range was greater, although the spread of the middle half of the data (the interquartile range) was slightly greater for the women.
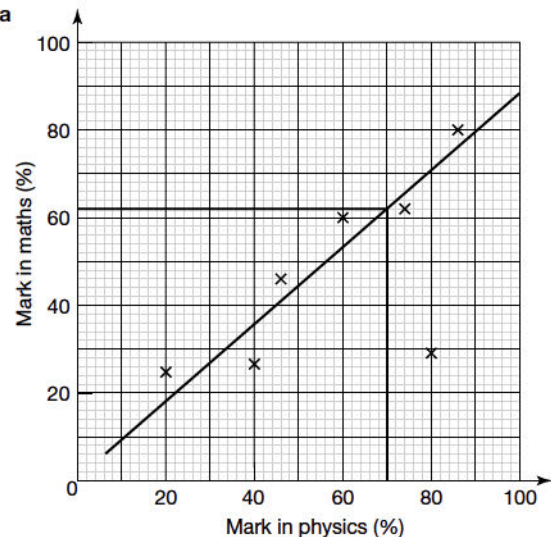
Scatter graphs

**1 a, b**
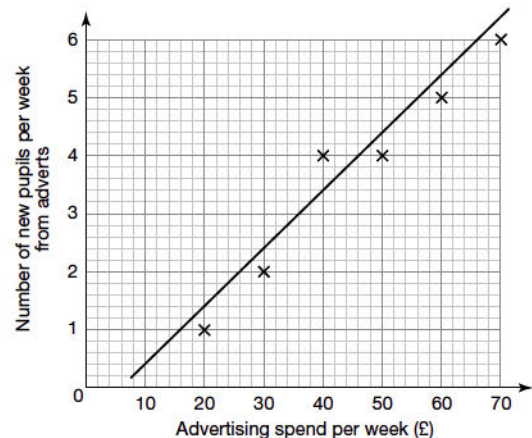
c positive correlation

2 a

b Reading from graph: 62%. A suitable range would be 60–63%.

c The data only goes up to a score of 86% in physics, and the score at 80% in physics is an outlier, so the line may not be accurate for higher scores in physics.
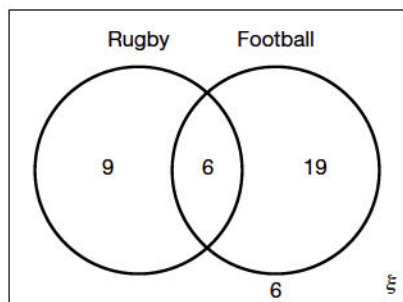
Practice paper (non-calculator)

1 a 267 000

   b 2.4×10^{-8}

   c $(4 \times 10^3) \div (8 \times 10^{-2}) = 4 \div 8 \times 10^{3-(-2)}$
   $$= 0.5 \times 10^5$$
   $$= 5 \times 10^4$$

2 $(2x^5y^2)^3 = 2^3x^{5\times3}y^{2\times3}$
   $$= 8x^{15}y^6$$

3 $(3x + 2)(2x - 3)(x - 4) = (3x + 2)(2x^2 - 11x + 12)$
   $$= 6x^3 - 29x^2 + 14x + 24$$

4 $y = \dfrac{2x + 1}{x - 4}$

   $y(x - 4) = 2x + 1$

   $xy - 4y = 2x + 1$

   $xy - 2x = 4y + 1$

   $x(y - 2) = 4y + 1$

   $x = \dfrac{4y + 1}{y - 2}$

5 Try $x = 2$: $x^3 = 8 > x$, so statement is true for this value.

   Try $x = -1$: $x^3 = -1 = x$, so statement is false.

   Try $x = \frac{1}{2}$: $x^3 = \frac{1}{8} < x$, so statement is false.
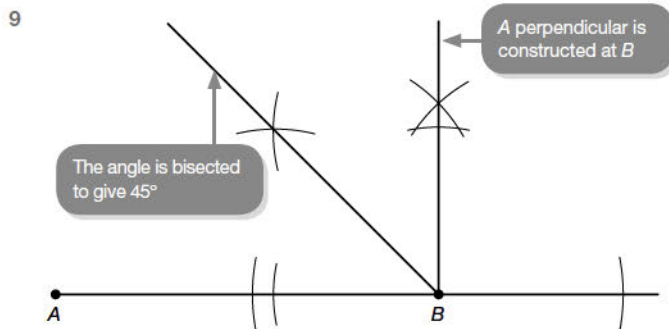
   John is incorrect.

6 $\dfrac{5 - 2\sqrt{3}}{3 + \sqrt{3}} = \dfrac{(5 - 2\sqrt{3})(3 - \sqrt{3})}{(3 + \sqrt{3})(3 - \sqrt{3})}$

   $$= \dfrac{15 - 6\sqrt{3} - 5\sqrt{3} + 6}{9 - 3}$$

   $$= \dfrac{21 - 11\sqrt{3}}{6}$$

7

8 a
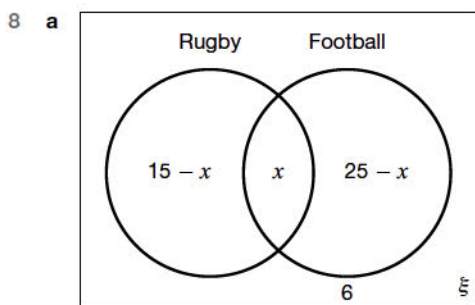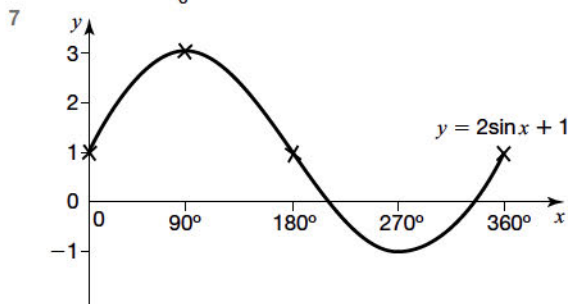
   $15 - x + x + 25 - x + 6 = 40$

   $46 - x = 40$

   $x = 6$

b P(rugby-playing boy picked at random also plays football)

   $= \dfrac{6}{15}$

   $= \dfrac{2}{5}$

9

   A perpendicular is constructed at B

   The angle is bisected to give 45°

10 a $(2 + \sqrt{3})^2 = 4 + 4\sqrt{3} + 3$
    $$= 7 + 4\sqrt{3}$$
    so $a = 7$ and $b = 4$

    b $8^{-\frac{2}{3}} = \dfrac{1}{\sqrt[3]{8^2}} = \dfrac{1}{\sqrt[3]{64}} = \dfrac{1}{4}$

    c $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

11 gradient of $OA = \dfrac{4}{3}$

    gradient of $BC = -\dfrac{3}{4}$

    equation of BC is

    $y - y_1 = m(x - x_1)$

    $y - 4 = -\dfrac{3}{4}(x - 3)$

    $4y - 16 = -3x + 9$

    $3x + 4y - 25 = 0$

12 a P(6 only on third attempt) $= \dfrac{3}{4} \times \dfrac{3}{4} \times \dfrac{1}{4} = \dfrac{9}{64}$

    b P(at least one 6) $= 1 - $ P(no 6s)

       P(no 6s) $= \dfrac{3}{4} \times \dfrac{3}{4} \times \dfrac{3}{4} = \dfrac{27}{64}$

       P(at least one 6) $= 1 - \dfrac{27}{64} = \dfrac{37}{64}$
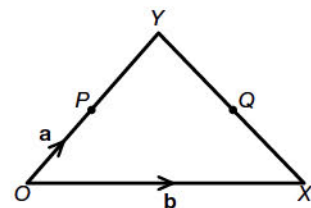
       $\dfrac{37}{64} > \dfrac{1}{2}$, so Amy is right.

13 a $8n < 11n - 9$
       $-3n < -9$
       $3n > 9$
       $n > 3$

    b smallest integer $= 4$

14 $\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY}$
    $$= -\mathbf{b} + 2\mathbf{a}$$
    $$= 2\mathbf{a} - \mathbf{b}$$

    $\overrightarrow{XQ} = \frac{1}{2}\overrightarrow{XY} = \frac{1}{2}(2\mathbf{a} - \mathbf{b}) = \mathbf{a} - \dfrac{\mathbf{b}}{2}$

    $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OX} + \overrightarrow{XQ}$
    $$= -\mathbf{a} + \mathbf{b} + \mathbf{a} - \dfrac{\mathbf{b}}{2}$$
    $$= \dfrac{\mathbf{b}}{2}$$

    $\overrightarrow{OX} = \mathbf{b}$, so $\overrightarrow{PQ}$ and $\overrightarrow{OX}$ have the same vector part ($\mathbf{b}$) and are therefore parallel.

15 $\dfrac{3}{x} + \dfrac{4}{x - 1} = \dfrac{3(x - 1) + 4x}{x(x - 1)}$

    $$= \dfrac{3x - 3 + 4x}{x(x - 1)}$$

    $$= \dfrac{7x - 3}{x(x - 1)}$$

16 a $f(-2) = 5 \times (-2)^2 + 4 = 20 + 4 = 24$

    b Let $y = 5x^2 + 4$

 $\dfrac{y - 4}{5} = x^2$

       $x = \sqrt{\dfrac{y - 4}{5}}$

       $f^{-1}(x) = \sqrt{\dfrac{x - 4}{5}}$

    c $fg(x) = f(x + 1)$
       $$= 5(x + 1)^2 + 4$$
       $$= 5(x^2 + 2x + 1) + 4$$
       $$= 5x^2 + 10x + 5 + 4$$
       $$= 5x^2 + 10x + 9$$

17 a $245 = 5 \times 49 = 5 \times 7^2$

b Shared prime factors are 5 and 7, so highest common factor $= 5 \times 7 = 35$

18 ratio of radii $= \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$

ratio of surface areas $= \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

19 $P(\text{green}) = \frac{x-3}{x+x-3+2x} = \frac{x-3}{4x-3}$

$\frac{x-3}{4x-3} = \frac{1}{5}$

$5(x-3) = 4x - 3$

$5x - 15 = 4x - 3$

$x = 12$

total number of counters $= 4x - 3 = 4 \times 12 - 3 = 45$

number of blue counters $= 2x = 24$

$P(\text{blue}) = \frac{24}{45} = \frac{8}{15}$

20 number of ways of selecting a starter and a main $= 5 \times 8 = 40$

number of ways of selecting a main and a dessert $= 8 \times 6 = 48$

number of ways of selecting a starter, a main and a dessert $= 5 \times 8 \times 6 = 240$

total number of ways of selecting a meal $= 40 + 48 + 240 = 328$

21 a angle $STQ = 180 - (45 + 35) = 100°$ (angles on a straight line add up to 180°)

b angle $TSQ = 35°$ (angle in the alternate segment)

c angle $QRS = 80°$ (opposite angles of a cyclic quadrilateral add up to 180°)

d angle $RQS = 180 - (60 + 80) = 40°$ (angles in a triangle add up to 180°)

22 a

Time for call to be answered (t seconds)	Frequency	Cumulative frequency
$0 < t \le 10$	12	12
$10 < t \le 20$	18	30
$20 < t \le 30$	25	55
$30 < t \le 40$	10	65
$40 < t \le 50$	2	67
$50 < t \le 60$	1	68

b
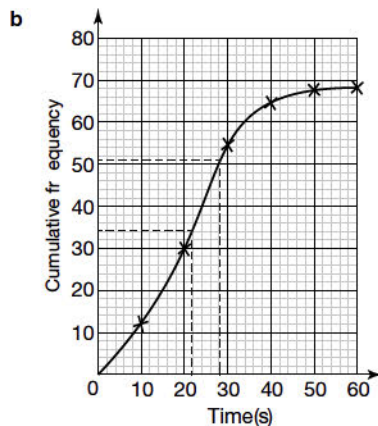
c Reading off from 34 on the cumulative frequency axis gives a time of 22 seconds.

median $= 22\,\text{s}$

d 75% of $68 = 0.75 \times 68 = 51$

Reading off from 51 on the cumulative frequency axis gives a time of 28 seconds.

75% of calls are answered within 28 seconds, so the target is being met.

23 Let cost of adult ticket $= £x$ and cost of child ticket $= £y$

$2x + 4y = 122$ (1)

$4x + 3y = 154$ (2)

equation (1) $\times$ 2:

$4x + 8y = 244$ (3)

(3) $-$ (2):

$5y = 90$

$y = 18$

Substituting into equation (1):

$2x + 72 = 122$

$2x = 50$

$x = 25$

Checking in (2):

$4 \times 25 + 3 \times 18 = 154$

cost of adult ticket $= £25$

cost of child ticket $= £18$

24 a $9\sqrt{3} \times \sqrt{3} = 9 \times 3 = 27$

b first term ($n = 1$): $(\sqrt{3})^2$

second term ($n = 2$): $(\sqrt{3})^3$

third term ($n = 3$): $(\sqrt{3})^4$

fourth term ($n = 4$): $(\sqrt{3})^5$

nth term ($n = n$): $(\sqrt{3})^{n+1}$

c $(\sqrt{3})^{20+1} \div (\sqrt{3})^{16+1} = (\sqrt{3})^{21} \div (\sqrt{3})^{17}$

$= \frac{(\sqrt{3})^{21}}{(\sqrt{3})^{17}}$

$= (\sqrt{3})^4$
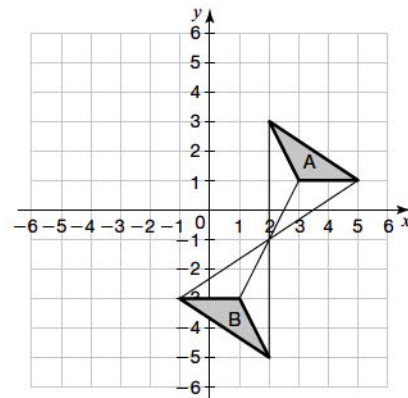
$= 9$

25 a i highest possible attendance $= 34\,849$

ii lowest possible attendance $= 34\,750$

b $3.46 < l < 3.47$

The number cannot be as low as 3.46 as it originally had more decimal places. (On the answer line in the Exam Practice Book, the first sign should have been <.)*

26 $\tan 30° + \tan 60° + \cos 30° = \frac{1}{\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{3} + \sqrt{3} + \frac{\sqrt{3}}{2}$

$= \frac{2\sqrt{3} + 6\sqrt{3} + 3\sqrt{3}}{6}$

$= \frac{11\sqrt{3}}{6}$

27 a

b A rotation of 180° (clockwise or anticlockwise) about the point $(2, -1)$.

28 Write the equation in the form $y = a(x + p)^2 + q$ by completing the square:

$y = 2x^2 - 5x + 4$

$= 2\left(x^2 - \frac{5}{2}x + 2\right)$

$= 2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16} + 2\right]$

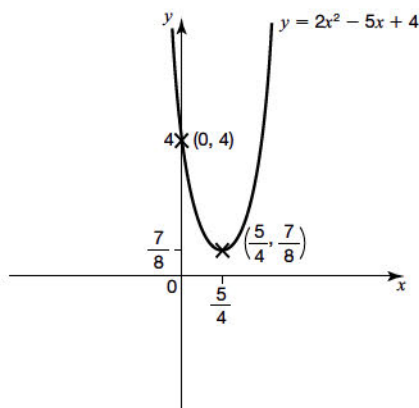$= 2\left[\left(x - \frac{5}{4}\right)^2 + \frac{7}{16}\right]$

$= 2\left(x - \frac{5}{4}\right)^2 + \frac{7}{8}$

The turning point of the graph is $(-p, q)$, which is $\left(\frac{5}{4}, \frac{7}{8}\right)$

To find the y-intercept, put $x = 0$:

$y = 2 \times 0^2 - 5 \times 0 + 4 = 4$

The curve cuts the y-axis at the point $(0, 4)$.

*This is an error in the first edition of our Exam Practice Book. It has been re-checked and will be corrected in future editions.

Graph: $y = 2x^2 - 5x + 4$, with points $(0, 4)$, $\frac{7}{8}$ on y-axis, $\frac{5}{4}$ on x-axis, and turning point $\left(\frac{5}{4}, \frac{7}{8}\right)$.

ii Let 'doesn't work Saturday' = C.
Let 'doesn't work Sunday' = D.
Let 'doesn't work either day' = C∩D.
$P(C) = 1 - P(A) = 0.6$
$P(C∩D) = 0.12$ (given in question)

$$P(D \text{ given } C) = \frac{P(C∩D)}{P(C)}$$

$$= \frac{0.12}{0.6}$$

$$= 0.2$$

You could also do the working above to find
P(B' given A'), with:
C = A', D = B' and
C ∩ D = (A ∪ B)'.

b

works Saturday   works Sunday

c From tree diagram:
P(works Saturday but not Sunday) = 0.4 × 0.5 = 0.2
P(does not work Saturday but works Sunday) = 0.6 × 0.8
= 0.48
P(only works one day over weekend) = 0.2 + 0.48 = 0.68

Practice paper (calculator)

1 $12x^2 - 75 = 3(4x^2 - 25)$
$$= 3(2x + 5)(2x - 5)$$

2 a $\dfrac{x^2 - 25}{2x^2 + 13x + 15} = \dfrac{(x + 5)(x - 5)}{(2x + 3)(x + 5)}$
$$= \dfrac{x - 5}{2x + 3}$$

b $x^2 + 6x + 12 = (x + 3)^2 - 9 + 12$
$$= (x + 3)^2 + 3$$
So $a = 3$ and $b = 3$

3 $\dfrac{1}{f} = \dfrac{1}{v} + \dfrac{1}{u}$

$\dfrac{1}{f} - \dfrac{1}{v} = \dfrac{1}{u}$

$\dfrac{v - f}{fv} = \dfrac{1}{u}$

$u = \dfrac{fv}{v - f}$

4 Say the cube has side 10 cm originally.
volume of original cube = 10 × 10 × 10 = 1000 cm³
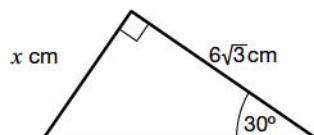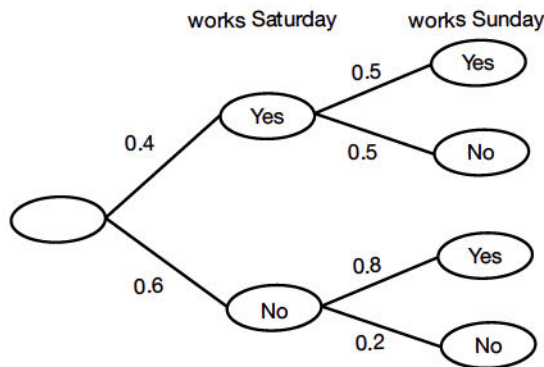Increasing the sides by 10% will make each side 11 cm.
volume of enlarged cube = 11 × 11 × 11 = 1331 cm³
increase in volume = 331 cm³
% increase in volume = $\frac{331}{1000}$ × 100 = 33.1%
So Seema is wrong.

5

Let x be the base of the right-angled triangle.
$$\dfrac{x}{6\sqrt{3}} = \tan 30°$$

$\tan 30° = \dfrac{1}{\sqrt{3}}$

So $\dfrac{x}{6\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$x = \dfrac{6\sqrt{3}}{\sqrt{3}}$
$$= 6 \text{ cm}$$
Area of triangle = $\frac{1}{2}$ × base × height
$$= \frac{1}{2} × 6 × 6\sqrt{3}$$
$$= 18\sqrt{3} \text{ cm}^2$$

6 a i Let 'works Saturday' = A.
Let 'works Sunday' = B.
So 'works both days' = A∩B.
$P(A) = 0.4$, $P(A∩B) = 0.2$ (given in question)

$P(B \text{ given } A) = \dfrac{P(A∩B)}{P(A)}$

$= \dfrac{0.2}{0.4}$
$= \dfrac{1}{2}$
$= 0.5$

7 a $x^2 + x - 12 > 0$
Factorising $x^2 + x - 12 = 0$ gives
$(x - 3)(x + 4) = 0$: $x = 3$ or -4
As the coefficient of x^2 is positive, the graph of
$y = x^2 + x - 12$ is U-shaped.
$x^2 + x - 12 > 0$ for the region above the x-axis (i.e. where
$y > 0$).
$x > 3$ or $x < -4$

b Equation of curve is $y = (x - 2)^2 - 3$
$$= x^2 - 4x + 4 - 3$$
$$= x^2 - 4x + 1$$
So $a = -4$ and $b = 1$

8 a Let $x = 0.13\dot{6} = 0.136363636...$
$$10x = 1.36363636...$$
$$1000x = 136.363636$$
$$1000x - 10x = 136.363636... - 1.36363636...$$
$$990x = 135$$
$$x = \frac{135}{990} = \frac{27}{198} = \frac{3}{22}$$

b $27^{\frac{1}{3}} × \left(\frac{1}{3}\right)^2 = \sqrt[3]{27} × \frac{1}{9}$
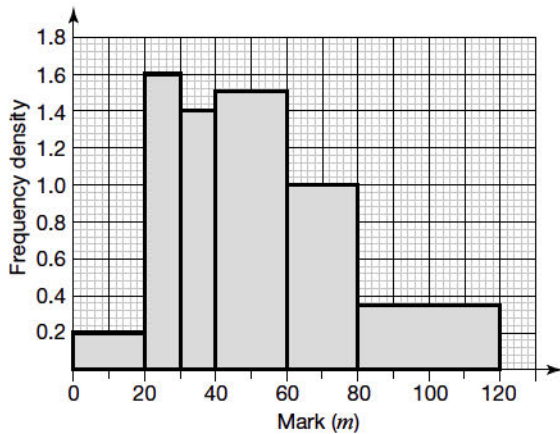$$= 3 × \frac{1}{9}$$
$$= 3 × \frac{1}{3}$$
$$= 0.\dot{3}$$

9

Mark (m)	Frequency	Frequency density
$0 < m \leq 20$	4	0.2
$20 < m \leq 30$	16	1.6
$30 < m \leq 40$	14	1.4
$40 < m \leq 60$	30	1.5
$60 < m \leq 80$	20	1.0
$80 < m \leq 120$	14	0.35

12 a

b There is one root of $x^3 - x - 2 = 0$, where the graphs of $y = x^3$ and $y = x + 2$ intersect.

c $x^1 = (1.5 + 2)^{\frac{1}{3}} = 1.5182945$ (7 d.p.)

$x^2 = 1.5209353$

$x^3 = 1.5213157$

$x^4 = 1.5213705$

$= 1.521$ (to 3 d.p.)

10
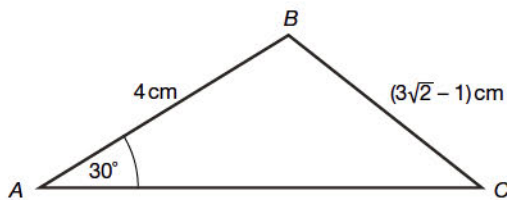
Using the sine rule:

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$\dfrac{3\sqrt{2} - 1}{\sin 30°} = \dfrac{4}{\sin ACB}$

$\sin ACB = \dfrac{4 \sin 30°}{3\sqrt{2} - 1}$

$= \dfrac{2}{3\sqrt{2} - 1}$

$= \dfrac{2(3\sqrt{2} + 1)}{(3\sqrt{2} - 1)(3\sqrt{2} + 1)}$

$= \dfrac{6\sqrt{2} + 2}{18 - 1}$

$= \dfrac{2 + 6\sqrt{2}}{17}$

So $m = 6$ and $n = 17$

11 a angle XYZ = angle TUZ (corresponding angles)

angle YXZ = angle UTZ (corresponding angles)

angle XZY = angle TZU (same angle)

Hence triangles XYZ and TUZ have corresponding angles equal, so the two triangles are similar.

$\dfrac{XY}{TU} = \dfrac{YZ}{UZ}$

$\dfrac{XY}{3} = \dfrac{15}{5}$

$XY = 9$ cm

angle YUT = angle YZW (corresponding angles)

angle YTU = angle YWZ (corresponding angles)

angle TYU = angle WYZ (same angle)

Hence triangles YTU and YWZ have corresponding angles equal, so the two triangles are similar.

$\dfrac{WZ}{TU} = \dfrac{YZ}{YU}$

$\dfrac{WZ}{3} = \dfrac{15}{10}$

$WZ = 4.5$ cm

b angle YTX = angle WTZ (vertically opposite angles)

angle YXZ = angle XZW (alternate angles)

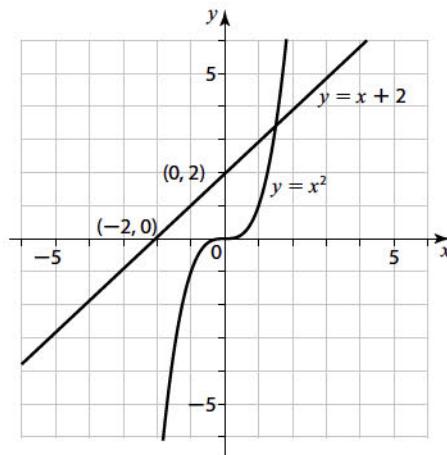Two angles in both triangles are equal so the third angles of both triangles are also equal.

Hence triangles TYX and TWZ are similar.

Area of TYX as a fraction/multiple of TWZ:

$= \left(\dfrac{XY}{WZ}\right)^2 = \left(\dfrac{9}{4.5}\right)^2 = 2^2 = 4$

Ratio of the areas of TYX and TWZ = 4 : 1

13 a The 25% should be applied to the sale price and not to the original price.

b Take a typical price (e.g. £100).

20% reduction on £100 = 0.8 × 100 = £80

25% reduction on £80 = £60

Total reduction = 100 − 60 = £40

£40 as a percentage of £100 = 40%

Hence, the original prices have been reduced by 40%.

14 January 2018: 20 000 × 1.05 − 2000 = £19 000

January 2019: 19 000 × 1.05 − 2000 = £17 950

January 2020: 17 950 × 1.05 − 2000 = £16 847.50

January 2021: 16 847.50 × 1.05 − 2000 = £15 689.88

Pascal will have £15 689.88 (to nearest penny) in the account in January 2021.

15

Number of households	Frequency	Midpoint	Frequency × midpoint
0–2	22	1	22
3–5	12	4	48
6–8	x	7	$7x$
9–11	3	10	30

Total number of pets $\approx 22 + 48 + 7x + 30 = 100 + 7x$

Total number of households $= 22 + 12 + x + 3 = 37 + x$

estimate of mean $= \dfrac{\text{total number of pets}}{\text{total number of households}} = \dfrac{100 + 7x}{37 + x}$

$\dfrac{100 + 7x}{37 + x} = 3.25$

$100 + 7x = 3.25(37 + x)$

$100 + 7x = 120.25 + 3.25x$

$3.75x = 20.25$

$x = \dfrac{20.25}{3.75}$

$x = 5.4$ pets (5 to nearest whole number)

The answer 5.4 is not realistic for a frequency (which should be an integer).* The workings given here are correct for the figures in the book.

16 3 + 5 = 8 parts

8 parts = 500 g so 1 part = 62.5 g

metal A: mass = 3 × 62.5 = 187.5 g

cost $= \dfrac{187.5}{1000} \times 50 = £9.375$

metal B: mass = 5 × 62.5 = 312.5 g

cost $= \dfrac{312.5}{1000} \times 140 = £43.75$

total cost = 9.375 + 43.75 = 53.125 = £53.13 (to nearest penny)

*This is an error in the question in the first edition of the Exam Practice Book. The question has been re-checked and will be corrected in future editions.

17 a multiplier $= 1 - \dfrac{\% \text{ evaporation rate}}{100}$

$= 1 - \dfrac{3}{100} = 0.97$

volume at the end of 4 weeks $= A_0 \times (\text{multiplier})^n$

$= 30 \times (0.97)^4$

$= 26.6\,\text{m}^3$ (to 3 s.f.)

b Need to aim for a volume of $15\,\text{m}^3$

Try 14 weeks: volume $= 30 \times (0.97)^{14} = 19.6$

Try 20 weeks: volume $= 30 \times (0.97)^{20} = 16.3$

Try 24 weeks: volume $= 30 \times (0.97)^{24} = 14.4$

Try 23 weeks: volume $= 30 \times (0.97)^{23} = 14.9$

Try 22 weeks: volume $= 30 \times (0.97)^{22} = 15.3$

Answer is 23 weeks.

18 a
$$y = \frac{k}{x^2}$$
$$10 = \frac{k}{0.25^2}$$
$$k = 0.625$$

Formula is $y = \dfrac{0.625}{x^2}$

b $y = \dfrac{0.625}{x^2}$

When $x = 0.1$, $y = \dfrac{0.625}{0.1^2} = 62.5$

19 a $(x + 1)(x + 2)(2x - 3) = (x + 1)(2x^2 + x - 6)$

$= 2x^3 + 3x^2 - 5x - 6$

b $10x - 8 = 2 - 5x$

$15x = 10$

$x = \dfrac{10}{15} = \dfrac{2}{3}$

c $6 - x > 5$

$-x > -1$

$x < 1$

20 $8^2 + 15^2 = 64 + 225 = 289$

$17^2 = 289$

Pythagoras' theorem is obeyed, so the triangle is right-angled.

21 $\tan 52° = \dfrac{BC}{8}$

$BC = 8 \tan 52°$

$= 10.24\,\text{cm}$

$CD = BD - BC$

$= 13 - 10.24$

$= 2.76\,\text{cm}$ (to 2 d.p.)

22 a i acceleration $= \dfrac{16}{10} = 1.6\,\text{m/s}^2$

ii acceleration $= 0\,\text{m/s}^2$

b total distance travelled = area under speed−time graph

= area of trapezium

$= \dfrac{1}{2}(30 + 15) \times 16$

$= 360\,\text{m}$

c $55.5 \le v < 56.5$

23 Using the cosine rule:

$6^2 = 14^2 + 16^2 - (2 \times 14 \times 16 \times \cos BAC)$

$36 = 196 + 256 - (448 \times \cos BAC)$

$-416 = -448 \cos BAC$

$\cos BAC = \dfrac{-416}{-448}$

$= \dfrac{13}{14}$

$= 0.92857$ (5 d.p.)

angle $BAC = 21.7868°$ (4 d.p.)

area of triangle $= \dfrac{1}{2} \times 14 \times 16 \times \sin 21.7868°$

$= 41.569$ (3 d.p.)

$= 41.6\,\text{cm}^2$ (3 s.f.)