AQA Higher Practice paper (calculator 2): full worked solutions

1 1×10^9 – this is 1 followed by 9 zeros.

2 A – because it goes from negative to positive and has both its turning points at the origin.

3 5 – because $(180 - 108) \times 5 = 360°$

4 $7\,600\,000\,\text{cm}^3$

$1\,\text{m}^3 = 100 \times 100 \times 100\,\text{cm}^3 = 1\,000\,000\,\text{cm}^3$

$7.6\,\text{m}^3 = 7.6 \times 1\,000\,000\,\text{cm}^3 = 7\,600\,000\,\text{cm}^3$

5 $6x - 5 > 1 - 2x$

$8x > 6$

$x > \frac{6}{8}$

$x > \frac{3}{4}$

6 Multiply the coefficient of x^2 by the number term:

$4 \times (-4) = -16$

Look for a factor pair of -16 that add to give the coefficient of x:

$16 + (-1) = 15$

Therefore, $4x^2 + 15x - 4 = 4x^2 + 16x - x - 4$

$= 4x(x + 4) - (x + 4)$

$= (4x - 1)(x + 4)$

7 $\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 18}{10} = \frac{\sin x}{25}$

$10 \sin x = 25 \sin 18$

$\sin x = \frac{25 \sin 18}{10}$

$x = \sin^{-1}\left(\frac{25 \sin 18}{10}\right)$

$x = 50.58275...$

The question states x is obtuse, so $x = 180 - 50.58275 = 129.41725$

$x = 129.4°$ (to 1 d.p.)

8 Volume of a prism = area of cross section × length

To find the area of a cross section of the prism use:

Area of a triangle $= \frac{1}{2} \times$ base × height

Volume $= \frac{1}{2} \times$ base × height × length

$960 = \frac{1}{2} \times 10 \times h \times 12$

$960 = 60 \times h$

$h = 16\,\text{cm}$

9 $\frac{27^{\frac{2}{3}}}{8^{\frac{2}{3}}} = \frac{(\sqrt[3]{27})^2}{(\sqrt[3]{8})^2} = \frac{3^2}{2^2} = \frac{9}{4}$

10 $P = \frac{k}{V}$

$1.6 = \frac{k}{100}$

$k = 160$

$P = \frac{160}{V}$

When $P = 40$, $V = \frac{160}{40} = 4$

Therefore $a = 4$

11 Write the equation in the form $y = a(x + p)^2 + q$ by completing the square:

$y = 2x^2 - 5x + 4$

$= 2(x^2 - \frac{5}{2}x + 2)$

$= 2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16} + 2\right]$

$= 2\left[\left(x - \frac{5}{4}\right)^2 + \frac{7}{16}\right]$

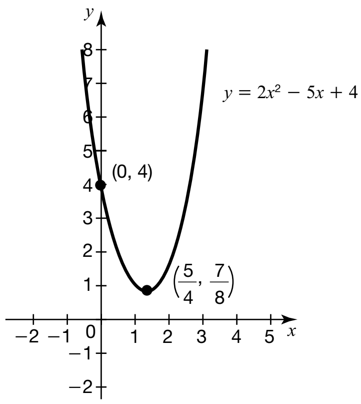
$= 2\left(x - \frac{5}{4}\right)^2 + \frac{7}{8}$

The turning point of the graph is $(-p, q)$, which is $\left(\frac{5}{4}, \frac{7}{8}\right)$

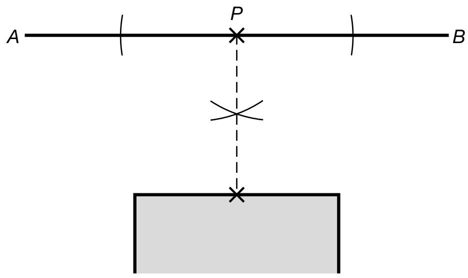
To find the y-intercept, put $x = 0$:

$y = 2 \times 0^2 - 5 \times 0 + 4 = 4$

The curve cuts the y-axis at the point (0, 4)



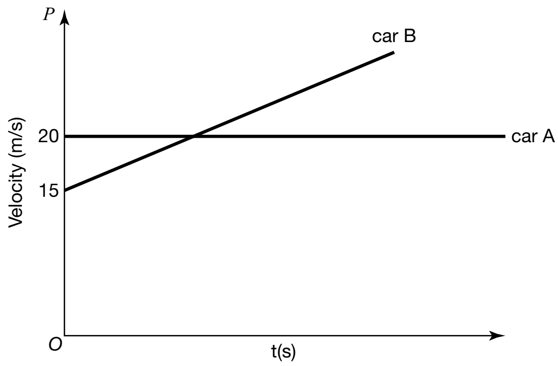
12 Assuming the pipe is parallel to the side of the house, a perpendicular bisector of the line *AB*, which joins with the edge of the house, should be constructed (as shown below).



13 A

You need a right-angled triangle containing an angle of 60°. A is an equilateral triangle, so all the angles are 60°. Half of it is a right-angled triangle with the lengths of sides shown in A, enabling sin 60° to be found.

14 **a**



b Suppose the cars are level after time t.

distance = area under the velocity–time graph for car A = 600

$20 \times t = 600$

$t = 30\,\text{s}$

c Acceleration $= \frac{\text{change in velocity}}{\text{change in time}}$

$= \frac{20 - 15}{30}$

$= 0.1\dot{6}$

$= 0.17\,\text{m/s}^2$ (2 d.p.)

15 $y = 3x + 6$ and $y = x^2 - 2x + 1$

$3x + 6 = x^2 - 2x + 1$

$x^2 - 5x - 5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-5)}}{2(1)}$

$= \frac{5 \pm \sqrt{25 + 20}}{2}$

$= \frac{5 \pm \sqrt{45}}{2}$

$= \frac{5 + \sqrt{45}}{2}$ or $\frac{5 - \sqrt{45}}{2}$

$= 5.85$ or -0.85 (to 2 d.p.)

16 $b = 3a - 2$

$c = 3b - 2$

$= 3(3a - 2) - 2$

$= 9a - 6 - 2$

$= 9a - 8$

17 **a** The 25% is applied to the sale price and not to the original price.

b Percentage of original price after the first reduction = $100 - 20 = 80\%$

Second reduction = 25% of 80 = 20%

Therefore, the overall percentage reduction is $20 + 20 = 40\%$

18 **a** $(1, -2)$ (translation of 2 units to the left)

b $(3, 0)$ (translation of 2 units up)

c $(3, 2)$ (Reflection in the x-axis)

19 If $a = -1$ and $b = -2$, then $a^2 = 1$ and $b^2 = 4$

For this example, $b^2 > a^2$ but $b < a$

Hence, the statement is disproved.

20 B

If you did not recognise the shape you could substitute $x = 0$ into the equation: then, $y = 2^0 = 1$ and the curve passes through $(0, 1)$. B is the only graph that shows this.

21 **a** $uf + vf = uv$ (multiplying both sides by vuf)

$vf - uv = -uf$ (terms containing v on the same side)

$v(f - u) = -uf$ (factorising)

$v = \frac{(-uf)}{(f - u)}$

or $v = \frac{uf}{(u - f)}$ (multiply top and bottom by -1)

b Comparing with $ax^2 + bx + c = 0$, $a = 2$, $b = -7$ and $c = 4$.

Using the quadratic formula:

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(4)}}{2(2)}$

$= \frac{7 \pm \sqrt{(49 - 32}}{4} = \frac{7 \pm \sqrt{17}}{4} = \frac{7 + \sqrt{17}}{4}$ or $\frac{7 - \sqrt{17}}{4}$

Hence $x = 2.78$ or 0.72 (2 d.p.)

22 January 2018: $(20\,000 \times 1.05) - 2000 = 19\,000$

January 2019: $(19\,000 \times 1.05) - 2000 = 17\,950$

January 2020: $(17\,950 \times 1.05) - 2000 = 16\,847.5$

January 2021: $(16\,847.5 \times 1.05) - 2000 = 15\,689.875$

Giovanni will have £15 689.88 (to the nearest penny) in the account in January 2021.

23 **a** Let $f(x) = x^3 - 4x + 2$

$f(1) = 1^3 - 4(1) + 2 = -1$

$f(0) = 0^3 - 4(0) + 2 = 2$

As there is a sign change, the root lies between 0 and 1.

b $x^3 - 4x + 2 = 0$

$4x = x^3 + 2$

$x = \frac{x^3 + 2}{4}$

$x = \frac{x^3}{4} + \frac{1}{2}$

c $x_1 = \frac{0.5^3}{4} + \frac{1}{2} = 0.53125$ (5 d.p.)

$x_2 = \frac{0.53125^3}{4} + \frac{1}{2} = 0.53748$ (5 d.p.)

$x_3 = \frac{0.53748^3}{4} + \frac{1}{2} = 0.53882$ (5 d.p.)

$x_4 = \frac{0.53882^3}{4} + \frac{1}{2} = 0.53911$ (5 d.p.)

An estimate for one of the roots of the equation is:

0.539 (to 3 d.p.)

24 **a** Total number of parts = $4 + 5 = 9$

Number of counters per part = $\frac{36}{9} = 4$

Number of red counters = $4 \times 4 = 16$

b i P(red then red) = $\frac{16}{36} \times \frac{15}{35} = \frac{4}{21}$

ii Number of blue counters = $36 - 16 = 20$

P(different colours) = P(red then blue) + P(blue then red)

$= \left(\frac{16}{36} \times \frac{20}{35}\right) + \left(\frac{20}{36} \times \frac{16}{35}\right) = \frac{32}{63}$

25 **a** $f(-2) = 5 \times (-2)^2 + 4 = 24$

b Let $y = 5x^2 + 4$

$\frac{y - 4}{5} = x^2$

$x = \sqrt{\frac{y - 4}{5}}$

$f^{-1}(x) = \sqrt{\frac{x - 4}{5}}$

c $fg(x) = f(x + 1)$

$= 5(x + 1)^2 + 4$

26 **a** $l = \frac{\theta}{360} \times 2\pi r = \frac{35}{360} \times 2\pi \times 8 = 4.89$ cm (to 2 d.p.)

b $A = \frac{\theta}{360}\pi r^2 = \frac{35}{360}\pi \times 8^2 = 19.55$ cm^2 (to 2 d.p.)

27 Multiples of 12: 12, 24, **36**, 48, 60, 72, 84, 96, 108, 120, 132, **144**, ...

Multiples of 16: 16, 32, 48, 64, 80, 96, 112, 128, **144**, ...

Multiples of 18: 18, **36**, 54, 72, 90, 108, 126, **144**, ...

a The first number to appear in 2 lists is 36.

36 days

b The first number to appear in all 3 lists is 144.

144 days

28 $\frac{1}{3x^2 + 5x - 2} \div \frac{1}{9x^2 - 1} = \frac{1}{3x^2 + 5x - 2} \times \frac{9x^2 - 1}{1}$

$= \frac{1}{(3x - 1)(x + 2)} \times (3x - 1)(3x + 1)$

$= \frac{3x + 1}{x + 2}$

$a = 3$, $b = 1$, $c = 1$, $d = 2$