Probability Skills Edexcel Maths Higher GCSE 9–1

Full worked solutions

Revision answers

The basics of probability p.8

1 There are two possible even numbers (i.e. 2, 4)

Probability of landing on an even number

$= \frac{\text{Number of ways something can happen}}{\text{Total of number of possible outcomes}} = \frac{2}{5}$

Probability of getting an even number on each of three spins $= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$

2 a As there are no white counters, the probability of picking one = 0

b Probability of picking a black counter $= \frac{4}{20} = \frac{1}{5}$

c Probability of picking a green counter $= \frac{9}{20}$

Prob of picking a green counter + prob of not picking a green counter = 1

Prob of not picking a green counter

$= 1 - \frac{9}{20} = \frac{11}{20}$

Probability experiments p.10

1 a Relative frequency for a score of 3 $= \frac{25}{120}$

= 0.21 (2 d.p.)

b Relative frequency for a score of 6 $= \frac{17}{120}$

= 0.14 (2 d.p.)

c Sean is wrong. 120 spins is a small number of spins and it is only over a very large number of spins that the relative frequencies may start to be nearly the same.

2 a Estimated probability = relative frequency

$= \frac{\text{Frequency}}{\text{Total Frequency}} = \frac{20}{500} = 0.04$

b Number of cans containing less than 330 ml = 0.04 × 15 000 = 600

Another way to do this is to see how many times 500 divides into 15 000. This is 30. So there will be 30 × 20 = 600 cans containing less than 30 ml.

3 Expected frequency = Probability of the event × number of events

$= \frac{3}{40} \times 600$

= 45 apples

The AND and OR rules p.12

1 a Independent events are events where the probability of one event does not influence the probability of another event occurring. Here it means that the probability of the first set of traffic lights being red does not affect the probability of the second set being red.

b P(A AND B) = P(A) × P(B)

P(stopped at first AND stopped at second)
= P(stopped first) × P(stopped second)

= 0.2 × 0.3

= 0.06

c P = 0.8 × 0.7 = 0.56

You can work out the probability of lights not being on red by subtracting the probability of being red from 1. So the probability of not being red at the first set is 1 – 0.2 = 0.8 and for the second set it is 1 – 0.3 = 0.7

2 a Probability of all events taking place $= \frac{1}{4} \times \frac{2}{3} \times \frac{7}{8}$

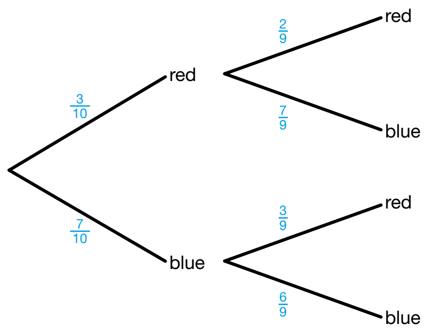
$= \frac{2 \times 7}{12 \times 8} = \frac{7}{48}$

b Probability of none of the events taking place

$= \frac{3}{4} \times \frac{1}{3} \times \frac{1}{8} = \frac{3}{3 \times 32} = \frac{1}{32}$

Tree diagrams p.16

1 a The following tree diagram is drawn.



P(red AND red) $= \frac{3}{10} \times \frac{2}{9} = \frac{6}{90} = \frac{1}{15}$

b P(red AND blue) = P(RB) + P(BR)

Note that red and blue does not specify an order. There are two paths that need to be considered on the tree diagram.

$= \frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{3}{9}$

$= \frac{7}{30} + \frac{7}{30}$

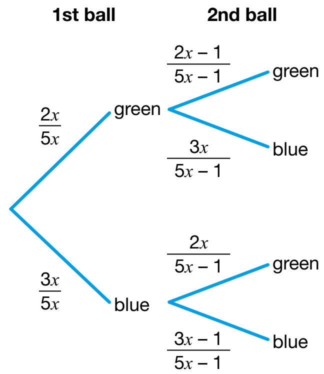
$= \frac{14}{30}$

$= \frac{7}{15}$

Remember to fully cancel fractions. Use your calculator to help you.

2 Let the number of green balls in the bag be $2x$. Let the number of blue balls be $3x$. So the total number of balls in the bag is $5x$.

Put these values into a tree diagram:



P(blue AND blue) $= \frac{3x}{5x} \times \frac{3x-1}{5x-1} = \frac{33}{95}$

Divide both sides by $\frac{3}{5}$ and cancel the x top and bottom on the left.

So $\frac{3x-1}{5x-1} = \frac{11}{19}$

$19(3x - 1) = 11(5x - 1)$

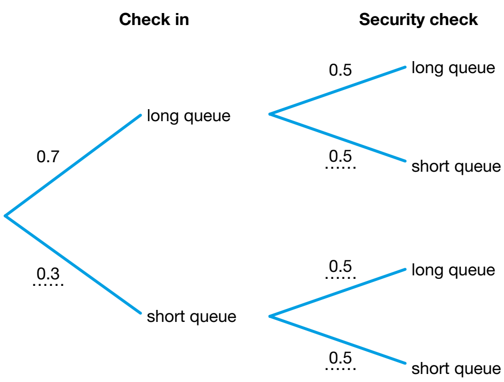
$57x - 19 = 55x - 11$

$2x = 8$

$x = 4$

Hannah put $5x = 5 \times 4 = 20$ balls into the bag.

3 a



b Probability = 0.3 × 0.5 = 0.15

c Probability = 1 – probability of short queue at both
= 1 – 0.15 = 0.85

4 a Total number of students in school = 450 + 500
= 950

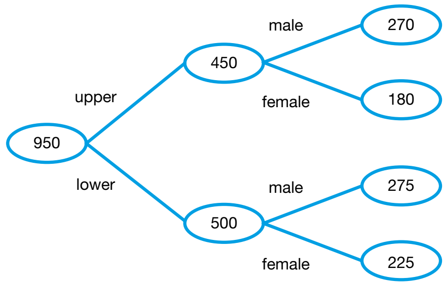
Number of male students in upper school
= 0.6 × 450 = 270

Number of female students in upper school
= 450 – 270 = 180

Number of male students in lower school
= 0.55 × 500 = 275

Number of female students in lower school
= 500 – 275 = 225

This information is added to the frequency tree.



b Total males = 270 + 275

= 545

P(student is male) $= \frac{545}{950}$

= 0.57

Venn diagrams and probability p.20

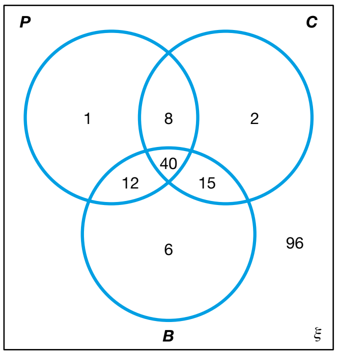
1 a i A $B = \{1, 3, 4, 5, 8, 9, 10, 11\}$

ii $A \cap B = \{8, 9\}$

iii $A' = \{2, 5, 6, 10, 13\}$

b $P(B') = \frac{7}{11}$

2 a Complete the diagram, using the information you are given to work out the unknown areas.

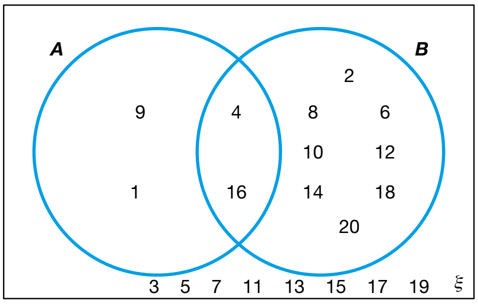


b P(all 3 sciences) $= \frac{40}{84} = \frac{10}{21}$

c P(only one science) $= \frac{(1+2+6)}{84} = \frac{9}{84} = \frac{3}{28}$

d P(chemistry if study physics) $= \frac{(8+40)}{(8+40+12+1)} = \frac{48}{61}$

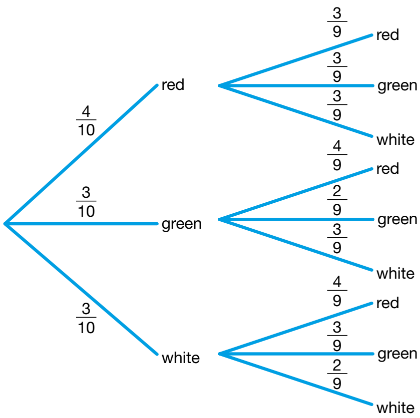
3 a



b P(A B)' $= \frac{8}{20} = \frac{2}{5}$

Review it! p.21

1 a



b Probability the same colour

= P (2 red) + P(2 green) + P(2 white)

$= \left(\frac{4}{10} \times \frac{3}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right)$

$= \frac{12}{90} + \frac{6}{90} + \frac{6}{90}$

$= \frac{24}{90}$

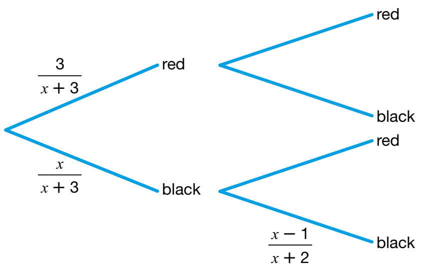
$= \frac{4}{15}$

c Probability different colours = 1 – probability of the same colour

$= 1 - \frac{4}{15}$

$= \frac{11}{15}$

2 a



Probability two black balls chosen $= \left(\frac{x}{x+3}\right) \times \left(\frac{x-1}{x+2}\right)$

Also, Probability two black balls chosen $= \frac{7}{15}$

$\left(\frac{x}{x+3}\right) \times \left(\frac{x-1}{x+2}\right) = \frac{7}{15}$

$15x(x - 1) = 7(x + 3)(x + 2)$

$15x^2 - 15x = 7x^2 + 35x + 42$

$8x^2 - 50x - 42 = 0$

$4x^2 - 25x - 21 = 0$

b Solving the quadratic equation

$4x^2 - 25x - 21 = 0$

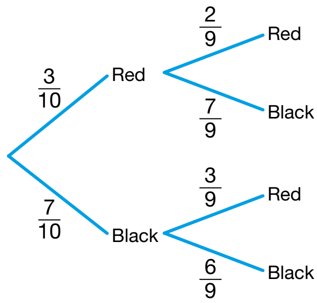
$(4x + 3)(x - 7) = 0$

$x = -\frac{3}{4}$ (which is impossible as x has to be a positive integer) or $x = 7$.

Hence $x = 7$

Total number of balls in the bag = 3 + 7 = 10

c Producing a new tree diagram now that x is known.



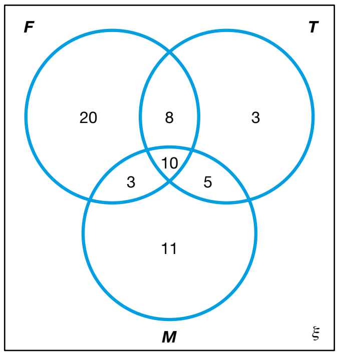
Probability of two different colours

$= \left(\frac{3}{10} \times \frac{7}{9}\right) + \left(\frac{7}{10} \times \frac{3}{9}\right)$

$= \frac{21}{90} + \frac{21}{90}$

$= \frac{7}{15}$

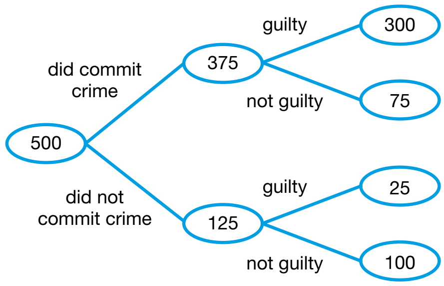
3



a P(only liked motor racing) $= \frac{11}{60}$

b P(student who liked motor racing also liked tennis) $= \frac{15}{29}$

4 a



b P(random defendant found guilty) $= \frac{300 + 25}{500}$

$= \frac{13}{20}$

c P(defendant who did not commit crime found guilty) $= \frac{25}{125} = \frac{1}{5}$

Exam practice answers

The basics of probability p.23

1 **a**

Dice 1

Dice 2

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b P(score of 12) = $\frac{1}{36}$

c P(prime number) = $\frac{15}{36} = \frac{5}{12}$

d 7, as there are six 7s which is more than all the other scores.

2 **a** total number of chocolates = $2x + 1 + x + 2x = 5x + 1$

P(mint) = $\frac{x}{5x+1} = \frac{4}{21}$

$21x = 20x + 4$

$x = 4$

total number of chocolates = $5x + 1 = 5 \times 4 + 1 = 21$

b number of truffles = $2 \times 4 + 1 = 9$

P(truffle) = $\frac{9}{21} = \frac{3}{7}$

3

Bethany

Amy

	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

a number of possible outcomes = 36

number of pairs the same = 6

P(scores are equal) = $\frac{6}{36} = \frac{1}{6}$

b The possible scores where Amy's score is higher are:

(2, 1)

(3, 1), (3, 2)

(4, 1), (4, 2), (4, 3)

(5, 1), (5, 2), (5, 3), (5, 4)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5)

P(Amy's score is higher) = $\frac{15}{36} = \frac{5}{12}$

Probability experiments p.24

1 **a** He is wrong because 100 spins is a very small number of trials. To approach the theoretical probability you would have to spin many more times. Only when the number of spins is extremely large will the frequencies start to become similar.

b relative frequency = $\frac{\text{frequency of event}}{\text{total frequency}} = \frac{22}{100} = \frac{11}{50}$

c Spinning 100 times gives a frequency of 19.

Spinning 500 times gives an estimate for the frequency = $5 \times 19 = 95$.

2 **a** $3x + 0.05 + 2x + 0.25 + 0.20 + 0.1 = 5x + 0.6$

The relative frequencies have to add up to 1.

So $5x + 0.6 = 1$

$5x = 0.4$

$x = 0.08$

b relative frequency for a score of 1 = $3 \times 0.08 = 0.24$

c number of times = $0.20 \times 80 = 16$

The AND and OR rules p.26

1 **a** P(picture card) = $\frac{12}{52} = \frac{3}{13}$

P(2 picture cards) = $\frac{3}{13} \times \frac{3}{13} = \frac{9}{169}$

b P(ace) = $\frac{4}{52} = \frac{1}{13}$

P(ace and picture card) = $\frac{1}{13} \times \frac{3}{13} = \frac{3}{169}$

c P(queen of hearts and queen of diamonds) = $\frac{1}{52} \times \frac{1}{52}$

$= \frac{1}{2704}$

2 **a** When an event has no effect on another event, they are said to be independent events. Here the colour of the first marble has no effect on the colour of the second marble.

b P(red) = $\frac{3}{10}$

P(2 reds) = $\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$

c P(red then blue) = $\frac{3}{10} \times \frac{5}{10} = \frac{15}{100}$

P(blue then red) = $\frac{5}{10} \times \frac{3}{10} = \frac{15}{100}$

P(red and blue) = $\frac{15}{100} + \frac{15}{100} = \frac{30}{100} = \frac{3}{10}$

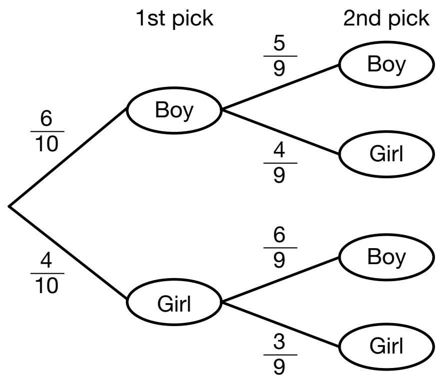
3 **a** P(homework in all 3 subjects) = $\frac{3}{5} \times \frac{3}{7} \times \frac{1}{4} = \frac{9}{140}$

b P(homework not given in any of the subjects)

$= \frac{2}{5} \times \frac{4}{7} \times \frac{3}{4} = \frac{24}{140} = \frac{6}{35}$

Tree diagrams p.27

1



a P(2 girls) = $\frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$

b P(boy and girl) = P(BG) + P(GB)

$= \left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9}\right)$

$= \frac{8}{15}$

2 **a** number of boys = $0.56 \times 450 = 252$

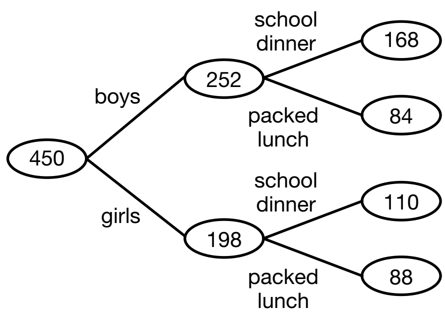
number of girls = $450 - 252 = 198$

number of boys who have a packed lunch = $\frac{1}{3} \times 252 = 84$

number of boys who have a school dinner = $252 - 84 = 168$

number of girls who have a school dinner = $\frac{5}{9} \times 198 = 110$

number of girls who have a packed lunch = $198 - 110 = 88$



b P(girl who has a school dinner) = $\frac{110}{450} = \frac{11}{45}$

c P(boy who has a school dinner) = $\frac{168}{450}$

P(school dinner) = $\frac{110}{450} + \frac{168}{450} = \frac{278}{450} = \frac{139}{225}$

3 **a** Possible ways of one marble of each colour:

RGY RYG GYR GRY YGR YRG

P(RGY) = $\frac{5}{9} \times \frac{3}{8} \times \frac{1}{7} = \frac{15}{504}$

P(RYG) = $\frac{5}{9} \times \frac{1}{8} \times \frac{3}{7} = \frac{15}{504}$

P(one of each colour) = $6 \times \frac{5}{9} \times \frac{3}{8} \times \frac{1}{7} = 6 \times \frac{15}{504} = \frac{90}{504}$

$= \frac{5}{28}$ = 0.179 (to 3 d.p.)

b P(no green) = $\frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{120}{504} = \frac{5}{21}$ = 0.238 (to 3 d.p.)

c P(all red) = $\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{5}{42}$

P(all green) = $\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} = \frac{1}{84}$

P(same colour) = $\frac{5}{42} + \frac{1}{84} = \frac{11}{84}$ = 0.131 (to 3 d.p.)

Venn diagrams and probability p.29

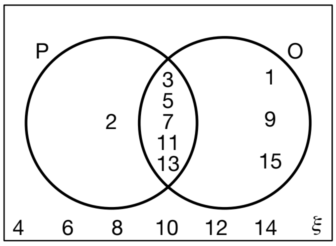
1 **a** 9, 8

b 1, 2, 3, 5, 7, 8, 9, 12, 15

c 1, 3, 4, 10, 12, 15

d 4, 10

2 **a**



b P(number in P O) = $\frac{5}{15} = \frac{1}{3}$

3 **a** P(team sports) = P(not only individual) = $1 - \frac{15}{100} = \frac{85}{100}$

$= \frac{17}{20}$

Alternatively, you could find the total of all those who play team sports.

b P(student playing large team also plays small team sports)

$= \frac{18}{73}$