Ratio Skills Edexcel Maths Higher GCSE 9–1

Full worked solutions

Revision answers

Introduction to ratios p.8

1 **a** 2 : 6 = 1: 3 (divide both sides by 2)

b 25 : 60 = 5 : 12 (divide both sides by 5)

c 1.6 : 3.6 = 4 : 9 (divide both sides by 0.4, or multiply by ten then divide by four)

2 **a** 250 g : 2 kg = 250 g : 2000 g = 250 : 2000 = 1 : 8

b 25 m : 250 mm = 25 000 mm : 250 mm
= 25 000 : 250 = 100 : 1

c 2 cl : 1 l = 2 cl : 100 cl = 2 : 100 = 1 : 50

3 Ratio = 3.5 : 2.1 = 35 : 21 = 5 : 3

Total shares = 5 + 3 = 8

1 share = £$\frac{400}{8}$ = £50

5 shares = 5 × £50 = £250

3 shares = 3 × £50 = £150

4 4 parts = 180

1 part = 45 (Dividing both sides by 4)

3 parts = 3 × 45
= 135

Hence there are 180 + 135 = 315 members of the gym.

5 The ratio is 21 : 25 : 29

Total shares = 21 + 25 + 29 = 75

One share = £$\frac{150\,000}{75}$ = £2000

Youngest daughter receives 21 × 2000 = £42 000

6 Total number of parts in the ratio = 5 + 2 = 7

Now pick a number that is divisible by 7. We will choose 70.

Dividing this into the ratio 5 : 2 gives 50 male guests and 20 female guests.

60% of male guests are under 40 and 70% of female guests are under 40.

60% of 50 = 30 (males under 40)

70% of 20 = 14 (females under 40)

So if there were 70 guests, 44 of them would be under 40 years old. Use this information to work out the correct percentage, whatever the number of guests:

$\frac{44}{70} \times 100 = 62.9\%$

7 There are 2 more parts of the ratio for 20p coins, and 6 more 20p coins.

So 2 parts = 6 coins

1 part = 3 coins.

Hence there are 5 × 3 = 15 10p coins, and 7 × 3 = 21 20p coins

Total amount (£) in the money box = 15 × 0.1 + 21 × 0.2
= 1.5 + 4.2 = £5.70

8 Let x be the number of yellow marbles.

So $5x$ = number of red marbles, and $2 \times 5x = 10x$ = number of blue marbles.

Hence the ratio of blue to red to yellow marbles = $10x : 5x : x$ = 10 : 5 : 1

Scale diagrams and maps p.9

1 First convert 150 km to cm.

150 km = 150 000 m = 15 000 000 cm

500 000 cm is equivalent to 1 cm on the map.

Distance in cm on the map = $\frac{15\,000\,000}{500\,000}$ = 30 cm

2 **a** Distance between the ship and the port = 2 cm.

This is measured using a ruler on the map.

We now need to get the units the same.

10 km = 10 000 m = 1 000 000 cm

The scale is 2 : 1 000 000

Dividing both sides of the ratio by 2 gives

1: 500 000

b Measuring the actual distance between the two ships gives 1.2 cm

Actual distance = 1.2 × 500 000 = 600 000 cm

Divide this by 100 and then 1000 to give the actual distance in km.

600 000 cm = 6 km

Percentage problems p.12

1 $\frac{8}{300} \times 100 = 2.\dot{6} = 2.67\%$ to 2 d.p.

2 Increase = Final earnings − Initial earnings
= 1 100 000 − 600 000 = £500 000

% increase = $\frac{\textit{Increase}}{\textit{original value}} \times 100$

$= \frac{500\,000}{600\,000} \times 100$

= 83.3% to 1 d.p.

3 $3.5\% = \frac{3.5}{100} = 0.035$

Add 1 to create a multiplier for the original number: 1.035

New salary = $1.035 \times 38\,000 =$ £39 330

4 $18\% = \frac{18}{100} = 0.18$

$1 - 0.18 = 0.82$

82% of original price = £291.92

1% of original price = $\frac{291.92}{82} = 3.56$

100% of original price = $3.56 \times 100 = 356$

Original price = £356

5 Amount of interest in one year = 3.5% of £12 000

$= \frac{3.5}{100} \times 12000 =$ £420

Total interest paid over 6 years = $6 \times$ £420 = £2520

Direct and inverse proportion p.15

1 Inverse proportion means that if one quantity doubles the other quantity halves.

2 **a** $y = kx$

b $8 = k \times 3$ giving $k = \frac{8}{3}$

$y = \frac{8}{3}x$

When $x = 4$, $\frac{8}{3} \times 4 = \frac{32}{3} = 10.7$ (1 d.p.)

3 **a** Find the equivalent price in £

€$\frac{120}{1.27}$ = £94.49

The sunglasses are cheaper in the UK.

b £94.49 – £89 = £5.49

4 $V \quad r^3$ so $V = kr^3$

When $V = 33.5$, $r = 2$ so $33.5 = k \times 2^3$

$k = \frac{33.5}{8} = 4.1875$

Substituting this value of k back into the equation gives

$V = 4.1875r^3$

When $r = 4$, $V = 4.1875 \times 4^3 = 268$ cm^3

5 $P \quad \frac{1}{V}$ so $P = \frac{k}{V}$

Hence $100\,000 = \frac{k}{1}$, giving $k = 100\,000$

Formula is $P = \frac{100\,000}{V}$

When $V = 3$,

$P = \frac{100\,000}{3}$

$= 33\,333$ (to the nearest whole number)

6 $a = kb^2$

When $a = 96$, $b = 4$ so $96 = k \times 4^2$ giving $k = 6$

Hence $a = 6b^2$ and when $b = 5$, $a = 6 \times 5^2 = 150$

7 **a** A square of side x cm has an area of x^2.

$A = kx^2$

As there are 6 faces to a cube, surface area = $6x^2$

Hence $A = 6x^2$

So constant of proportionality $k = 6$

b $A = 6x^2$

When $x = 4$ cm

$A = 6 \times 4^2 = 96$ cm^2

Graphs of direct and inverse proportion and rates of change p.18

1 Graph C

2 Graph B

3 As P and V are inversely proportional, $P = \frac{k}{V}$

At point A, when $P = 12$, $V = 3$ so $12 = \frac{k}{3}$ hence $k = 36$

Substituting this value of k back into the equation we have $P = \frac{36}{V}$

When $V = 6$, $P = \frac{36}{6} = 6$

Hence $a = 6$

4 As x and y are inversely proportional, $y \quad \frac{1}{x}$, so $y = \frac{k}{x}$

When $x = 1$, $y = 4$ so $4 = \frac{k}{1}$ so $k = 4$

The equation of the curve is now $y = \frac{4}{x}$

When $x = 4$, $y = \frac{4}{4} = 1$ so $a = 1$

When $y = 0.8$, $0.8 = \frac{4}{x}$ giving $x = 5$ so $b = 5$.

Hence $a = 1$ and $b = 5$.

5 $y = kx^2$

When $x = 2$, $y = 16$ so $16 = k \times 2^2$ giving $k = 4$.

$y = 4x^2$

Hence when $y = 36$,

$36 = 4x^2$ so $x = 3$ or -3

Since a is positive,

$a = 3$

Growth and decay p.21

1 For a, b and c, multiplier is $\frac{100\% + \text{percentage given}}{100}$

a 1.05

b 1.25

c 1.0375

d Multiplier is $\frac{100\% - \text{percentage given}}{100} = 0.79$

2 Multiplier $= 1 - \frac{18}{100} = 0.82$

Value at the end of n years = $A_0 \times (\text{multiplier})^n$ where A_0 is the initial value.

Value at the end of 3 years = $9000 \times (0.82)^3 =$ £4962.312

= £4962 (nearest whole number)

3 Number of restaurants after n years, $R_n = R_0 \times (\text{multiplier})^n$

Multiplier = 1.25, $n = 3$, $R_n = 4000$

So $R_0 = \frac{4000}{(1.25)^3} = 2048$

There were 2048 restaurants 3 years ago.

Ratios of lengths, areas and volumes p.24

1 Scale factor $= \frac{6}{9} = \frac{2}{3}$

$x = \frac{2}{3} \times 10 = \frac{20}{3} = 6\frac{2}{3}$ cm

2 As the shapes are similar $\frac{V_A}{V_B} = \left(\frac{r_A}{r_B}\right)^3$

Cube rooting both sides gives $\frac{r_A}{r_B} = \sqrt[3]{\frac{V_A}{V_B}}$

$\frac{r_A}{r_B} = \sqrt[3]{\frac{27}{64}}$

$= \frac{3}{4}$

$\frac{\text{radius of cylinder A}}{\text{radius of cylinder B}} = \frac{3}{4}$

$\frac{\text{surface area of cylinder A}}{\text{surface area of cylinder B}} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

So surface area of cylinder A = surface area of cylinder B $\times \frac{9}{16}$

Surface area of cylinder A = $96 \times \frac{9}{16} = 54\,\text{cm}^2$

3 **a** As *BE* is parallel to *CD* all the corresponding angles in both triangles are the same so triangles *ABE* and *ACD* are similar.

$\frac{BE}{8} = \frac{5}{10}$ giving $BE = 4$ cm

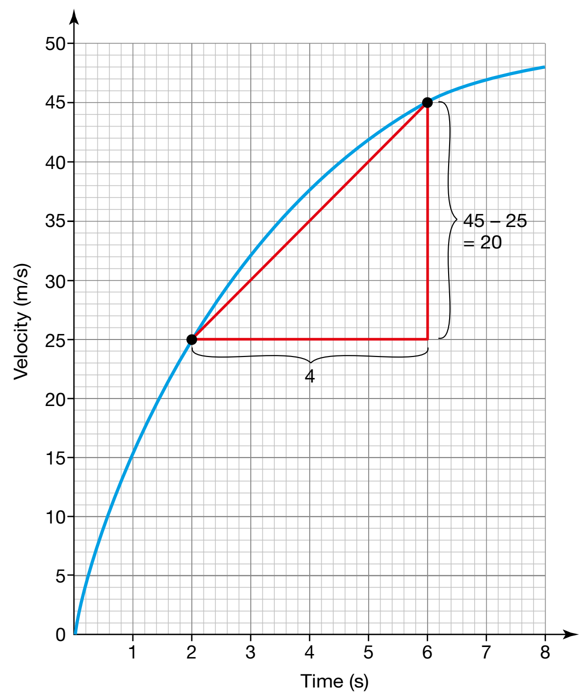
b $BE = 4$ cm and $AB = 5$ cm. Angle *ABE* is a right angle because angle *ACD* is a right angle and the triangles are similar.

Hence area $ABE = \frac{1}{2} \times 4 \times 5 = 10\,\text{cm}^2$

Gradient of a curve and rate of change p.27

1 **a** The gradient represents the acceleration.

b Find the gradient between when time = 2 s and time = 6 s.



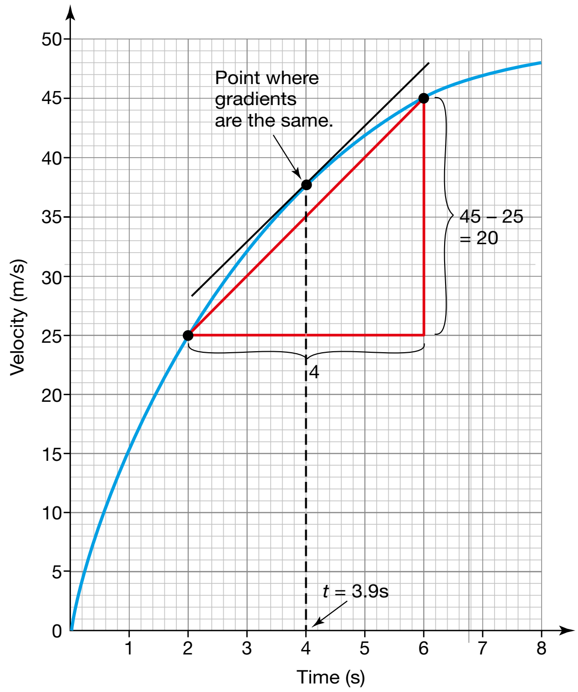
From the graph, acceleration between time = 2 s and time = 6 s

$= \frac{45 - 25}{6 - 2}$

$= \frac{20}{4}$

$= 5\,\text{m/s}^2$.

c We need to find a point on the graph where the gradient equals that of the tangent already drawn ($5\,\text{m/s}^2$).



Time when the instantaneous acceleration is the same as the average acceleration = 3.9 s

Converting units of areas and volumes, and compound units p.30

1 **a** Surface area $= 2 \times 4 \times 6 + 2 \times 4 \times 5 + 2 \times 6 \times 5$
$= 148\,\text{cm}^2$

i $1\,\text{cm}^2 = 10 \times 10\,\text{mm}^2 = 100\,\text{mm}^2$

$148\,\text{cm}^2 = 148 \times 100\,\text{mm}^2 = 14\,800\,\text{mm}^2$

ii $1\,\text{m}^2 = 100 \times 100\,\text{cm}^2 = 10\,000\,\text{cm}^2$

$148\,\text{cm}^2 = 148 \div 10\,000\,\text{m}^2 = 0.0148\,\text{m}^2$

b Volume $= 6 \times 5 \times 4 = 120\,\text{cm}^3$

$1\,\text{m}^3 = 100 \times 100 \times 100\,\text{cm}^3 = 1\,000\,000\,\text{cm}^3$

$120\,\text{cm}^3 = 120 \div 1\,000\,000 = 0.00012\,\text{cm}^3$

2 Density $= \frac{\text{mass}}{\text{volume}} = \frac{1159}{600} = 1.932\,\text{g/cm}^3$ (3 d.p.)

3 First get the units the same.

$1\text{m}^3 = 100 \times 100 \times 100\,\text{cm}^3 = 1\,000\,000\,\text{cm}^3$

Number of ball bearings $= \frac{1\,000\,000}{0.5} = 2\,000\,000$
(i.e. 2 million)

4 50 km/h = 50 000 m/h

1 h = 60 × 60 s = 3600 s

Now as fewer m/s will be covered compared to m/h we divide by 3600 to convert the speed to m/s.

Hence 50 000 m/h $= \frac{50\,000}{3600} = 13.89$ m/s (2 d.p.)

5 **a** Distance = speed × time = 70 × 4 = 280 km

Mary's speed = 280/5 = 56 km/h

b If Mary's route was longer, her average speed would increase. If her route was shorter, her average speed would decrease.

Review it! p.31

1 80% of the original price = £76.80

10% of the original price = $\frac{76.80}{8}$ = £9.60

100% of the original price = £9.60 × 10 = £96

2 **a** $A_1 = 1.02 \times A_0$

$= 1.02 \times 5$

$= 5.10$ m² (2 d.p.)

b $A_2 = 1.02 \times A_1 = 1.02 \times 5.1 = 5.202$ m²

$A_3 = 1.02A_2 = 1.02 \times 5.202 = 5.31$ m² (2 d.p.)

3 $y = \frac{k}{x}$

$4 = \frac{k}{2.5}$

$k = 10$

Hence $y = \frac{10}{x}$

When $x = 5$, $y = \frac{10}{5} = 2$

4 2 parts of the ratio = 54 students, so 1 part = 27 students.

Students in the whole year = 2 + 7 parts = 9 parts.

In Year 11, there are 9 × 27 = 243 students.

5 The price of the house has more than doubled.

Multiplier = 1 + 1.2 = 2.2

So £220 000 × 2.2 = £484 000

The value of the house = £485 000 to the nearest £5000

6 **a** Simple interest: One year = 2000 × 0.025 = £50

50 × 5 = 250

After 5 years, there will be £2250 in the account.

b Compound interest: 2000×1.025^5 = £2262.82

7 **a** 400 × 8.55 = 3420 yuan

b Travel agent: 800 ÷ 8.6 = £93.02

Commission = 93.02 × 0.025 = £2.33

So Tom would get £93.02 – £2.33 = £90.69

From the post office, Tom would get 800 ÷ 8.9 = £89.89

Tom will get a better deal from the travel agent.

8 The difference between Luke's and Amy's payouts was £4000. The difference in parts is 7 – 5 = 2.
So 2 parts = £4000 and 1 part = £2000

The profits are split into 3 + 5 + 7 = 15 parts.

Total profits = 15 × 2000 = £30 000

9 $\frac{\text{Surface area A}}{\text{Surface area B}} = \left(\frac{\text{Length A}}{\text{Length B}}\right)^2$

So $\frac{25}{4} = \frac{5^2}{2^2}$

Similarly, $\frac{\text{Volume A}}{\text{Volume B}} = \left(\frac{\text{Length A}}{\text{Length B}}\right)^3$

$\frac{10}{\text{Volume B}} = \frac{5^3}{2^3}$

So Volume B $= \frac{10 \times 8}{125} = \frac{80}{125} = 0.64$ cm³

10 7 + 4 = 11 parts in the ratio.

Let this be 1100 members.

There would then be 700 male club members: 25% of this is 0.25 × 700 = 175 junior members.

There would be 400 female club members: 10% of this is 0.1 × 400 = 40 junior members.

In total, there would be 175 + 40 = 215 junior members.

As a percentage, this is $\frac{215}{1100} \times 100 = 19.54$
= 20% to the nearest integer.

11 Each cm³ of the alloy contains 9 parts (= 0.9 cm³) copper and 1 part (= 0.1 cm³) tin.

So the mass of copper in 1 cm³ = 8.9 × 0.9 = 8.01 g

The mass of tin in 1 cm³ = 7.3 × 0.1 = 0.73 g

Mass of 1 cm³ of alloy = 8.01 + 0.73 = 8.74 g

So the density of the alloy = 8.7 g/cm³ (1 d.p.)

Exam practice answers

Introduction to ratios p.34

1 3 parts = 18 black balls so 1 part = 6 black balls

There are 3 + 2 = 5 parts in total, so the total number of balls

= 5 × 6 = 30.

2 total shares = 15 + 17 + 18 = 50

50 shares = £25 000

1 share = $\frac{£25\,000}{50}$ = £500

15 year old will get 15 × £500 = £7500.

17 year old will get 17 × £500 = £8500.

18 year old will get 18 × £500 = £9000.

3 40% of 800 = = 320 acres

remainder of land area = 800 – 320 = 480 acres

total shares = 9 + 7 =16

1 share = $\frac{480}{16}$ = 30

area devoted to sheep = 7 shares = 7 × 30 = 210 acres

4 $\frac{3x + 1}{x + 4} = \frac{2}{3}$

$3(3x + 1) = 2(x + 4)$

$9x + 3 = 2x + 8$

$7x = 5$

$x = \frac{5}{7}$

5 For 2 oak trees there are 3 ash trees, so if there are 8 oak trees there would be 12 ash trees.

pine : oak: ash = 5 : 8 : 12

There are a total of 5 + 8 + 12 = 25 parts.

1 part = $\frac{300}{25}$ = 12

number of ash trees = 12 × 12 = 144

Scale diagrams and maps p.35

1 10 cm on map = 10 × 50 000 cm = 500 000 cm actual distance

500 000 cm = 5000 m = 5 km

Towns are 5 km apart.

2 **a** length of road = 2.3 × 40 000 cm

= 92 000 cm

= 920 m

= 0.92 km

b length of road = 3 × 40 000 mm

= 120 000 mm

= 12 000 cm

= 120 m

= 0.12 km

3 5 cm : 40 m = 5 cm : 4000 cm

= 5 : 4000

= 1 : 800

scale of drawing = 1 : 800

4 length measured on map = 6 cm

Scale is:

6 cm : 12 km = 6 : 12 × 1000 × 100

= 6 : 1 200 000

= 1 : 200 000

Percentage problems p.36

1 increase = £35

percentage increase = $\frac{\text{increase}}{\text{original price}} \times 100$

$= \frac{35}{350} \times 100$

= 10%

2 increase = final earnings – initial earnings

= 1 100 000 – 600 000

= £500 000

% increase = $\frac{\text{increase}}{\text{original value}} \times 100$

$= \frac{500\,000}{600\,000} \times 100$

= 83.3%

3 multiplier = 1 – 0.28 = 0.72

value of car = 0.72 × 25 000

= £18 000

4 88% of the original price = £14 300

1% of the original price = $\frac{14\,300}{88}$

100% of the original price = $\frac{14\,300}{88} \times 100$ = 16 250

original price = £16 250

5 interest earned = $\frac{2.8}{100} \times 8000 \times 4$

= £896

Direct and inverse proportion p.37

1 **a** $P = kT$

b $P = kT$ so $k = \frac{P}{T} = \frac{200\,000}{540} = 370.370$

$P = 370.370T = 370.370 \times 200 = 74\,074$ pascals (to nearest whole number)

2 $C \quad r^2$

$C = kr^2$

$480 = k \times 3^2$

$k = 53.33$

$C = 53.33r^2$

cost = 53.33 × 4² = £853.28 = £853 (to nearest whole number)

3 **a** $c \quad \frac{1}{h}$ so $c = \frac{k}{h}$

$3 = \frac{k}{12}$, so $k = 36$

$c = \frac{36}{h}$

b $c = \frac{36}{h}$

$= \frac{36}{15}$

= 2.4

4 **a** 350 × 1.15 = €402.50

b 80 ÷ 1.11 = £72.07 (to nearest penny)

c The €80 she had left cost her 80 ÷ 1.15 = £69.57 before her holiday.

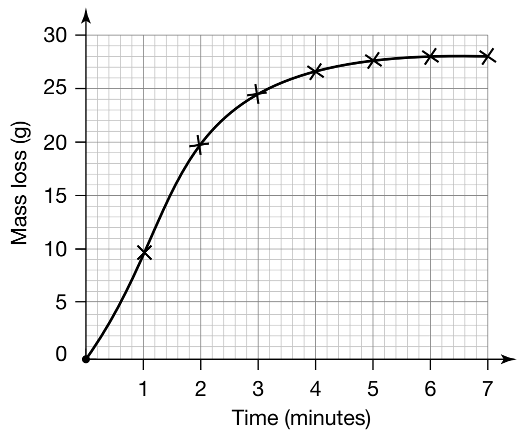
If she hadn't changed this amount, she would have saved 72.07 – 69.57 = £2.50

Graphs of direct and inverse proportion and rates of change p.39

1 straight line through the origin: B

2 curve that gets close to but does not cross either axis: B

3 a



b i initial rate of change = gradient over first 2 minutes

$= \frac{19.6}{2}$

= 9.8 g/minute

ii initial rate of change = $\frac{9.8}{60}$ = 0.16 g/second (to 2 d.p.)

Growth and decay p.40

1 a multiplier = $1 + \frac{\%\ \text{increase}}{100} = 1 + \frac{6}{100} = 1.06$

population after n years = $A_0 \times (\text{multiplier})^n$,

where A_0 = initial population and n = number of years

population after 3 years = $150\,000 \times 1.06^3 = 178\,652$

b Try n = 5 years: population = $150\,000 \times 1.06^5 = 200\,733$

Try n = 4 years: population = $150\,000 \times 1.06^4 = 189\,372$

After 5 years, the population will have risen to over 200 000.

2 multiplier = $1 - \frac{\%\ \text{decrease}}{100} = 1 - \frac{12}{100} = 0.88$

value of car after 4 years = $21\,000 \times 0.88^4$ = £12 594

3 multiplier = $1 - \frac{\%\ \text{decrease for each time unit}}{100} = 1 - \frac{50}{100} = 0.5$

2 minutes = 120 seconds

number of time intervals, $n = \frac{120}{12} = 10$

amount at the end of n time intervals = $A_0 \times (\text{multiplier})^n$

$= 100 \times 0.5^{10}$

= 0.1 (to 1 s.f.)

Ratios of lengths, areas and volumes p.41

1 a scale factor for enlargement $\left(\frac{big}{small}\right)^3 = \left(\frac{12}{8}\right)^3 = 3.375$ or $\frac{27}{8}$

b $\frac{\text{area of triangle of large prism}}{\text{area of triangle of small prism}} = \left(\frac{12}{8}\right)^2$

area of triangle of large prism = $\left(\frac{12}{8}\right)^2 \times 10 = 22.5\ \text{cm}^2$

c $\frac{\text{volume of small prism}}{\text{volume of large prism}} = \left(\frac{8}{12}\right)^3$

volume of small prism = $\left(\frac{8}{12}\right)^3 \times 450$

= 133.33

= 133 cm^3 (to nearest whole number)

2 $\frac{\text{volume of larger cuboid}}{\text{volume of smaller cuboid}} = \left(\frac{h}{12}\right)^3$

$\frac{\text{volume of larger cuboid}}{\text{volume of smaller cuboid}} = 1.953$

$\left(\frac{h}{12}\right)^3 = 1.953$

$h^3 = 3374.784$

h = 15 cm (to nearest cm)

3 a i angle XYZ = angle TUZ (corresponding angles)

angle ZXY = angle ZTU (corresponding angles)

angle XZY = angle TZU (same angle)

Hence triangles XYZ and TUZ are similar.

$\frac{XY}{TU} = \frac{YZ}{UZ}$

$\frac{XY}{3} = \frac{15}{5}$

XY = 9 cm

ii angle YUT = angle YZW (corresponding angles)

angle YTU = angle YWZ (corresponding angles)

angle TYU = angle WYZ (same angle)

Hence triangles YUT and YZW are similar.

$\frac{WZ}{TU} = \frac{YZ}{YU}$

$\frac{WZ}{3} = \frac{15}{10}$

WZ = 4.5 cm

b angle YTX = angle ZTW (vertically opposite angles)

angle YXT = TZW (alternate angles)

Two angles of both triangles are equal so the third angles must also be equal.

Hence triangles XYT and ZWT are similar.

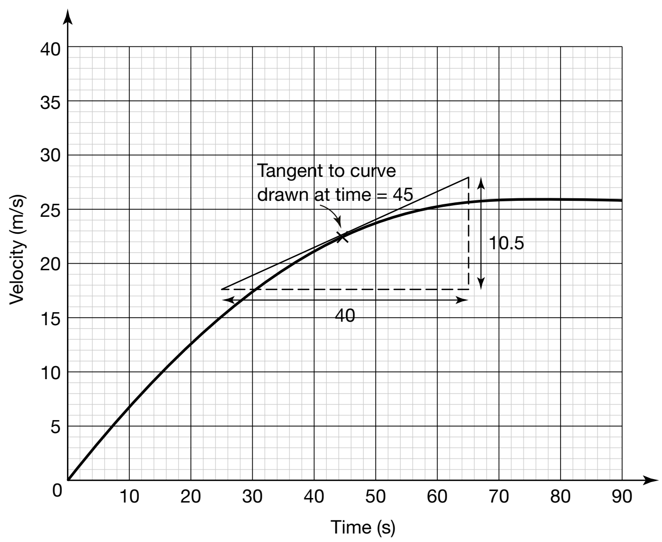
ratio of the areas of TYX and TWZ = $\left(\frac{XY}{WZ}\right)^2 = \left(\frac{9}{4.5}\right)^2 = 4$

ratio of areas = 4 : 1

Gradient of a curve and rate of change p.43

1 a acceleration = gradient of straight line = $\frac{10-0}{15-0} = \frac{2}{3}$ m/s^2

b Instantaneous acceleration = gradient of the tangent to the curve at 45 s.



Gradient = $\frac{10.5}{40}$ = 0.2625

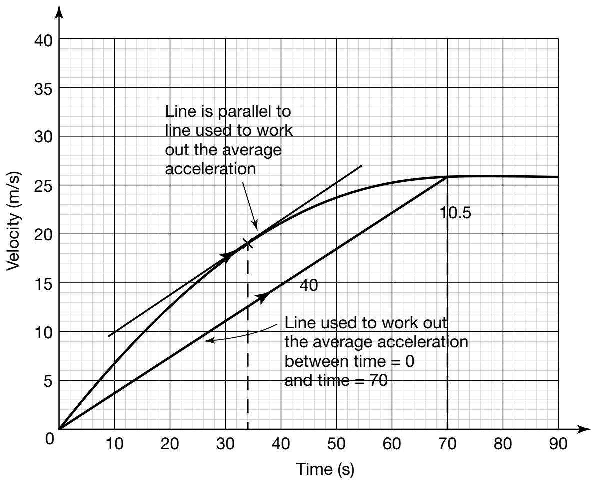
Instantaneous acceleration at 45 s = 0.26 m/s^2 (to 2 d.p.)

c average acceleration over the first 70 s = gradient of the line from (0, 0) to (70, 26)

gradient = $\frac{26-0}{70-0}$ = 0.3714…

acceleration = 0.37 m/s^2 (to 2 d.p.)

d



gradients of the two lines are parallel when time = 34 s

Converting units of areas and volumes, and compound units p.44

1 pressure = $\frac{\text{force}}{\text{area}} = \frac{200}{0.4} = 500\,\text{N/m}^2$

2 $1\,\text{m}^2 = 100 \times 100\,\text{cm}^2 = 10\,000\,\text{cm}^2$

$1\,\text{cm}^2 = \frac{1}{10000}\,\text{m}^2$

$200\,\text{cm}^2 = \frac{200}{(10\,000)}\,\text{m}^2 = 0.02\,\text{m}^2$

pressure = $\frac{\text{force}}{\text{area}} = \frac{500}{0.02} = 25\,000\,\text{N/m}^2$

3 density = $\frac{\text{mass}}{\text{volume}}$

mass = volume × density

= 12 × 8.92

= 107.04 g

mass of wire = 107 g (to nearest g)

4 He has worked out the area in m² by dividing the area in cm² by 100, which is incorrect.

There are 100 × 100 = 10 000 cm² in 1 m², so the area should have been divided by 10 000.

Correct answer:

area in m² = $\frac{9018}{10\,000}$

= 0.9018

= 0.90 m² (to 2 d.p.)

5 distance in first two hours = time × speed

= 2 × 60

= 120 km

distance in next three hours = time × speed

= 3 × 80

= 240 km

total distance = 120 + 240

= 360 km

average speed = $\frac{\text{distance}}{\text{time}}$

= $\frac{360}{5}$

= 72 km/h