AQA Foundation Mathematics Exam Practice Book

Full worked solutions

Number

Factors, multiples and primes

1 $18 = (2) \times (3) \times 3$

$24 = (2) \times 2 \times 2 \times (3)$

Find the factors that they both share (2 and 3) and multiply together:

$2 \times 3 = 6$, so HCF is 6.

2 17, 19, 23 are the only numbers in this range with only 2 factors (1 and the number itself).

3 $60 = 20 \times 3$

$= 2 \times 2 \times 5 \times 3$

$= 2^2 \times 3 \times 5$

4 Drummer 1 hits her drum at: 6 12 18 24 30 36 42 48 54 60 seconds

Drummer 2 hits his drum at: 8 16 24 32 40 48 56 seconds

They hit their drums at the same time twice (two times), after 24 seconds and after 48 seconds.

Ordering integers and decimals

1 Negative numbers are smaller than zero

-12 is further left on the number line than -8, -1 is larger than -8 (and -12) so it appears next.

Then comes 0, then 2.

So the order is:

$-12, -8, -1, 0, 2$

2 First look at the place value for 10ths: 0.32 and 0.3 have the higher number of 10ths.

Now compare their 100ths. 0.32 has 2 100ths but 0.3 doesn't have any, so it's smaller.

Similarly, 0.23 and 0.203 both have 2 10ths, but 0.23 is bigger than 0.203 because it has 3 100ths while 0.203 only has 3 1000ths.

So the order is:

0.32, 0.3, 0.23, 0.203

3 **a** $-4 < 0.4$ (the negative number is smaller)

b $4.200 < 4.3$ (the larger number has more 10ths)

c $-0.404 > -0.44$ (because they are both negative, the one with more 100ths is smaller)

d $0.33 < 0.4$ (the larger number has more 10ths)

Calculating with negative numbers

1 **a** $-7 + -3 = -7 - 3 = -10$

b $-7 - -3 = -7 + 3 = -4$

c $8 + -5 - -2 = 8 - 5 + 2 = 5$

d $-4 - -6 + -1 = -4 + 6 - 1 = 1$

2 **a** -18

b $-12 \div -3 = 12 \div 3 = 4$

c $-4 \times -2 \times 5 = 4 \times 2 \times 5 = 40$

d $(-24 \div 3) \times 2 = -8 \times 2 = -16$

3 $-7 - 4 = -11$, $-11 \times -2 = 22$

4 Let a = number of correct answers, b = number of incorrect answers

$3a - 2b = -5$ (1)

There are five questions, so $a + b = 5$ and $b = 5 - a$ (2)

Substituting this for b in (1): $3a - 2(5 - a) = -5$

$3a - 10 + 2a = -5$

$5a = 5$

$a = 1$

Substituting this in (2):

$b = 5 - 1$

$b = 4$

Sally got 1 correct answer and 4 incorrect answers in the test.

Multiplication and division

1 **a**

$$\begin{array}{r} 357 \\ \times \quad 6 \\ \hline 2142 \end{array}$$

2142

b

$$\begin{array}{r} 261 \\ \times \quad 43 \\ \hline 783 \ (=261 \times 3) \\ 10440 \ (=261 \times 40) \\ \hline 11223 \end{array}$$

11223

c

$$6\overline{)5^{5}5^{1}2} = 092$$

92

d

$$\begin{array}{r} 052 \\ 13\overline{)676} \\ -65 \\ \hline 026 \end{array}$$

52

2 **a**

$$24\overline{)30^{6}0} = 012 \text{ remainder } 12$$

So 12 boxes are filled.

b $24 \times 12 = 20 \times 12 + 4 \times 12 = 240 + 48 = 288$

$300 - 288 = 12$

There are 12 books left over.

3 $12500 - 440 = 12060$

$$36\overline{)1^{1}2^{1}0^{1}0^{1}0} = 00335$$

$$\begin{array}{r} -108 \\ \hline 12^{1}16 \\ -108 \\ \hline 0180 \end{array}$$

Each repayment is £335.

4 $52 - 6 = 46$ weeks $\quad 46 \times 26 = 1196$ hours

Calculating with decimals

1 First note the combined number of decimal places in both numbers (2).

Remove the decimal points to do the calculation:

$$\begin{array}{r} 92 \\ \times \quad 83 \\ \hline 276 \ (=92 \times 3) \\ 7360 \ (=92 \times 80) \\ \hline 7636 \\ 1 \end{array}$$

Now you've got the digits right put the decimal point back, counting in from the right 2 places, to give a number with 2 decimal places:

76.36

2

$$\begin{array}{r} 22.50 \\ +\ 19.99 \\ \hline 42.49 \end{array}$$

$$\begin{array}{r} {}^{4}5{}^{1}\cancel{0}{}^{9}\cancel{.}{}^{1}\cancel{0}{}^{1}\cancel{0} \\ -\ 42.49 \\ \hline 07.51 \end{array}$$

She should get £7.51 change.

3

$$6\overline{)2^{2}2^{4}9.7^{5}4} \quad 038.29$$

4 Kirsty raises $\frac{172.50}{5+1}$

Kirsty raises £28.75 × 5 = 28.75 × 10 ÷ 2 = 287.5 ÷ 2 = 143.7

$$6\overline{)1^{1}7^{5}2.5^{3}0} \quad 028.75$$

Kirsty raises
£143.75

$$\begin{array}{r} {}^{6}1\cancel{7}{}^{11}\cancel{2}{}^{14}\cancel{.}{}^{1}50 \\ -\ 143.75 \\ \hline 028.75 \end{array}$$

Flo raises £28.75

Rounding and estimation

1 **a** The first non-zero digit is 7, so round the digit after the 8. This is below 5, so the 8 doesn't change.
0.798

b Look at the digit in the third decimal place. It is 5 or above (8), so round the 9 up to 10 and the 7 up to 8.
0.80

2 $\frac{9.74 \times 4.02}{7.88} \approx \frac{10 \times 4}{8} = 5$

3 **a** 40 × 500 = 20 000
20 000 − 12 500 = £7500

b Overestimate, because the concert ticket price and number of tickets sold were rounded up, and so the amount of income was estimated more than it really is.

Converting between fractions, decimals and percentages

1 **a** There are $0 \times 10^{\text{ths}}$, $7 \times 100^{\text{ths}}$ and $1 \times 1000^{\text{ths}}$
$= 071 \times 1000^{\text{ths}}$ so: $\mathbf{\frac{71}{1000}}$

b 63 ÷ 100 = 0.63

c 0.4 × 100 = 40%

d 32% = $\frac{32}{100} = \frac{8}{25}$

2 **a** 5 ÷ 16 = 0.3125

b To convert a number to a percentage, multiply its decimal value by 100.
0.3125 × 100 = 31.25%

3 $\frac{5}{8}$ = 0.625, 60% = 0.6, so 0.65 is the largest.

Ordering fractions, decimals and percentages

1 **a** $\frac{1}{2} = \frac{5}{10}$ = 0.5, so

$\frac{1}{2} < 0.6$

b $\frac{3}{4}$ = 3 ÷ 4 = 0.75, so

$\frac{3}{4} > 0.7$

c $\frac{-3}{10}$ = −0.3, so

$\frac{-3}{10} < 0.2$

2 **a** LCM of 12, 15 and 20 is 60

$\frac{5}{12} = \frac{25}{60}$

$\frac{7}{15} = \frac{28}{60}$

$\frac{9}{20} = \frac{27}{60}$

So order from lowest to highest is $\frac{5}{12}$, $\frac{9}{20}$, $\frac{7}{15}$

b 45% = $\frac{45}{100}$ = 0.45

$\frac{1}{25} = \frac{4}{100}$ = 0.04

0.04 < 0.4 < 0.45

So order is:

$\frac{1}{25}$, 0.4, 45%

3 Shop C is cheapest $\left(\frac{2}{5} = 40\%\right)$, then Shop A $\left(\frac{1}{3} = 33.3...\%\right)$, and Shop B offers the least discount at 30%.

4 $\frac{5}{9} = 0.\dot{5}$

38.5% = 0.385

$\frac{3}{10}$ = 0.3

So the order is $\frac{5}{9}$, 38.5%, 0.38, $\frac{3}{10}$

Calculating with fractions

1 $\frac{1}{5} + \frac{4}{9} = \frac{9}{45} + \frac{20}{45} = \frac{9+20}{45} = \frac{29}{45}$

2 $2\frac{3}{4} - 2\frac{2}{3} = \frac{11}{4} - \frac{8}{3} = \frac{33}{12} - \frac{32}{12} = \frac{1}{12}$

3 $1\frac{5}{6} \times \frac{2}{7} = \frac{11}{6} \times \frac{2}{7} = \frac{22}{42} = \frac{11}{21}$

4 $6 \div \frac{3}{5} = 6 \times \frac{5}{3} = \frac{30}{3} = 10$

Jo can make 10 necklaces.

Percentages

1 $\frac{40}{100} \times 25 = 10$

2 16 × 0.85 = £13.60

3 12450 × 1.14 = 14193

4 40 × 7 × 3 = £840

840 × 1.2 = £1008

Order of operations

1 $3^2 - 6 \div (2 + 1) = 9 - \frac{6}{3} = 9 - 2 = 7$

2 $2^3 + 3x\sqrt{25} = 8 + (3 \times 5) = 8 + 15 = 23$

3 $(1.7 - 0.12)^2 + \sqrt[3]{4.096} = 4.0964$

Exact solutions

1 Area of triangle = $\frac{1}{2}$ × base × vertical height = 0.5 × 0.76 × 0.35 = 0.133 cm²

2 $\left(1\frac{1}{3}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9} = 1\frac{7}{9}$ m²cm²

3 $\sqrt{2} \times \sqrt{6} = \sqrt{12} = 2\sqrt{3}$ cm²

4 Area of a circle = πr^2
The fraction of the circle shown = $\frac{3}{4}$
The area of the circle shown = $\frac{3}{4} \times \pi r^2$
The radius = 2 cm
So area of shape shown = $\frac{3}{4} \times \pi \times 2^2 = 3\pi$ cm²

Indices and roots

1 **a** $7 \times 7 \times 7 \times 7 = 7^4$

b $\frac{1}{5 \times 5 \times 5} = \frac{1}{5^3} = 5^{-3}$

2 **a** $2^4 = 2 \times 2 \times 2 \times 2 = 16$

b $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$

3 $2^3 = 2 \times 2 \times 2 \times 2 = 8$

$3^{-2} = \frac{1}{9}$

$\sqrt[3]{27} = 3$

$\sqrt{25} = 5$ or -5

Assuming the square root of 25 is positive, the answer is:

$3^{-2}, \sqrt[3]{27}, \sqrt{25}, 2^3$

If it were negative, the answer would be:

$\sqrt{25}, 3^{-2}, \sqrt[3]{27}, 2^3$

4 $\frac{9^5}{9^3 \times 9^2} = \frac{9^5}{9^5} = 1$

Standard form

1 2750

2 1.5×10^8

3 Move the decimal point three places to the right to give 6.42×10^{-3}

4 $(1.4 \times 10^{-5}) \times 20 = (2.8 \times 10^{-5}) \times 2 \times 10 = 2.8 \times 10^{-4}$ km

Listing strategies

1 259, 295, 529, 592, 925, 952

2 **a**

		4-sided spinner			
		0	**1**	**2**	**3**
3-sided spinner	**1**	1	2	3	4
	2	2	3	4	5
	3	3	4	5	6

b 3

3

		Gift tag				
		1	2	3	4	5
Wrapping paper	Red	Red, 1	Red, 2	Red, 3	Red, 4	Red, 5
	Blue	Blue, 1	Blue, 2	Blue, 3	Blue, 4	Blue, 5
	Green	Green, 1	Green, 2	Green, 3	Green, 4	Green, 5

Terrence can use 15 different combinations of wrapping paper and gift tags.

4 spj; spi; sfj; sfi ; bpj; bpi; bfj; bfi

Algebra

Understanding expressions, equations, formulae and identities

1 **a** identity **b** equation **c** expression

2 **a** Equation, because it has an equals sign and can be solved.

b Formula, because it has letter terms, an equals sign and the values of the letters can vary.

c Expression, because it has letter terms and no equals sign.

d Formula, because it has letter terms, an equals sign and the values of the letters can vary.

3 **a** Any of: $2x + 10$ or $10x + 2$ or $x + 210$ or $x + 102$

b Any of: $2x = 10$ or $10x = 2$

Simplifying expressions

1 $8x$

2 **a** $6a \times 8a = (6 \times 8) \times (a \times a) = 48 \times a^2 = 48a^2$

b $2p \times 3p \times 5p = (2 \times 3 \times 5) \times (p \times p \times p)$
$= 30 \times p^3 = 30p^3$

3 $35yz \div 7z = (35 \div 7) \times (yz \div z) = 5 \times y = 5y$

4 $\frac{32uv}{4v} = \frac{32}{4} \times \frac{uv}{v} = 8 \times u = 8u$

Collecting like terms

1 **a** $7m + 6n - 4m - 2n = (7 - 4)m + (6 - 2)n = 3m + 4n$

b $9q - 5r - 12q + 3r = (9 - 12)q + (3 - 5)r = -3q - 2r$

2 **a** $11a + 5b - 10a + 8b = (11 - 10)a + (5 + 8)b = a + 13b$

b $6c - 4d - 7c + 5d = (6 - 7)c + (5 - 4)d = -c + d$

c $m + m + m + n \times n = 3m + n^2$

3 **a** $9p^3 + p - 4p^3 = (9 - 4)p^3 + p = 5p^3 + p$

b $12 - 5x^2 + 3x - 2x^2 = 12 - (5 + 2)x^2 + 3x$
$= -7x^2 + 3x + 12$

c $m + m + m + n \times n = 3m + n^2$

4 $3\sqrt{5} - f - 8\sqrt{5} + 2f = (3 - 8)\sqrt{5} + (2 - 1)f = -5\sqrt{5} + f$

Using indices

1 **a** $p^3 \times p = p^{(3+1)} = p^4$

b $4y^2 \times 3y^3 = (4 \times 3) \times y^{(2+3)} = 12 \times y^5 = 12y^5$

c $2a^4b \times 5ab^2 = (2 \times 5) \times a^{(4+1)} \times b^{(1+2)}$
$= 10 \times a^5 \times b^3 = 10a^5b^3$

2 **a** $q^{-2} \times q^{-4} = q^{(-2-4)} = q^{-6}$

b $(u^{-3})^2 = u^{((-3)\times 2)} = u^{-6}$

c $x^{-1} \times x = x^{-1} \times x^1 = x^{(-1+1)} = x^0 = 1$

3 **a** $b^4 \div b^3 = b^{(4-3)} = b^1 = b$

b $\frac{f^5}{f^2} = f^{(5-2)} = f^3$

c $\frac{xy^3}{x^2y} = x^{(1-2)} \times y^{(3-1)} = x^{-1} \times y^2 = \frac{1}{x} \times y^2 = \frac{y^2}{x}$

4 Let the first box $= x$ and the second box $= y$

$(xm^3)^y = x^y m^{3y} = 8m^9$

comparing terms, $3y = 9$

$y = 3$

Substitute in the y value: $(xm^3)^3 = 8m^9$

$x^3 = 8$

$x = \sqrt[3]{8} = 2$

Therefore, the completed expression is $(2m^3)^3$

Expanding brackets

1 **a** $4(m + 3) = (4 \times m) + (4 \times 3) = 4m + 12$

b $2(p - 1) = (2 \times p) + (2 \times -1) = 2p - 2$

c $10(3x - 5) = (10 \times 3)x + (10 \times -5) = 30x - 50$

2 **a** $3(m + 2) + 5(m + 1) = 3m + 6 + 5m + 5 = 8m + 11$

b $6(x - 1) - 2(x - 4) = 6x - 6 - 2x + 8 = 4x + 2$

3 **a** $(y + 3)(y + 7) = y^2 + 7y + 3y + 21 = y^2 + 10y + 21$

b $(b + 2)(b - 4) = b^2 - 4b + 2b - 8 = b^2 - 2b - 8$

c $(x - 4)(x - 6) = x^2 - 6x - 4x + 24 = x^2 - 10x + 24$

4 **a** $(q + 1)^2 = (q + 1)(q + 1) = q^2 + q + q + 1 = q^2 + 2q + 1$

b $(c - 3)^2 = (c - 3)(c - 3) = c^2 - 3c - 3c + 9 = c^2 - 6c + 9$

c $(4m - n)(3m + n) = 12m^2 + 4mn - 3mn - n^2$
$= 12m^2 + mn - n^2$

Factorising

1 Divide the expression by the highest common factor (HCF) of both terms to find the bracket, and then place the HCF outside of the bracket to give the full factorisation.

a $(4x + 8) \div 4 = x + 2$

factorisation: $4(x + 2)$

b $(3d - 15) \div 3 = d - 5$

factorisation: $3(d - 5)$

c $(8y - 12) \div 4 = 2y - 3$

factorisation: $4(2y - 3)$

2 Divide the expression by the common term to find the bracket, and then place the common term outside of the bracket to give the full factorisation.

a $(q^2 + q) \div q = q + 1$

factorisation: $q(q + 1)$

b $(a^2 + 6a) \div a = a + 6$

factorisation: $a(a + 6)$

c $(10z^2 + 15z) \div 5z = (2z + 3)$

factorisation: $5z(2z + 3)$

3 Find which factors of the number term add together to give the coefficient of the x term.

a $12 = 3 \times 4$

$7 = 3 + 4$

factorisation: $(x + 3)(x + 4)$

b $-16 = (-2) \times 8$
$6 = -2 + 8$
factorisation: $(x - 2)(x + 8)$

c $24 = (-6) \times (-4)$
$-10 = (-6) + (-4)$
factorisation: $(a - 6)(a - 4)$

4 a Write $y^2 - 4$ in the form of $a^2 - b^2$:
$y^2 - 2^2$
Using the formula for the difference of two squares, the factorisation is
$(y + 2)(y - 2)$

b Write $x^2 - 9$ in the form of $a^2 - b^2$:
$x^2 - 3^2$
Using the formula for the difference of two squares, the factorisation is
$(x + 3)(x - 3)$

c Write $p^2 - 100$ in the form of $a^2 - b^2$:
$p^2 - 10^2$
Using the formula for the difference of two squares, the factorisation is
$(p + 10)(p - 10)$

Substituting into expressions

1 $4x + 5y = 4 \times 3 + 5 \times (-2) = 12 - 10 = 2$

2 $s = ut + \frac{1}{2}at^2$
$= 12 \times 2 + \frac{1}{2} \times 10 \times 2^2$
$= 12 \times 2 + \frac{1}{2} \times 40$
$= 24 + 20$
$s = 44$

3 a $f = 3c - 2(c - d)$
$= 3 \times 7 - 2 \times (7 - (-5))$
$= 21 - 2 \times (12)$
$= 21 - 24$
$f = -3$

b $f = -c(d^2 - 3c)$
$= -7 \times ((-5)^2 - 3 \times 7)$
$= -7 \times (25 - 21)$
$= -7 \times 4$
$f = -28$

c $f^2 = 7c - 3d$
$= 7 \times 7 - 3 \times (-5)$
$= 49 + 15$
$= 64$
$f = \sqrt{64}$
$f = \pm 8$

4 $\frac{p(q-3)}{r} = \frac{6(7-3)}{-8}$
$= \frac{6 \times 4}{-8}$
$= \frac{24}{-8}$
$= -3$

Writing expressions

1 a $n + 3$ **b** $(n \times 2) - 9 = 2n - 9$

2 a $x + y$ **b** $5 \times x = 5x$

c $(12 \times x) + (11 \times y) = 12x + 11y$

3 $2 \times 9p + 2(5p + 2) = 18p + 10p + 4 = 28p + 4$

4 The area of the rectangle is given by height × length, which is $s \times (5s + 1) = s(5s + 1)$.

Solving linear equations

1 a $x = 12 - 5$
$x = 7$

b $x = 10 + 3$
$x = 13$

c $x = \frac{20}{4}$
$x = 5$

d $x = 6 \times 3$
$x = 18$

2 a $2x + 3 = 15$
$2x = 12$
$x = 6$

b $3x - 5 = 16$
$3x = 21$
$x = 7$

c $\frac{x}{5} + 3 = 8$
$\frac{x}{5} = 5$
$x = 25$

d $7 - 2x = 1$
$7 = 2x + 1$
$6 = 2x$
$x = 3$

3 a $3(x + 9) = 30$
$3x + 27 = 30$
$3x = 3$
$x = 1$

b $5(p - 2) = 10$
$5p - 10 = 10$
$5p = 20$
$p = 4$

c $2(10 - 3m) = 8$
$20 - 6m = 8$
$12 = 6m$
$m = 2$

d $4(6 - 2q) + 10 = 2$
$24 - 8q + 10 = 2$
$32 = 8q$
$q = 4$

4 a $4x - 6 = x + 9$
$3x - 6 = 9$
$3x = 15$
$x = 5$

b $2y + 5 = 4y - 3$
$5 = 2y - 3$
$8 = 2y$
$y = 4$

c $4(2x + 3) = 11x + 3$
$8x + 12 = 11x + 3$
$9 = 3x$
$x = 3$

d $3(n + 4) = 2(2n + 3)$
$3n + 12 = 4n + 6$
$12 = n + 6$
$n = 6$

Writing linear equations

1 Sum of the angles in a triangle are 180°
$(2x + 3) + 81 + (3x - 4) = 180$
$5x + 80 = 180$
$5x = 100$
$x = 20$

2 Let Jamie's age = x years. Sophie's age = $\frac{x}{2}$

$x + \frac{x}{2} = 18$

$\frac{3x}{2} = 18$

$3x = 36$

$x = 12$

Jamie is 12 years old

3 Let width = x so length = $x + 3$

Perimeter = $2x + 2(x + 3)$

$46 = 4x + 6$

$x = 10$

Length = 10cm and width = 13cm

Area = $10 \times 13 = 130\text{cm}^2$

4 Opposite angles are equal so $3x + 10 = 5x - 20$

$30 = 2x$ giving $x = 15$

Also $3x + 10 + 7x + 5y = 180$

$10x + 10 + 5y = 180$

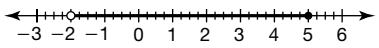
Now $x = 15$ so $150 + 10 + 5y = 180$

Solving this gives $y = 4$

Linear inequalities

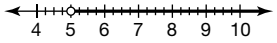
1 **a** $-1, 0, 1, 2, 3, 4, 5$

b



2 **a** $4x > 20$

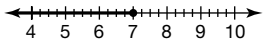
$x > 5$



b $3x - 8 \leq 13$

$3x \leq 21$

$x \leq 7$

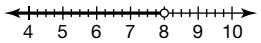


c $2(x - 3) < 10$

$2x - 6 < 10$

$2x < 16$

$x < 8$



3 **a** $2 \leq 3x + 5$

$-3 \leq 3x$

$-1 \leq x$

$3x + 5 < 11$

$3x < 6$

$x < 2$

Hence $-1 \leq x < 2$

b $-4 > 5x + 6$

$-10 < 5x$

$-2 < x$

$5x + 6 \leq 6$

$5x \leq 0$

$x \leq 0$

Hence $-2 < x \leq 0$

4 $n + n + 3 < 15$

$2n + 3 < 15$

$2n < 12$

$n < 6$

Possible integer values of n = 1, 2, 3, 4, 5

Formulae

1 **a** $t = (40 \times 2) + 20 = 100$ minutes = 1 hour 40 minutes

b $t = (40 \times 1.5) + 20 = 80$ minutes = 1 hour 20 minutes

The chicken should be put in the oven 1 hour and 20 minutes earlier than 1.30 pm, which is a time of 12.10 pm.

2 **a** $C = l + kn$

b $C = 90 + 6.5 \times 3$

$C = £109.50$

3 **a** $p = \frac{qs}{3}$

$3p = qs$

$q = \frac{3p}{s}$

b $p = \frac{q}{r} + t$

$p - t = \frac{q}{r}$

$q = rp - rt$ or $r(p - t)$

c $p = 3(q + r)$

$\frac{p}{3} = q + r$

$q = \frac{p}{3} - r = \frac{p - 3r}{3}$

d $p = \sqrt{2q}$

$p^2 = 2q$

$q = \frac{p^2}{2}$

Linear sequences

1 **a** The term in position 1 is $1 \times 5 + 1 = 6$

The term in position 2 is $2 \times 5 + 1 = 11$

The term in position 3 is $3 \times 5 + 1 = 16$

The term in position 4 is $4 \times 5 + 1 = 21$

b The term in position 50 is $50 \times 5 + 1 = 251$

2 **a** Each pattern has 2 more dots than the last, so pattern 7 will have 8 more dots than pattern 3. Pattern 7 will have 19 dots.

b No, Rachel is not correct, because the number of triangles is not the pattern number multiplied by 2. Instead, it is the pattern number plus 2, so there will be 6 triangles in pattern 4.

3 **a** Common difference = 11, so $11n$ is in the sequence.

When $n = 1$:

$11n = 11$, but the 1st term is 3.

$3 = 11n - 8$

So the expression for the sequence is $11n - 8$

b Assume 100 is in the sequence. Then:

$11n - 8 = 100$

$11n = 108$

$n = 108 \div 11 = 9$ remainder 9

But n must be a whole number, and it is not; so 100 is not in this sequence.

Non-linear sequences

1 **a** Rule is multiply by 2.

$8 \times 2 = 16$

$16 \times 2 = 32$

So terms are 16, 32.

b Rule is divide by 10.

$1 \div 10 = 0.1$

$0.1 \div 10 = 0.01$

So terms are 0.1, 0.01

c Rule is multiply by -2.

$-12 \times -2 = 24$

$24 \times -2 = -48$

So terms are 24, -48.

d They involve multiplying and dividing, not adding and subtracting, so they are geometric.

2 **a** Next term = 6 + 9 =15

b 5th term = 6 + 9 = 15

6th term = 9 + 15 = 24

7th term = 15 + 24 = 39

8th term = 24 + 39 = 63

9th term = 39 + 63 = 102

The 9th term is the first term in the sequence over 100

3 **a**

Day	Mon	Tue	Wed	Thu	Fri
Number of ladybirds	2	8 (= 2 × 4)	32 (= 8 × 4)	32 × 4 = 128	128 × 4 = 512

The gardener is correct. There will be more than 500 ladybirds.

b Saturday, because 512 × 4 = 2048.

4 **a** First term: $\frac{1}{2} \times 1^2 = \frac{1}{2}$

Second term: $\frac{1}{2} \times 2^2 = 2$

Third term: $\frac{1}{2} \times 3^2 = \frac{9}{2} = 4\frac{1}{2}$

b If 32 is in the sequence, then:

$\frac{1}{2}n^2 = 32$

$n^2 = 64$

$n = 8$

This gives n as a whole number, 8, so 32 is the 8th term in the sequence.

Show that...

1 LHS = $2x + 1$; RHS = $2x + 1$; LHS = RHS. Therefore, $2\left(x + \frac{1}{2}\right) \equiv x + x + 1$

2 LHS = $x^2 - 25 + 9 = x^2 - 16$; RHS = $x^2 - 16$

3 Let the three consecutive numbers be n, $n + 1$ and $n + 2$.

$n + n + 1 + n + 2 = 3n + 3 = 3(n+1)$. Therefore, the sum of three consecutive numbers is a multiple of 3.

4 **a** Width of pond = $x - y + x + x + x - y = 4x - 2y$

Length of pond = $4x$

Perimeter = $4x - 2y + 4x - 2y + 4x + 4x = 16x - 4y$

b Yes Sanjit is correct, because $16x - 4y = 4(4x - y)$, showing that when x and y are whole numbers, the perimeter is always a multiple of 4.

Number machines

1 **a** when $x = 3$, $y = 3 \times 4 - 1 = 11$

b when $y = 23$, $4x - 1 = 23$, therefore $x = (23 + 1) \div 4 = 6$

c To get y you multiply x by 4 and subtract 1, so $y = 4x - 1$

2

x	Operations	y
−2	(−2) × 2 + 3	−1
0	0 × 2 + 3	3
3	(9 − 3) ÷ 2	9

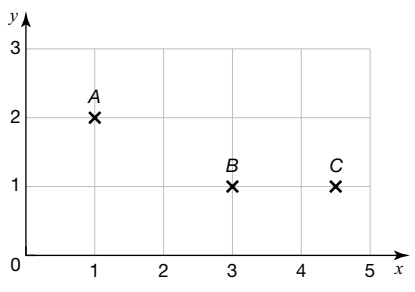
3

x	Operations	y
−2	(−2) ÷ 2 + 1	0
1	(1) ÷ 2 + 1	$1\frac{1}{2}$
8	(5 − 1) × 2	5

Coordinates and midpoints

1 **a** 1 along the x-axis and 2 up the y-axis: (1, 2)

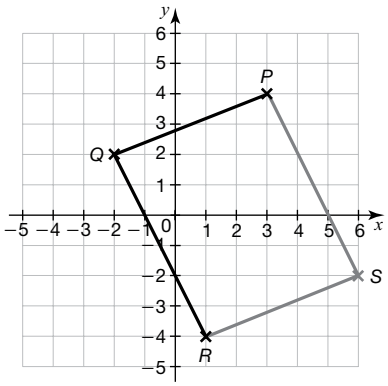
b



c $4\frac{1}{2}$ along the x-axis and 1 up the y-axis: $(4\frac{1}{2}, 1)$

2 **a** 1 along the x-axis and −4 'up' the y-axis: (1, −4)

b



S = (6, −2) to make a parallelogram

3 **a** x coordinate = $\frac{4 + (-2)}{2} = 1$

y coordinate = $\frac{5 + 1}{2} = 3$

Midpoint of XY is (1, 3).

b Midpoint of $XZ = \left(\frac{4 + 4}{2}, \frac{5 + (-4)}{2}\right) = (4, \frac{1}{2})$

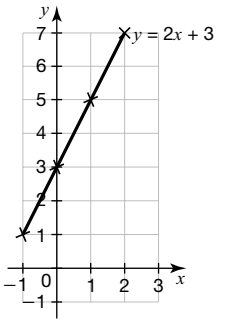
c Midpoint of $YZ = \left(\frac{(-2) + 4}{2}, \frac{1 + (-4)}{2}\right) = (1, -1\frac{1}{2})$

Straight line graphs

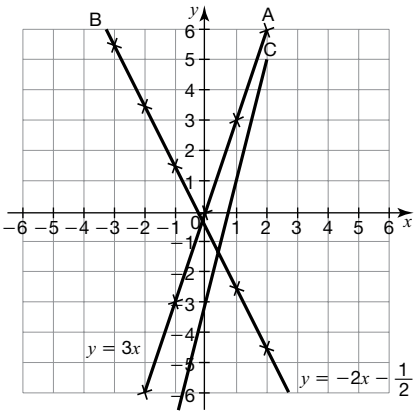
1 **a** For $y = 2x + 3$

x	−1	0	1	2
Operations	2 × (−1) + 3	2 × (0) + 3	2 × (1) + 3	2 × (2) + 3
y	**1**	**3**	**5**	**7**

b



2 **a** and **b**



a For $y = 3x$

x	−2	−1	0	1	2
Operations	3 × (−2)	3 × (−1)	3 × (0)	3 × (1)	3 × (2)
y	**−6**	**−3**	**0**	**3**	**6**

b Rearrange the equation to give $y = -2x - \frac{1}{2}$

x	Operations	y
-3	$-2 \times (-3) - \frac{1}{2}$	$5\frac{1}{2}$
-2	$-2 \times (-2) - \frac{1}{2}$	$3\frac{1}{2}$
-1	$-2 \times (-1) - \frac{1}{2}$	$1\frac{1}{2}$
0	$-2 \times (0) - \frac{1}{2}$	$-\frac{1}{2}$
1	$-2 \times (1) - \frac{1}{2}$	$-2\frac{1}{2}$
2	$-2 \times (2) - \frac{1}{2}$	$-4\frac{1}{2}$

c Line C goes through points (0, -3), (1, 1) and (2, 5)

The y intercept is -3.

The gradient is $\frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}} = \frac{5-1}{2-1} = 4$

The equation of line C is $y = 4x - 3$.

3 **a** B and C, because they have the same gradient of 2.

b A and B, because they both have a y-intercept at (0, 1).

4 The gradient is $\frac{2-(-6)}{3-(-1)} = \frac{8}{4} = 2$

Using point (3, 2) and gradient m $= 2$:

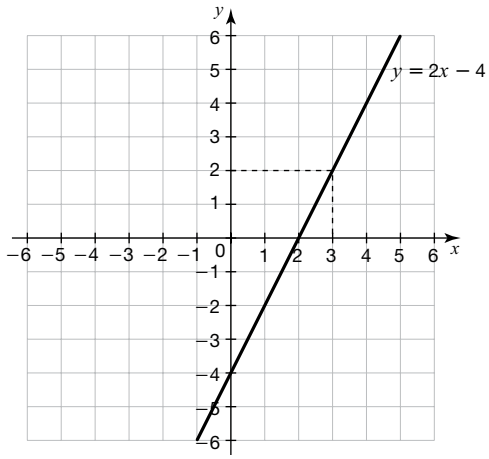
$y = 2x + c$

$2 = 2 \times 3 + c$

$c = -4$

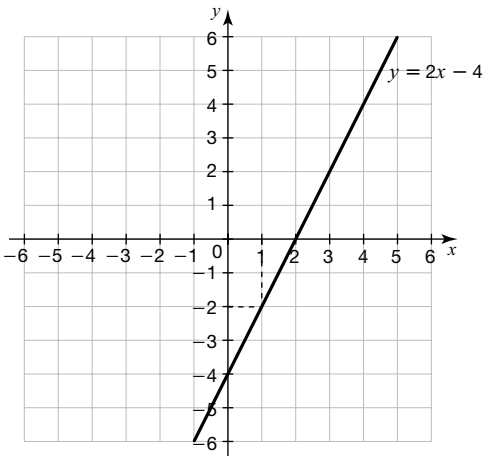
Equation of the line is $y = 2x - 4$

5 **a**



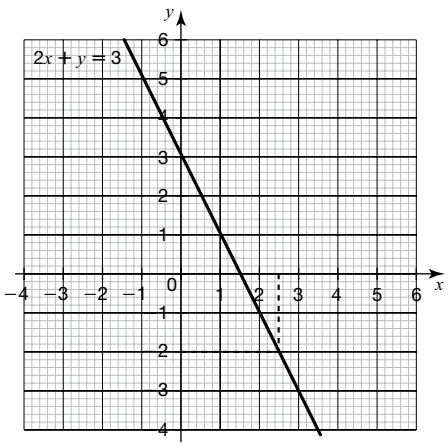
Draw a vertical line up from $x = 3$ to the graph, and then a horizontal line to the y-axis to read off the result: $y = 2$.

b



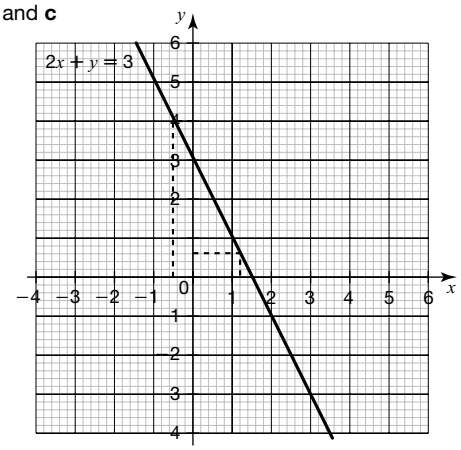
Draw a horizontal line across from $y = -2$ to the graph, and then a vertical line up to the x-axis to read off the result: $x = 1$.

6 **a**



Draw a horizontal line across from $y = -2$ to the graph, and then a vertical line up to the x-axis to read off the result: $x = 2.5$.

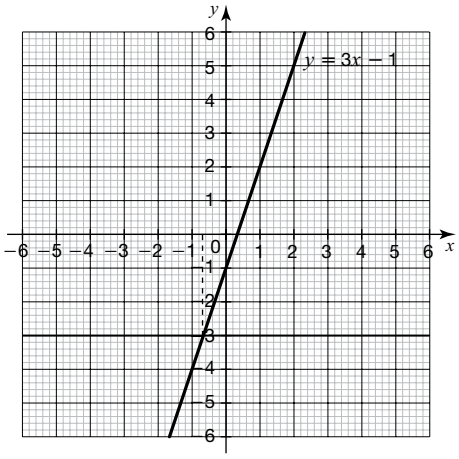
b and **c**



b Draw a vertical line up from $x = -0.5$ to the graph, and then a vertical line across to the y-axis to read off the result: $y = 4$.

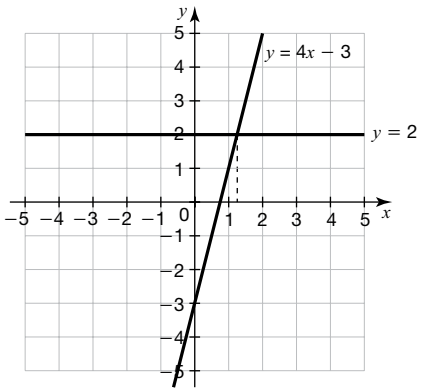
c Reading up from $x = 1.2$ and then across to y axis gives $y = 0.6$. Any value from 0.6 to 0.75 is acceptable.

7 **a** and **b**



b Where the graphs cross, draw a vertical line up to the x-axis. It meets the axis two thirds of the way between $x = -1$ and $x = 0$, so the solution is approximately $x = -0.67$. Any value from -0.6 to -0.7 is acceptable.

8



Compare the equations $y = 4x - 3$ and $4x - 3 = 2$.
y has been replaced with 2, so add line $y = 2$ to the graph.

The intersection point of the two graphs gives the solution to the equation $4x - 3 = 2$

$x = 1.25$. Any answer between 1.2 and 1.3 is acceptable.

Solving simultaneous equations

1 a Substituting $y = 2x$ into the first equation gives

$3x + 2x = 15$

$5x = 15$

$x = 3$

When $x = 3$, $y = 2 \times 3 = 6$

b $3x = 12$

$x = 4$

Substituting $x = 4$ into the first equation gives

$8 + y = 9$

$y = 1$

c $5x = 10$

$x = 2$

Substituting $x = 2$ into the first equation gives

$6 + y = 4$

$y = -2$

2 a $2x + 2y = 14$ (1)

$3x + y = 11$ (2)

(2) × 2 $6x + 2y = 22$

(3) − (1) $4x = 8$

$x = 2$

Substitute into (1) $4 + 2y = 14$

$2y = 10$

$y = 5$

Solution is $x = 2$, $y = 5$

b $4x - 2y = 2$ (1)

$2x - 3y = 7$ (2)

(2) × 2 $4x - 6y = 14$ (3)

(3) − (1) $-4y = 12$

$y = -3$

Substitute into (1) $4x + 6 = 2$

$4x = -4$

$x = -1$

Solution is $x = -1$, $y = -3$

c $2x + 3y = 20$ (1)

$3x + 2y = 15$ (2)

(1) × 2 $4x + 6y = 40$ (3)

(2) × 3 $9x + 6y = 45$ (4)

(4) − (3) $5x = 5$

$x = 1$

Substituting $x = 1$ into equation (1) $2 + 3y = 20$

$3y = 18$

$y = 6$

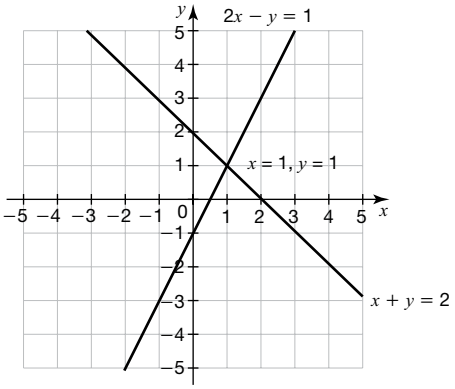
Solution is $x = 1$, $y = 6$

3 a $x + y = 2$ $2x - y = 1$

x	0	2
y	2	0

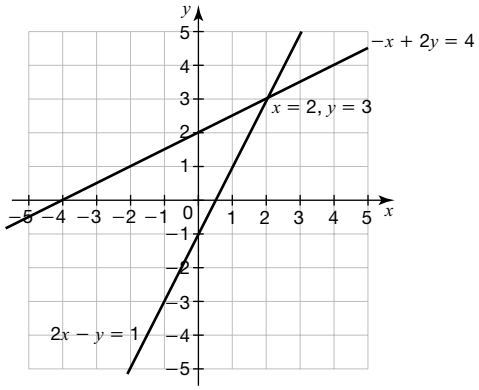
x	0	$\frac{1}{2}$
y	−1	0

b



From the intersection of the two lines, $x = 1$, $y = 1$.

c



From the intersection of the two lines, $x = 2$, $y = 3$.

Quadratic graphs

1 a C and D – they are straight lines so they are linear.

b E – it is quadratic, with a positive multiplier for x^2, and is symmetrical about the origin.

c A – it is quadratic, with a negative multiplier for x^2, and is symmetrical about the origin.

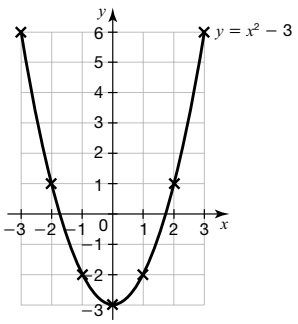
d D – the x coordinates are all different, but all the y coordinates on this line are 1.

e B – it is the same as E except that it has been moved 1 unit up the y-axis.

2 a

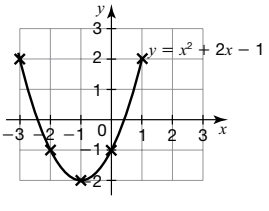
x	−3	−2	−1	0	1	2	3
y	6	1	−2	−3	−2	1	6

b



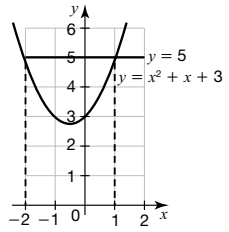
c $x = 0$ **d** (0, −3)

3 a



b $x = -1$ **c** $(-1, -2)$

4 a

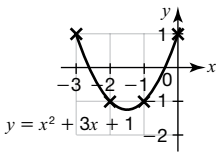


b $x = -2$ or $x = 1$

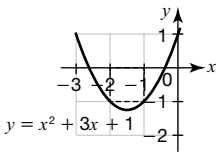
5 $x = -3$ or $x = 1$

6 a

x	−3	−2	−1	0
$x^2 + 3x + 1$	$(-3)^2 + 3\times(-3) + 1$	$(-2)^2 + 3 \times (-2) + 1$	$(-1)^2 + 3 \times (-1) + 1$	$(0)^2 + 3\times(0) + 1$
y	**1**	**−1**	**−1**	**1**



b Compare the equations $y = x^2 + 3x + 1$ and $x^2 + 3x + 1 = -1$. y has been replaced with -1, so the solutions to the equation $x^2 + 3x + 1 = -1$ are where $y = -1$.



$x = -2$ and $x = -1$

c Compare the equations $y = x^2 + 3x + 1$ and $x^2 + 3x + 1 = 0$. y has been replaced with 0, so the solutions to the equation $x^2 + 3x + 1 = 0$ are where the graph crosses the x-axis.

$x = -2.6$ and $x = -0.38$ (any answer close to -0.4 is acceptable)

Solving quadratic equations

1 a $x(x + 6) = 0$
$x = 0$ or -6

b $y(y - 11) = 0$
$y = 0$ or 11

c $3d(d - 3) = 0$
$d = 0$ or 3

2 a $(x + 4)(x - 4) = 0$
$x = 4$ or -4

b $(a + 9)(a - 9) = 0$
$a = 9$ or -9

c $(z - 10)(z + 10) = 0$
$z = 10$ or -10

3 a $(x + 3)(x + 2) = 0$
$x = -2$ or -3

b $(x + 5)(x - 2) = 0$
$x = -5$ or 2

c $(x - 7)(x - 2) = 0$
$x = 2$ or 7

4 a $0 = x(x - 3)$
$x = 0$ or 3

b $0 = (x - 5)(x + 5)$
$x = 5$ or -5

c $0 = (x + 6)(x - 3)$
$x = -6$ or 3

Cubic and reciprocal graphs

1 a A, C and D – they are not continuous curves with two turning points (s-shaped curves).

b B – it has two turning points and has rotational symmetry about the origin.

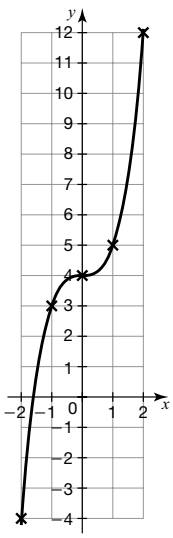
c E – it has two turning points and is a reflection of B, raised up one unit on the y-axis.

d D – this is the form for a reciprocal graph.

2 a

x	−2	−1	0	1	2
y	−4	3	4	5	12

b

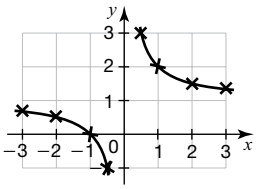


3 a cubic **b** (0, −8) **c** (2, 0)

4 a

x	−3	−2	−1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	3
y	$\frac{2}{3}$	$\frac{1}{2}$	0	−1	3	2	$1\frac{1}{2}$	$1\frac{1}{3}$

b



Drawing and interpreting real-life graphs

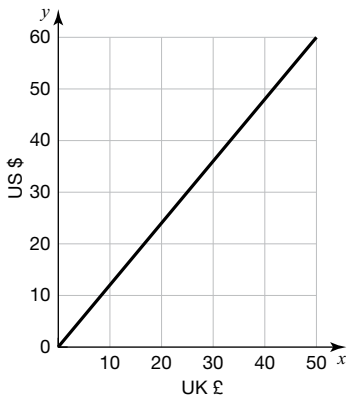
1 a The initial charge is the value at 0 miles, where the graph cuts the y-axis (the y-intercept): \$3.

b The charge per mile is given by the gradient of the graph.

Gradient $= \frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}} = \frac{23 - 3}{8 - 0} = \frac{20}{8} = 2.5$

Charge per mile is \$2.50

c



d From the conversion graph, £15 = \$18.

From the graph of the cost of a taxi in New York, \$18 allows you to travel 6 miles.

2 a You can read this from the highest point of the graph: 16 m.

b Read from the highest point down to the value on the horizontal axis: 4 seconds.

c This is when the ball reaches 0 height for the second time: 8 seconds.

d Read from 12 on the vertical axis across to the curve: 2 seconds and 6 seconds.

3 **a** This is the highest value on the vertical axis: 10 m/s.

b The cyclist is travelling at a constant speed of 10 m/s.

c The cyclist is decelerating, so it will be a negative value.

Acceleration $= \frac{-10}{20} = -\frac{1}{2}$ m/s^2

d The speed returns to 0: the cyclist stops.

4 **a** Assuming that water pours into the container at a constant rate,

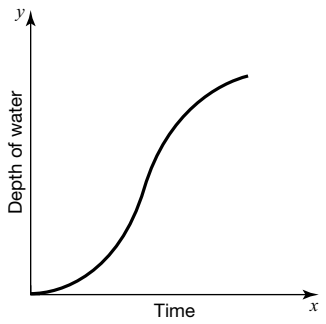
A: depth goes up increasingly slowly as the container widens, so 2.

B: depth rises steadily and fairly slowly in a broad container of consistent diameter, so 4.

C: depth rises quickly at first, then more slowly as the container widens, then more quickly as it gets narrow towards the top, so 3.

D: depth rises steadily and quickly in a narrow container of consistent diameter, so 1.

b



Ratio, proportion and rates of change

Units of measure

1 **a** 4 (m) × 100 = 400 cm

b 500 (g) ÷ 1000 = 5 kg

c 1.5 (*l*) × 1000 = 1500 ml

d 8250 (m) ÷ 1000 = 8.25 km

2 6 (litres) × 1000 = 6000 ml

6000 − 3500 = 2500

Sally had **2500 ml** of lemonade left.

3 **a** Luke: 240 seconds; Adam: 3 × 60 + 47 = 227 seconds. Adam arrived first.

or Luke: 240 ÷ 60 = 4 minutes; Adam: 3 minutes 47 seconds. Adam arrived first.

b 4 minutes − 3 minutes 47 seconds = 13 seconds. Adam waited **13 seconds** for Luke to arrive at school.

4 Ben = 1.25 m = 3.2 + 0.8 feet = 4 feet. Tom is taller.

or Tom = 4.8 feet = 3.2 + 1.6 feet = 1 + 0.5 metres = 1.5 metres. Tom is taller.

Ratio

1 12 stories and 8 colouring

Ratio of story: colouring = 12 : 8 = 3 : 2

2 Total parts = 1 + 2 = 3

1 part $= \frac{15}{3} = 5$

Spent = 2 × 5 = £10

3 Total parts = 1 + 2 + 3 = 6

1 part $= \frac{60}{6} = 10$

Amount given to charity = 3 × 10 = £30

4 Ratio of blue to yellow required is 3 : 7.

There are 3 + 7 = 10 parts. He needs to make 5 litres.

10 parts = 5000 ml

1 part = 500 ml

Phil needs 3 × 500 ml = 1500 ml = 1.5 litres of blue paint. He has 2 litres of blue paint.

Phil needs 7 × 500 ml = 3500 ml = 3.5 litres of yellow paint. He has 3 litres of yellow paint.

Phil has enough blue paint, but does not have enough yellow paint.

Scale diagrams and maps

1 1 cm on the map is **10000 cm** in real life.

This means 1 cm on the map is **100 m** in real life.

2 1 cm on the map represents 50 m in real life.

3 × 50 m = 150 m, so the bus stop is **3 cm** from the village shop on the map.

3 Measure the distance between the trees on the diagram = 5 cm

1 cm on the diagram represents 4 m in real life.

5 × 4 = 20

The trees are **20 m** apart.

4 A scale of 1 : 400 means 1 cm on the model represents 4 m (= 400 cm) in real life.

96 ÷ 4 = 24

The scale mode is **24 cm** tall.

Fractions, percentages and proportion

1 1 + 3 = 4 parts so Bess receives $\frac{3}{4}$

2 **a** 1 : 3 : 6

b 1 + 3 + 6 = 10 items in the basket

Fruit $= \frac{3}{10}$

c Tins $= \frac{6}{10} = 60\%$

3 $\frac{50}{4000} = \frac{1}{80}$

4 Total parts = 3 + 8 + 14 = 25

$\frac{8}{25} = \frac{32}{100} = 32\%$

Direct proportion

1 **a** One ticket costs £80 ÷ 5 = £16

b Nine tickets cost 9 × £16 = £144

2 **a** Read up from 6 packs on the horizontal axis, to the line, then across to the vertical axis to find the cost: £1.20

b There are 10 pencils in a pack, so 1 pencil is 0.1 of a pack. Reading off the graph using this value, the price is 2p.

c It is a straight-line graph; the graph passes through the origin (0, 0).

3 **a** Sally needs to make 28 ÷ 4 = 7 lots of the recipe.

She will need 1 × 7 = 7 teaspoons of turmeric, 2 × 7 = 14 teaspoons of chilli powder and $2\frac{1}{2} \times 7 = 17\frac{1}{2}$ teaspoons of cumin.

b Sally has 75 g of chilli powder. That is 75 ÷ 3 = 25 teaspoons.

Sally needs 14 teaspoons to make the curry for her class. She does have enough.

Inverse proportion

1 **a** Start from 5 on the x-axis, read up to the graph, then left to the scale on the y-axis.

5 winners will each get **£400**.

b Start from 200 on the y-axis, read right to the graph, then down to the scale on the x-axis. 10 winners each get £200, so there are **9** other winners.

c Multiply the number of winners by the amount each one gets to find the total prize money. e.g. 5 × £400 = **£2000**.

d Compare 2 points on the graph. (Use your answers to parts **a** and **b**.) If the number of winners goes up, the prize money they each receive goes down. The prize money and the number of winners are in inverse proportion.

2 **a** The total time needed to decorate the room is 3 × 2 = 6 hours.

b 6 hours ÷ 12 people = 0.5 hours = **30 minutes** (or 0.5 hours).

3 **a** y decreases from 1 to $\frac{1}{8}$ as x increases from 1 to 8 (when $x = 1$, $y = 1$; when $x = 8$, $y = \frac{1}{8}$)

b x is inversely proportional to y, because as x increases, y decreases.

4 The printer can print $240 \div 4 = 60$ pages per minute.

$600 \div 60 = 10$, so it would take **10 minutes** to print the larger document.

Working with percentages

1 $125 - 75 = 50$

$\frac{50}{125} \times 100 = 40\%$

2 **a** Amount of increase $= 24 - 15 =$ £9 million

$\frac{9}{15} \times 100 =$ **60%** increase in sales for Company X

b $125\% =$ £35 million

$35 \div 125 \times 100 =$ **£28 million** sales in 2006.

3 **a** $\frac{2}{100} \times 265 = 5.30$ and $3 \times 5.30 = 15.90$

$15.90 + 265 =$ £280.90

b $265 \times (1.02)^3 =$ £281.22

Compound units

1 $\frac{3\text{ km}}{\text{minute}} = \frac{3000\text{ m}}{\text{minute}} = \frac{3000\text{ m}}{60\text{ seconds}} = 50\text{ m/s}$

2 $20 \div 5 = 4$ minutes to fill the tank.

3 Pressure $= \frac{300}{0.05} = 6000$ Newtons/m^2 (**or** 6000 N/m^2)

4 On Saturday Sami drove $4 \times 50 = 200$ miles; on Sunday Sami drove $356 \div 8 \times 5 = 222.5$ miles. Sami drove further on Sunday.

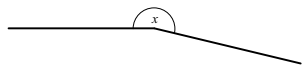
Geometry and measures

Measuring and drawing angles

1 **a** 123°

b 42°

c 331°

2 

3 **a** 100°, 120°, 140°, 160°

b First angle + second angle = 87°. This means both angles are less than 87°, and so they both must be acute.

Using the properties of angles

1 **a** Angle $ACB = 52°$ (Angles in a triangle add up to 180°)

$x = 128°$ (Angles on a straight line add up to 180°)

b Angle $ADC = 86°$ (Angles in a quadrilateral add up to 360°)

$x = 94°$ (Angles on a straight line add up to 180°)

2 **a** Angle $BED = 39°$ (Alternate angles are equal)

Angle $BDE = 39°$ (Base angles in an isosceles triangle are equal)

$x = 102°$ (Angles in a triangle add up to 180°)

b Angle $DCF = 98°$ (Vertically opposite angles are equal)

$x = 98°$ (Corresponding angles are equal)

3 Angle $CFG = 62°$ (Co-interior angles add up to 180°)

$x = 66°$ (Angles on a straight line add up to 180°)

4 $x + 40 + 3x + 5x - 40 = 180°$

$9x = 180°$

$x = 20°$

Angle $BAC = x + 40 = 20 + 40 = 60°$

Angle $ACB = 3x = 3 \times 20 = 60°$

Angle $ABC = 5x - 40 = 5 \times 20 - 40 = 60°$

Triangle ABC has equal angles of 60°. Therefore, it is an equilateral triangle.

Using the properties of polygons

1 **a** $180° \times (6 - 2) = 720°$

b $720° \div 6 = 120°$

c $180° - 120° = 60°$ or $360° \div 6 = 60°$

2 **a** It is an octagon because it has eight sides.

b All angles are equal; all sides are equal.

c $180° \times (8 - 2) = 1080°$

$1080° \div 8 = 135°$

or $180° - (360° \div 8) = 135°$

3 exterior angle $= 180° - 144° = 36°$

number of sides $= 360 \div 36 = 10$

Therefore it is a decagon.

Using bearings

1 **a** This is the angle measured clockwise from North at A: 065°.

b $180 - 138 = 42°$ This is the acute angle at C.

Bearing of B from $C = 360 - 42° = 318°$

c $180 - 65 = 115°$ This is the angle between the north line at B and AB, measured anticlockwise.

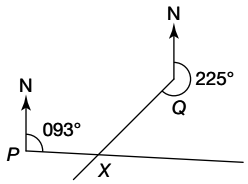
Bearing of A from $B = 360 - 115 = 245°$

2 To find a reciprocal bearing, subtract 180 from the original bearing (or add 180 to it).

The bearing of O from $X = 276 - 180 = 096°$

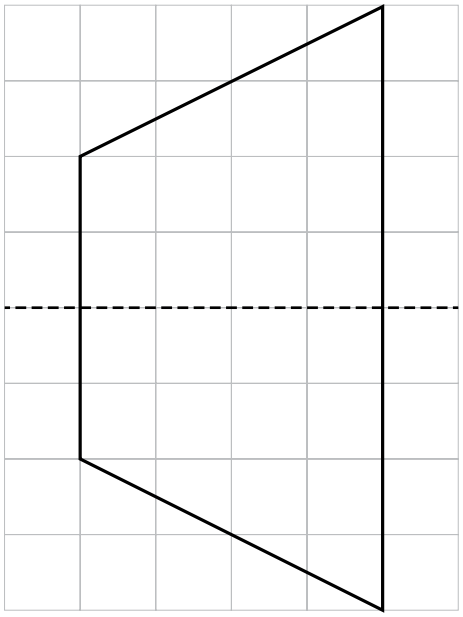
3 **a** Draw a North line at P, then join P to Q and measure the angle between the North line and this line: 060° (any value 058° to 062° accepted).

b



Properties of 2D shapes

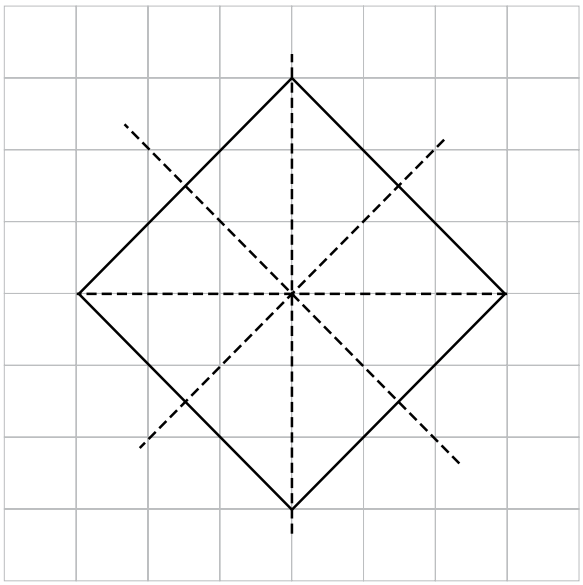
1 **a**



b trapezium

c one pair of parallel sides

2 a



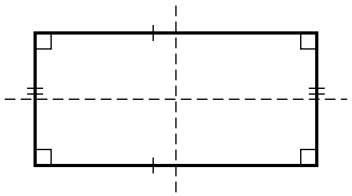
b 4

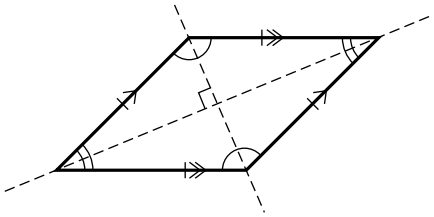
c square

d Two from: all sides equal in length; all angles are 90°; diagonals are equal; diagonals bisect each other at 90°.

3 a rectangle, rhombus

b





Congruent shapes

1 D and F are exactly the same as A – they are the same size and shape (it doesn't matter that they are rotated). E is similar to A, not congruent – it is smaller.

2 If the triangles are congruent, all three angles must be the same in both. You know that two of the angles are 35° and 82°, so $x = 180 - 35 - 82 = 63°$.

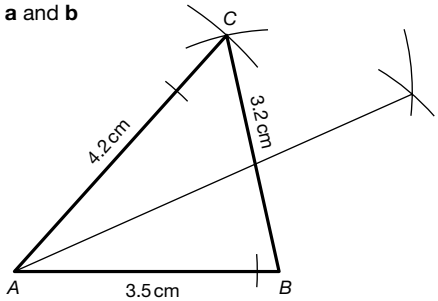
3 a Identify what values match: SAS (side, angle, side – two sides and the angle between them).

b Identify what values match: ASA (angle, side angle – two angles and a corresponding side).

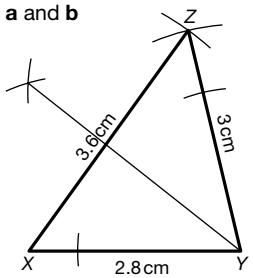
4 No, they are not congruent. They have the same angles, but the sides may not be the same size (one triangle could be an enlargement of the other).

Constructions

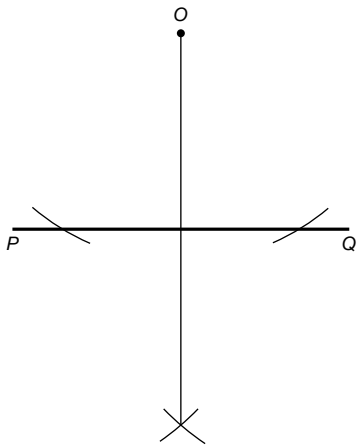
1 a and **b**



2 a and **b**



3 a



b Distance on the diagram = 2.5 cm

$2.5 \times 100 = 250$ cm in real life = 2.5 m

Drawing circles and parts of circles

1 a–d

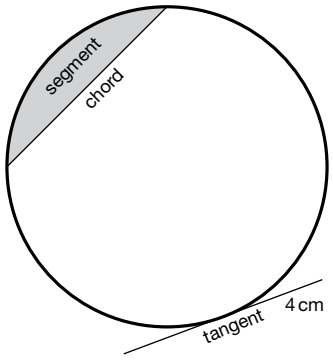


Diagram not to scale

2 a and **b**

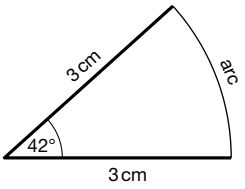
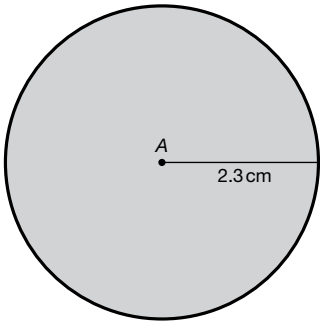


Diagram not to scale

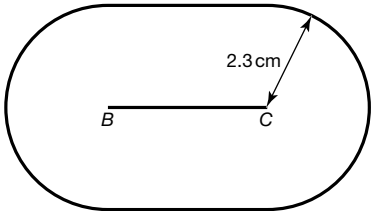
3 No, Donald is not correct. A segment of a circle is the area enclosed by a chord and an arc; a sector of a circle is the area enclosed by two radii and the arc between them.

Loci

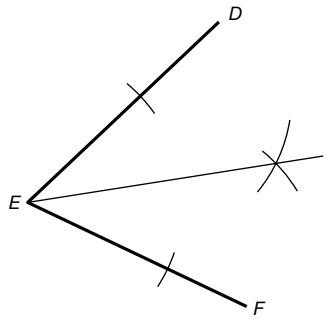
1 a



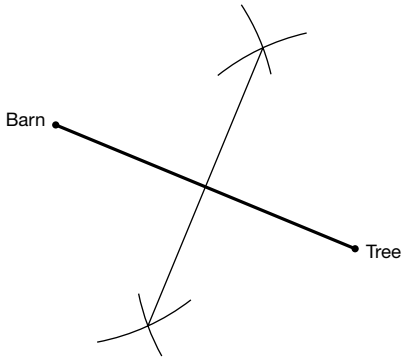
b



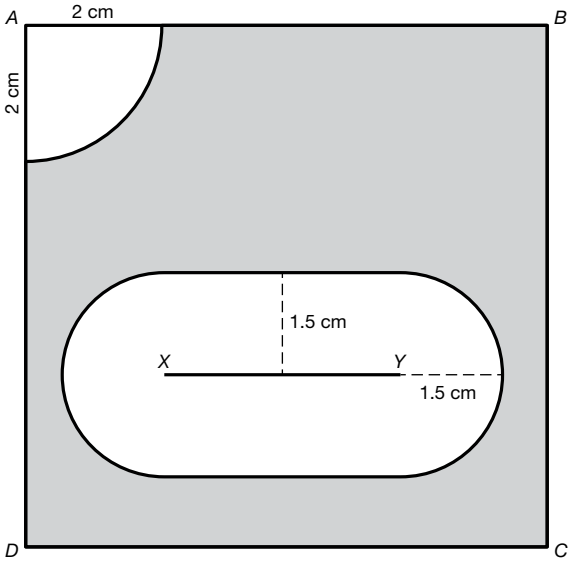
c



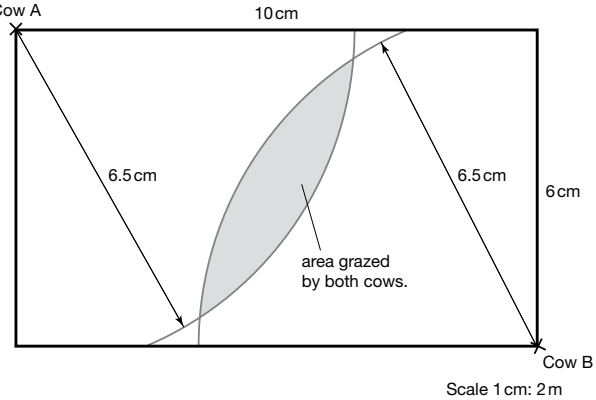
2



3



4



Scale 1 cm: 2 m

Perimeter

1 A hexagon has 6 sides so perimeter $= 6 \times 9 = 54$ cm

2 Missing vertical length $= 20 - 5 - 5 - 4 = 6$ mm

Missing horizontal lengths are all equal $= 25 - 13 = 12$ mm each

Perimeter $= 20 + 25 + 6 + 12 + 4 + 12 + 5 + 12 + 5 + 13$
$= 114$ mm

$114 \div 10 = 11.4$ cm

3 Perimeter of cushion $= \frac{1}{2} \times 2 \times \pi \times 24 + 30 + 48 + 30$

$= 183$ cm (to nearest cm)

$= 1.83$ m. So no, Greta does not have enough lace.

Area

1 **a** Area $= 12 \times 6 = 72$ cm^2

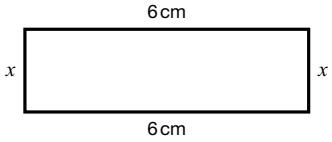
b Area $= \frac{1}{2} \times (3 + 8) \times 4 = 22$ cm^2

c Area of rectangle $= 2 \times 10 = 20$ cm^2

Area of trapezium $= \frac{1}{2}(a + b)h = \frac{1}{2}(2.5 + 10)10$
$= 62.5$ cm^2

Area of shape $= 20 + 62.5 = 82.5$ cm^2

2 First draw a diagram. Two sides are equal, and are 6 cm. The two other sides are equal, and are x cm.



$x + x + 6 + 6 = 16$

$2x + 12 = 16$

$x = \frac{16 - 12}{2} = 2.$

Area $= 6 \times 2 = 12$ cm^2

3 Radius of circle $= 3.5$ cm

Area of circle $= \pi r^2 = \pi \times 3.5^2 = 38.48$ cm^2

Area of square $= 49$ cm^2

Area of shaded part $= \frac{49 - 38.48}{4} = 2.63$ cm^2

Sectors

1 **a** $\frac{120}{360} = \frac{1}{3}$

b Area $= \frac{1}{3} \times \pi \times 3^2 = 3\pi$ cm^2

2 **a** Area $= \frac{1}{4} \times \pi \times 2.8^2 = 6.2$ cm^2 (1 d.p.)

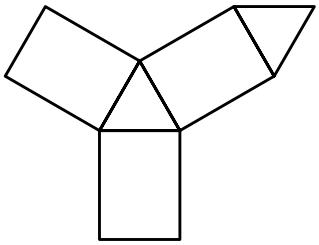
b Perimeter $= 2.8 + 2.8 + \frac{1}{4} \times 2 \times \pi \times 2.8$
$= 10.0$ cm (1 d.p.)

3 **a** Area $= \frac{40}{360} \times \pi \times 5^2 = 8.73$ cm^2 (2 d.p.)

b Arc $AB = \frac{40}{360} \times 2 \times \pi \times 5 = 3.49$ cm (2 d.p.)

3D shapes

1 **a**

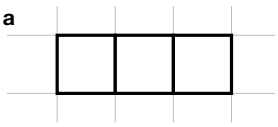


b triangular prism

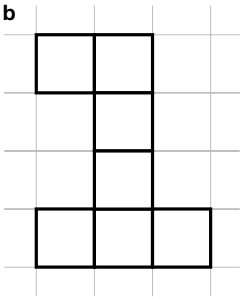
c

Number of faces	Number of edges	Number of vertices
5	9	6

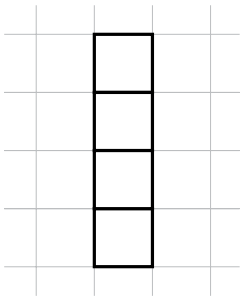
2 a



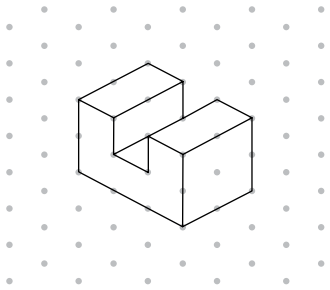
b



c



3 a



3 b The front elevation shows 5 cubes and the side shows that the shape is 2 cubes deep. $5 \times 2 = 10$, so 10 cubes make up the shape.

Volume

1 The front elevation shows 5 cubes and the side shows that the shape is 4 cubes deep.Volume $= 5 \times 4 = 20\ \text{cm}^3$

2 a Volume = area of cross-section × length $= \pi \times 5^2 \times 12$
$= 942\,\text{cm}^3$ (to 3 s.f.)

b Volume $= \frac{1}{3} \times$ area of cross-section × length
$= \frac{1}{3} \times \pi \times 7^2 \times 15 = 770\,\text{cm}^3$ (to nearest cm)

3 Volume $= \frac{1}{2} \times \frac{4}{3} \pi r^3 = \frac{2}{3} \times \pi \times 8^3 = 1072.33\,\text{cm}^3$ (to 2 d.p.)

4 Volume of tank $= 40 \times 40 \times 60 = 96\,000\,\text{cm}^3$

Volume of water in tank, 80% full $= 0.8 \times 96\,000$
$= 76\,800\,\text{cm}^3$

Height of water in pond (1st fill) $= 76\,800 \div (80 \times 60) = 16\,\text{cm}$

Height of water in pond (2nd fill) $= 16 \times 2 = 32\,\text{cm}$

Height of water in pond (3rd fill) $= 16 \times 3 = 48\,\text{cm}$

Three tanks of water are needed to fill the pond.

Alternative method: divide volume of pond by volume of water in tank.

$\frac{80 \times 60 \times 48}{76\,800} = 3$

Surface area

1 a 6 faces

b Surface area $= 60 + 60 + 5 + 5 + 3 + 3 = 136\ \text{cm}^2$

2 surface area = area of triangular side × 4 + area of square base

$= \left(\frac{1}{2} \times 6 \times 5\right) \times 4 + 6^2$

$= (15 \times 4) + 36$

Surface area $= 96\ \text{cm}^2$

3 a Surface area of a sphere $= 4\pi r^2 = 4 \times \pi \times 14^2$
$= 2463.01\,\text{cm}^2$ (to 2 d.p.)

b Surface area of a cone $= \pi rl + \pi r^2 = \pi \times 6 \times 10 + \pi \times 6^2$
$= 301.59\,\text{cm}^2$ (to 2 d.p.)

4 Area of cylinder $= 2\pi rh = 2\pi \times 6 \times 1.5 = 56.55\,\text{cm}^2$

Area of circular base $= \pi r^2 = \pi \times 6^2 = 113.10\,\text{cm}^2$

Area of curved surface area of cone $= \pi rl = \pi \times 6 \times 11.5$
$= 216.77\,\text{cm}^2$

Total surface area $= 56.55 + 113.10 + 216.77 = 386.42\,\text{cm}^2$ (to 2 d.p.)

Using Pythagoras' theorem

1 $x^2 = 3^2 + 4^2$
$= 9 + 16$
$= 25$
$x = \sqrt{25}$
$= 5\ \text{cm}$

$15^2 = y^2 + 12^2$
$225 = y^2 + 144$
$81 = y^2$
$y = \sqrt{81}$
$= 9\,\text{cm}$

2 $6^2 = 4.5^2 + w^2$
$36 = 20.25 + w^2$
$w = \sqrt{15.75}$
$= 3.97\,\text{cm}$

Area $= l \times w = 4.5 \times 3.97 = 17.9\,\text{cm}^2$

3 $AB^2 = 2^2 + 4^2$
$= 4 + 16$
$= 20$
$AB = \sqrt{20}$
$= \sqrt{4 \times 5}$
$= 2\sqrt{5}$ units

4 Square of diagonal of doorway $= 70^2 + 190^2 = 41\,000$

Diagonal of doorway $= \sqrt{41\,000}$

$= 202.48\,\text{cm} = 2.0248\,\text{m} = 2.02\,\text{m}$ (2 d.p.)

Yes, the artwork will fit through the diagonal of the doorway.

Trigonometry

1 $\tan x = \frac{8}{13}$
$x = 31.6°$

2 $\cos 42 = \frac{17}{AC}$
$AC = \frac{17}{\cos 42} = 22.9\,\text{cm}$

3 $\sin 49 = \frac{h}{6}$
$h = 6 \sin 49$
$= 4.53\,\text{m}$

Exact trigonometric values

1 a $\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{1} = 1$

b $x = \tan^{-1}(1) = 45°$

2 $\cos 30 = \frac{\sqrt{3}}{PR}$
$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{PR}$
$PR = 2\,\text{cm}$

3 $\frac{YZ}{20} = \sin 30$
$YZ = 20 \sin 30$
$= 20 \times \frac{1}{2}$
$= 10\,\text{cm}$

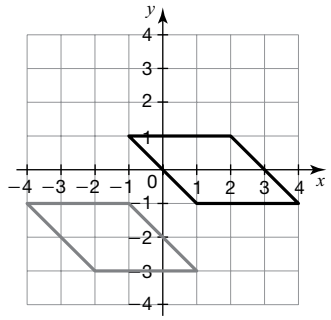
4 $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$

So the second angle in the triangle must be 45°.

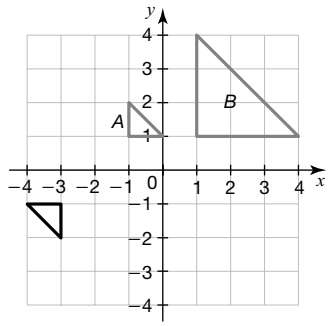
The third angle will be $180 - 90 - 45 = 45°$.

Transformations

1

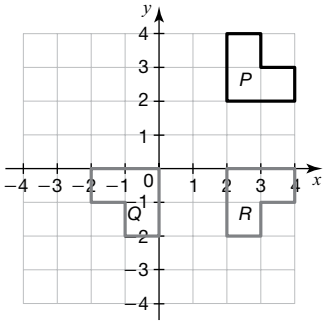


2



3 Rotation 90° clockwise about (1, −1); or rotation 270° anticlockwise about (1, −1).

4 **a** and **b**



c Reflection in $y = 1$

Similar shapes

1 **a** *YZW*

b Scale of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{6.3}{2.1} = 3$

c $WZ = 4 \times 3 = 12\,\text{cm}$

2 **a** 37.5°

b Scale of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{5}{2.5} = 2$

Length of $AB = 8 \div 2 = 4$ cm

c They are isosceles, because they have two equal sides and two equal angles. Note that the diagrams are not drawn to scale, as is common practice in maths questions – you have to go by the numbers.

d Length of BC = length of $AC = 2.5\,\text{cm}$

3 **a** Scale of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{4}{6} = \frac{2}{3}$

b Length of $RT = 4.5 \times \frac{2}{3} = 3\,\text{cm}$

4 **a** Scale of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{2.2}{4.4} = \frac{1}{2}$

b Length of CE = length of $CD \div \frac{1}{2} = 2.8 \times 2 = 5.6\,\text{cm}$

c Angle $ACE = 180 - 70 - 64 = 46°$

Vectors

1 $\mathbf{a} = \begin{pmatrix}3\\2\end{pmatrix}$ $\mathbf{b} = \begin{pmatrix}-3\\3\end{pmatrix}$ $\mathbf{c} = \begin{pmatrix}2\\-4\end{pmatrix}$ $\mathbf{d} = \begin{pmatrix}-4\\-2\end{pmatrix}$

2 **a**

b $-\mathbf{p} = \begin{pmatrix}6\\-1\end{pmatrix}$

c $\mathbf{2p} = \begin{pmatrix}-12\\2\end{pmatrix}$

d $\mathbf{2p} + \mathbf{p} = \begin{pmatrix}-12\\2\end{pmatrix} + \begin{pmatrix}-6\\1\end{pmatrix} = \begin{pmatrix}-18\\3\end{pmatrix}$

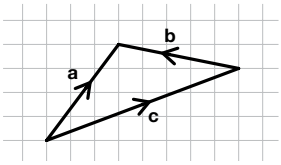
$\mathbf{3p} = 3 \times \begin{pmatrix}-6\\1\end{pmatrix} = \begin{pmatrix}-18\\3\end{pmatrix}$

Therefore, $\mathbf{2p} + \mathbf{p} = \mathbf{3p}$

3 **a** $\mathbf{a} + \mathbf{b} = \begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}-5\\1\end{pmatrix} = \begin{pmatrix}3-5\\4+1\end{pmatrix} = \begin{pmatrix}-2\\5\end{pmatrix}$

b $\mathbf{c} = \mathbf{a} - \mathbf{b} = \begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}-5\\1\end{pmatrix} = \begin{pmatrix}3-(-5)\\4-1\end{pmatrix} = \begin{pmatrix}8\\3\end{pmatrix}$

c



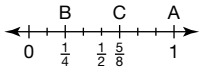
Probability

Basic probability

1 Probability $= \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}} = \frac{1}{10}$ (**or** 0.1 **or** 10%)

2 P(not rain) = 1 − P(rain) = 1 − 0.6 = 0.4

3 **a b c**



a P(a number from 1 to 8) $= \frac{8}{8} = 1$

b P(a multiple of 3) $= \frac{2}{8} = \frac{1}{4}$

c P(a number greater than 3) $= \frac{5}{8}$

4 $2p - 0.1 + 2p + 0.1 + p = 1$

$5p = 1$

$p = 0.2$

Outcome	Red	Blue	Green
Probability	$2p - 0.1$ $= 2 \times 0.2 - 0.1$ $= 0.3$	$2p + 0.1$ $= 2 \times 0.2 + 0.1$ $= 0.5$	$p = 0.2$

Blue is most likely.

Two-way tables and sample space diagrams

1 Work out the missing values one by one, for example in the order shown from first to seventh. (There is more than one order you can do it in.)

	Single	Double	King	Totals
Oak	2	Fourth: 42 − 12 − 14 = 16	Fifth: 30 − 16 − 2 = 12	30
Pine	First: 54 − 14 − 17 = 23	14	17	54
Walnut	1	12	Sixth: 32 − 12 − 17 = 3	Seventh: 1 + 12 + 3 = 16
Totals	Second: 2 + 23 + 1 = 26	Third: 100 − 26 − 32 = 42	32	100

2 **a**

		Spinner			
		1	2	3	4
Coin	Heads	1, H	2, H	3, H	4, H
	Tails	1, T	2, T	3, T	4, T

b $P(1, T) = \frac{1}{8}$

c $P(2, H) + P(3, H) + P(4, H) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

3 a

	Study sciences	Do not study sciences	Totals
Boys	$\frac{1}{5} \times 120 = 24$	$45 - 24 = 21$	$\frac{3}{8} \times 120 = 45$
Girls	$75 - 40 = 35$	$\frac{1}{3} \times 120 = 40$	$120 - 45 = 75$
Totals	$24 + 35 = 59$	$21 + 40 = 61$	120

b 21 boys do not study science, so probability $= \frac{21}{120}$ **or** $\frac{7}{40}$

c There are 75 girls and 35 of them study science, so probability $= \frac{35}{75} = \frac{7}{15}$

Sets and Venn diagrams

1 a $\xi = \{21, 22, 23, 24, 25, 26, 27, 28, 29\}$

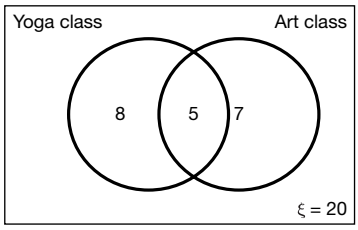
b $A = \{21, 24, 27\}$

c $B = \{24, 28\}$

d $A \cup B = \{21, 24, 27, 28\}$ – A 'union' B means **all** the values in A and **all** the values in B

e $A \cap B = \{24\}$ – A 'intersect' B means only those value that are in **both** A and B.

2 a



b There are 7 adults who only go to art class, so probability $= \frac{7}{20}$

c 8 adults only go to yoga class and 7 adults only go to art class.
Probability $= \frac{8 + 7}{20} = \frac{3}{4}$

3 a Total number of respondents $= \xi$
$= 25 + 11 + 2 + 12 + 0 + 7 + 3 = 60$
P(supermarket only) $= \frac{25}{60} = \frac{5}{12}$

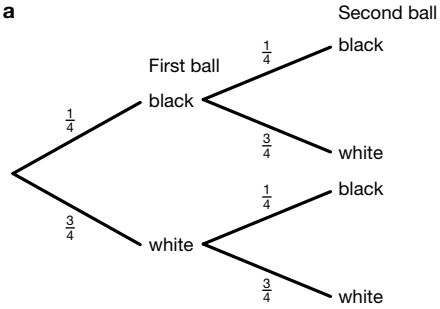
b $P(F) = \frac{12 + 2 + 7 + 3}{60} = \frac{24}{60} = \frac{2}{5}$

c $P(L \cap F) = \frac{2 + 7}{60} = \frac{9}{60} = \frac{3}{20}$

d $P(S') = \frac{0 + 7 + 3}{60} = \frac{10}{60} = \frac{1}{6}$

Frequency trees and diagrams

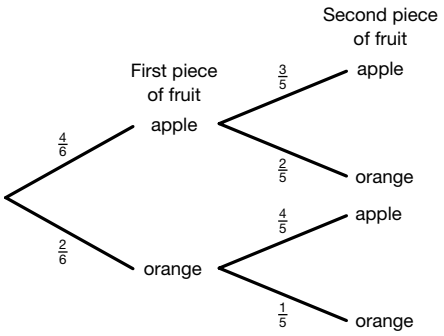
1 a



b $P(W, B) = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$

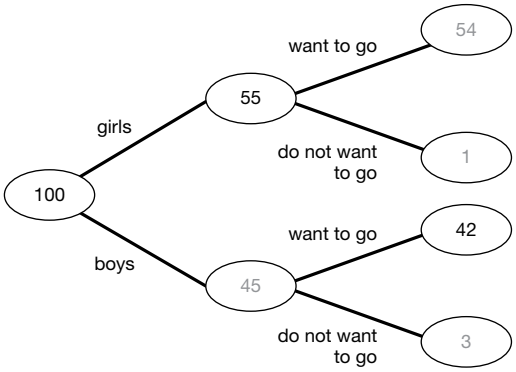
c $P(B, B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

2 a



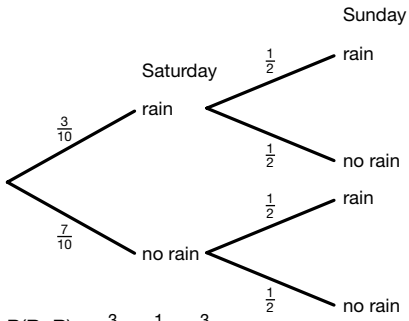
b $P(O, O) + P(A, A) = \left(\frac{2}{6} \times \frac{1}{5}\right) + \left(\frac{4}{6} \times \frac{3}{5}\right) = \frac{2}{30} + \frac{12}{30} = \frac{14}{30}$
$= \frac{7}{15}$

3 a



b One boy chosen at random:
P (does not want to go) $= \frac{3}{45} = \frac{1}{15}$

4 a



b $P(R, R) = \frac{3}{10} \times \frac{1}{2} = \frac{3}{20}$

c $P(R, R') + P(R', R) = \left(\frac{3}{10} \times \frac{1}{2}\right) + \left(\frac{7}{10} \times \frac{1}{2}\right)$
$= \frac{3}{20} + \frac{7}{20} = \frac{10}{20} = \frac{1}{2}$
The probability of one day having rain and one day having no rain is 50%.

Expected outcomes and experimental probability

1 a The spinner was spun $12 + 13 + 10 + 15 = 50$ times.

b Estimated probability of blue $= \frac{13}{50}$

c Estimated probability of yellow $= \frac{15}{50} = \frac{3}{10}$

d You would expect $\frac{10}{15} \times 100 =$ **20** green outcomes from 100 spins.

2 $0.75 \times 20 =$ **15** students would be expected to pass the exam.

3 a Total number of customers $= 26 + 20 + 6 + 5 + 3 = 60$
Estimated probability that someone will buy stamps $= \frac{20}{60} = \frac{1}{3}$

b $\frac{1}{3} \times 450 =$ **150** customers buy stamps each day

c $\frac{6}{60} \times 450 =$ **45** customers buy foreign currency each day

d $450 \times 6 =$ **2700** customers each week
$\frac{5}{60} \times 2700 =$ **225** customers use the post office for banking each week

Statistics

Data and sampling

1 65, because that is 10% of 650 (the entire population).

2 $\frac{50}{25\,000} \times 100 = 0.2\%$. The sample is not big enough.

People in the town centre may not be the only ones using buses. For example, some people may take buses to the local train station, school or hospital.

3 **a** $\frac{9}{45} \times 400 = 80$ people like carrot cake

Sam needs to make 80 cakes.

b Assumptions

Assumed that these are individual carrot cakes. If instead each cake is a large one divided into 8 slices, then only $80 \div 8 = 10$ carrot cakes would be needed.

Assumed the sample is representative of the population; this could affect the answer because not all the 400 people who have accepted the invitation may turn up.

Frequency tables

1 **a** $1 + 11 + 9 + 6 + 1 + 2 = 30$ tables

b $(1 \times 1) + (2 \times 11) + (3 \times 9) + (4 \times 6) + (5 \times 1) + (6 \times 2) = 91$ people

2

Number of electronic devices	Tally	Frequency
0–1	|||	3
2–3	𝍸 |||	10
4–5	𝍸	5
6–7	𝍸	5
8–9	|	1

3 **a** Continuous

b

Mass, m (kg)	Tally	Frequency
$50 \leq m < 60$	|||	3
$60 \leq m < 70$	𝍸	5
$70 \leq m < 80$	|||	4
$80 \leq m < 90$	𝍸	5
$90 \leq m < 100$	|||	3

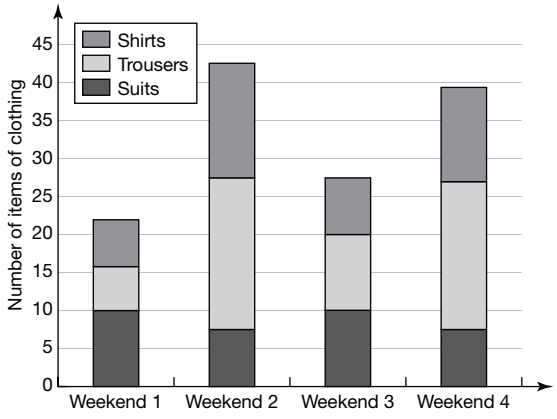
4 There were 12 people who spent 10 or more minutes, but fewer than 14 minutes, with the GP. However, it could be that none of them spent exactly 10 minutes with the GP.

Bar charts and pictograms

1 **a** $9 - 4 =$ **5** more boys than girls prefer squash

b $15 + 6 + 9 + 7 + 4 =$ **41** girls were surveyed

2



3 **a** $3 + 3 + 3 + 3 + 2 = 14$ endangered species

b Blue Mountain has 9 endangered species, Salt Springs has 7 endangered species. Blue Mountain has 2 more endangered species than Salt Springs.

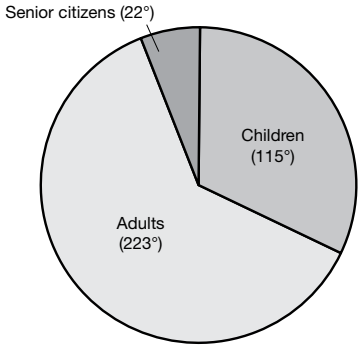
Pie charts

1 **a** $\frac{150}{360} \times 300 =$ **125** people last saw an action movie

b Romance = 90°, so $\frac{3}{4} \times 300 =$ **225** people did not see a romance movie.

2

	Children	Adults	Senior citizens
Calculation	$\frac{80}{250} \times 360$ $= 115.2$	$\frac{155}{250} \times 360$ $= 223.2$	$\frac{15}{250} \times 360$ $= 21.6$
Angle	115°	223°	22°



3 Angle for salad = 72°

$\frac{360}{72} \times 12 =$ **60** children were surveyed

Or:

72° = 12 children

$\frac{72}{12} = 6° = 1$ child

$\frac{360}{6} = 60$ children were surveyed

Measures of central tendency: mode

1 There are two modes: 3 minutes and 4 minutes, since each appears twice. An alternative correct answer is to say that there is no mode.

2 This is the one with the highest frequency: $12 < a \leq 13$.

3 This is the number with the biggest slice of the pie: 3.

4 The mode is the interest which represents the highest number of students: Sport (boys); Crafts (girls).

Measures of central tendency: median

1 Ages in order: 11 11 12 13 13 13 13 14 15 15 16 16 17 17 18 18

Median = $14\frac{1}{2}$ years old

2 Total frequency = $5 + 12 + 17 + 10 + 6 = 50$

Median = $\frac{50 + 1}{2} = 25.5$th person

Median class = $12 < a \leq 13$

Measures of central tendency: mean

1 Mean age = $\frac{6 + 7 + 11 + 13 + 18}{5} = 11$ years old

2 Number of bedrooms = $(1 \times 4) + (2 \times 7) + (3 \times 13) + (4 \times 17) = 125$

Number of houses = $4 + 7 + 13 + 17 = 41$

Mean number of bedrooms = $125 \div 41 = 3.05 \approx 3$ bedrooms

3 Number of holidays = $(0 \times 4) + (1 \times 21) + (2 \times 9) + (3 \times 2) = 45$

Number of employees = $4 + 21 + 9 + 2 = 36$

Mean number of holidays = $45 \div 36 = 1.25 \approx 1$ holiday

4

Age of patients, a	Midpoint	Frequency	Midpoint × frequency
$0 < a \leq 10$	5	3	15
$10 < a \leq 20$	15	18	270
$20 < a \leq 30$	25	6	150
$30 < a \leq 40$	35	11	385
$40 < a \leq 50$	45	10	450
$50 < a \leq 60$	55	19	1045
$60 < a \leq 70$	65	16	1040
$70 < a \leq 80$	75	17	1275
		Total = 100	Total = 4630

Mean age of patients $= \frac{4630}{100} = 46.3 \approx 46$ years old

Range

1 **a** Range of boys' ages $= 4 - 2 = 2$ years

b Range of boys' ages $= 4 - 1 = 3$ years

2 **a** Range in temperatures for Resort A $= 25 - 16 = 9°C$

b Range in temperatures for Resort B $= 28 - 13 = 15°C$

3 **a** Business A: Range $= 45\,816 - 23\,561 = £22\,255$

Mean profit $= \frac{23\,561 + 30\,485 + 39210 + 45\,816}{4} = £34\,768$

b Business B: Range $= 63\,248 - 17\,894 = £45\,354$

Mean profit $= \frac{32\,820 + 40\,328 + 17\,894 + 63\,248}{4} = £38\,572.50$

c **Either** Business A because its range in profit is lower and the profit is increasing each year, and so it shows a more consistent performance.

or Business B because its mean profit is higher, and its most recent profit (in Year 4) is £17 432 more than Business A.

Comparing data using measures of central tendency and range

1 **a** Mean time for bus journey $= \frac{32 + 30 + 39 + 32 + 43 + 31}{6} =$ 34.5 minutes

b Mean time for train journey $= \frac{16 + 24 + 18 + 26 + 70 + 17}{6} =$ 28.5 minutes

c Range for bus journey $= 43 - 30 = 13$ minutes

Range for train journey $= 70 - 16 = 54$ minutes

The train journey has the bigger range.

d **Either:** The bus is better because although it takes longer (on average), the range is lower, and so you can predict the time it takes for the journey.

Or: The train is better because it is quicker than the bus (on average), although the range suggests it may be less reliable.

2 **a** Mean = 13. This does not represent the age of the people using the playground. In fact, those using the playground are small children (under 10) and their parents (over 25).

b There are five modes (3, 4, 5, 7, 8), and so the mode does not represent the age of the people using the playground.

c Ages in order: 3 3 4 4 5 5 7 7 8 8 26 30 33 39

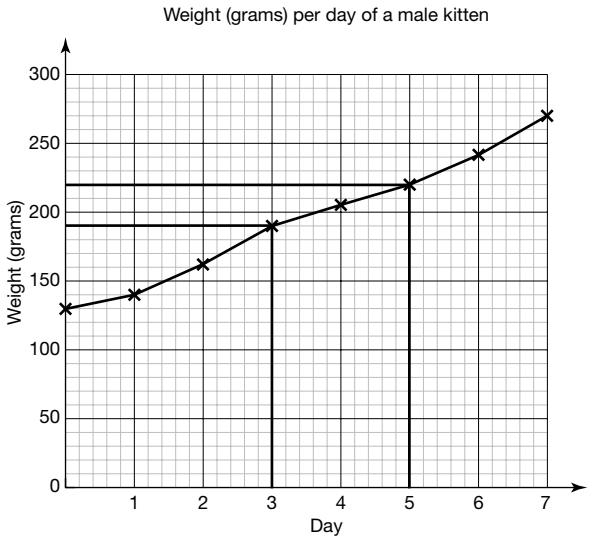
Median position $= \frac{14 + 1}{2} = 7.5$th value

Median age = 7 years old

3 Mode = 0; Median = 0; Mean = 2 days. Mode or median are the best averages to use, because the mean is skewed by the student who is absent due to sickness for 24 days.

Time series graphs

1 **a** and **b**



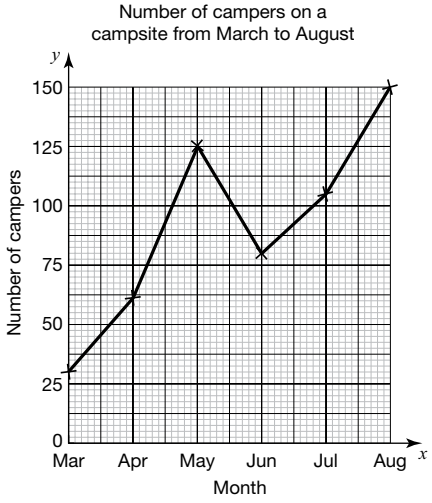
a Start at Day 3, read up to the graph line then left to the weight scale: 190 g

b Start at 220 g, read across to the graph line then down: 5 days old

c Mean weight gained $= \frac{\text{total weight gained}}{\text{number of days}} = \frac{270 - 130}{7}$

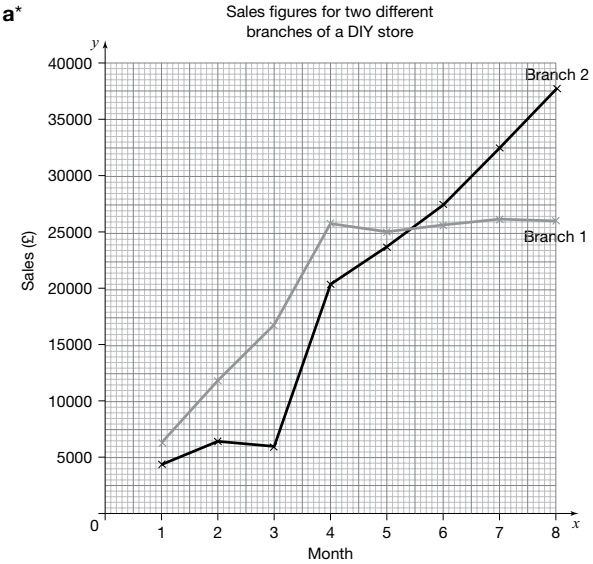
$= 20$ g per day

2 **a**



b There is an increase in campers in May. This may be due to May bank holidays, or May half-term, or perhaps there was some very sunny weather.

3 **a***



*This answer differs from the one in the Exam Practice Book due to an error in our first edition. This answer has now been re-checked and corrected.

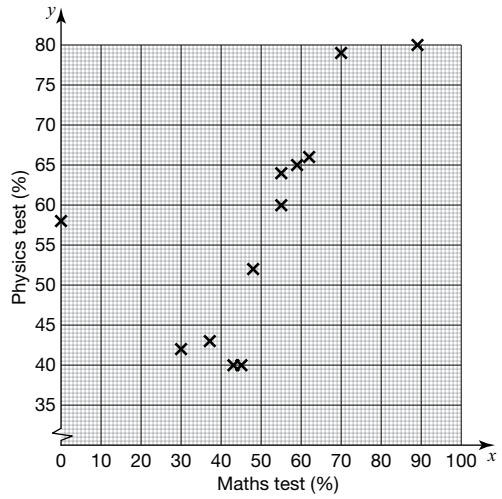
b Branch 1 had a steady increase in sales for the first four months. Then sales levelled off to stay at around £25 000. Branch 2 had a slow start to its sales in the first three months. Then perhaps it had a promotion, because sales increased a lot in month 4. Sales have been increasing ever since.

Scatter graphs

1 a Positive correlation. This means as the temperature rises, more pairs of flip flops are sold.

b Negative correlation. This means as the temperature rises, fewer wellington boots are sold.

2 a

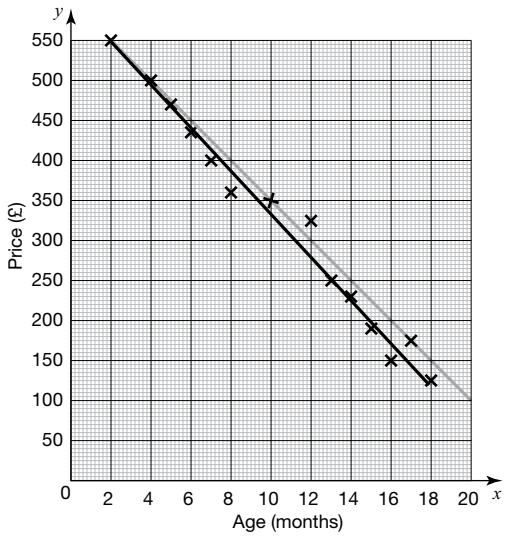


b The scatter diagram shows a positive correlation between students' maths and physics test percentages. Therefore, the students who got a low percentage in the maths test got the lower percentages in the physics test; the students who got a high percentage in the maths test got the higher percentages in the physics test.

c The outlier is the point marked at (0, 58).

d The student was absent for the maths test.

3 a



The black line shows the line of best fit.

The grey line shows the line where a laptop loses £150 every 6 months.

The shop owner is not correct. The line of best fit shows on average a laptop loses approximately £159/£160 every 6 months.

b The line of best fit cannot make a prediction outside the available data. The data only goes as far as 18 months.

Practice papers

Non-calculator

1 Digits move one place value to the left:

3.4

2 If you rotate C 90° anticlockwise it is an exact reflection of A. It has the same area, proportions, dimensions and shape (in reflection) so it is congruent.

C

3 $\frac{10}{14} = \frac{5}{7}$

$\frac{12}{21} = \frac{4}{7}$ (original fraction cancelled by 3)

$\frac{16}{28} = \frac{4}{7}$ (original fraction cancelled by 4)

$\frac{24}{42} = \frac{4}{7}$ (original fraction cancelled by 6)

$\frac{10}{14}$

4 $4x - (2 - 3x)$

$= 4x - 2 + 3x$

$= 7x - 2$

5 $7.93 \approx 8$

$12.1 \approx 12$

$3.86 \approx 4$

Estimate:

$\frac{8 + 12}{4}$

$= \frac{20}{4}$

$= 5$

6 You can cancel the second fraction as follows:

$\frac{8a}{10a} = \frac{4a}{5a} = \frac{4}{5}$

So the first two fractions are the same (and are equal to 80%)

8% is not equivalent to the others.

7 Its opposite sides are equal (indicated by the pairs of slashed lines across the opposite sides).

It is a parallelogram (because it has two pairs of equal opposite sides).

Note that:

- we have no evidence that its vertices are right angles, so we should **not** assume it is a rectangle
- its diagonals do not bisect at right angles, because its sides are **not** equal, so it is not a rhombus.

8 $\frac{3}{8} \times 2400 =$

$3 \times \frac{2400}{8}$

$= 3 \times 300$

$= 900$

9 5 : 7

Total number of parts = 12

7 parts are boys

$\frac{7}{12}$ are boys

10 a Factor pairs of 36:

1×36

2×18

3×12

4×9

6×6

Multiples of 6:

6, 12, 18, 24, 30, 36

Numbers in both groups are:

6, 12, 18, 36

From list:

12 and 18

b $36 = 2 \times 18$
$= 2 \times 2 \times 9$
$= 2^2 \times 3^2$
Therefore the only prime factors of 36 are 2 and 3.

c No. A prime number has exactly two factors: 1 and itself. A multiple of 6 would have at least four factors: 1, 2, 3, 6.

11 Let the coins be:
$2q + r + s + t$
(Two are the same, represented by $2q$.)
Working in pence:
$2q + r + s + t = 170$
Ways of making 70p with 4 coins, with no more than 2 being the same:
50p + 10p + 5p + 5p
20p + 20p + ? (no way of making 30p without repeating coins)
Without using a 50p or a 20p you can't make 70 out of 4 coins (even if you repeated the largest one, 10p, more than once).
So $q = 5$, $r = 50$, $s = 10$
$(2 \times 5) + 50 + 10 + t = 170$
$t = 170 - 10 - 50 - 10 = 170 - 70 = 100$
So $t = 100\text{p} = £1$
The coins are:
£1, 50p, 10p, 5p, 5p

12 60 minutes in an hour
As a fraction, 25 minutes is:
$\frac{25}{60} = \frac{5}{12}$

13 $a + 2b = 6 + 2 \times 4 = 14$ (even)
$2(a - b) = 2(6 - 4) = 2 \times 2 = 4$ (even)
$\frac{2a}{b} = \frac{2 \times 6}{4} = \frac{12}{4} = 3$ (odd)
Therefore $\frac{2a}{b}$ is the expression which gives an odd number.
Or:
even + 2(even) = even + even = even
even(even − even) = even × even = even
$\frac{2 \times \text{even}}{\text{even}} = \frac{\text{even}}{\frac{1}{2} \times \text{even}} = \frac{\text{even}}{\text{even}}$ **or** $\frac{\text{even}}{\text{odd}}$
so the answer could be either even or odd.
Therefore $\frac{2a}{b}$ is the expression which gives an odd number.

14 Area of rectangle: $4 \times 1.5 = 6\text{m}^2$
Perimeter of triangle $= 3 \times 6 = 18\text{m}$
$BC = 18 - (5.1 + 3.4)$
$= 18 - 8.5$
$= 9.5\text{m}$
Note that you cannot use Pythagoras' theorem because *BAC* is not a right angle.

15 **a** Ratio 2 : 3
16 = 2 parts
1 part = 16 ÷ 2 = 8
8 × 3 = 24
There are 24 children.

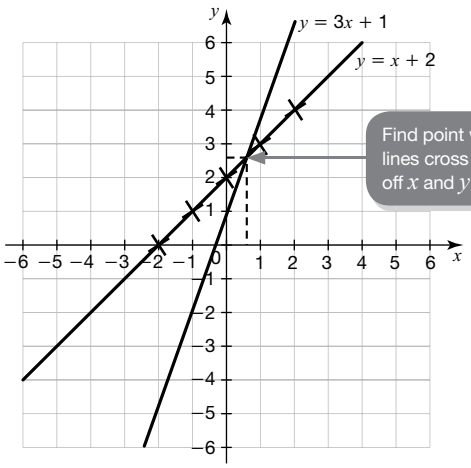
b 16 + 4 = 20 adults
ratio of adults to children = 20 : 24
Divide by 4:
5 : 6

16 The width of the tin is 4 cm and each crayon is 0.8 cm wide.
4 ÷ 0.8 = 5
So you could fit the widths of 5 crayons in.
The height of the tin is 1 cm and the height of each crayon is 1 cm.
So you could fit 1 layer of crayons in.
The length of tin is 20 cm and the length of each crayon is 10 cm.
20 ÷ 10 = 2
So you could fit 2 rows of crayons in.
Total crayons that will fit in the tin:
5 × 1 × 2 = 10 crayons

17 Animals that are not sheep: (10 + 12 + 18) = 40%
240 animals (sheep) = 60%
10% = 240 ÷ 6 = 40
100% = 10 × 40 = 400
There are 400 animals on the farm.

18 Let price of coffee = x and price of cake = y.
$2x + y = 4.4$ (1)
$x + y = 3.2$ (2)
Multiply (2) by 2:
$2x + 2y = 6.4$
Subtract (1) from this:
$y = 6.4 - 4.4 = 2$
The cake was £2.

19 **a** and **b**



Reading from graph:
$y = 2.5$
$x = 0.5$
(Acceptable alternative readings from graph in book would be: 2.4 and 0.4.)
Check answer using the two equations:
$3 \times 0.5 + 1 = 1.5 + 1 = 2.5$
$0.5 + 2 = 2.5$

20 Let fare = f and number of kilometres = d
In pounds:
$f = 2.25d + 3.5$
$d = \frac{f - 3.5}{2.25}$
When $f = 12.5$:
$d = \frac{12.5 - 3.5}{2.25} = \frac{9}{2.25} = 4$
Abbie travelled 4 km.

21 Ali has not included labels for 'phone call' or 'letter'. This is inconsistent and does not enable someone to know which is which. A better way to do it would be to include a key for the chart.
The 'phone call' and 'letter' angles should not be equal sizes. The phone call angle should be twice as big as the letter angle (40°, and 20° for letter).

22 **a** **i** $-2 \leq a < 3$
a is a whole number that can be as low as -2 but is less than 3:
$-2, -1, 0, 1, 2$

ii $-5 < b \leq 2$
b is a whole number that is greater than -5 but can be as high as 2:
$-4, -3, -2, -1, 0, 1, 2$

b The largest integer in each list is 2:

$2 \times 2 = 4$

However, the smallest integers in the lists are $a = -2$ and $b = -4$, and to get ab we multiply them:

$-2 \times -4 = 8$

So the largest possible whole number value for $ab = 8$.

23 $1.5 \times 3 \div 10\,000$

$= 4.5 \times 10^{-4}$

24 **a** From the graph:

books that are not fiction = (100 − 60)

= 40%

Or:

Fiction books in library A = 60% × 1500 = $\frac{1500 \times 6}{10}$ = $150 \times 6 = 900$

Books that are not fiction: $1500 - 900 = 600$

P(not fiction) = $\frac{600}{1500} = \frac{6}{15} = \frac{2}{5}$ or 40% or 0.4

b Local history books in library A = (88 – 60)% = 28%

Total local history books in A = $\frac{1500 \times 28}{100} = 15 \times 28 = 1.5 \times 280 = 420$

Local history books in library B = (56 − 40)% = 16%

Total local history books in B = $\frac{3200 \times 16}{100}$

$= 32 \times 16 = 30 \times 16 + 2 \times 16$

$= 480 + 32$

$= 512$

$512 > 420$

Library B has more local history books.

25 $3\frac{3}{4} + 1\frac{7}{8}$

$= \frac{15}{4} + \frac{15}{8}$

$= \frac{30}{8} + \frac{15}{8}$

$= \frac{45}{8}$

$= 5\frac{5}{8}$

26 **a** $3(x + 2) = 5x - 4$

$3x + 6 = 5x - 4$

$6 + 4 = 2x$

$10 = 2x$

$x = 5$

b $3x^2 + 27x$

$= x(3x + 27)$

$= 3x(x + 9)$

27 $A = \frac{1}{2}bh$

Base (b) = 6 cm

Height (h) = $\sqrt{10^2 - 6^2}$

$= \sqrt{100 - 36}$

$= \sqrt{64}$

$= 8$ cm

Therefore, $A = \frac{1}{2} \times 6 \times 8$

$= 24\,\text{cm}^2$

28 **a** $5n + 1 = 36$

$5n = 35$

$n = 7$

It is the 7th term.

b The value of n needs to be an integer if 36 is in the sequence. Testing them:

$3n - 1 = 36$

$3n = 37$

$n = \frac{37}{3}$ (not an integer)

$2n + 3 = 36$

$2n = 33$

$n = \frac{33}{2}$ (not an integer)

$5n - 8 = 36$

$5n = 44$

$n = \frac{44}{5}$ (not an integer)

$7n + 1 = 36$

$7n = 35$

$n = 5$

$7n + 1$ is a sequence that also includes 36.

29 **a** tan = $\frac{\text{opposite}}{\text{adjacent}}$

From second triangle, tan 55° = $\frac{14.3}{10} = 1.43$ (3 s.f.)

b sin = $\frac{\text{opposite}}{\text{hypotenuse}}$

From first triangle, sin 55° = $\frac{8.19}{10} = 0.819$ (3 s.f.)

30 Area of trapezium = $\frac{1}{2}(a + b)h$

$\frac{1}{2}(2 + 8) \times 3$

$= \frac{10 \times 3}{2}$

$= 15\,\text{cm}^2$

Area of semicircle = $\frac{1}{2} \times \pi r^2$

$\frac{1}{2} \times \pi \left(\frac{8}{2}\right)^2$

$= \frac{1}{2} \times 4^2\pi$

$= 8\pi$

Total area = $8\pi + 15\,\text{cm}^2$

Calculator

1 A prime number has exactly 2 factors: 1 and itself.

1 has only 1 factor (1).

$8 = 1 \times 8$ and $8 = 2 \times 4$

$11 = 1 \times 11$

$15 = 1 \times 15$ and $15 = 3 \times 5$

Only 11 has exactly 2 factors. 11 is a prime number.

2 All the triangles are right-angled (they are on a grid).

Those that are similar will have the same ratio between their shorter sides (meaning the angles of both triangles are the same).

Ratios of shorter sides are as follows.

A has ratio 3 : 4

B has ratio 3 : 6

C has ratio 6 : 8 = 3 : 4

D has ratio 4 : 6

So A and C have the same ratio and are similar.

3 $-10.2 + 7 = -3.2$

4 $11.5 \times 37 = 425.5$

£425.50 < £462.50

B is the better rate of pay.

5 2 pm is a useful time to work from. The bus departed at (21 − 10) minutes and (25 − 0) seconds before 2 pm (11 minutes, 25 seconds before 2 pm).

In 24-hour clock times, it departed at:

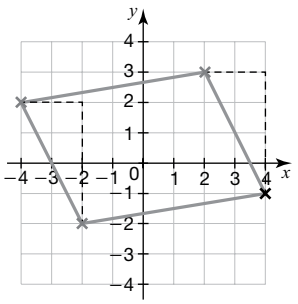
= 14:00 hours − 11 minutes − 25 seconds

= 13.49 hours − 25 seconds

There are 60 seconds in a minute, so:

departure = 13.48 and 35 seconds,
or 1.48 pm and 35 seconds

6 Plot the points you are given on the grid.



To move between points (−4, 2) and (−2, −2) you go 2 spaces along to the right and −4 spaces up (4 spaces down).

Moving in the same way from (2, 3) to the final point gives coordinates:

((2 + 2), (3 − 4))

= (4, −1)

7 **a** Add all the scores together and divide by the number of scores.

mean $= \frac{18 + 16 + 19 + 2 + 18 + 17}{6} = \frac{90}{6} = 15$

b Arrange them in ascending order: 2, 16, 17, 18, 18, 19

median is between middle two values: $\frac{17 + 18}{2} = 17.5$

c The **median** is better because it represents the typical score – the low score of 2 is an outlier and skews the value of the mean.

8 P(snow) + P(not snow) = 100%

P(not snow) = 100% – P(snow)

= (100 – 20)%

= 80%

= 0.8

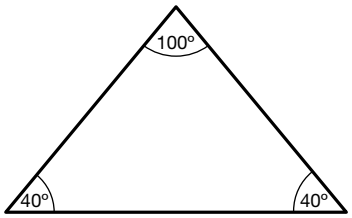
9 **a** $5x + 2x + 2x = 180°$

$9x = 180°$

$x = 20°$

$2x = 40°$

$5x = 100°$



b The triangle has two equal angles (and two equal sides) and is therefore **isosceles**.

10 String used to tie plants:

$4 \times \frac{3}{8} = \frac{3}{2}$ m

Remaining string:

$4 - \frac{3}{2} = \frac{8}{2} - \frac{3}{2} = \frac{5}{2}$ m

String used to lay out flower bed:

$\frac{5}{2} \times \frac{4}{5} = \frac{5 \times 2}{1 \times 5} = \frac{1 \times 2}{1 \times 1} = 2$ m

Total string used $= \frac{3}{2} + 2 = \frac{3}{2} + \frac{4}{2} = \frac{7}{2}$ m

String left $= 4 - \frac{7}{2} = \frac{8}{2} - \frac{7}{2} = \frac{1}{2}$ m

$\frac{1}{2}$ m = 50 cm

11 (Note that the 10 mark is missing from the vertical axis of the graph in the book.)

a Profit = sales – costs

= 74 000 − 40 000

= 34 000

£34 000

b Branch 2 sales: 90 000

Branch 2 profit: 90 000 − 52 000 = 38 000

Branch 3 sales: 44 000

Branch 3 profit: 44 000 − 20 000 = 24 000

Double the profit of Branch 3 = 24 000 × 2 = 48 000

38 000 < 48 000

The Branch 2 manager is incorrect.

12 **a** $\frac{x}{5} = 20$

$x = 5 \times 20$

$x = 100$

b $3a^2 + 2ab$ or $a(3a + 2b)$

c Divide the cost of the box (p) by the number of tins of beans (t).

$c = \frac{p}{t}$

13 Bill before VAT:

£70 + (3 × £35) = £175

Using multiplier for 20% increase, bill after VAT:

£175 × 1.2 = £210

14 Let Roman's age = r, Freddie's age = f and Claudia's age = c.

$r + f + c = 33$ (1)

$r + f = 27$

So $c + 27 = 33$

$c = 6$ (2)

Roman is 9 years older than Claudia, so:

$r - 9 = 6$

$r = 15$ (3)

From (1), (2) and (3):

$15 + f + 6 = 33$

$f = 33 - 15 - 6$

$f = 12$

Freddie is 12.

15 **a** $x + y = 360 - 2 \times 129$ (angles in a quadrilateral = 360°)

$= 360 - 258$

$= 102°$

$x + y = 102$ (1)

$2x = y$ (2)

By substitution in (1): $x + 2x = 102$

$3x = 102°$

$x = 34°$

$y = 2 \times 34 = 68°$

b It has two equal opposite angles, and two different angles. It has two pairs of equal adjacent sides, and one line of symmetry. So it is a kite.

16 Let number of 10p coins be p, number of 20p coins be q, and number of 50p coins be r.

$r = 4q$ (check it's the right way round: r will be a bigger number than q)

$p = \frac{1}{2}q$ (check it's the right way round: p will be a smaller number than q)

$2p = q$

$p = 8$, so eliminate p:

$2 \times 8 = q$

$q = 16$

From first equation:

$r = 4 \times 16 = 64$

So $p = 8$, $q = 16$ and $r = 64$.

Amount in meter, in pence = (8 x 10) + (16 x 20) + (64 x 50)

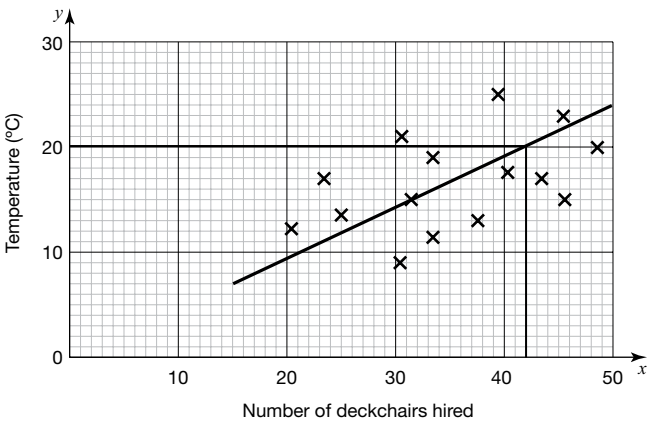
= 80 + 320 + 3200

= 3600 pence

= £36

17 **a** There is a loose **positive correlation**: there tend to be more deckchairs hired on warmer days.

b Draw a line of best fit on the graph and read off the value at 20°.



Jeff can expect to hire out approximately 42 deckchairs.

(Depending on how you draw your line of best fit, any answer up to 46 is acceptable.)

18 **a** Number taking just French and German:

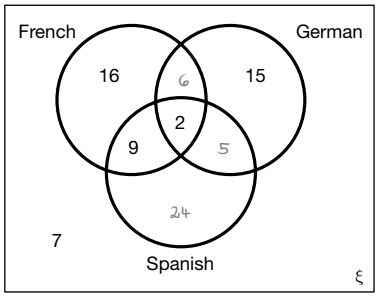
$33 - 16 - 9 - 2 = 6$

Number taking just German and Spanish:

$28 - 15 - 2 - 6 = 5$

Number taking only Spanish:

$40 - 9 - 2 - 5 = 24$



b Total $= \xi = 7 + 16 + 9 + 2 + 6 + 15 + 5 + 24 = 84$

c $2 + 5 = 7$ students take both German and Spanish.

$P(\text{German} \cap \text{Spanish}) = \frac{7}{84} = \frac{1}{12}$

19 The price has been reduced by $\frac{2}{9}$, so the sale price is $\frac{7}{9}$.

350 represents 7 parts of original price

$350 \div 7 = 50$

50 is 1 part $= \frac{1}{9}$ of original price

Original price:

$50 \times 9 = 450$

£450

20 Volume for sculpture:

$\frac{4}{3}\pi 3^3 + \frac{4}{3}\pi 2^3$

$= \frac{4 \times 27}{3}\pi + \frac{4 \times 8}{3}\pi$

$= 36\pi + \frac{32}{3}\pi$

$= 46\frac{2}{3}\pi$

$= 146.61\,\text{m}^3$ to 2 d.p.

21 $\begin{pmatrix} -3 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -3+1 \\ 2+4 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 6 \end{pmatrix}$*

22 **a** Write an equation showing the relationship between dollars (d) and pounds (p):

$d = 1.272p$

$p = \frac{d}{1.272}$

Lucy buys $500:

$500 = 1.272p$

$p = \frac{500}{1.272}$

$= 393.08$ (2 d.p.)

It costs Lucy £393.08.

b Lucy brings home $500 - 427 = \$73$

$p = 0.789d$

$= 0.789 \times 73$

$= 57.60$ (2 d.p.)

Lucy gets back £57.60.

23 $\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$

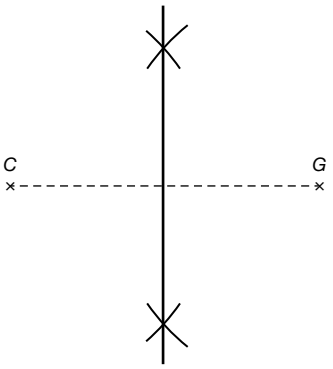
$\cos 42° = \frac{7}{XZ}$

$XZ = \frac{7}{\cos 42°}$

$= 9.42$ cm (2 d.p.)

24 x is a negative number because negative × negative × negative = negative, **or**: when multiplying you need an even number of negative signs to make a positive, and with a cube number you are multiplying 3 numbers that are exactly the same. So if the original number is negative, the cube will be negative too.

25 **a** Construct the perpendicular bisector of the line CG.



b Distance measured on page is 4.1 cm

1 cm represents 200 m, so:

4.1 cm represents $200 \times 4.1 = 820$ m

$= 0.82$ km (0.8 km to 1 d.p)

26 **a** $4.927^2 + \sqrt[3]{8.135} = 26.2865163054$ (10 d.p. from calculator display)

$= 26.287$ to 3 d.p.

b $4.927 \approx 5$

$5^2 = 25$

$8.135 \approx 8$

$\sqrt[3]{8.135} \approx \sqrt[3]{8} = 2$

$25 + 2 = 27$, so the answer to part a is sensible.

27 Depreciation of 18% means each year it has $100 - 18 = 82\%$ of its original value.

Whole amount $= 1 \times 17\,000$

Multiplier for depreciation $= 0.82$

$17\,000 \times 0.82^3 = 9373.256$

Value after 3 years = £9373 to nearest pound.

28 **a** y-intercept (c) $= 2$

gradient (m) $= \frac{-12}{6} = -2$

Comparing with $y = mx + c$:

$y = -2x + 2$

b Parallel line must have the same gradient (m) $= -2$

$y = -2x$

*This differs from the question in the Exam Practice book due to an error in our first edition. This question has now been re-checked and corrected.