Answers

For full worked solutions, visit: www.scholastic.co.uk/gcse

Number

Integers, decimals and symbols

1 a 200.1 b 2.001 c 2.3 d 87
2 a 140.94 b 1.4094 c 290 d 4.86
3 −0.5, 0, 0.012, 0.12, 12
4 a $\frac{5}{0.5} = 10$ b $1\frac{5}{9} > \frac{4}{3}$ c $-3 < -1$
5 a 5 c −15 e −4
b −8 d 6

Addition, subtraction, multiplication and division

1 a 1561 c 69.93
b 3047 d 23.923
2 a 2819 c 8.185
b 287 d 5.401
3 a 29798 b 29.26 c 40.768
4 a 46 b 343 c 35.4

Using fractions

1 a $\frac{16}{5} = 3\frac{1}{5}$ c $\frac{5}{8}, \frac{3}{4}, \frac{9}{10}, 1\frac{1}{5}, \frac{16}{5}$
b $1\frac{1}{5} = \frac{6}{5}$ d $4\frac{2}{5}$ e $2\frac{23}{40}$
2 $\frac{15}{45}, \frac{4}{12}, \frac{16}{48}$
3 a $7\frac{1}{3}$ b $3\frac{1}{2}$
4 $\frac{13}{60}$

Different types of number

1 a 16 b 5 c 16
2 $2 \times 2 \times 3 \times 5 \times 5$
3 every 144 days
4 a $2^2 \times 3^3 \times 7$ b 36

Listing strategies

1 12 2 180

The order of operations in calculations

1 a 18 b 13 c 25
2 a 10 b 23 c 5

Indices

1 a 7^{10} b 3^{-6} c 5^{20}
2 a 5^7 c $2^{10} \times 5^{-3}$
b 6^{-3} d $7^{10} \times 11^{-1}$
3 a 1 c 16 e 18
b 10 d $\frac{1}{5}$ (or 0.2)
4 $x = 1$

Surds

1 a $\sqrt{6}$ b 5 c 18 d 20
2 $a = 2$
3 a $3\sqrt{5}$ b $6\sqrt{2}$
4 a $\frac{16}{3\sqrt{2}} = \frac{16\sqrt{2}}{3\sqrt{2}\sqrt{2}} = \frac{16\sqrt{2}}{3 \times 2} = \frac{8\sqrt{2}}{3}$ b $8 - 2\sqrt{7}$
5 a −4 b $7 + 4\sqrt{3}$ c $5 + 3\sqrt{3}$

Standard form

1 a 0.005 b 565 000
2 a 2.5×10^4 c 5×10^2
b 1.25×10^{-3} d 1.4×10^{-2}
3 a 9×10^{-4} c 2×10^2
b 2.4×10^3 d 8.04×10^4
4 a 1.33×10^{10} pounds
b 26 600 000 people
5 a 1.55×10^4 c 5×10^2
b 655 000 d 4×10^3

Converting between fractions and decimals

1 a 0.43 b 0.375 c 0.55
2 a $\frac{4}{5}$ b $\frac{9}{20}$ c $\frac{73}{125}$
3 a $\frac{7}{9}$ b $\frac{2}{45}$ c $\frac{21}{22}$
4 a $\frac{14}{27}$ b $\frac{19}{25}$
5 1

Converting between fractions and percentages

1 a $\frac{1}{4}$ b $\frac{17}{20}$ c $\frac{17}{25}$
2 maths: 81.25%
Charlie did better at maths.
3 a 30% b 16% c 42.9%

Fractions and percentages as operators

1 a £480 b £4.50 c 76 kg
2 School A: 336, School B: 455

Standard measurement units

1 a 9700 g b 0.85 litres c 205 000 cm
2 8.64×10^4 seconds
3 £81.60

Rounding numbers

1 a 1260 c 0.000308 e 1.81×10^{-4}
b 14.9 d 9 080 000
2 a 10.6 c 0.03 e 0.002
b 123.977 d 3.971 f 4.10
3 a 2000 b 2000 c 1990
4 0.0004 (4 decimal places)

Estimation

1 a 2400 b 3
2 a A e A i B
b C f B j A
c B g C
d B h A
3 a 6.7 – accept 6.5 to 6.9
b 10.2 – accept 10.1 to 10.3
c 12 – accept 10 to 14
d 5 – accept 4 to 6

Upper and lower bounds

1 a 144.5 cm
b 145.5 cm
c $144.5 \leq l < 145.5$ cm

Review it!

1 a 24 647.515 b 21.5 (to 1 d.p.)
2 a 6.5 c 81
b 4 d 5.94
3 a $5\frac{1}{3}$ b $9\frac{7}{13}$
4 $4\frac{3}{4} + \frac{1}{2}$
5 a lower bound = 10.113 cm^2 (to 3 d.p.)
upper bound = 10.180 cm^2 (to 3 d.p.)
b 10 cm^2
6 a 3.34×10^{23} molecules
b 2.99×10^{-26} kg
7 0.16
8 a 1 c $\frac{1}{64}$
b 3 d $\frac{1}{4}$
9 $5.55 \leq y < 5.65$
10 $\frac{1-\sqrt{2}}{1+\sqrt{2}} = \frac{1-\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-2\sqrt{2}+2}{1-2} = \frac{3-2\sqrt{2}}{-1} = 2\sqrt{2} - 3$
11 $111.5 \leq a < 112.5$
12 $\frac{8}{11}$
13 a $0.287996 \leq c < 0.289272$
b 0.29 (to 2 s.f.)
14 a $\frac{2}{3}$
b 80
15 a 4.5×10^{-7}
b 1.2×10^{7}
c 5.64×10^{3}
16 3.2×10^{-1}
17 a 1, 2, 4, 8, 16, 32, 64
b 4

Algebra

Simple algebraic techniques

1 a formula b identity c expression
d identity e equation
2 a $10x^2 + 4x$ c $-3x^2 + 10xy$
b $3a - b$ d $3x^3 - x - 5$
3 16
4 4

Removing brackets

1 a $2x + 8$ c $x - 1$ e $3x^2 + 3x$
b $63x + 21$ d $3x^2 - x$ f $20x^2 - 8x$
2 a $5x + 12$ c $4x^2 + 2x$
b $3x + 45$ d $3x^2 - 10x + 8$
3 a $t^2 + 8t + 15$ c $6y^2 + 41y + 63$
b $x^2 - 9$ d $4x^2 - 4x + 1$
4 a $2x^3 + 21x^2 + 55x + 42$
b $24x^3 - 46x^2 + 29x - 6$

Factorising

1 a $6(4t + 3)$ c $5y(x + 3z)$
b $a(9 - 2b)$ d $6xy^2(4x^2 + 1)$
2 a $(x + 7)(x + 3)$ c $(2x + 5)(3x + 2)$
b $(x + 5)(x - 3)$ d $(2x + 7)(2x - 7)$
3 $\frac{1}{2x + 3}$

Changing the subject of a formula

1 a $r = \sqrt{\frac{A}{\pi}}$ b $r = \sqrt{\frac{A}{4\pi}}$ c $r = \sqrt[3]{\frac{3V}{4\pi}}$
2 a $c = y - mx$ d $s = \frac{v^2}{2a}$
b $u = v - at$ e $u = \sqrt{v^2 - 2as}$
c $a = \frac{v-u}{t}$ f $t = \frac{2s}{u + v}$

Solving linear equations

1 a $x = 3$ b $x = 3$ c $x = 20$
2 a $x = 5$ b $x = 18$ c $x = 20$
3 a $x = -2$ b $m = 1$ c $x = \frac{6}{5}$, $1\frac{1}{5}$ or 1.2

Solving quadratic equations using factorisation

1 a $x = -2$ or $x = -3$
b $x = -3$ or $x = 4$
c $x = -\frac{7}{2}$ or $x = -5$
2 a Area = $\frac{1}{2}$ × base × height
$\frac{1}{2}(2x + 3)(x + 4) = 9$
$\frac{1}{2}(2x^2 + 11x + 12) = 9$
$2x^2 + 11x + 12 = 18$
$2x^2 + 11x - 6 = 0$
b $x = \frac{1}{2}$ c base = 4 cm, height = 4.5 cm

3 By Pythagoras' theorem $(x + 1)^2 + (x + 8)^2 = 13^2$

$x^2 + 2x + 1 + x^2 + 16x + 64 = 169$

$2x^2 + 18x - 104 = 0$

Dividing through by 2 gives

$x^2 + 9x - 52 = 0$

$(x - 4)(x + 13) = 0$

So $x = 4$ or -13 (disregard $x = -13$ as x is a length)

Hence $x = 4$ cm

Solving quadratic equations using the formula

1 $x = 2.14$ or $x = -1.64$ (to 3 s.f.)

2 **a** $\frac{2x + 3}{x + 2} = 3x + 1$

$2x + 3 = (3x + 1)(x + 2)$

$2x + 3 = 3x^2 + 7x + 2$

$0 = 3x^2 + 5x - 1$

$3x^2 + 5x - 1 = 0$

b $x = -1.85$ or $x = 0.18$ (to 2 d.p.)

Solving simultaneous equations

1 **a** $x = 2, y = -1$

b $x = 4, y = 2$

2 $x = \frac{1}{5}, y = -2\frac{3}{5}$ or $x = \frac{1}{2}, y = -2$

3 Equating the y values gives

$x^2 + 5x - 4 = 6x + 2$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or -2

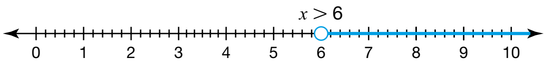
When $x = 3$, $y = 6 \times 3 + 2 = 20$

When $x = -2$, $y = 6 \times (-2) + 2 = -10$

Points are (3, 20) and (−2, −10)

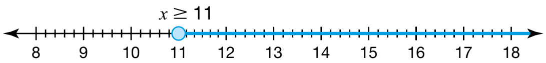
Solving inequalities

1 **a** $x > 6$



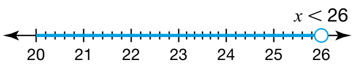
$\{x: x > 6\}$

b $x \geq 11$



$\{x: x \geq 11\}$

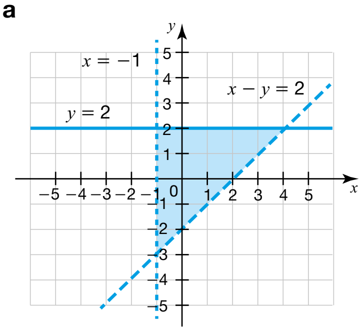
c $x < 26$



$\{x: x < 26\}$

2 **a** $x > 10$ **b** $x < 0.4$ or $\frac{2}{5}$ **c** $x \leq 8$

3 **a**



b (0, 2), (0,1), (0, 0), (0, −1), (1, 2), (1, 1), (1, 0), (2, 2), (2, 1), (3, 2)

4 $x < -2$ and $x > 5$

Problem solving using algebra

1 26, 51 **2** 65 **3** 9 cm by 3 cm

Use of functions

1 **a** -1 **b** $-\frac{2}{3}$ **c** $\frac{x + 1}{x}$

2 **a** $\sqrt{x^2 + 8x + 7}$ **c** 4

b $\sqrt{(x^2 - 9)} + 4$

Iterative methods

1 1.521

Let $f(x) = x^3 - x - 2$

$f(1.5215) = (1.5215)^3 - 1.5215 - 2 = 0.0007151$

$f(1.5205) = (1.5205)^3 - 1.5205 - 2 = -0.005225$

As there is a change in sign, $a = 1.521$ to 3 decimal places.

Equation of a straight line

1 **a** 2 **b** $-\frac{1}{2}$ **c** $y = -\frac{1}{2}x + 5$

2 $y = 3x - 3$

3 $2x - y + 2 = 0$

4 **a** $\frac{1}{2}$ **b** (2, 2) **c** **i** -2

ii $y = -2x + 6$

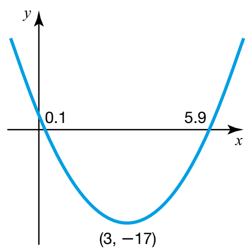
Quadratic graphs

1 **a** $2(x - 3)^2 - 17$

b **i** (3, −17)

ii $x = 0.1$ and $x = 5.9$

c



2 **a** $y = x^2 - 4x - 5$ **b** $y = -x^2 + 9x - 14$

3 **a** $x^2 + 12x - 16 = (x + 6)^2 - 52$

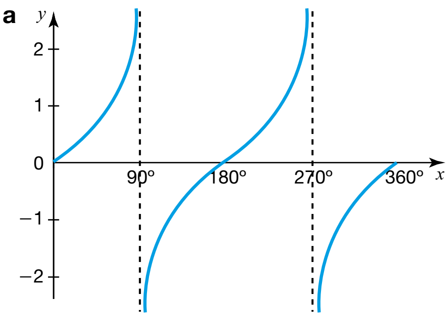
b (−6, −52)

Recognising and sketching graphs of functions

1 **a** B **c** E **e** D

b F **d** A **f** C

2 **a**



b $x = 240°$

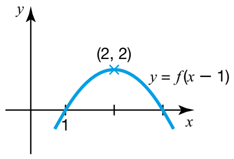
3 **a** A **b** G **c** F **d** E

Translations and reflections of functions

1 **a** (3, 5) **c** (2, −5)

b (−1, 5) **d** (−2, 5)

2 **a**



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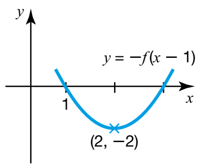
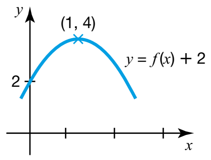
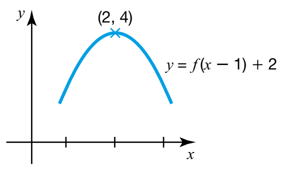


b



d



Equation of a circle and tangent to a circle

1 **a** (0, 0) **b** 7

2 **a** $x^2 + y^2 = 100$

b Gradient of radius to (8, 6) $= \frac{6}{8} = \frac{3}{4}$

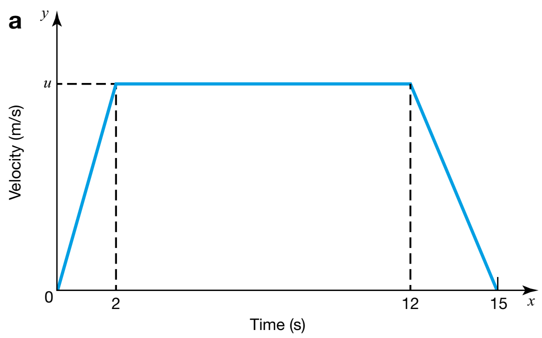
Gradient of tangent $= -\frac{4}{3}$

c $y = -\frac{4}{3}x + 16\frac{2}{3}$

Real-life graphs

1 **a** 5 km/h **b** 0.25 hours **c** 24 km/h

2 **a**



b $u = 4$ m/s **c** 1.33 m/s^2

Generating sequences

1 **a** 17 **c** −12 **e** $\frac{1}{48}$

b 3.0 **d** 432 **f** $-\frac{1}{16}$

2 17, 290

3 2.25, 5.5

The *n*th term

1 **a** 47, 44, 41

b no **c** −1

2 **a** 6, 18, 54, 162

b Both 2 and 3 are factors, so 6 must also be a factor.

3 **a** nth term $= 2n - 3$ **b** $x = 31$

4 nth term $= 4n^2 + n - 1$

Arguments and proofs

1 **a** True: $2n$ is always even as it is a factor of 2. Adding 1 to an even number always gives an odd number.

b False: $x^2 = 9$, so $x = \sqrt{9} = \pm 3$.

c False: n could be a decimal such as 4.25, so squaring it would not give an integer.

d False: if $n \leq 1$ this is not true.

2 Let the four consecutive numbers be n, $n + 1$, $n + 2$ and $n + 3$.

Sum $= n + (n + 1) + (n + 2) + (n + 3)$

$= 4n + 6$

$= 2(2n + 3)$

Since the sum is a multiple of 2, it is always even.

Therefore the sum of four consecutive numbers is always even.

3 Let the consecutive integers be x, $x + 1$ and $x + 2$.

Sum of the integers $= x + x + 1 + x + 2 = 3x + 3 = 3(x + 1)$

As 3 is a factor, the sum must be a multiple of 3.

4 **a** The numerator is larger than the denominator so the fraction will always be greater than 1. Statement is false.

b As a is larger than b, squaring a will result in a larger number than squaring b. Hence $a^2 > b^2$ so the statement is false.

c The square root of a number can have two values one positive and the other negative so this statement is false.

Review it!

1 **a** $-9x + 12$ **b** $6x + 4$ **c** $6x^3 + 25x^2 + 16x - 15$

2 **a** $(2x - 1)(x + 4)$ **b** $x = \frac{1}{2}$ or $x = -4$

3 **a** $8x^6y^3$ **b** $6x$ **c** $\frac{5}{b}$

4 $x = 2$, $y = 1$

5 **a** $\frac{3}{x + 7} = \frac{2 - x}{x + 1}$

$3(x + 1) = (2 - x)(x + 7)$

$3x + 3 = 2x + 14 - x^2 - 7x$

$3x + 3 = -x^2 - 5x + 14$

$x^2 + 8x - 11 = 0$

b $x = 1.20$ or $x = -9.20$ (to 2 d.p.)

6 $x = \frac{3y - 2z}{az + 1}$

7 **a** $f^{-1}(x) = 3x - 15$ or $f^{-1}(x) = 3(x - 5)$

b $k = 7$

8 **a** 26, 22, 18 **b** 8th term $= -2$

9 The point lies outside the circle.

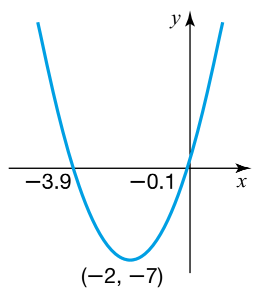
10 $x - 9y$

11 **a** $2(x + 2)^2 - 7$

b **i** (−2, −7)

ii $x = -3.9$ and $x = -0.1$

c



12 30 cm^2

13 $y = 7x + 12$

14 $x = -2, y = 0$ or $x = \frac{6}{5}, y = \frac{8}{5}$

Ratio, proportion and rates of change

Introduction to ratios

1 **a** 1:3 **b** 5:12 **c** 4:9

2 **a** 1:8 **b** 100:1 **c** 1:50

3 £250, £150

4 315 members

5 £42 000

6 62.9%

7 £5.70

8 10:5:1

Scale diagrams and maps

1 30 cm

2 **a** 1:500 000 **b** 6 km

Percentage problems

1 2.67% **3** £39 330 **5** £2520

2 83.3% **4** £356

Direct and inverse proportion

1 Inverse proportion means that if one quantity doubles the other quantity halves.

2 **a** $y = kx$ **b** 10.7

3 **a** cheaper in the UK

b £5.49 cheaper in the UK

4 268 cm^3

5 33 333 (to nearest whole number)

6 150

7 **a** $A = 6x^2, k = 6$ **b** 96 cm^2

Graphs of direct and inverse proportion and rates of change

1 Graph C

2 Graph B

3 $a = 6$

4 Equation connecting x and y is $y = \frac{k}{x}$

When $x = 1, y = 4$ so $4 = \frac{k}{1}$ so $k = 4$

The equation of the curve is now $y = \frac{4}{x}$

When $x = 4, y = \frac{4}{4} = 1$ so $a = 1$

When $y = 0.8, 0.8 = \frac{4}{x}$ giving $x = 5$ so $b = 5$.

Hence $a = 1$ and $b = 5$.

5 $y = kx^2$

When $x = 2, y = 16$ so $16 = k \times 2^2$ giving $k = 4$.

$y = 4x^2$

Hence $36 = 4x^2$ so $x = 3$

$a = 3$

Growth and decay

1 **a** 1.05 **c** 1.0375

b 1.25 **d** 0.79

2 £4962 (to nearest whole number)

3 2048

Ratios of lengths, areas and volumes

1 $6\frac{2}{3}$ cm

2 $\frac{V_A}{V_B} = \frac{27}{64} = (\text{scale factor})^3$, so scale factor $= \frac{3}{4}$

$\frac{A_A}{A_B} = (\text{scale factor})^2 = \frac{9}{16}$

So $A_A = \frac{9}{16} \times 96 = 54\text{ cm}^2$

3 **a** Triangles *ABE* and *ACD* must be proved similar:

BE parallel to *CD*, all the corresponding angles in both triangles are the same.

BE = 4 cm

b 10 cm^2

Gradient of a curve and rate of change

1 **a** The gradient represents the acceleration.

b 5 m/s^2

c 3.9 s

Converting units of areas and volumes, and compound units

1 **a** **i** 14 800 mm^2

ii 0.0148 m^2

b 0.000 12 m^3

2 1.932 g/cm^3 (to 3 d.p.)

3 2 000 000

4 13.8 m/s (to 2 d.p.)

5 **a** 56 km/h

b The distance will not be the same, so the average speed will be different.

Review it!

1 £96

2 **a** 5.10 m^2

b 5.31 m^2

3 $y = 2$

4 243 students

5 £485 000

6 **a** £2250

b £2262.82

7 **a** 3420 yuan

b travel agent

8 £30 000

9 0.64 cm^3

10 20%

11 8.7 g/cm^3

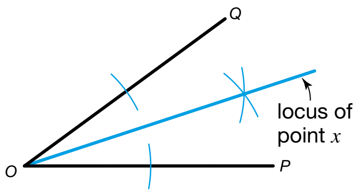
Geometry and measures

2D shapes

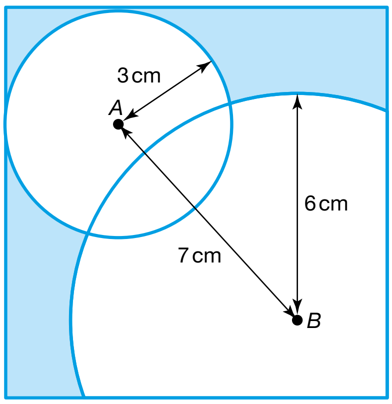
1 a true c true e true
  b false d false f true

Constructions and loci

1



2 This is a reduced version of the answer.



Properties of angles

1 144°

2 a 9 b 1260°

3 20°

4 a x = angle *EBC* = 55° (alternate angles)

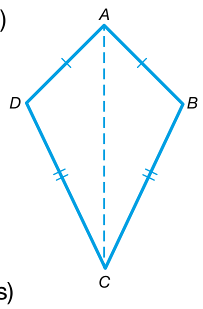
  b Angle *EHI* = 180 − (85 + 55) = 40° (angle sum in a triangle)

  Angle *DEH* = angle *EHI* = 40° (alternate angles)

5 77°, 103°, 77°, 103° (a = 24).

Congruent triangles

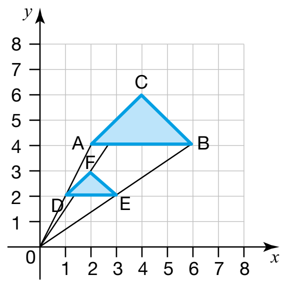
1 *AC* = *AC* (common to both triangles)
AB = *AD* (given)
BC = *CD* (given)
Triangles *ACD* and *ACB* are congruent (SSS).
Hence angle *ABC* = angle *ADC*



2 *AB* = *AC* (given)
BM = *MC* (M is the midpoint of *BC*)
AM = *AM* (common to both triangles)
Triangles *ABM* and *ACM* are congruent (SSS).
Hence angle *AMB* = angle *AMC*
Angle *AMB* + angle *AMC* = 180°, so angle
AMB = $\frac{180}{2}$ = 90°

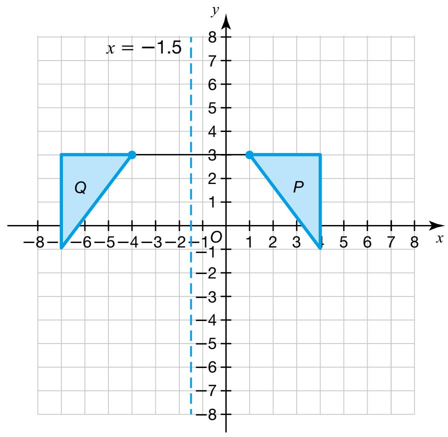
Transformations

1 a, b

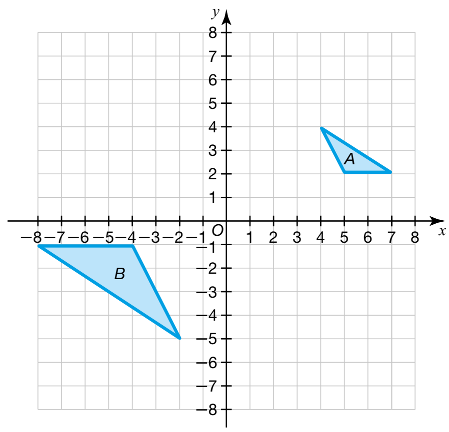


c Enlargement, scale factor 2, centre of enlargement (0, 0)

2



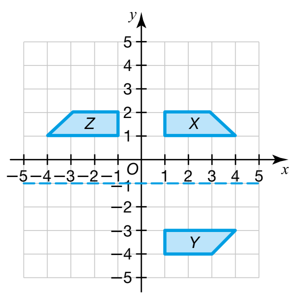
3



4 Translation by $\begin{pmatrix}5\\-4\end{pmatrix}$

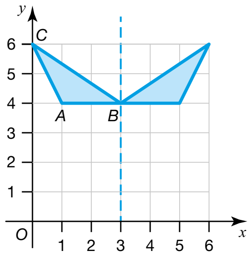
Invariance and combined transformations

1 a, b, c



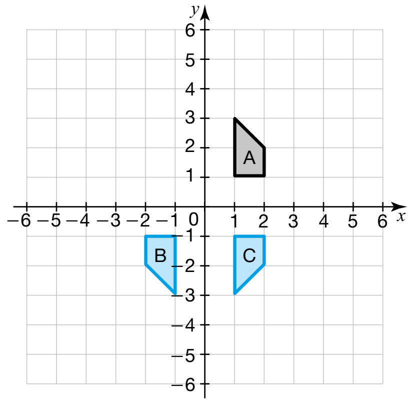
d Reflection in the y-axis

2 a, b



c (3, 4)

3 a, b



c A reflection in the x-axis.

3D shapes

1

Shape	Number of vertices	Number of faces	Number of edges
Triangular-based pyramid	4	4	6
Cone	1	2	1
Cuboid	8	6	12
Hexagonal prism	12	8	18

2 $V = 16$, $F = 10$, $E = 24$; $V + F - E = 16 + 10 - 24 = 2$

Parts of a circle

1 **a** radius **c** minor arc **e** minor sector
b chord **d** minor segment

Circle theorems

1 **a** angle ACB = angle YAB = 30° (alternate segment theorem)
b angle ABC = 90° (angle in a semicircle)
c angle ADC = 90° (angle in a semicircle)

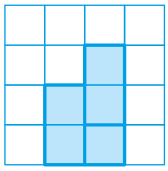
2 **a** angle OAX = 90° (angle between tangent and radius)
b angle AOX = 180 − (90 + 30) = 60° (angle sum in a triangle)
c angle ACB = 30° (angle at centre twice angle at circumference)

3 **a** angle ADB = 40° (angles on same arc)
b angle EDB = angle EDA + angle ADB = 50 + 40 = 90°
So BD is a diameter of the circle (angle between tangent and diameter is 90°)
angle BAD = 90° (angle in a semicircle)

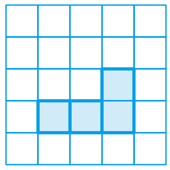
4 Angle ACB = 46° (angles bounded by the same chord in the same segment are equal)
Angle ABC = 90° (angle in a semi-circle is a right-angle)
Angle BAC = 180 − (90 + 46) = 44° (angles in a triangle add up to 180°)

Projections

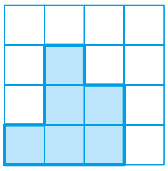
1 **a**



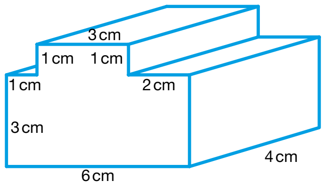
c



b



2



Bearings

1 245°

2 **a** 225° **b** 320°

Pythagoras' theorem

1 **a** 10.3 cm (to 1 d.p.) **b** 8.8 cm (to 1 d.p.)

2 height = 10.91 cm
area = 54.54 cm^2

3 x = 8.12 cm (to 2 d.p.)

4 Let the perpendicular height of the triangle = h
Area of triangle = $\frac{1}{2} \times 14 \times h$
Hence $\frac{1}{2} \times 14 \times h = 90$
Solving gives h = 12.8571 cm

By Pythagoras' theorem, $AC^2 = 7^2 + 12.8571^2$
Solving gives AC = 14.6 cm (3 significant figures)

Area of 2D shapes

1 £1596

2 $\frac{5}{8}$

3 **a** area of semicircle = $\frac{1}{2} \times \pi \times x^2 = \frac{\pi x^2}{2}$
area of rectangle = $4x \times 2x = 8x^2$
area of shape = $\frac{\pi x^2}{2} + 8x^2 = x^2(8 + \frac{\pi}{2})$
b $x(10 + \pi)$

Volume and surface area of 3D shapes

1 $r = \sqrt[3]{\frac{3}{2}a^3}$

2 **a** 12.5 m^2 **b** 62.5 m^3 **c** 10 hours (to nearest hour)

3 1200 m^2 (to 3 s.f.)

Trigonometric ratios

1 **a** 17.32 cm (to 2 d.p.)

b 9.19 cm (to 2 d.p.)

2 **a** 73.3° (to 0.1°)

b 50.3° (to 0.1°)

3 $\sin\theta = \frac{b}{c}$ and $\cos\theta = \frac{a}{c}$

$\frac{\sin\theta}{\cos\theta} = \frac{\frac{b}{c}}{\frac{a}{c}} = \frac{b}{c} \times \frac{c}{a} = \frac{b}{a} = \tan\theta$

4 **a** 13.60 cm (to 2 d.p.)

b 63.8° (to 1 d.p.)

c 8.56 cm (to 2 d.p.)

Exact values of sin, cos and tan

1 $\sqrt{3}\tan 30° + \cos 60° = \sqrt{3} \times \frac{1}{\sqrt{3}} + \frac{1}{2} = 1 + \frac{1}{2} = \frac{3}{2}$

$a = 3$, $b = 2$

2 $\sin 30 = \frac{1}{2}$ and $\cos 30 = \frac{\sqrt{3}}{2}$

$\sin^2 30 = \frac{1}{4}$ and $\cos^2 30 = \frac{3}{4}$

$\sin^2 30 + \cos^2 30 = \frac{1}{4} + \frac{3}{4} = 1$

Sectors of circles

1 47.7° (to 1 d.p.)

2 **a** 7.99 cm **b** 3.94 cm^2

3 Radius OA = 10.3 cm

4 Area of sector = 109 cm^2

Sine and cosine rules

1 **a** 15 cm^2

b 15.5 cm (to 3 s.f.)

2 27.7° (to 0.1°)

3 **a** 38.6°

b If angle XZY is not obtuse, then angle XYZ can be obtuse. An alternative answer would be 141.4°.

Vectors

1 **a** $\begin{pmatrix}-6\\8\end{pmatrix}$ **b** $\begin{pmatrix}2\\0\end{pmatrix}$

2 **a** $\mathbf{b} - \mathbf{a}$ **b** $\frac{3}{5}(\mathbf{b} - \mathbf{a})$ **c** $\frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$

3 **a** $-\mathbf{a} - 3\mathbf{b}$

b $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BC} = \frac{1}{2}(-\mathbf{a} - 3\mathbf{b})$

$\overrightarrow{PM} = \overrightarrow{PA} + \overrightarrow{AB} + \overrightarrow{BM} = 2\mathbf{b} + \mathbf{a} + \frac{1}{2}(-\mathbf{a} - 3\mathbf{b})$

$= \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{a} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$

$\overrightarrow{MD} = \overrightarrow{MB} + \overrightarrow{BD} = -\frac{1}{2}(-\mathbf{a} - 3\mathbf{b}) + \mathbf{a} = \frac{3}{2}\mathbf{a} + \frac{3}{2}\mathbf{b}$

$= \frac{3}{2}(\mathbf{a} + \mathbf{b})$

$\overrightarrow{PM}$ and $\overrightarrow{MD}$ have the same vector part ($\mathbf{a} + \mathbf{b}$), therefore they are parallel. Both lines pass through M and parallel lines cannot pass through the same point unless they are the same line. Hence PMD is a straight line.

Review it!

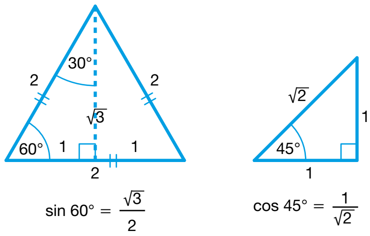
1 Angle ADC = $(180 - x)°$ (opposite angles of cyclic quadrilateral)

Angle ADE = $y°$ (alternate segment theorem)

Angle CDF = 180 − angle ADE − angle ADC

$= 180 - y - (180 - x) = (x - y)°$

2



$\cos 45° + \sin 60° = \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}$

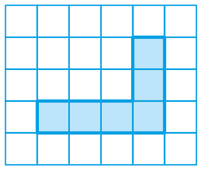
$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}$

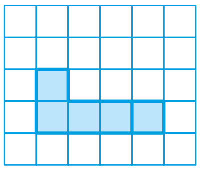
$= \frac{1}{2}(\sqrt{2} + \sqrt{3})$

3 5, 12 and 13

4 **a**

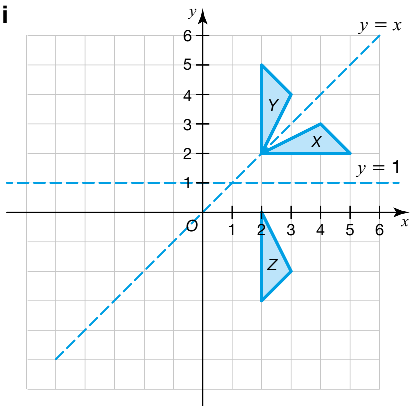


b



5 **a**

b **i**



b **ii** (2, 2)

c See triangle Z on grid above

d Rotation of 90° clockwise about (1, 1)

6 **a** 19.4 km (to 1 s.f.)

b 280°

Probability

The basics of probability

1 $\frac{8}{125}$

2 **a** 0

b $\frac{1}{5}$

c $\frac{11}{20}$

Probability experiments

1 **a** 0.21 (to 2 d.p.)

b 0.14 (to 2 d.p.)

c Sean is wrong. 120 spins is a small number of spins and it is only over a very large number of spins that the relative frequencies may start to be nearly the same.

2 **a** 0.04 **b** 600 cans

3 45 apples

The AND and OR rules

1 **a** Independent events are events where the probability of one event does not influence the probability of another event occurring. Here it means that the probability of the first set of traffic lights being red does not affect the probability of the second set being red.

b 0.06 **c** 0.56

2 **a** $\frac{7}{48}$ **b** $\frac{1}{32}$

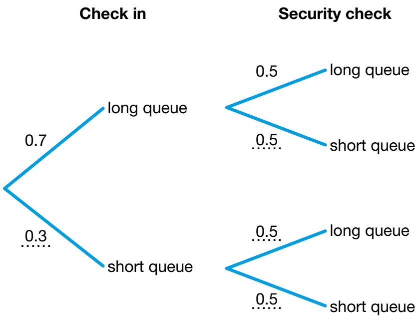
Tree diagrams

1 **a** $\frac{1}{15}$

b $= \frac{7}{15}$

2 20 balls

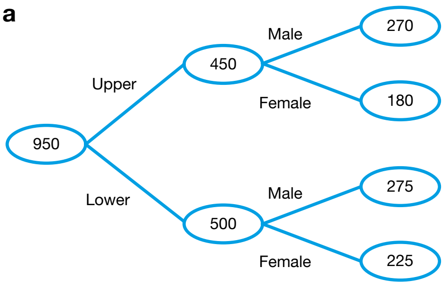
3 **a**



b 0.15

c 0.85

4 **a**



b $\frac{109}{190}$ or 0.57

Venn diagrams and probability

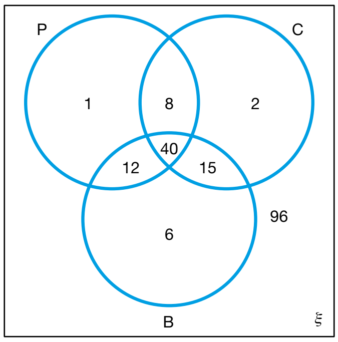
1 **a** **i** {1, 3, 4, 5 8, 9, 10, 11}

ii {8, 9}

iii {2, 5, 6. 10, 13}

b $\frac{7}{11}$

2

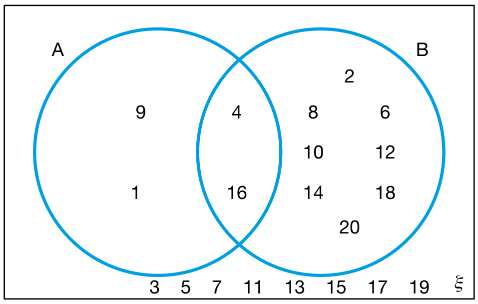


a $\frac{10}{21}$

b $\frac{3}{28}$

c $\frac{48}{61}$

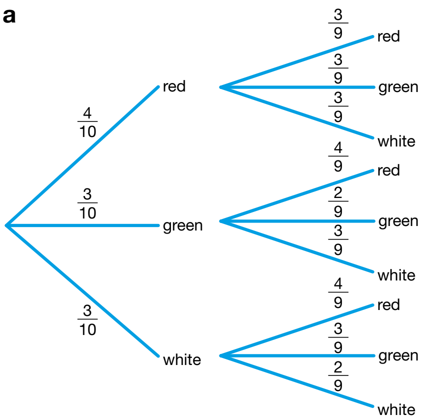
3 **a**



b $P = \frac{8}{20} = \frac{2}{5}$

Review it!

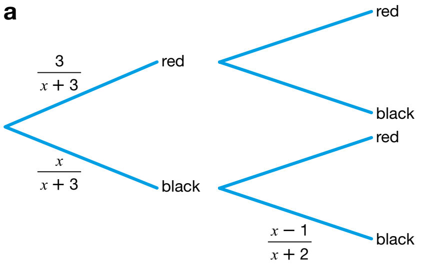
1 **a**



b $\frac{4}{15}$

c $\frac{11}{15}$

2 a



P(two black) $= \left(\frac{x}{x+3}\right) \times \left(\frac{x-1}{x+2}\right) = \frac{7}{15}$

$15x(x - 1) = 7(x + 3)(x + 2)$

$15x^2 - 15x = 7x^2 + 35x + 42$

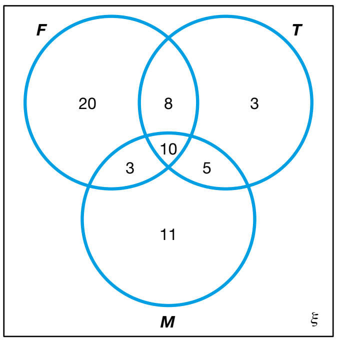
$8x^2 - 50x - 42 = 0$

$4x^2 - 25x - 21 = 0$

b 10 balls

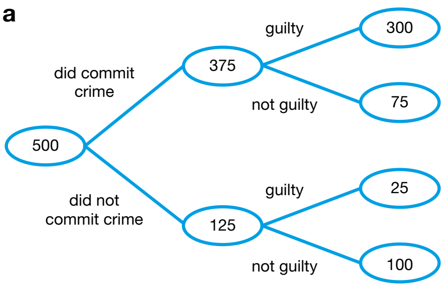
c $\frac{7}{15}$

3



a $\frac{11}{60}$ **b** $\frac{15}{29}$

4 a



b $\frac{13}{20}$ **c** $\frac{1}{5}$

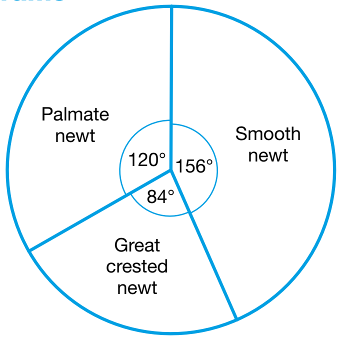
Statistics

Sampling

1 26 **2** 11

Two-way tables, pie charts and stem-and-leaf diagrams

1 a



b No. The other pond might have had more newts in total. The proportion of smooth newts in the second pond is lower, but there may be more newts.

2

2	2	3	3	4			
3	0	0	1	3	5	5	6
4	0	1	2	5	7		

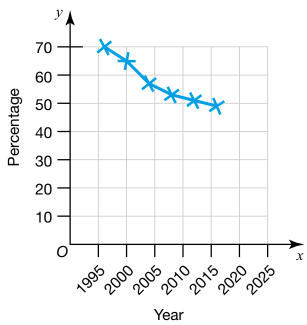
key 2|2 represents 22 mins

Line graphs for time series data

1 a 926, 904, 885, 854

b The trend is decreasing sales.

2 a



b The percentage of people using the local shop is decreasing.

c 44%

d There are no points so you would be trying to predict the future. There may be a change of ownership/or a refurbishment making it more popular. It could even close down before then.

Averages and spread

1 13.5 years

2 Mean $= \frac{\text{total number of marks}}{\text{number of students}}$

Total mark for boys $= 50 \times 10 = 500$

Total mark for girls $= 62 \times 15 = 930$

Total mark for class $= 500 + 930 = 1430$

Mean for class $= \frac{\text{total number of marks}}{\text{number of students}} = \frac{1430}{25} = 57.2\%$

Joshua is wrong, because he didn't take account of the fact that there was a different number of boys and girls.

3 a

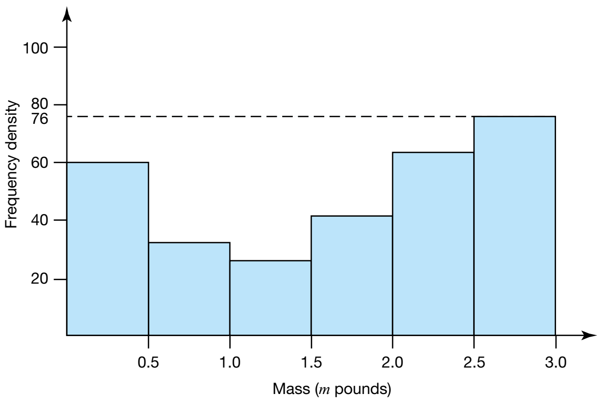
Cost (£C)	Frequency	Mid-interval value	Frequency × mid-interval value
$0 < C \le 4$	12	2	24
$4 < C \le 8$	8	6	48
$8 < C \le 12$	10	10	100
$12 < C \le 16$	5	14	70
$16 < C \le 20$	2	18	36

b £7.51 (to nearest penny)

Histograms

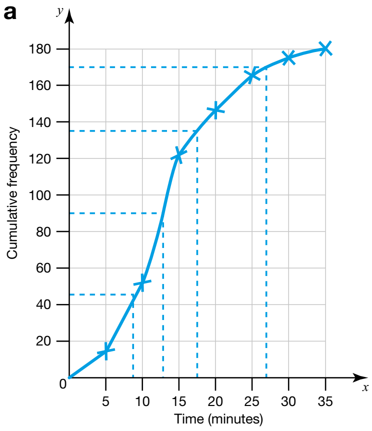
1

Mass (m pounds)	Frequency
$0.0 < m \le 0.5$	30
$0.5 < m \le 1.0$	**16**
$1.0 < m \le 1.5$	13
$1.5 < m \le 2.0$	21
$2.0 < m \le 2.5$	**32**
$2.5 < m \le 3.0$	38



Cumulative frequency graphs

1 a



b 13 minutes

c **i** 17.5 minutes

ii 9 minutes

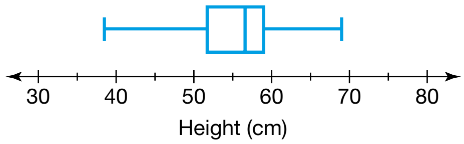
iii 8.5 minutes

d 5.6% (to 1 d.p.)

Comparing sets of data

1

	Height (cm)
Lowest height	38
Lower quartile	**52**
Median	**57**
Upper quartile	**59**
Highest height	69



2 a **i** 120 marks

ii 65 marks

iii 75 marks

iv 51 marks

v 24 marks

b For the girls: median mark = 74 marks, upper quartile = 89 marks, lower quartile = 58 marks and interquartile range = 31 marks

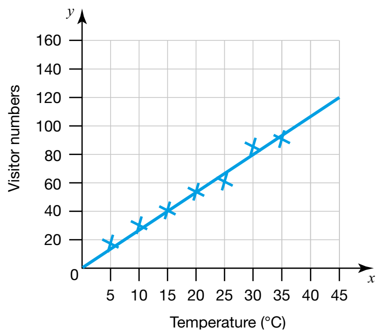
Comparison:

The median mark for the girls is higher (or higher average mark).

The interquartile range is lower for the boys showing that their marks are less spread out for the middle half of the marks.

Scatter graphs

1 a, b



c 71 visitors

d **i** 120 visitors

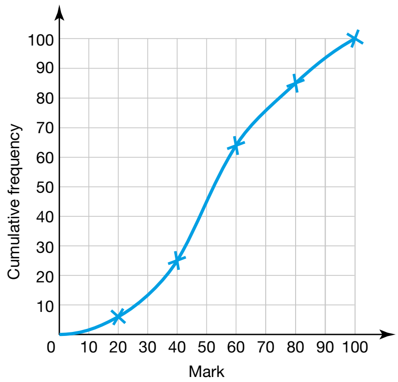
ii There are no points near this temperature so you cannot assume the trend continues.

Review it!

1 **a** 36 students **c** $4 < a \leq 6$

b £5.83 to the nearest penny

2 **a**



b 54 **c** 40

3 **a** D (the median mark is furthest to the right.)

b C (the largest gap between the quartiles.)

c D as the median mark is the highest and also the interquartile range is small which means 50% of pupils got near to the median mark.

4 **a** The sample needs to be representative of people living on the street: male/female, adults/children.

b 120 people