# Edexcel Foundation Mathematics Revision Guide

## Full worked solutions

### Number

#### Factors, multiples and primes

**1** **a** 5 **b** 1, 12 **c** 1, 5, 45

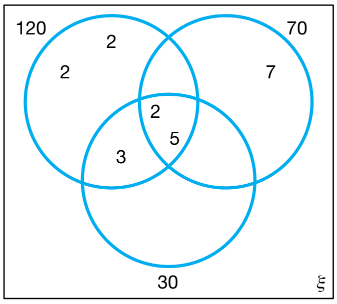
**2** $70 = 2 \times 5 \times 7$

$150 = 2 \times 3 \times 5 \times 5$

HCF = 10, LCM = 1050

**3** $2 \times 3^2 \times 5$

**4**




**a** $2 \times 5 = 10$ **b** $2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840$

**5** 12 and 18

#### Ordering integers and decimals

**1** **a** false **b** true **c** true **d** true **e** true

**2** −0.3, −1.5, −2.5, −4.2, −7.2

**3** 0.049, 0.124, 0.412, 0.442, 1.002

**4** **a** $<$ **b** $<$ **c** $>$

#### Calculating with negative numbers

**Stretch it!** Multiplying three negative numbers together always gives a negative answer.

**1** **a** $-8 - 3 = -11$ **b** 99 **c** −6 **d** $14 + 4 = 18$ **e** 0 **f** $12 + 15 - 2 = 25$

**2** −8 and 9

**3** 32°C

#### Multiplication and division

**Stretch it!**

$$\begin{array}{rl} 621 & \\ \times\ 239 & \\ \hline 5589 & \times 9 \\ 18630 & \times 30 \\ 124200 & \times 200 \\ \hline 148419 & \end{array}$$

148419

**1** **a**

$$\begin{array}{r} 235 \\ \times\ \ 9 \\ \hline 2115 \end{array}$$

2115

**b**

$$\begin{array}{rl} 924 & \\ \times\ 61 & \\ \hline 924 & \times 1 \\ 55440 & \times 60 \\ \hline 56364 & \end{array}$$

56 364

**2** **a** $5\overline{)235}$ = 047 47

**b** $8\overline{)4128}$ = 0516 516

**c**

$$\begin{array}{r} 0126 \\ 17\overline{)2146} \\ 17\ \ \\ \hline 44\ \\ 34\ \\ \hline 106 \\ 102 \\ \hline 4 \end{array}$$

126 remainder 4, or $126\frac{4}{17}$

**3** $8\overline{)265}$ = 033 remainder 1

**a** 33 boxes

**b** 1 pencil

**4** $4\overline{)365.00}$ = 091.25 £91.25

**5**

$$\begin{array}{r} 32 \\ \times\ 9 \\ \hline 288 \end{array}$$

£288

**6** $3\overline{)923.00}$ = 307.66 $307.\dot{6} = 307\frac{2}{3}$

**7**

$$\begin{array}{rl} 823 & \\ \times\ 35 & \\ \hline 4115 & \times 5 \\ 24690 & \times 30 \\ \hline 28805 & \end{array}$$

28 805

8

$$\begin{array}{r} 0\;3\;7 \\ 12\overline{)4\;5\;0} \\ 3\;6 \\ \hline 9\;0 \\ 8\;4 \\ \hline 6 \end{array}$$

37 remainder 6, so 37 boxes

9 He has not placed a zero in the ones column before multiplying through by 5. The $\times$ 50 line should have 5 digits: 36 300, so his final three rows of working should look like this:

$$\begin{array}{rl} 1\,4\,5\,2 & \times 2 \\ 3\,6\,3\,0\,0 & \times 50 \\ \hline 3\,7\,7\,5\,2 & \end{array}$$

## Calculating with decimals

**Stretch it!** $3.2 + 7.5 \times 2 = 3.2 + 15 = 18.2$

1 a

$$\begin{array}{r} 3\,.\,{}^{3}\cancel{4}{}^{1}0 \\ -1\,.\,0\,7 \\ \hline 2\,.\,3\,3 \end{array}$$

2.33

b

$$\begin{array}{r} 1\,9\cdot 3\;0\;0 \\ +\;\;5\cdot 0\;9\;1 \\ \hline 2\,4\cdot 3\;9\;1 \\ {}_{1}\quad\quad\quad\quad \end{array}$$

24.391

c $5 \times 7 = 35$

$0.05 \times 0.7 = 0.035$

d

$$\begin{array}{rl} 3\;2\;1 & \\ \times\;1\;9 & \\ \hline 2\;8\;8\;9 & \times 9 \\ 3\;\overset{1}{2}\;1\;0 & \times 10 \\ \hline 6\;0\;9\;9 & \\ {}_{1}\quad\quad & \end{array}$$

$3.21 \times 1.9 = 6.099$

e

$$\begin{array}{r} 1.5\;6\;3 \\ 5\overline{)7.{}^{2}8\;{}^{3}1\;{}^{1}5} \end{array}$$

1.563

2

$$\begin{array}{r} 6\;4 \\ \times\;2\;4 \\ \hline 2\;5\;6 \\ 1\;2\;8\;0 \\ \hline 1\;5\;3\;6 \\ {}_{1}\quad\quad \end{array}$$

$24 \times 64 = 1536$

$24 \times £0.64 = £15.36$

$£20 - £15.36 = £4.64$

3

$$\begin{array}{r} 2\;7.4\;6 \\ 3\overline{)8\,{}^{2}2\,.{}^{1}3\,{}^{1}8} \end{array}$$

Erica $2 \times 27.46 = £54.92$

Freya $82.38 - 54.92 = £27.46$

## Rounding and estimation

**Stretch it! a** 1.0 **b** 1.00 **c** 1.000

All the answers are 1

**Stretch it!** $6.5 \times 8.5 = 55.25\,\text{m}^2$ – an overestimate.

1 a 0.35 c 32.6

b 10 d 33 100

2 a $150 \le x < 250$ c $3.15 \le x < 3.25$

b $5.5 \le x < 6.5$ d $5.055 \le x < 5.065$

3 $\frac{30}{0.5 \times 6} = 10$

4 b is false since $18 \times 1 = 18$ so $18 \times 0.9$ cannot be 1.62

c is false because if you divide by a number smaller than 1, the answer will be larger.

5 Night-time low tariff: 2.32 units $\times$ 1.622p = 3.76p

7.151 units $\times$ 2.315p = 16.55p

20.31p

One tariff: $2.320 + 7.151 = 9.471$ units

9.471 units $\times$ 1.923p = 18.21p

Tarik should choose One tariff, since it is cheaper.

## Converting between fractions, decimals and percentages

**Stretch it!** $0.\dot{1}, 0.\dot{2}, 0.\dot{3}, \ldots 0.\dot{4}, 0.\dot{5}$. The number of ninths is the same as the digit that recurs. The exception is $\frac{9}{9}$ which is the same as 1.

1 a $\frac{32}{100} = \frac{8}{25}$ c $\frac{33}{100}$

b $1\frac{24}{100} = 1\frac{6}{25}$ d $\frac{95}{100} = \frac{19}{20}$

2 a

$$\begin{array}{r} 0\,.\,4\,1\,6\,6\,6\ldots \\ 12\overline{)5\,{}^{5}.\,0\,{}^{2}0\,{}^{8}0\,{}^{8}0\,{}^{8}0} \end{array}$$

$0.41\dot{6}$

b

$$\begin{array}{r} 0\,.\,3\,7\,5 \\ 8\overline{)3\,.\,{}^{3}0\,{}^{6}0\,{}^{4}0\,0} \end{array}$$

0.375

c 0.49

d 0.185

e

$$\begin{array}{r} 0\,.\,4\,2\,8\,5\,7\,1\,4\,2\ldots \\ 7\overline{)3\,.\,{}^{3}0\,{}^{2}0\,{}^{6}0\,{}^{4}0\,{}^{5}0\,{}^{1}0\,{}^{3}0\,{}^{2}0} \end{array}$$

$0.\dot{4}2857\dot{1}$

3 a $\frac{91}{100} = 91\%$

b $\frac{3}{10} = \frac{30}{100} = 30\%$

c $\frac{4}{5} = \frac{80}{100} = 80\%$

d $\frac{9}{15} = \frac{3}{5} = \frac{6}{10} = \frac{60}{100} = 60\%$

4 $\frac{3}{8}$

$$\begin{array}{r} 0\,.\,3\,7\,5 \\ 8\overline{)3\,.\,{}^{3}0\,{}^{6}0\,{}^{4}0\,0} \end{array}$$

$= 0.375 = 37.5\%$

5 0.35 $\frac{2}{5} = \frac{4}{10} = 0.4$ $30\% = 0.3$

$30\%, 0.35, \frac{2}{5}$

6 $\frac{15}{20} = \frac{75}{100} = 75\%$ – Amy

Rudi's mark was higher.

## Ordering fractions, decimals and percentages

1 $\frac{1}{3} = \frac{8}{24}$ $\frac{3}{8} = \frac{9}{24}$ $\frac{7}{12} = \frac{14}{24}$

$\frac{7}{12}, \frac{3}{8}, \frac{1}{3}$

2 $-2.2, 7, \frac{1}{5} = 0.2, -\frac{1}{10} = -0.1, 15\% = 0.15,$

$1\% = 0.01, 0.1$

In order, this is: $-2.2, -\frac{1}{10}, 1\%, 0.1, 15\%, \frac{1}{5}, 7$.

The middle value is 0.1

3 Yes. If the numerator of a fraction is half the denominator then the fraction is equivalent to $\frac{1}{2}$.

If the numerator is smaller than this the fraction must be less than $\frac{1}{2}$.

## Calculating with fractions

**Stretch it!** No – you could add the whole number parts, and then add the fraction parts.

**1** **a** $2\frac{3}{8} - \frac{3}{4} = \frac{19}{8} - \frac{3}{4} = \frac{19}{8} - \frac{6}{8} = \frac{13}{8} = 1\frac{5}{8}$

**b** $\frac{\cancel{15}^{3}}{17} \times \frac{2}{\cancel{5}_{1}} = \frac{6}{17}$

**c** $\frac{1}{7} \times 3\frac{1}{3} = \frac{1}{7} \times \frac{10}{3} = \frac{10}{21}$

**d** $2\frac{2}{5} + 5\frac{3}{4} = 7 + \frac{2}{5} + \frac{3}{4} = 7 + \frac{8}{20} + \frac{15}{20}$

$= 7\frac{23}{20}$

$= 8\frac{3}{20}$

**e** $\frac{1}{5} \div 2\frac{1}{2} = \frac{1}{5} \div \frac{5}{2} = \frac{1}{5} \times \frac{2}{5} = \frac{2}{25}$

**2** **a** $30 \div 5 = 6$

$6 \times 2 = 12$

**b** $40 \div 8 = 5$

$5 \times 7 = £35$

**c** $1818 \div 9 = 202$

$202 \times 4 = 808\,\text{mm}$

**3** $35 \div 7 = 5$

$3 \times 7 = 15$

$35 - 15 = 20$

**4** The number must be a multiple of 5, and $\frac{2}{5}$ of it must be a multiple of 2.

$\frac{2}{5}$ of 45 = 18

$\frac{2}{5}$ of 40 = 16

$\frac{2}{5}$ of 35 = 14

$\frac{2}{5}$ of 30 = 12

$\frac{2}{5}$ of the number must be greater than 12, so the number is 35

## Percentages

**1** **a** $18 \div 100 = 0.18\,\text{cm}$

$0.18 \times 10 = 1.8\,\text{cm}$

**b** $1.20 \div 100 = £0.012$

$0.012 \times 25 = £0.30$

**c** $200\,\text{ml} \div 100 = 2\,\text{ml}$

$2\,\text{ml} \times 2 = 4\,\text{ml}$

**2** **a** $1.1 \times 30 = 33$

**b** $1.08 \times 500 = 540$

**c** $1.12 \times 91 = 101.92$, so £101.92

**3** **a** $0.8 \times 600 = 480$

**b** $0.95 \times 140 = 133$

**c** $0.81 \times 18 = 14.58$, so £14.58

**4** $1.09 \times 2800 = 3052$

**5** $0.65 \times 22\,000 = £14\,300$

## Order of operations

**1** **a** 7

**b** $0.9 + 3.2 - \sqrt{36}$

$= 0.9 + 3.2 - 6$

$= -1.9$

**c** $(-1)^2 - 14$

$= 1 - 14$

$= -13$

**2** 30

**3** $(8 - 3 + 5) \times 4$

## Exact solutions

**1** **a** $\pi$   **b** $36\pi$   **c** $2\frac{1}{2}\pi$ or $\frac{5}{2}\pi$

**2** **a** $7\pi$   **b** $\frac{5}{8}\pi$

**3** Area $= \frac{2}{7} \times \frac{3}{4} = \frac{6}{28} = \frac{3}{14}\,\text{cm}^2$

Perimeter $= \left(2 \times \frac{3}{4}\right) + \left(2 \times \frac{2}{7}\right) = \frac{3}{2} + \frac{4}{7} = \frac{21 + 8}{14} = \frac{29}{14}$

$= 2\frac{1}{14}\,\text{cm}$

**4** **a** $2 \times 9 \times \pi = 18\pi\,\text{cm}$

**b** $12^2 \times \pi = 144\pi\,\text{cm}^2$

**5** Circumference $= 2 \times \pi \times 1 = 2\pi\,\text{cm}$

Length of one side of square $= 2\pi \div 4 = \frac{1}{2}\pi\,\text{cm}$

## Indices and roots

**1** **a** $\frac{1}{3}$   **b** $\frac{1}{0.4} = \frac{10}{4} = 2\frac{1}{2}$

**c** $\frac{1}{0.9} = \frac{10}{9} = 1\frac{1}{9}$

**2** $3^2 = 9$,  $1^3 = 1$,  $\sqrt[3]{27} = 3$,  $\sqrt[3]{8} = 2$

In order, this gives $1^3, \sqrt[3]{8}, \sqrt[3]{27}, 3^2$

**3** **a** $-8$   **b** 1   **c** 81   **d** 1

**4** **a** $\frac{1}{4}$   **b*** $\frac{1}{7^2} = \frac{1}{49}$

**c*** $\frac{1}{1^4} = 1$   **d** $\frac{1}{3}$

**5** $\frac{5^9}{5^5} = 5^4$

## Standard form

**1** **a** 45 000 000   **b** 0.091

**2** **a** $6.45 \times 10^8$   **b** $7.9 \times 10^{-8}$

**3** $350000 - 4200 = 345800$

**4** $3.2 \times 10^2 = 320$   $3.1 \times 10^{-2} = 0.031$

$3.09 \times 10 = 30.9$   $3 + (2.1 \times 10^2) = 213$

In order, this gives: $3.1 \times 10^{-2}$   $3.09 \times 10$

$3 + (2.1 \times 10^2)$   $3.2 \times 10^2$

**5** $3 \times 10^8$ m/s

**6** $200 \times 1.1 \times 10^{-4} = 2.2 \times 10^{-2} = 0.022\text{ m} = 2.2\,\text{cm}$

## Listing strategies

**Stretch it!**

red + small, red + medium, red + large,

green + small, green + medium, green + large,

blue + small, blue + medium, blue + large.

**1** 111,

112, 121, 211, 113, 131, 311,

222

221, 212, 122, 223, 232, 322

333

331 313 133 332 323 233

123 132 213 231 312 321

*These answers are labelled in the wrong order in the first edition of our Revision Guide.

2 444 446 449

464 466 469

494 496 499

3 Small A, Small B, Small C, Small D

Medium A, Medium B, Medium C, Medium D

Large A, Large B, Large C, Large D.

## Review it!

1 7 and 6 (or 11 and 2, where both are prime and 2 is also a factor of 12)

2 $630 = 2 \times 3 \times 3 \times 5 \times 7 = 2 \times 3^2 \times 5 \times 7$*

3 $18 = 2 \times 3 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

$40 = 2 \times 2 \times 2 \times 5$

HCF = 2

4 −11.5, −8.3, −3.5, −3.2, 1.4

5 a
$$\begin{array}{r} 32.99 \\ +18.74 \\ \hline 51.73 \\ \hline {}_{1\,1\;\;1} \end{array}$$
£51.73

b
$$3\overline{)5{}^{2}4.99}\quad 18.33$$
£18.33

6 a
$$\begin{array}{rl} 23 & \\ \times 14 & \\ \hline 92 & \times 4 \\ 230 & \times 10 \\ \hline 322 & \end{array}$$
$23 \times 0.14 = 3.22$

b
$$\begin{array}{rl} 149 & \\ \times 27 & \\ \hline 1043 & \times 7 \\ 2980 & \times 20 \\ \hline 4023 & \end{array}$$
4023

7 $81 \div 3 = 27$

8
$$11\overline{)3{}^{3}4{}^{1}5{}^{4}}\quad 031 \text{ remainder } 4$$
$31\frac{4}{11}$

9 a
$$8\overline{)3.{}^{3}0{}^{6}0{}^{4}00}\quad 0.375$$
0.375

b $0.7 \times 100 = 70\%$

10 a $70\% = \frac{70}{100} = \frac{7}{10}$

b $0.8 = \frac{8}{10} = \frac{4}{5}$

11 $\frac{1}{2} = \frac{2}{4}$   $\frac{1}{2}$ is larger

$\frac{2}{7} = \frac{8}{28}$   $\frac{1}{4} = \frac{7}{28}$   $\frac{2}{7}$ is larger

$\frac{3}{11} = \frac{12}{44}$   $\frac{1}{4} = \frac{11}{44}$   $\frac{3}{11}$ is larger

$\frac{2}{5} = \frac{8}{20}$   $\frac{1}{4} = \frac{5}{20}$   $\frac{2}{5}$ is larger

They all are

12 a $\frac{3}{5} + \frac{1}{7} = \frac{21}{35} + \frac{5}{35} = \frac{26}{35}$

b $2\frac{1}{5} - \frac{7}{10} = \frac{11}{5} - \frac{7}{10} = \frac{22}{10} - \frac{7}{10} = \frac{15}{10} = 1\frac{1}{2}$

c $\frac{2}{3} \div \frac{4}{9} = \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{9}^{3}}{\cancel{4}_{2}} = \frac{3}{2} = 1\frac{1}{2}$

13 $0.25 - 0.07 = 0.18 = \frac{18}{100} = \frac{108}{600}$

$\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6} = \frac{100}{600}$

$0.25 - 0.07$ is larger

14 $\frac{3}{\cancel{5}} \times \frac{\cancel{5}}{4} = \frac{3}{4}$

15 $\frac{45}{1000} = \frac{9}{200}$

16 $8.6 \div 100 = 0.086$

$0.086 \times 25 = £2.15$

17 a 9   b 5

18 a $3.4 \times 10^9$   b $3.04 \times 10^{-7}$

19 $37.55 \leq x < 37.65$

20 a 51   b 12, 15, 21, 51, 25, 52

21 a $200 \times 9 \times 10 = 18000 = £180.00$

b Underestimate since all numbers were rounded down.

22 40% of 600 = 240

$\frac{1}{5}$ of 600 = 120

$600 - (240 + 120) = 240$

23 More than 33%, less than 50%, multiple of 5.  35%

24 No, since 2 is a prime number and odd + odd + even = even

25 $0.8 \times 349 = £279.20$

26 a 3.1   b 3.05

27 a 325 000   b 320 000

28 $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$

29 a $26.25 + 18.23 + (4 \times 5.5) = £66.48$

$£66.48 \div 4 = £16.62$

30 $0.19 \times 18\,000 = 3420$

31 a 2010 and 2011

b $1.1 \times 102.3 = 112.53$

# Algebra

## Understanding expressions, equations, formulae and identities

1 a $3a + 6 = 10$ (It can be solved to find the value of $a$.)

b $C = \pi D$ (The value of $C$ can be worked out if the value of $D$ is known.)

c $3(a + 2)$ (It does not have an equals sign.)

d $3ab + 2ab = 5ab$ (Collecting the like terms on the left-hand side gives $5ab$ which is equal to the right-hand side.)

2 James is correct.

$4x - 2 = 2x$ can be solved to find the value of $x$ so it is an equation.

Or, the two sides of $4x - 2 = 2x$ are not equal for all values of $x$ so it cannot be an identity. For example, when $x = 2$:

(Left-hand side) $4x - 2 = 4 \times 2 - 2 = 6$

(Right-hand side) $2x = 2 \times 2 = 4$

$6 \neq 4$



*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

## Simplifying expressions

**Stretch it!**

The expressions must all contain algebra, so each part must include $t$.

There are four possible combinations that make $12t^3$: $12t \times t \times t$, $2t \times 6t \times t$, $2t \times 3t \times 2t$, $3t \times 4t \times t$.

**1** **a** $p^3$
**b** $4 \times b \times c \times 7 = 4 \times 7 \times b \times c = 28bc$
**c** $4a \times 3b = 4 \times 3 \times a \times b = 12ab$
**d** $5x \times 4x = 5 \times 4 \times x \times x = 20x^2$
**e** $2g \times (-4g) = 2 \times (-4) \times g \times g = -8g^2$
**f** $2p \times 3q \times r = 2 \times 3 \times p \times q \times r = 6pqr$

**2** **a** $10x \div 2 = \frac{10x}{2} = 5x$
**b** $\frac{14w}{-2} = -7w$
**c** $6p \div p = \frac{6p}{p} = 6$
**d** $8mn \div 2m = \frac{8mn}{2m} = 4n$
**e** $\frac{12xy}{3y} = 4x$
**f** $9abc \div bc = \frac{9abc}{bc} = 9a$

## Collecting like terms

**1** **a** $5f$
**b** $7b$
**c** $5mn$
**d** $3d + 4e + d - 6e = 3d + d + 4e - 6e = 4d - 2e$
**e** $2x + 5y + 3x - 2y - 2 = 2x + 3x + 5y - 2y - 2$
$= 5x + 3y - 2$
**f** $2a - b - 5a - 3 = 2a - 5a - b - 3 = -3a - b - 3$
**g** $2x^2$
**h** $2t^3 + 4 - t^3 - 4 = 2t^3 - t^3 + 4 - 4 = t^3$
**i** $7\sqrt{x}$
**j** $3\sqrt{x}$
**k** $7\sqrt{x}$

## Using indices

**1** **a** $x^5 \times x^4 = x^{5+4} = x^9$
**b** $p \times p^4 = p^{1+4} = p^5$
**c** $2m^4 \times 3m^4 = 2 \times 3 \times m^4 \times m^4 = 6 \times m^{4+4} = 6m^8$
**d** $3m^4n \times 5m^2n^3$
$= 3 \times 5 \times m^4 \times m^2 \times n \times n^3$
$= 15 \times m^{4+2} \times n^{1+3} = 15m^6n^4$
**e** $u^{-2} \times u^5 = u^{-2+5} = u^3$
**f** $t^7 \times t^{-6} = t^{7+(-6)} = t$

**2** **a** $x^4 \div x^2 = x^{4-2} = x^2$
**b** $\frac{y^7}{y^3} = y^{7-3} = y^4$
**c** $\frac{p^9}{p^8} = p^{9-8} = p$
**d** $8x^6 \div 4x^3 = \frac{8x^6}{4x^3}$
$(8 \div 4) \times (x^6 \div x^3) = 2 \times x^{6-3} = 2x^3$
**e** $m^3 \div m^5 = m^{3-5} = m^{-2} = \frac{1}{m^2}$
**f** $\frac{5x^8}{15x^4} = \frac{5}{15} \times \frac{x^8}{x^4} = \frac{1}{3} \times x^{8-4} = \frac{x^4}{3}$
**g** $3x^2 \div 9x = \frac{3x^2}{9x} = \frac{3}{9} \times \frac{x^2}{x} = \frac{1}{3} \times x^{2-1} = \frac{x}{3}$

**3** **a** $(x^2)^3 = x^{2\times3} = x^6$
**b** $(y^4)^4 = y^{4\times4} = y^{16}$
**c** $(p^5)^2 = p^{5\times2} = p^{10}$
**d** $(4m^5)^2 = 4^2 \times (m^5)^2 \times 16 \times m^{5\times2} = 16m^{10}$
**e** $(x^2)^{-3} = x^{2\times(-3)} = x^{-6} = \frac{1}{x^6}$
**f** $(n^{-4})^{-2} = n^{-4\times(-2)} = n^8$

**4** **a** $2x \times 3x^2 = 2 \times 3 \times x \times x^2 = 6 \times x^{1+2} = 6x^3$
**b** $\frac{x^8}{x} = x^{8-1} = x^7$
**c** $\frac{2x^3}{8x^2} = \frac{2}{8} \times \frac{x^3}{x^2} = \frac{1}{4} \times x^{3-2} = \frac{x}{4}$
**d** $(3y^3)^3 = 3^3 \times (y^3)^3 = 27 \times y^{3\times3} = 27y^9$
**e** $y^{-4} = \frac{1}{y^4}$
**f** $a^2b \times a^3b^2$
$= a^2 \times a^3 \times b \times b^2 = a^{2+3} \times b^{1+2} = a^5b^3$

## Expanding brackets

**Stretch it!**

**1** $(x+2)(x+4) = x^2 + 6x + 8$

**2** **a** $2x^2 + 8x + 6$  **b** $3x^2 + 10x - 8$
**c** $6x^2 + 7x - 3$

**1** **a** $3a + 6$  **b** $4b - 16$  **c** $10c + 25$
**d** $6 - 2e$  **e** $4x + 4y + 8$  **f** $-2y - 4$
**g** $x^2 - 2x$  **h** $2a^2 + 10a$

**2** **a** $2(2x + 3) + 4(x + 5) = 4x + 6 + 4x + 20 = 8x + 26$
**b** $3(3y + 1) + 2(4y - 3) = 9y + 3 + 8y - 6 = 17y - 3$
**c** $4(2m + 4) - 3(2m - 5) = 8m + 16 - 6m + 15 = 2m + 31$

**3** **a** $(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$
**b** $(y - 3)(y + 4) = y^2 + 4y - 3y - 12 = y^2 + y - 12$
**c** $(a + 3)(a - 7) = a^2 - 7a + 3a - 21 = a^2 - 4a - 21$
**d** $(m - 1)(m - 6) = m^2 - 6m - m + 6 = m^2 - 7m + 6$

**4** **a** $(x + 1)^2 = (x + 1)(x + 1)$
$= x^2 + x + x + 1 = x^2 + 2x + 1$
**b** $(x - 1)^2 = (x - 1)(x - 1)$
$= x^2 - x - x + 1 = x^2 - 2x + 1$
**c** $(m - 2)^2 = (m - 2)(m - 2)$
$= m^2 - 2m - 2m + 4 = m^2 - 4m + 4$
**d** $(y + 3)^2 = (y + 3)(y + 3)$
$= y^2 + 3y + 3y + 9 = y^2 + 6y + 9$

## Factorising

**Stretch it!**

The width of the rectangle $= x + 1$, since $x^2 + 3x + 2 = (x + 2)(x + 1)$

**1** **a** $3(a + 3)$  **b** $5(b - 2)$
**c** $7(1 + 2c)$  **d** $d(d - 2)$

**2** **a** $4(2a + 5)$  **b** $4(b - 3)$
**c** $9(2 + c)$  **d** $d(2d - 3)$

**3** **a** $2(2x - 3y)$  **b** $m(a + b)$
**c** $x(4x + 3y)$  **d** $n(2 - 9n)$
**e** $5x(1 + 2y)$  **f** $4p(q - 3)$

**4** **a** $(x + 1)(x + 7)$  **b** $(x - 1)(x + 5)$
**c** $(x + 2)(x - 4)$  **d** $(x - 2)(x - 3)$
**e** $(x - 3)(x - 3)$  **f** $(x + 3)(x + 4)$
**g** $(x - 2)(x + 5)$  **h** $(x + 4)(x - 5)$

5 a $x^2 - 16 = x^2 - 4^2 = (x + 4)(x - 4)$
b $x^2 - 36 = x^2 - 6^2 = (x + 6)(x - 6)$
c $x^2 - 81 = x^2 - 9^2 = (x + 9)(x - 9)$
d $y^2 - 100 = y^2 - 10^2 = (y + 10)(y - 10)$

## Substituting into expressions

1 $c = 5 \times 3 + 2 \times (-2) = 15 + (-4) = 11$
2 a $2 - 2 \times (-4) = 2 - (-8) = 10$
b $3 \times 2 \times (-4) = -24$
c $4 \times (-4) - 3 \times 2 = -16 - 6 = -22$
d $2^2 + (-4)^2 = 4 + 16 = 20$
e $2 \times 2 + 4(2 - (-4)) = 2 \times 2 + 4 \times 6 = 4 + 24 = 28$
f $\frac{1}{2}(2 + (-4)) = \frac{1}{2} \times -2 = -1$

3 False.
When $a = 3$: $3a^2 = 3 \times 3^2 = 3 \times 9 = 27$
4 a $10 \times \frac{1}{2} \times 4 = 20$
b $8 \times \left(\frac{1}{2}\right)^2 = 8 \times \left(\frac{1}{4}\right) = 2$
c $0.5 \times 4 \times 10 \times 10 \times 10 \times 10 \times 10 = 200\,000$
$(= 2 \times 10^5)$
d $(4 \times 3) \div (10 \times 10 \times 10) = 12 \div 1000 = 0.012$
e $2 \times 4 \times (10 \times 10 \times 10 \times 10 \times 10) \times 3 \div (10 \times 10 \times 10)$
$2 \times 4 \times 100\,000 \times 3 \div 1000$
$= 2\,400\,000 \div 1000$
$= 2400$
f $4^2 - 12 \times \frac{1}{2} = 16 - 12 \times \frac{1}{2} = 16 - 6 = 10$

## Writing expressions

1 a $4 - q$ b $n + m$ (or $m + n$)
c $8t$ d $xy$ (or $yx$)
e $p^2$ f $a^3$
2 $x + y$
3 $100n + 75b$
4 Perimeter $= 3a + 2a + 4 + 4a - 2 = 9a + 2$
5 Area $= \frac{1}{2} \times 4 \times (2a + 5) = 2 \times (2a + 5) = 4a + 10$

## Solving linear equations

1 a $5a = 35$
$a = \frac{35}{5}$
$a = 7$
b $b - 9 = 8$
$b = 8 + 9$
$b = 17$
c $\frac{c}{4} = 4$
$c = 4 \times 4$
$c = 16$
d $d + 4 = 2$
$d = 2 - 4$
$d = -2$
2 a $2x + 3 = 13$
$2x = 10$
$x = 5$
b $3y - 4 = 11$
$3y = 15$
$y = 5$
c $2p + 9 = 1$
$2p = -8$
$p = -4$
d $\frac{f}{3} - 7 = 4$
$\frac{f}{3} = 11$
$f = 33$
e $\frac{x+5}{2} = 8$
$x + 5 = 16$
$x = 11$
f $\frac{f-7}{3} = 4$
$f - 7 = 12$
$f = 19$
3 a $9 - m = 7$
$9 = 7 + m$
$2 = m$
b $10 - 3x = 1$
$10 = 1 + 3x$
$9 = 3x$
$3 = x$
c $7 - 2x = 2$
$7 = 2 + 2x$
$5 = 2x$
$\frac{5}{2} = x$
(Or $x = 2.5$, or $x = 2\frac{1}{2}$)
d $5 = 1 - 2f$
$5 + 2f = 1$
$2f = -4$
$f = -2$
4 Hannah has not subtracted 4 from *both* sides.
Correct working:
$2x + 4 = 8$
$2x = 4$
$x = 2$
5 a $3(a + 2) = 15$
$3a + 6 = 15$
$3a = 9$
$a = 3$
b $4(b - 2) = 4$
$4b - 8 = 4$
$4b = 12$
$b = 3$
c $3(4c - 9) = 9$
$12c - 27 = 9$
$12c = 36$
$c = 3$
d $2(d + 3) + 4 = 2$
$2d + 6 + 4 = 2$
$2d + 10 = 2$
$2d = -8$
$d = -4$

**e** $4(2x + 3) - 2 = 6$
$8x + 12 - 2 = 6$
$8x + 10 = 6$
$8x = -4$
$x = -\frac{4}{8} = -\frac{1}{2}$
(Or $x = -0.5$)

**6 a** $3m = m + 6$
$2m = 6$
$m = 3$

**b** $5t - 6 = 2t + 3$
$3t - 6 = 3$
$3t = 9$
$t = 3$

**c** $4x + 3 = 2x + 8$
$2x + 3 = 8$
$2x = 5$
$x = \frac{5}{2}$
(Or $x = 2.5$ or $x = 2\frac{1}{2}$)

**d** $3 - 2p = 6 - 3p$
$3 + p = 6$
$p = 3$

**e** $3y - 8 = 5y + 4$
$-8 = 2y + 4$
$-12 = 2y$
$-6 = y$

**7 a** $2(x + 5) = x + 6$
$2x + 10 = x + 6$
$x + 10 = 6$
$x = -4$

**b** $7b - 2 = 2(b + 4)$
$7b - 2 = 2b + 8$
$5b - 2 = 8$
$5b = 10$
$b = 2$

**c** $4(2y + 1) = 3(5y - 1)$
$8y + 4 = 15y - 3$
$4 = 7y - 3$
$7 = 7y$
$1 = y$

**d** $2x - 1 = 8 - 4x$
$6x - 1 = 8$
$6x = 9$
$x = \frac{9}{6} = \frac{3}{2}$
(Or $x = 1.5$ or $x = 1\frac{1}{2}$)

## Writing linear equations

**1 a** Perimeter $= 4 \times (2s + 3) = 8s + 12$
(Or, Perimeter $= 2s + 3 + 2s + 3 + 2s + 3 + 2s + 3 = 8s + 12$)

**b** $8s + 12 = 84$
$8s = 72$
$s = 9$ cm

**2 a** Angles in a quadrilateral add up to 360° so:
$x + 20 + 2x - 15 + x + 65 + 2x - 10 = 360$
$6x + 60 = 360$
$6x = 300$
$x = 50$

**b** Largest angle: $x + 65 = 50 + 65 = 115°$
(Other angles: $x + 20 = 50 + 20 = 70°$;
$2x - 15 = 2 \times 50 - 15 = 85°$;
$2x - 10 = 2 \times 50 - 10 = 90°$)

**3** Let $a$ = Karen's age
Monica is 4 years younger: $a - 4$
$a + a - 4 = 64$
$2a - 4 = 64$
$2a = 68$
$a = 34$
Karen is 34 years old.
$a - 4 = 34 - 4 = 30$
Monica is 30 years old.

**4** Let $n$ = number.
$2n + 4 = 16 - n$
$3n + 4 = 16$
$3n = 12$
$n = 4$
The number is 4.

**5** Let $l$ = length of rectangle.
Width is 2 cm smaller: $l - 2$
Perimeter $= 2l + 2(l - 2)$
$= 2l + 2l - 4 = 4l - 4$
$4l - 4 = 36$
$4l = 40$
$l = 10$
Length is 10 cm.
$l - 2 = 10 - 2 = 8$
Width is 8 cm.

**6** Base angles of an isosceles triangle are equal so:
$4a - 20 = 2a + 16$
$2a - 20 = 16$
$2a = 36$
$a = 18$
When $a = 18$: $4a - 20 = 4 \times 18 - 20 = 52$
So $2a + 16 = 52$
Angles in a triangle add up to 180° so:
$4b - 2a + 52 + 52 = 180$
$4b - 2a + 104 = 180$
$4b - 2a = 76$ (Substitute a = 18)
$4b - 2 \times 18 = 76$
$4b - 36 = 76$
$4b = 112$
$b = 28$

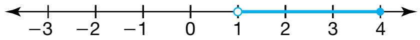
## Linear inequalities

**1 a** $x = 3, 4, 5$
**b** $x = 2, 3, 4, 5$
**c** $x = 0, 1, 2, 3$
**d** $x = -3, -2, -1, 0, 1$

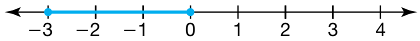
**2** **a** $x < 3$ **b** $x \geq -2$

**c** $-1 \leq x \leq 5$

**3** **a** 

**b** 

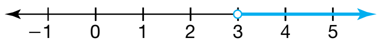
**c** 

**d** 

**4** **a** $2x - 2 > 4$

$2x > 6$

$x > 3$

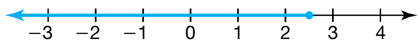


**b** $4x + 3 \leq 13$

$4x \leq 10$

$x \leq \frac{10}{4} = \frac{5}{2}$

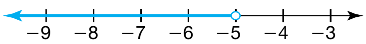
(Or $x \leq 2.5$ or $x \leq 2\frac{1}{2}$)



**c** $4x < 2x - 10$

$2x < -10$

$x < -5$



**d** $7x + 2 \geq 3x - 2$

$4x + 2 \geq -2$

$4x \geq -4$

$x \geq -1$

**5** Olivia has not multiplied all the terms in the bracket by the term outside.

Correct working:

$3(x + 4) > 22$

$3x + 12 > 22$

$3x > 10$

$x > \frac{10}{3}$

(Or $x > 3\frac{1}{3}$)

**6** **a** $2x + 3 < 4x + 8$

$3 < 2x + 8$

$-5 < 2x$

$-\frac{5}{2} < x$

(Or $x > -\frac{5}{2}$)

**b** Smallest integer value of $x$ that satisfies the inequality is $-2$

**7** No, Lee is not correct.

$6 \leq 2x + 4 < 16$

$-2 \leq 2x < 12$

$-1 \leq x < 6$

Inequality is not true when $x = 6$ so $x = 6$ is not a possible solution.

Alternative method:

When $x = 6$:

$6 \leq 2 \times 6 + 4 < 16$

$6 \leq 16 < 16$

Since 16 is not less than 16, $x = 6$ is not a possible solution.

**8** **a** $4 - x \leq 1$

$4 \leq x + 1$

$3 \leq x$ (Or $x \geq 3$)

Alternative method:

$4 - x \leq 1$

$-x \leq -3$

$x \geq 3$

**b** $6 - 3x > 9$

$6 > 3x + 9$

$-3 > 3x$

$-1 > x$ (Or $x < -1$)

Alternative method:

$6 - 3x > 9$

$-3x > 3$

$x < -1$

**c** $8 - 2x \geq 7$

$8 \geq 2x + 7$

$1 \geq 2x$

$\frac{1}{2} \geq x$ (Or $x \leq \frac{1}{2}$)

Alternative method:

$8 - 2x \geq 7$

$-2x \geq -1$

$x \leq \frac{1}{2}$

**d** $-2 < -x \leq 3$

$2 > x \geq -3$

## Formulae

**1** Pay $= 8 \times 35 + 25 = 280 + 25 = 305$

Pay = £305

**2** $P = 2(8 + 5.5) = 2 \times 13.5 = 27$

**3** $v = 10 + (-20) \times 5 = 10 + (-100) = -90$

**4** $C = 25d + 50$

**5** $A = l^2$

**6** **a** $P = 2a + 2(a + 3) = 2a + 2a + 6 = 4a + 6$

(Or $P = a + a + a + 3 + a + 3 = 4a + 6$)

**b** $P = 4 \times 6 + 6 = 24 + 6 = 30$

$P = 30\,\text{cm}$

**7** $-10 = \frac{D}{6.5}$

$-65 = D$

**8** **a** $v = u + at$

$v - u = at$

$\frac{v - u}{t} = a$

**b** $V = \frac{1}{3}Ah$

$3V = Ah$

$\frac{3V}{A} = h$

**c** $y = 3(x - 3)$

$y = 3x - 9$

$y + 9 = 3x$

$\frac{y+9}{3} = x$

Or: $x = \frac{y}{3} + 3$

**d** $v^2 = u^2 + 2as$

$v^2 - u^2 = 2as$

$\frac{v^2 - u^2}{2a} = s$

**e** $T = \sqrt{\frac{2s}{g}}$

$T^2 = \frac{2s}{g}$

$gT^2 = 2s$

$g = \frac{2s}{T^2}$

## Linear sequences

**1 a i** 2, 5, 8, 11, **14, 17**

**ii** 23, 19, 15, 11, **7, 3**

**iii** 3, 9, 15, 21, **27, 33**

**iv** 4, 9, 14, 19, **24, 29**

**b i** 2, 5, 8, 11, 14, 17, 20, 23, 26, **29**

(Or, 10th term $= 2 + (3 \times 9) = 29$)

**ii** 23, 19, 15, 11, 7, 3, −1, −5, −9, **−13**

(Or, 10th term $= 23 - (4 \times 9) = -13$)

**iii** 3, 9, 15, 21, 27, 33, 39, 45, 51, **57**

(Or, 10th term $= 3 + (6 \times 9) = 57$)

**iv** 4, 9, 14, 19, 24, 29, 34, 39, 44, **49**

(Or, 10th term $= 4 + (5 \times 9) = 49$)

**2 a** 1st term $= 1 \times 4 - 2 = 2$

2nd term $= 2 \times 4 - 2 = 6$

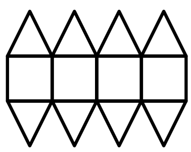
3rd term $= 3 \times 4 - 2 = 10$

4th term $= 4 \times 4 - 2 = 14$

**b** 20th term $= 20 \times 4 - 2 = 78$

**3** −25, −18, −11, −4, **3, 10**

**4 a**



**b** Number of triangles: 2, 4, 6, 8, 10, 12, 14, 16

So 16 triangles in pattern number 8

Or, $2 + 7 \times 2 = 16$ triangles

**c** No. The number of triangles forms an even number sequence and 35 is odd.

**5 a** 3, 7, 11, 15, 19

Common difference $= +4$

$4 \times$ term number $= 4, 8, 12, 16, 20$

$-1$ to get each term in the original sequence

So, $n$th term is $4n - 1$

**b** $4n - 1 = 99$

$4n = 100$

$n = 25$

Yes, 99 is a term in the sequence because 25 is an integer.

## Non-linear sequences

**1** 1, 3, 5, 7, 9, … Arithmetic sequence (Term-to-term rule is add 2)

1, 2, 4, 8, 16, … Geometric sequence (Term-to-term rule is multiply by 2, or double)

1, 4, 5, 9, 14, … Fibonacci-type sequence (Next term of sequence is found by adding the previous two terms together)

1, 4, 9, 16, 25, … Square-number sequence (Sequence of square numbers $1^2, 2^2, 3^2, 4^2$…)

**2 a** 4, 2, 1, $\mathbf{\frac{1}{2}}$, $\mathbf{\frac{1}{4}}$ (÷ 2)

**b** 5, 0.5, 0.05, **0.005, 0.0005** (÷ 10)

**c** $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\mathbf{\frac{1}{16}}$, $\mathbf{\frac{1}{32}}$ (÷ 2)

**d** $\frac{1}{9}$, $\frac{1}{3}$, 1, **3**, **9** (× 3)

**e** −0.1, −0.2, −0.4, **−0.8, −1.6** (× 2)

**f** 3, −6, 12, **−24, 48** (× −2)

**3** When $n = 5$:

$3 \times 5^2 - 4 = 3 \times 25 - 4 = 71$

**4 a** 1st term $= a$

2nd term $= b$

3rd term $= a + b$

4th term $= b + a + b = a + 2b$

5th term $= a + b + a + 2b = 2a + 3b$

**b** $b = 5$

$2a + 3b = 23$ (Substitute $b = 5$)

$2a + 3 \times 5 = 23$

$2a + 15 = 23$

$2a = 8$

$a = 4$

## Show that…

**1 a** LHS $= 4(x - 3) + 2(x + 5) = 4x - 12 + 2x + 10$
$= 6x - 2$

RHS $= 3(2x - 1) + 1 = 6x - 3 + 1 = 6x - 2$

LHS = RHS

So $4(x - 3) + 2(x + 5) \equiv 3(2x - 1) + 1$

**b** LHS $= (x + 2)(x - 2) = x^2 - 2x + 2x - 4 = x^2 - 4$

LHS = RHS

So $(x + 2)(x - 2) = x^2 - 4$

**2** Rod A $= n$

Rod B $= n + 1$

Rod C $= n + 2$

Rod A + Rod C $= n + n + 2$

$= 2n + 2$

$= \mathbf{2}(n + 1)$

Rod A + Rod C is 2 times the length of rod B.

## Functions

**1 a** $10 \times 3 - 3 = 30 - 3 = 27$

**b** $(9 + 3) \div 3 = 12 \div 4 = 4$

**c** If $x = y$,

$3x - 3 = x$

$2x - 3 = 0$

$2x = 3$

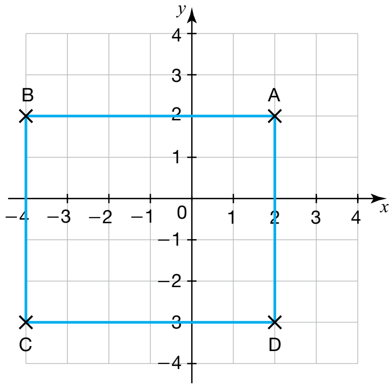
$x = \frac{3}{2}$

(Or $x = 1\frac{1}{2}$)

## Coordinates and midpoints

**1** **a** A(2, 2)

**b** and **c**




**d** B(−4, 2), D(2, −3)

$x$-coordinate: $2 + (-4) = -2$

$-2 \div 2 = -1$

$y$-coordinate: $-3 + 2 = -1$

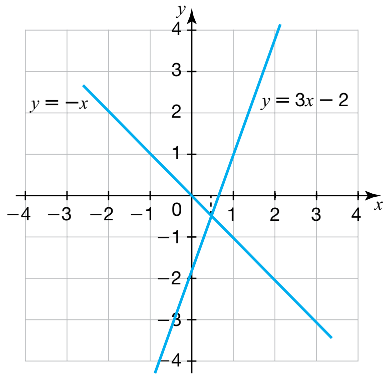
$-1 \div 2 = -0.5$

Midpoint is (−1, −0.5)

## Straight-line graphs

**Stretch it!**

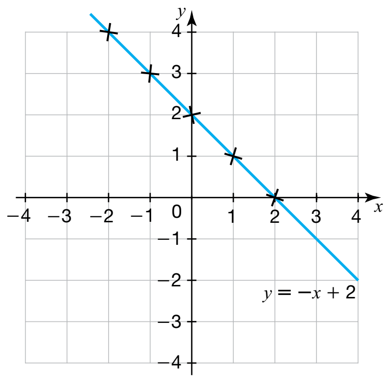
To solve the equation, you need to find where the graph of $y = 3x - 2$ intersects the graph of $y = -x$.




So the solution to $3x - 2 = -x$ is $x = 0.5$.

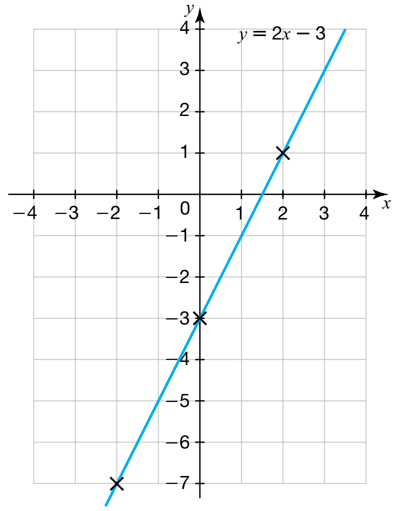
**1**

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 4 | 3 | 2 | 1 | 0 |

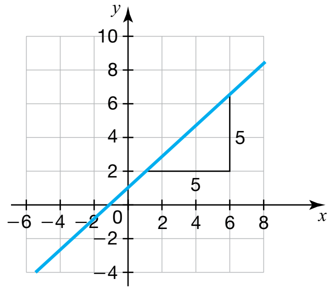



**2**

| $x$ | −2 | 0 | 2 |
|---|---|---|---|
| $y$ | −7 | −3 | 1 |




**3**




Gradient $= \dfrac{\text{difference in } y \text{ coordinate}}{\text{difference in } x \text{ coordinate}} = \dfrac{5}{5} = 1$

Gradient, $m = 1$

$y$-intercept, $c = 1$

Using the general form of the equation of a line $y = mx + $ c: the equation of the line is $y = x + 1$

**4** Gradient, $m = 2$

$y = mx + c$ (General form of the equation of a line)

$y = 2x + c$

For point (1, −2), $x = 1$, $y = -2$:

$-2 = 2 \times 1 + c$

$-2 = 2 + c$

$-4 = c$

So the equation of the line is $y = 2x - 4$

**5** Gradient $= \dfrac{\text{difference in } y \text{ coordinate}}{\text{difference in } x \text{ coordinate}} = \dfrac{4-2}{0-4} = -\dfrac{2}{4} = -\dfrac{1}{2}$

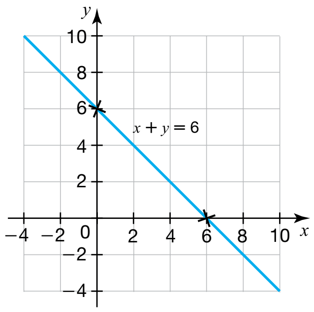
Gradient, $m = -\dfrac{1}{2}$

Given the point (0, 4): $y$-intercept, $c = 4$

So the equation of the line is $y = -\frac{1}{2}x + 4$

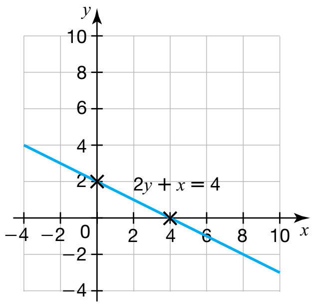
**6 a**

| $x$ | 0 | 6 |
|---|---|---|
| $y$ | 6 | 0 |




**b**

| $x$ | 0 | 4 |
|---|---|---|
| $y$ | 2 | 0 |




**7** A: $y = 4x + 1$

B: $4x + 4y = 4$

$4y = -4x + 4$

$y = -x + 1$

C: $x - 2y = 2$

$-2y = -x + 2$

$y = \frac{x}{2} - 1$

D: $2y = 4 + 8x$

$y = 2 + 4x$

$y = 4x + 2$

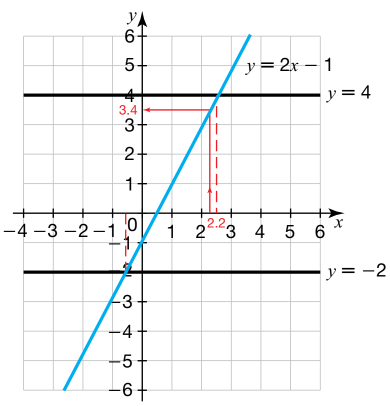
Lines A and D are parallel.

**8** $2y = x - 4$

$y = \frac{x}{2} - 2$

The $y$-intercept is $(0, -2)$

**9**




**a** $y = 3.4$ **b** $x = 0.5$

**c** $x \approx 2.5$ **d** $x \approx -0.5$

## Solving simultaneous equations

**1 a** $2x + y = 4$ (1)

$3x - y = 1$ (2)

(1) + (2): $5x = 5$

$x = 1$

Substitute $x = 1$ in (1)

$2 \times 1 + y = 4$

$2 + y = 4$

$y = 2$

Solution: $x = 1$, $y = 2$

**b** $x - y = 5$ (1)

$2x + y = 4$ (2)

(1) + (2): $3x = 9$

$x = 3$

Substitute $x = 3$ into (1)

$3 - y = 5$

$3 = y + 5$

$y = -2$

Solution: $x = 3$, $y = -2$

**c** $2x + y = 8$ (1)

$x + y = 2$ (2)

(1) − (2): $x = 6$

Substitute $x = 6$ into (2)

$6 + y = 2$

$y = -4$

Solution: $x = 6$, $y = -4$

**d** $4x - y = 10$ (1)

$x + 2y = 7$ (2)

(1) × 2: $8x - 2y = 20$ (3)

(2) + (3): $9x = 27$

$x = 3$

Substitute $x = 3$ into (2):

$3 + 2y = 7$

$2y = 4$

$y = 2$

Solution: $x = 3$, $y = 2$

**e** $2x + y = 7$ (1)

$x - 4y = 8$ (2)

(1) × 4: $8x + 4y = 28$ (3)

(2) + (3): $9x = 36$

$x = 4$

Substitute $x = 4$ into (1):

$2 \times 4 + y = 7$

$8 + y = 7$

$y = -1$

Solution: $x = 4$, $y = -1$

**f** $2x + 3y = 7$ (1)

$3x - 2y = 4$ (2)

(1) × 2: $4x + 6y = 14$ (3)

(2) × 3: $9x - 6y = 12$ (4)

(3) + (4): $13x = 26$

$x = 2$

Substitute $x = 2$ into (1)

$2 \times 2 + 3y = 7$

$4 + 3y = 7$

$3y = 3$

$y = 1$

Solution: $x = 2$, $y = 1$

**2** $x + y = 21$ (1)

$x - y = 7$ (2)

(1) + (2): $2x = 28$

$x = 14$

Substitute $x = 14$ into (1)

$14 + y = 21$

$y = 7$

The two numbers are 7 and 14.

**3** Let $b$ = burger and $c$ = cola.

$3b + 2c = 505$ (1)

$3b + 4c = 725$ (2)

(2) − (1): $2c = 220$

$c = 110$

Substitute $c = 110$ into (1)

$3b + 2 \times 110 = 505$

$3b + 220 = 505$

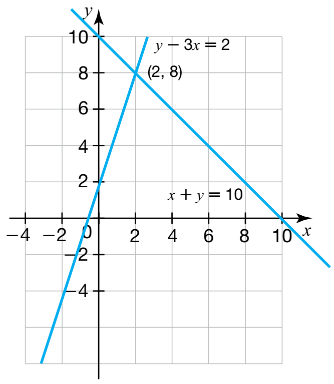
$3b = 285$

$b = 95$

A burger costs 95p.

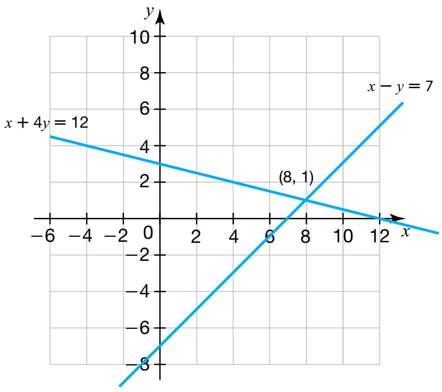
A cola costs £1.10.

**4** **a**




$x = 2$, $y = 8$

**b**



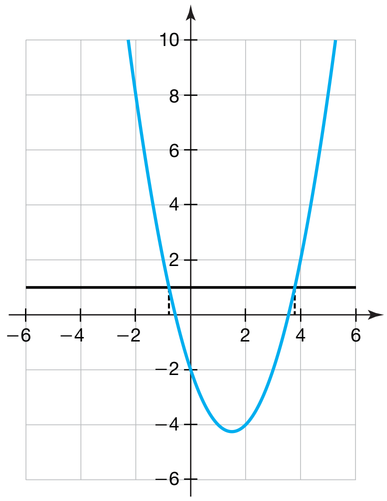

$x = 8$, $y = 1$

## Quadratic graphs

**Stretch it!**

Rearrange $x^2 - 3x = 3$, to give $x^2 - 3x - 2 = 1$

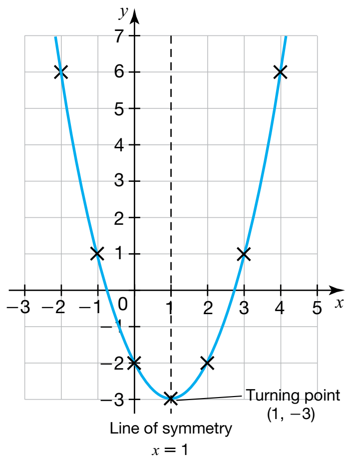
You can solve this graphically by finding where the lines $y = x^2 - 3x - 2$ and $y = 1$ intersect.



So the solutions to the equation $x^2 - 3x = 3$ are $x = 3.8$ and $x = -0.8$. Acceptable readings from the graph would be in the range 3.6 to 3.9 and −0.6 to −0.9.

**1** **a**

| $x$ | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | 6 | 1 | −2 | −3 | −2 | 1 | 6 |

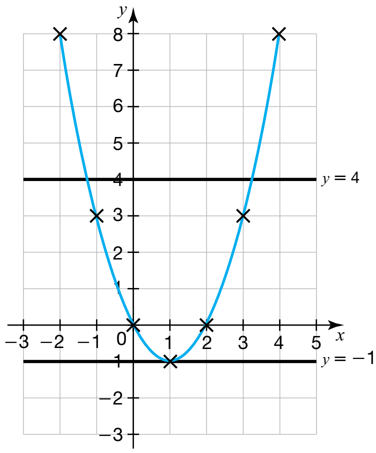



**b** **i** $x = 1$

**ii** (1, −3)

**2**

| $x$ | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y$ | 8 | 3 | 0 | −1 | 0 | 3 | 8 |



**a** $x = 0$ and $x = 2$

**b** $x \approx -1.2$ and $x \approx 3.2$

**c** $x = 1$

## Solving quadratic equations

**Stretch it!**

$\frac{x^2}{2} = 8$

$x^2 = 16$

$x = \sqrt{16}$

So $x = 4$ or $x = -4$

$2x^2 = 50$

$x^2 = 25$

$x = \sqrt{25}$

So $x = 5$ or $x = -5$

**1** **a** $x^2 - 4x = 0$

$x(x - 4) = 0$

Either $x = 0$ or $x - 4 = 0$

$x = 4$

So $x = 0$ or $x = 4$

**b** $x^2 + 7x = 0$

$x(x + 7) = 0$

Either $x = 0$ or $x + 7 = 0$

$x = -7$

So $x = 0$ or $x = -7$

**c** $x^2 - 16 = 0$ ($x^2 - 16 = x^2 - 4^2$, Factorise)

$(x + 4)(x - 4) = 0$

Either $x + 4 = 0$ or $x - 4 = 0$

$x = -4$ $\quad x = 4$

So $x = -4$ or $x = 4$

**d** $x^2 + 10x + 9 = 0$

$(x + 1)(x + 9) = 0$

Either $x + 1 = 0$ or $x + 9 = 0$

$x = -1$ $\quad x = -9$

So $x = -1$ or $x = -9$

**e** $x^2 + x - 12 = 0$

$(x - 3)(x + 4) = 0$

Either $x - 3 = 0$ or $x + 4 = 0$

$x = 3$ $\quad x = -4$

So $x = 3$ or $x = -4$

**f** $x^2 - 6x - 16 = 0$

$(x + 2)(x - 8) = 0$

Either $x + 2 = 0$ or $x - 8 = 0$

$x = -2$ $\quad x = 8$

So $x = -2$ or $x = 8$

**2** **a** $y = x^2 - 49$ (Set $y = 0$)

$x^2 - 49 = 0$ ($x^2 - 49 = x^2 - 7^2$, Factorise)

$(x + 7)(x - 7) = 0$

Either $x + 7 = 0$ or $x - 7 = 0$

$x = -7$ $\quad x = 7$

So $x = -7$ or $x = 7$

**b** $y = x^2 - 3x$ (Set $y = 0$)

$x^2 - 3x = 0$

$x(x - 3) = 0$

Either $x = 0$ or $x - 3 = 0$

$x = 3$

So $x = 0$ or $x = 3$

**c** $y = x^2 + 7x + 6$ (Set $y = 0$)

$x^2 + 7x + 6 = 0$

$(x + 1)(x + 6) = 0$

Either $x + 1 = 0$ or $x + 6 = 0$

$x = -1$ $\quad x = -6$

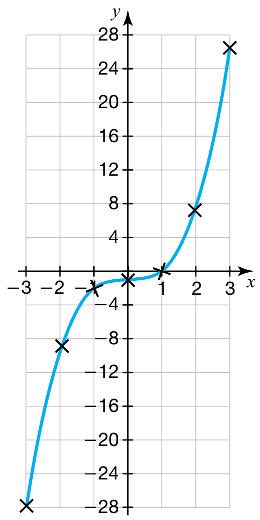
So $x = -1$ or $x = -6$

## Cubic and reciprocal graphs

**1** **a**

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | $-28$ | $-9$ | $-2$ | $-1$ | $0$ | $7$ | $26$ |

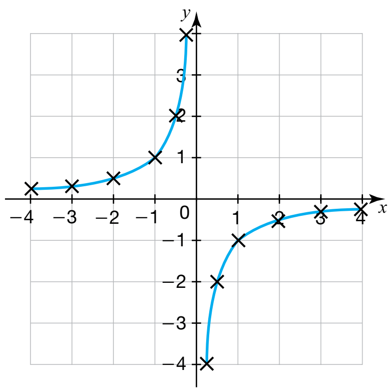
**b**




**2** **a**

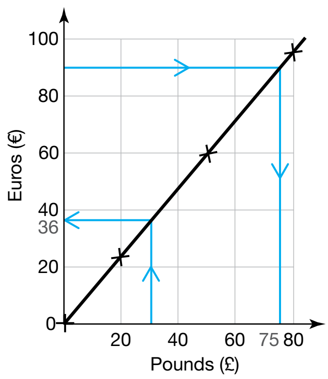
| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $-\frac{1}{2}$ | $-\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $1$ | $2$ | $4$ | $-4$ | $-2$ | $-1$ | $-\frac{1}{2}$ | $-\frac{1}{3}$ | $-\frac{1}{4}$ |

**b**




## Drawing and interpreting real-life graphs

**1** **a**



**b** **i** €36

**ii** £75

**c** From the graph: £30 = €36

So £90 = €36 × 3 = €108

Ring is cheaper in France.

**2** **a** Monthly charge = £10 (cost of 0 minutes from the graph)

**b** Gradient $= \frac{30}{240} = 0.125$

Charge per minute of calls is 13p.

**3** A: The temperature is steadily increasing.

B: The temperature remains constant.

C: The temperature rises steadily for a period and then remains constant.

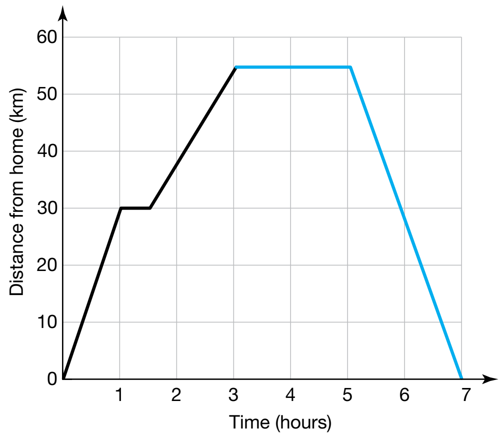
**4** **a** 30 minutes (Horizontal line on graph)

**b** 55 km

**c** Speed before break $= \frac{\text{distance (km)}}{\text{time (hours)}} = \frac{30}{1} = 30$ km/hr

Speed after break $= \frac{\text{distance (km)}}{\text{time (hours)}} = \frac{25}{1.5} = 16.7$ km/hr

**d**



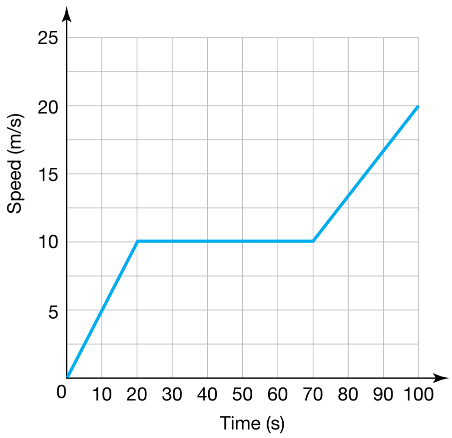

**5** **a** 6 m/s

**b** 4 seconds

**c** 6 seconds

**d** Acceleration $= \frac{\text{change in speed (m/s)}}{\text{time (s)}} = \frac{6}{4} = 1.5$ m/s$^2$

**6** **a**




**b** Acceleration $= \frac{\text{change in speed (m/s)}}{\text{time (s)}} = \frac{20-10}{30} = \frac{10}{30}$

$= 0.\dot{3}$ m/s$^2$

**7** **a** The maximum depth of water in the bath before the person got in was 35cm

**b** Between C and D, the person was taking their bath.

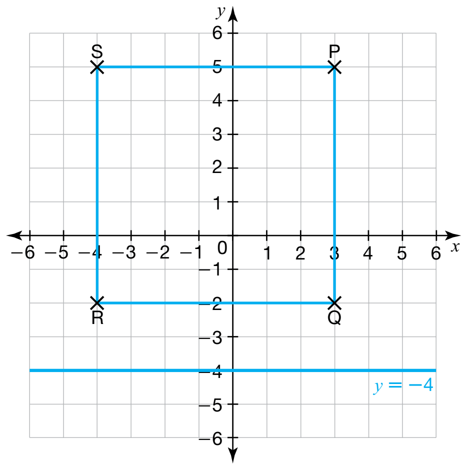
**c** Between D and E, the person got out of the bath.

**d** Running water into the bath was quicker. The slope of the line between O and A (filling the bath) is steeper than the slope of the line between E and F (emptying the bath).

## Review it!

**1** **a** P(3, 5)

**b**, **c** and **e**




**d** Q(3, −2), S(−4, 5)

$x$-coordinate: −4 + 3 = −1

−1 ÷ 2 = −0.5

$y$-coordinate: 5 + (−2) = 3

3 ÷ 2 = 1.5

Midpoint is (−0.5, 1.5)

**2** **a** $2x + 8 = 4$

$2x = -4$

$x = -2$

**b** 4 × 4 − 3 × 3 = 16 − 9 = 7

**c** Millie is correct.

When $x = 4$, $3x^2 = 3 \times 4^2 = 3 \times 16 = 48$

(George has worked out $(3x)^2$ instead.)

**3** **a** $4(2x + 3)$

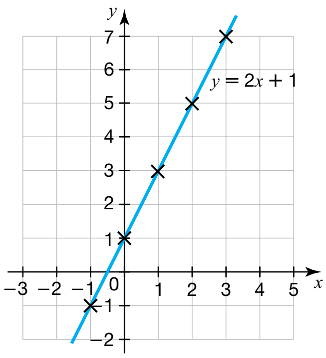
**b** $m^4 \times m = m^{4+1} = m^5$

**c** $\frac{x^8}{x^3} = x^{8-3} = x^5$

**4** **a**

| $x$ | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y$ | −1 | 1 | 3 | 5 | 7 |

**b**




**c** Compare $y = 2x + 1$ with $y = mx + c$ (general form of the equation of a line):

Gradient, $m = 2$

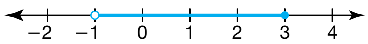
**5** $4x + 4 = x + 13$

$3x + 4 = 13$

$3x = 9$

$x = 3$

**6 a** $x \le 2$

**b** 


**c** $x = 1, 2, 3$

**d** $4x + 2 \le 2x + 5$

$2x + 2 \le 5$

$2x \le 3$

$x \le \frac{3}{2}$

(Or $x \le 1\frac{1}{2}$)

Largest integer value of $x$ that satisfies the inequality = 1

**7 a** 3, 9, 15, 21, 27, **33**

**b** No. This is a sequence of odd numbers and 44 is even.

**c** When $n = 5$:

$2n^2 - 3 = 2 \times 5^2 - 3 = 2 \times 25 - 3 = 47$

**8** $(x + 3)(x + 4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

**9** The opposite sides of a rectangle are equal in length so:

$5x - 8 = 2x + 4$

$3x - 8 = 4$

$3x = 12$

$x = 4$

$14 - 2y = 4y + 2$

$14 = 6y + 2$

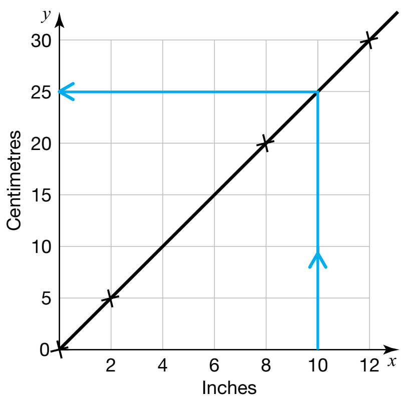
$12 = 6y$

$2 = y$

**10 a** $12m$

**b** $3p \times 4p = 3 \times 4 \times p \times p = 12p^2$

**c** $12x \div 2 = \frac{12x}{2} = 6x$

**11 a** 


**b i** 25 cm

**ii** From the graph: 25 cm = 10 inches

So 50 cm = 10 × 2 = 20 inches

**c** From the graph: 10 inches = 25 cm

So 60 inches = 25 × 6 = 150 cm

Cost of beading = 150 × 2 = 300p

Cost = £3.00

**12 a** $T = 12.50x + 10$

**b** $72.50 = 12.50x + 10$

$62.50 = 12.50x$

$5 = x$

Suzanne hired the costume for 5 days.

**13 a** $4x + 2 \le 8$

$4x \le 6$

$x \le \frac{6}{4} = \frac{3}{2}$

(Or $x \le 1\frac{1}{2}$)

**b** $3x - 4 < 17$

$3x < 21$

$x < 7$

$4x + 2 \ge 22$

$4x \ge 20$

$x \ge 5$

If $x < 7$ and $x \ge 5$ then $x = 6$ and $x = 5$ satisfy both.

**14** Ollie has squared each term inside the brackets rather than squaring the whole bracket.

Correct working:

$(x + 4)^2 = (x + 4)(x + 4) = x^2 + 4x + 4x + 16 = x^2 + 8x + 16$

**15** $P = \frac{Q}{4} + R$

$P - R = \frac{Q}{4}$

$4(P - R) = Q$

**16 a** $m(m + 8)$

**b** $(x + 3)(x + 4)$

**17 a** 2, 5, 8, 11, 14

Common difference = +3

3 × term number: 3, 6, 9, 12, 15

− 1 to get each term in the original sequence

So $n$th term = $3n - 1$

**b** $2n - 3 = 112$

$2n = 115$

$n = 57.5$

No, Kadena is incorrect.

112 cannot be a term in the sequence because 57.5 is not an integer.

**18 a** $4(x + 5) - 3(2x - 1) = 4x + 20 - 6x + 3 = -2x + 23$

**b** $4a^3b^2 \times 5a^2b = 4 \times 5 \times a^3 \times a^2 \times b^2 \times b$

$= 20 \times a^{3+2} \times b^{2+1} = 20a^5b^3$

**19** Perimeter = $3x - 2 + 2x + 1 + 3x + 5 + 2x = 10x + 4$

$10x + 4 = 49$

$10x = 45$

$x = 4.5$

**20 a** 1st term: $a$

2nd term: $b$

3rd term: $a + b$

4th term: $b + a + b = a + 2b$

5th term: $a + b + a + 2b = 2a + 3b$

6th term: $a + 2b + 2a + 3b = 3a + 5b$

7th term: $2a + 3b + 3a + 5b = 5a + 8b$

**b** $a + b = 5$ (1)

$5a + 8b = 34$ (2)

(1) × 5: $5a + 5b = 25$ (3)

(2) − (3): $3b = 9$

$b = 3$

Substitute b = 3 into (1)

$a + 3 = 5$

$a = 2$

# Ratio, proportion and rates of change

## Units of measure

1 a 3000 m

b 75 mins

c 13 000 $cm^2$

d 3.52 litres

e 7200 seconds

f 14 kg

2 4.5 − 0.325 = 4.175 kg or 4500 − 325 = 4175 g

3 $5 \div 2.2 = 2.\dot{2}\dot{7}$ kg

## Ratio

**Stretch it!** 31 + 25 = 56, fraction male = $\frac{31}{56}$

1 a 1 : 4 b 1 : 3 : 4 c 4 : 5

2 35 : 5 = 7 : 1

3 a 7 : 1

b 800 ÷ (7 + 1) = 100

1 × 100 = 100 tickets

4 a 3 : 2

b $\frac{3}{5}$ of 200 = (200 ÷ 5) × 3 = 120

5 1500 g ÷ (2 + 3) = 300
2 × 300 = 600 g or 0.6 kg

6 9 ÷ 3 = 3

4 × 3 = 12 cm

5 × 3 = 15 cm

7 $s = 20t$

8 5 − 2 = 3

60 ÷ 3 = 20 g

2 × 20 g = 40 g

## Scale diagrams and maps

**Stretch it!** 50 miles on ground = $\frac{50}{x}$ miles on map

1 mile = 1610 m = 161 000 cm

50 miles on ground = $\frac{50}{x}$ × 161 000 cm* on map

1 A, B, F

2 a 3 × 12 = 36 km

b 15 ÷ 12 = 1.25 cm

3 12 × 1000 = 12 000 cm = 120 m

4 a 2 cm: 2 × 50 000 = 100 000 cm = 1 km

(Any answer within the range of 1 km – 1.1 km is acceptable.)

b 250°

## Fractions, percentages and proportion

1 $\frac{20}{3500} = \frac{1}{175}$

2 2 + 3 + 8 = 13 hours

24 − 13 = 11 hours

$\frac{11}{24}$ of the day remaining

3 a $\frac{15}{20} = \frac{3}{4}$

b $1 - \frac{3}{4} = \frac{1}{4} = 25\%$

4 1 + 2 + 7 = 10, $\frac{1}{10} = 10\%$

5 School A: 125 : 145 = 25 : 29

School B: 100 : 120 = 5 : 6

No since the ratios are not equivalent.

6 150 ÷ 100 = 1.5

1.5 × 22 g = 33 g

33 g ÷ 8 = 4.125 g

## Direct proportion

**Stretch it!**

For two values to be in direct proportion, when one is 0 the other must be 0. Here, when distance is 0 miles, the fee is £2.

1 A and E

2 a i 20 meringues = 2 eggs, divide both by 2 to give: 10 meringues = 1 egg

3 eggs: 3 × 10 = 30 meringues

ii 20 meringues = 120 g of sugar, divide both by 2 to give: 10 meringues = 60 g of sugar. Multiply both by 10 to give 100 meringues

b 20 meringues = 2 eggs, divide both by 2 to give 10 meringues = 1 egg, multiply both by 7 to give 70 meringues = 7 eggs

3 675 ÷ 4.5 = 150 minutes = 2 hours 30 minutes

4 A, D

## Inverse proportion

1 D

2 At 60 miles it takes 15 minutes.

$60 \times \frac{2}{3} = 40$

$15 \div \frac{2}{3} = 22.5$ mins

3 2 × 3 = 6 decorators

$5 \div 3 = 1\frac{2}{3}$ of a day

4 a 2

b The age of the chicken and the number of eggs it lays are in inverse proportion, this means that as the age of the chicken increases, the number of eggs it lays decreases.

## Working with percentages

**Stretch it!** £128

**Stretch it!** Let percentage rate = $x$

$$\left(1 + \frac{x}{100}\right)^5 \times £100 = £110$$

$$\left(1 + \frac{x}{100}\right)^5 = \frac{110}{100}$$

$$1 + \frac{x}{100} = \sqrt[5]{\frac{110}{100}}$$

$$1 + \frac{x}{100} = 1.02$$

$$\frac{x}{100} = 0.02$$

$$x = 2$$

Percentage interest is 2%

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

**1** **a** $1.03 \times 50 = £51.50$
**b** $2.48 \times 400 = 992$
**c** $0.195 \times 64 = 12.48$
**2** $45 - 40 = 5$, $\frac{5}{40} \times 100 = 12.5\%$
**3** $24 \div 115 = 0.209$, $0.209 \times 100 = 20.9°C$
**4** $15\,000 \times 1.20^3 = 25920$
**5** 20% is $\frac{1}{5}$ of the price.
$30 \times 5 = £150$

## Compound units

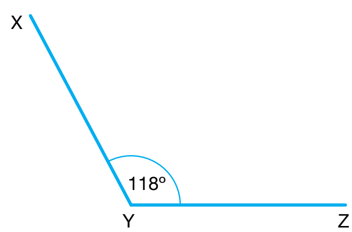
**Stretch it!** $\frac{100}{x}$ mph
**1** $29.50 \div 0.18 = 164$ or $2950 \div 18 = 164$ units
**2** Time $= \frac{80}{120} = \frac{2}{3}$ hour $= 40$ minutes
**3** Density $= \frac{0.72}{3} = 0.24$ g/cm$^3$
**4** Pressure $= \frac{12}{2} = 6$N/m$^2$
**5** 3 m/s $= 3 \times 60$ m/minute $= 3 \times 60 \times 60$ m/hour
$= 10800$ m/hour $= 10.8$ km/hour
**6** 0.6 litres per second $= 0.6 \times 60$ litres per minute
$= 0.6 \times 60 \times 60$ litres per hour
$= 2160$ litres per hour.
$2160 \div 4.55 = 475$ gallons
475 gallons per hour (to the nearest whole number)
**7** Bolt: 100 m in 9.58 seconds $= 10.4$ m/s
Cheetah: 120 km/h $= 120000$ m/hour
$= 120000 \div 60$ m/min
$= 2000$ m/min
$= 2000 \div 60$ m/sec $= 33.3$ m/s
The Cheetah is fastest.

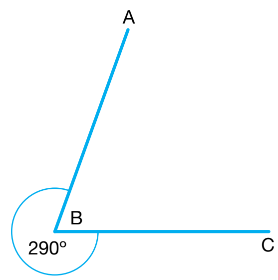
## Review it!

**1** **a** $3.2 \times 1000 = 3200$ m
**b** $9 \times 60 = 540$ seconds
**c** $0.4 \times 1000 = 400$ ml
**2** $4600 \div 1000 = 4.6$ km
**3** $2.5 \times 60 = 150$ minutes
**4** $1.1 \times 0.32 = 0.352$ m$^2$ or $110 \times 32 = 3520$ cm$^2$
**5** $3 \times 10000 = 30000$ cm$^2$
**6** $\frac{5}{12}$
**7** $26:18 = 13:9$
**8** $100 - 85 = 15$, $15 \div 3 = 5$ minutes
**9** density $= \frac{345}{0.15} = 2300$ kg/m$^3$
**10** $10 - 8 = 2$ km, $\frac{2}{8} \times 100 = 25\%$
**11** $25 - 13 = 12$, $\frac{12}{25}$ OR 48%
**12** $15 + 5 + 3 = 23$ mins
$\frac{23}{90}$
**13** $20 \div \left(\frac{4}{5}\right) = 25$ hours $= 1$ day and 1 hour
**14** **a** $50 \div 5 = 10$, Josie: $1 \times 10 = 10$ marbles, Charlie: $4 \times 10 = 40$ marbles, Charlie has 30 more.
**b** $C = 4J$
**15** $\frac{100}{360} \times 100 = 28\%$
**16** $0.8 \times 1200 = £960$
$0.9 \times 960 = £864$
**17** 1 cm: 50 000 cm
50 000 cm $= 0.5$ km
3 km $\div 0.5 = 6$
6 cm
**18** $1.02^3 \times 1500 = £1591.81$
**19** $32\,000 \div 4 = 8000$ people
**20** $393 \div 125 = 3.144$ hours $= 3$ hours 9 minutes
**21** $2.50 + 1.90 + (2 \times 5.30) = £15$
$1.05 \times £15 = £15.75$
**22** $37 + 15 + 4 + 19 = 75$
$\frac{15}{75} \times 100 = 20\%$
**23** $0.045 \times 3000 = £135$
$3000 + (5 \times 135) = £3675$
**24** $30 \div 3 = 10$
boys $= 2 \times 10 = 20$
Girls $= 1 \times 10 = 10$
Boys $= 20 - 2 = 18$
Girls $= 10 + 3 = 13$
18 : 13
**25** Men to women is $7:6 = 35:30$
Ratio of women to children is $15:2 = 30:4$
Ratio of men to women to children is $35:30:4$
$35 + 30 + 4 = 69$
$3450 \div 69 = 50$
$35 \times 50 = 1750$ men
**26** No – for two things to be in direct proportion when one is zero the other must be zero; the graph does not go through the origin so this is not the case.
**27** Neither, since the time taken to cook increases as the weight increases it is not in indirect proportion. It is not in direct proportion since a graph to illustrate the relationship would not go through the origin.
**28** speed $= \frac{\text{distance}}{\text{time}} = \frac{0.05}{17} = \frac{1}{340}$ hours $= \frac{3}{17}$ mins
$= 11$ seconds
**29** She is incorrect since the ratio of females to males must be the same for them to have equivalent proportions: 35 : 60 is not equivalent to 12 : 37.

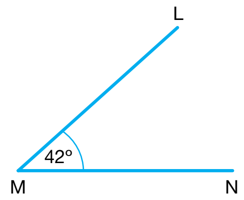
# Geometry and measures

## Measuring and drawing angles

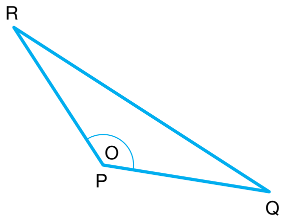
**1** **a** 43° **b** Acute
**2** **a**


**b**


**c**




**3 a**




**b** 18°

## Using the properties of angles

**1** Angles around a point add up to 360° so:

$a + 112 + 88 + 106 = 360$

$a + 306 = 360$

$a = 54°$

**2 a i** $a = (180 - 40) \div 2 = 70°$

**ii** Base angles of an isosceles triangle are equal.

**b** Exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices so:

$b = 70 + 40$

$b = 110°$

**Or**, angles on a straight line add up to 180° so:

$b = 180 - 70 = 110°$

**3** Angles around a point add up to 360° so:

$5x + 9x + 108 = 360$

$14x + 108 = 360$

$14x = 252$

$x = 18°$

**4 a i** $x = 180 - 126 = 54°$

**ii** Angles on a straight line add up to 180°.

**b** Angles in a quadrilateral add up to 360° so:

$y + 135 + 54 + 88 = 360$

$y + 277 = 360$

$y = 83°$

**5 a** Angles on a straight line add up to 180° so:

$x = 180 - 84$

$x = 96°$

**b i** $y = 96°$

**ii** Use the fact that corresponding angles are equal, then the fact that vertically opposite angles are equal.

**Or**, use the fact that alternate angles are equal, then use angles on a straight line add up to 180°.

**6 a** Base angles of an isosceles triangle are equal so $a = 58°$.

**b** Angles in a triangle add up to 180° so:

$b = 180 - 58 - 58$

$b = 64°$

**c** Alternate angles are equal so $c = 58°$ (since angle $a$ = angle $c$).

Or, since opposite angles of a parallelogram are equal:

$b + c = 122$

$64 + c = 122$

$c = 58°$

**7** Angle $BAD = 62°$ (Opposite angles of a parallelogram are equal)

Angle $ADE = 62°$ (Alternate angles are equal)

$x = 180 - 62 - 62$ (Base angles of an isosceles triangle are equal)

$x = 56°$

**8** Angle $ACB = 36°$ (Base angles of an isosceles triangle are equal)

Angle $ABC = 180 - 36 - 36$ (Angles in a triangle add up to 180°)

Angle $ABC = 108°$

$x = 108°$ (Alternate angles are equal)

## Using the properties of polygons

**Stretch it!**

**1** The angle sum of a triangle is 180°.

Sum of interior angles of a hexagon = 4 × 180° = 720°.

**2**

| Polygon | Number of sides ($n$) | Number of triangles formed | Sum of interior angles |
|---|---|---|---|
| Triangle | 3 | 1 | 180° |
| Quadrilateral | 4 | 2 | 360° |
| Pentagon | 5 | 3 | 540° |
| Hexagon | 6 | 4 | 720° |
| Heptagon | 7 | 5 | 900° |
| Octagon | 8 | 6 | 1080° |
| Decagon | 10 | 8 | 1440° |

**3** $n - 2$

**4** $180 \times (n - 2)$

**Stretch it!** Exterior angle of a regular hexagon = 360 ÷ 6 = 60°

Interior angle = 180 − 60 = 120°

Three hexagons meet at a point, so 120 + 120 + 120 = 360°

Similarly, interior angle of an octagon = 180 − (360 ÷ 8) = 135°

Interior angle of a square = 90°, so 135 + 135 + 90 = 360°.

Regular pentagons have an interior angle of 108°. This does not divide equally into 360°, so these shapes will not fit together at a point in this way.

**1** Regular decagon has 10 equal sides.

Exterior angle = 360° ÷ 10 = 36°

**2 a** Number of sides = 360° ÷ 15° = 24

**b** Angles on a straight line add up to 180° so:

Interior angle + exterior angle = 180

Interior angle + 15 = 180

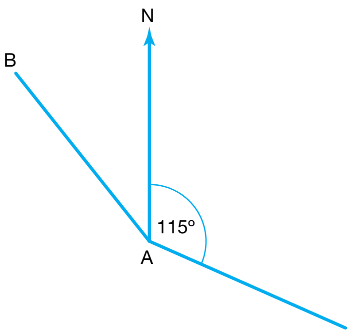
Interior angle = 165°

Sum of interior angles = 24 × 165 = 3960°

3 Exterior angle = 360° ÷ 8 = 45°
Angles on a straight line add up to 180° so:
$x$ = 180° − 45° = 135°

## Using bearings

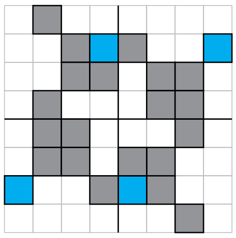
1 a 360 − 45 (acute angle) = 315°
b



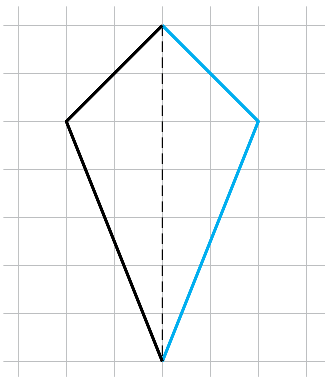

2 Bearing of P from Q = 180° + 164° = 344°
3 Kirsty is correct.
The bearing is 314° (360° − 46°) as it must be measured clockwise from north.

## Properties of 2D shapes

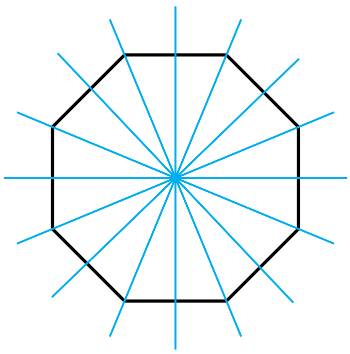
**Stretch it!**



1 a



b kite
2 a 8 possible lines of symmetry:



b 8

3 a A rectangle has rotational symmetry of order **2.**
b A **rhombus** has all sides equal and rotational symmetry of order 2.
c A kite has **1** line of symmetry and **no** rotational symmetry.
d The diagonals of a **square** and a **rhombus** bisect each other at 90°.

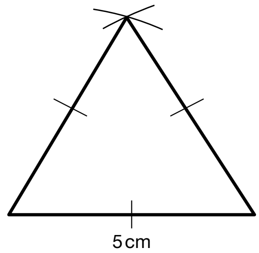
## Congruent shapes

1 Any accurate copy of shape A, in any orientation.
2 a Corresponding angles are equal so $x$ = 120°
b Corresponding sides are the same length so $y$ = 12 cm
3 a SSS (each triangle has equal sides: 3 cm, 3 cm, 2.5 cm)
b ASA (two angles, 70° and 60°, and the included side, 8 cm, are equal)

## Constructions

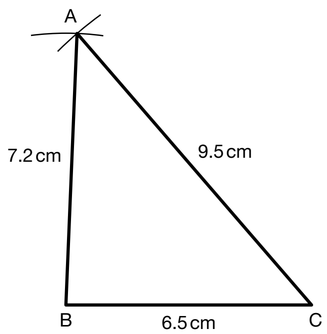
**Stretch it!**

A triangle with sides of 5cm with constructions lines indicating the use of compasses

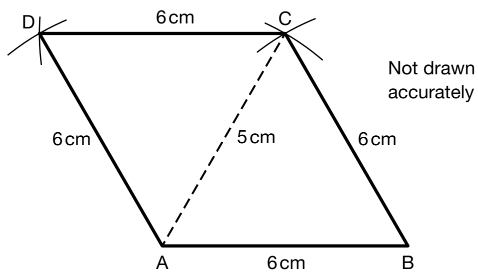



Angle size 60°

1




2



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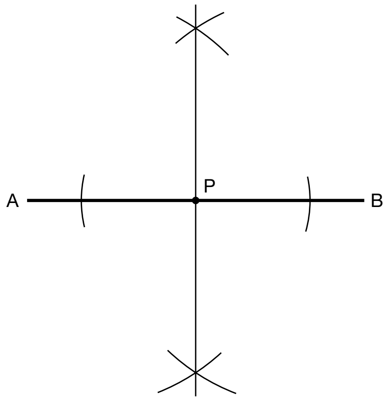


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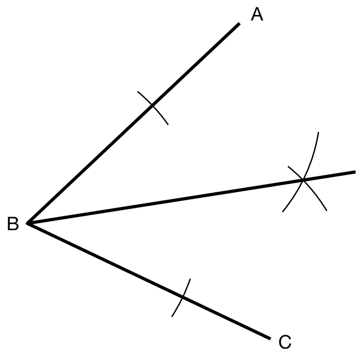


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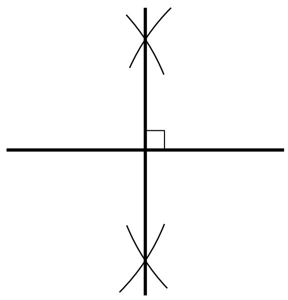


## Drawing circles and parts of circles

**1** **a** A **chord** is a straight line that does not pass through the centre of a circle but touches the circumference at each end.

**b** A **tangent** is a straight line that touches the outside of a circle at one point only.

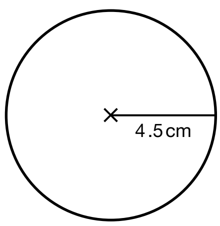
**c** A **diameter** is a straight line through the centre of a circle that touches the circumference at each end.

**d** An **arc** is part of the circumference of a circle.

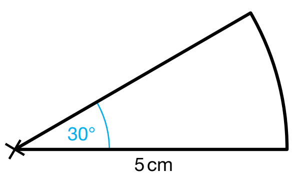
**e** A **radius** is a straight line from the centre of a circle that is half the length of the diameter.

**f** The part of a circle that has a chord and an arc as its boundary is called a **segment.**

**2**

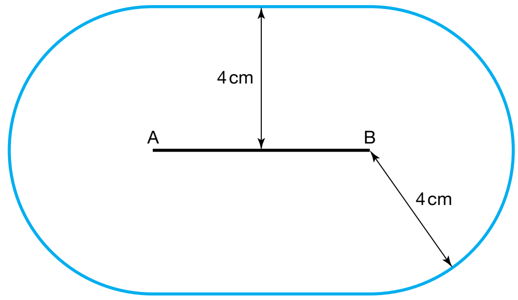



**3**

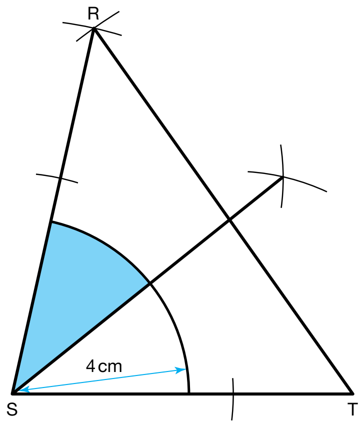



## Loci

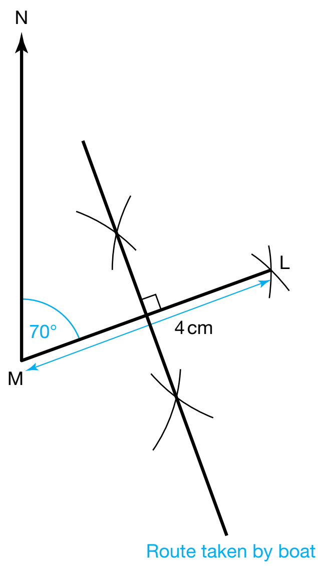
**1**




**2**




**3** **a** and **b**




## Perimeter

**1** $4 \times 7.2 = 28.8$ cm

**2** $7 + 9 + 9 + 5 + 5 + 7 = 42$ cm

**3** Curved edge $= 2\pi r \div 2 = (2 \times \pi \times 4) \div 2 = 4\pi$

Perimeter $= 4\pi + 2 \times r = 4\pi + 8$ cm

So $k = 4$ and $b = 8$

**4** Perimeter $= (\pi \times 30) + 100 + 100 = 200 + 30\pi$ m

**5** Perimeter $= \left(\frac{1}{2} \times \pi \times 32\right) + 32 + 32 = 16\pi + 64$ cm

Ribbon $= 16\pi + 64 + 5 = 16\pi + 69$ cm $= 119.3$ cm

120 cm must be bought

$12 \times £0.15 = £1.80$

## Area

**Stretch it!** Area of a semicircle $= \frac{\pi r^2}{2}$, area of a quarter circle $= \frac{\pi r^2}{4}$

1 a $4.5 \times 2 = 9.0\,\text{cm}^2$

b $3 \times 1.5 = 4.5\,\text{cm}^2$

c $\frac{(5+9)}{2} \times 4 = 28.0\,\text{cm}^2$

d $\frac{1}{2} \times 2 \times 5 = 5.0\,\text{cm}^2$

e $\pi \times 4.5^2 = 63.6\,\text{cm}^2$

2 Length of side $= 12 \div 4 = 3\,\text{cm}$

Area $= 3^2 = 9\,\text{cm}^2$

3 Shaded triangles would fit together to form one triangle with base $10 - 6 = 4$.

So area of shaded triangles $= \frac{1}{2} \times 4 \times 7 = 14\,\text{cm}^2$

Area of trapezium $= \frac{(6+10)}{2} \times 7 = 56\,\text{cm}^2$

Fraction of the shape that is shaded $= \frac{14}{56} = \frac{1}{4}$

4 Area of whole shape $= 6 \times 8 = 48\,\text{cm}^2$

Fraction shaded $= \frac{6}{16} = \frac{3}{8}$

Area shaded $= \left(\frac{3}{8}\right) \times 48 = 18\,\text{cm}^2$

5 Area of square $= 46 \times 46 = 2116\,\text{cm}^2$

Each circle has radius $= 11.5\,\text{cm}$

Area of four circles $= 4 \times \pi \times 11.5^2 = 1661.9\,\text{cm}^2$

Shaded area $= 2116 - 1661.9 = 454.1\,\text{cm}^2$

## Sectors

1 Area $= \frac{1}{2} \times \pi \times 5^2 = 39.3\,\text{cm}^2$

Perimeter $= \frac{1}{2} \times \pi \times 10 + 10 = 25.7\,\text{cm}$

2 Area $= \frac{3}{4} \times \pi \times 4^2 = 12\pi\,\text{cm}^2$

3 Area $= \frac{1}{2} \times \pi \times 3^2 = 14.1\,\text{m}^2$

$14.1 \div 2 = 7.05$, so 8 bags needed.

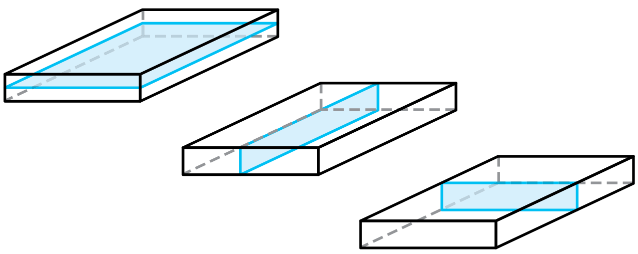
$8 \times 14.99 = £119.92$

## 3D shapes

**Stretch it!**

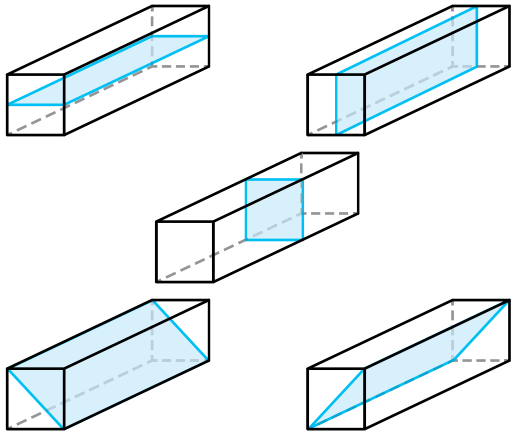
| 3D shape | Faces | Edges | Vertices |
|---|---|---|---|
| Cube | 6 | 12 | 8 |
| Cuboid | 6 | 12 | 8 |
| Square-based pyramid | 5 | 8 | 5 |
| Tetrahedron | 4 | 6 | 4 |
| Triangular prism | 5 | 9 | 6 |
| Hexagonal prism | 8 | 18 | 12 |

**Stretch it!** There are three planes of symmetry for the first cuboid:

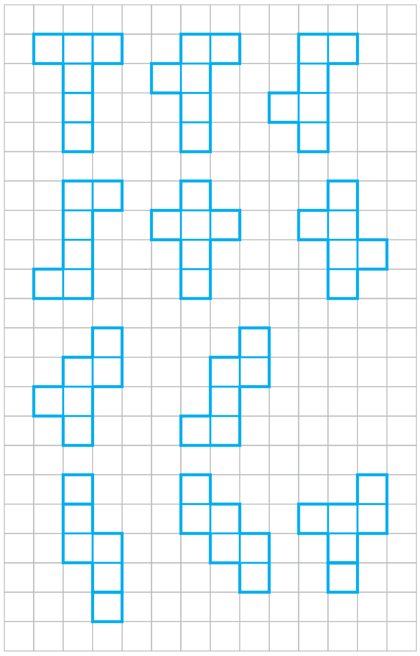


*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

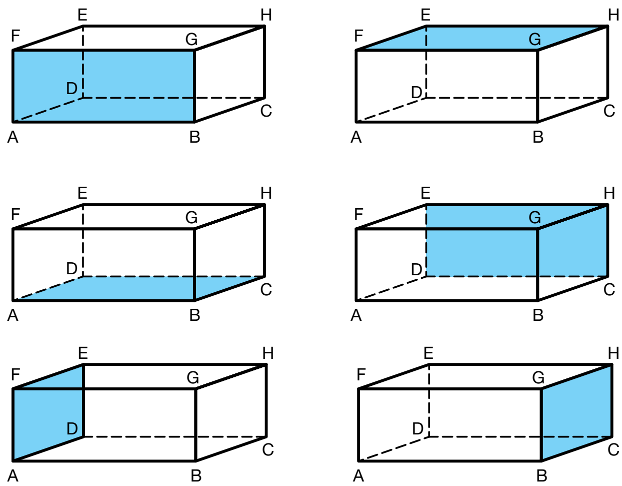
There are 5 planes of symmetry for the second cuboid: the same 3 planes as the first cuboid, plus two more planes along the diagonals of the square faces.



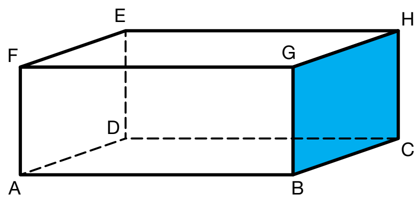
**Stretch it!***



1 a 6 possible rectangular faces:

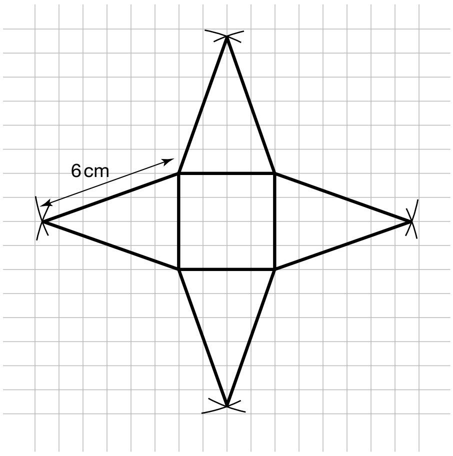



b

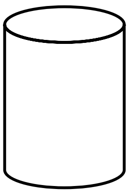


**c** Kelli has not counted the hidden edges.

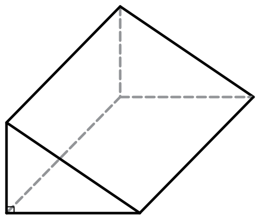
**2*** Draw a square in the middle with sides of 4 units (1 unit represents 1 cm). Set your compasses to 6 units and draw pairs of intersecting arcs from the corners of the square. These are the apices (top points) of the triangular sides. Draw lines for the sides of the triangles.



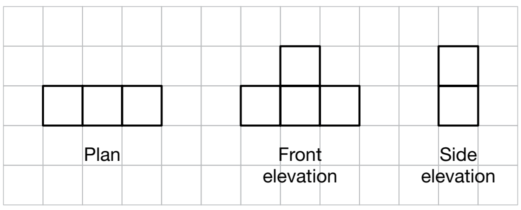

**3** **a**



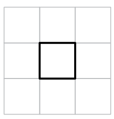
**b**



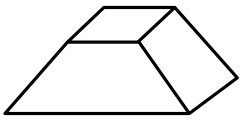
**4**




**5** **a**



**b**



## Volume

**1** $\frac{4}{3} \times \pi \times 4.5^3 = 381.7 = 382\,\text{cm}^3$ (to 3 s.f.)

**2** $\pi r^2 h + \frac{1}{3}\pi r^2 h = \pi \times 0.5^2 \times 2 + \frac{1}{3} \times \pi \times 0.5^2 \times 1.5$

$= 0.625\,\pi = 1.96\,\text{m}^3$

**3** $\frac{1}{3} \times \pi \times 6^2 \times 22 = \frac{1}{3} \times 792 \times \pi = 264\pi\,\text{cm}^3$

$k = 264$

**4** Volume of water $= 18 \times 7 \times 7 = 882\,\text{cm}^3$

$882 = 7 \times 20 \times h$

$882 = 140 \times h$

$h = 6.3\,\text{cm}$

## Surface area

**1** $6 \times (5 \times 5) = 150\,\text{cm}^2$

**2** $4\pi r^2 = 4 \times \pi \times 3^2 = 36\pi\,\text{cm}^2$

**3** $18 - 4 = 14\,\text{cm}^2$

**4** sloping surface $= \pi \times 14 \times 45 = 630\pi\,\text{cm}^2$

Base $= \pi \times 14^2 = 196\pi\,\text{cm}^2$

Total surface area $= 196\pi + 630\pi = 826\pi$

Percentage yellow $= \frac{630}{826} \times 100 = 76.3\%$

## Using Pythagoras' theorem

**1** Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$AC^2 = AB^2 + BC^2$

$15^2 = 11^2 + BC^2$

$BC^2 = 15^2 - 11^2 = 104$

$BC = \sqrt{104}$

$BC = 10.2\,\text{cm}$ (to 3 s.f.)

**2** $c^2 = a^2 + b^2$

$6^2 = 3.6^2 + b^2$

$b^2 = 6^2 - 3.6^2 = 23.04$

$b = \sqrt{23.04}$

$b = 4.8$

The ladder reaches 4.8 m up the wall.

**3** Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$XZ^2 = XY^2 + YZ^2$

$15^2 = XY^2 + 9^2$

$XY^2 = 15^2 - 9^2 = 144$

$XY = \sqrt{144}$

$XY = 12\,\text{cm}$

Area $= \frac{1}{2}bh = \frac{1}{2} \times 9 \times 12$

Area $= 54\,\text{cm}^2$

**4** If the triangle is right-angled, $PQ^2 = PR^2 + RQ^2$

$PQ^2 = 13^2 = 169$

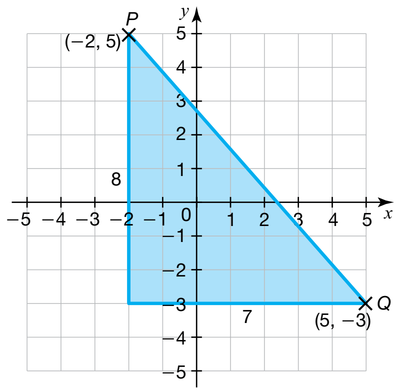
$PR^2 + RQ^2 = 8^2 + 5^2 = 64 + 25 = 89$*

$PQ^2 \neq PR^2 + RQ^2$

Claudia is not correct.

Notice that $PR + RQ = 8 + 5 = 13\,\text{cm}$ = length of PQ, so PQR isn't a triangle at all, it is just a straight line!

**5** P(−2, 5), Q(5, −3)



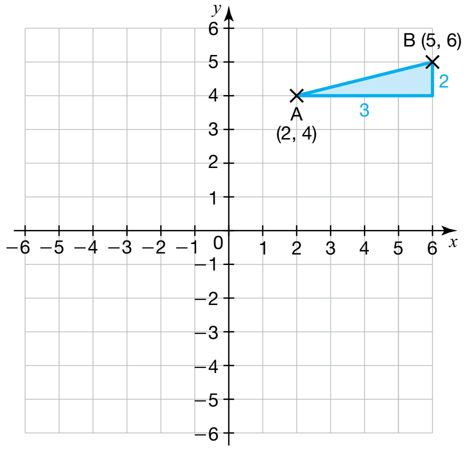

$PQ^2 = 8^2 + 7^2 = 113$

$PQ = \sqrt{113}$

$PQ = 10.63$ (to 2 d.p.)

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

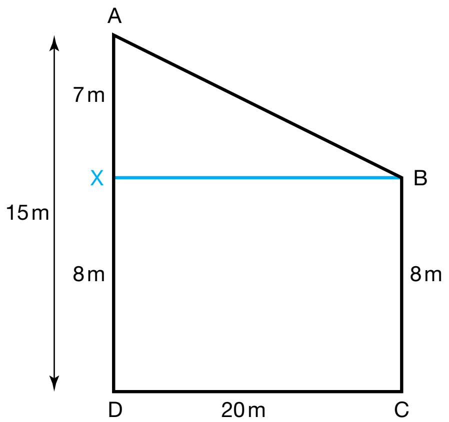
**6** A(2, 4), B(5, 6)




AB$^2$ = $2^2 + 3^2 = 13$

AB = $\sqrt{13}$

**7**




Using Pythagoras' theorem $c^2 = a^2 + b^2$:

AB$^2$ = AX$^2$ + BX$^2$

AB$^2$ = $7^2 + 20^2 = 449$

AB = $\sqrt{449}$

AB = 21.2 (to 3 s.f.)

Perimeter of field ABCD = 15 + 20 + 8 + 21.2
= 64.2 ≈ 65m

Cost of fencing = 65 × £14 = £910

## Trigonometry

**Stretch it!**

Opposite could have been 1 m, hypotenuse could have been 2 m. They could be any lengths that keep opposite and hypotenuse in the ratio 1:2.

**1** **a** 0.4 **b** 0.6 **c** 1.0
**d** 26.6 **e** 48.6 **f** 54.7

**2** Cos 72° = $\frac{\text{MN}}{15}$ MN = 15 cos 72° = 4.6 cm

**3** Tan ABC = $\frac{6}{7}$

ABC = $\tan^{-1}\left(\frac{6}{7}\right)$

ABC = 40.6°

**4** Let $x$ be the depth of water.

sin15° = $\frac{x}{10}$

$x$ = 10 sin 15°

$x$ = 2.6 m

## Exact trigonometric values

**1** **a** 0.5 **b** 0 **c** 0
**d** $\frac{1}{\sqrt{2}}$ **e** $\sqrt{3}$

**2** tan 45° = 1 = $\frac{\text{opposite}}{\text{adjacent}} = \frac{4}{\text{AC}}$

Therefore AC = 4 cm

cos 45° = $\frac{1}{\sqrt{2}} = \frac{4}{\text{BC}}$

BC = $4\sqrt{2}$

Therefore BC = $4\sqrt{2}$ cm

**3** Since: tan 30° = $\frac{1}{\sqrt{3}}$ one angle must be 30° and therefore the other is 60°

**4** sin 30° = $\frac{1}{2}$ therefore ABC = 30°

**5** cos 30° = $\frac{\sqrt{3}}{2}$ = 0.866 (3 d.p.) tan 45° = 1

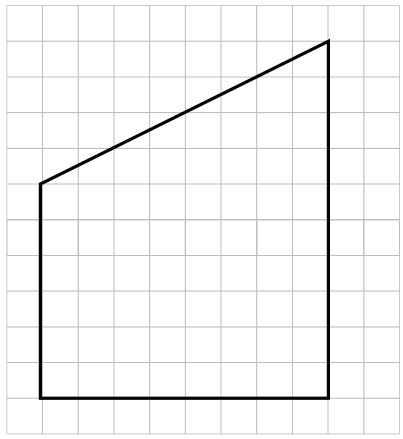
Smallest to largest = 0.5, $\frac{3}{4}$, cos 30°, tan 45°

## Transformations

**Stretch it!**

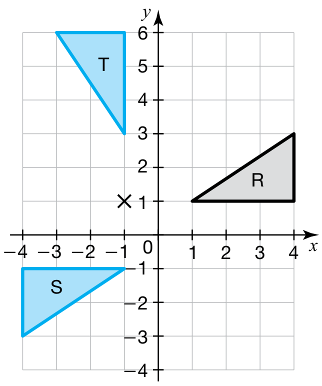
Yes. Reflection in the $x$ axis followed by reflection in the $y$ axis (or vice versa) will always produce a rotation of 180°.

**1**



**2** Translation by vector $\begin{pmatrix}-4\\-2\end{pmatrix}$

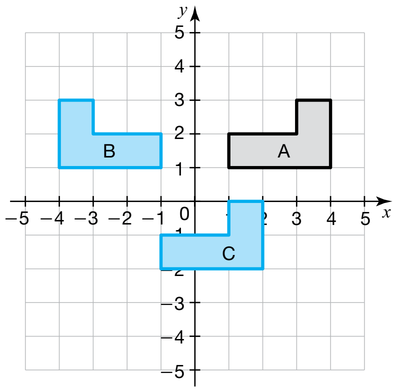
**3** **a** and **b**




**4** Reflection in the $y$-axis

**5** Enlargement by scale factor $\frac{1}{2}$, centre (3, 3)

**6 a** and **b**




**c** Rotation of 90° clockwise about (0, 0)

## Similar shapes

**Stretch it!**

Perimeter of ABC = 3 + 6 + 5 = 14 cm

Perimeter of DEF = 6 + 12 + 10 = 28 cm

The perimeter of a shape enlarged by scale factor 2 will also be enlarged by scale factor 2.

In general, all lengths on an enlarged shape, including the perimeter, are enlarged by the same scale factor.

**1 a** Angle DFE = 30° (Corresponding angles are the same)

**b** Scale factor of enlargement = $\frac{\text{enlarged length}}{\text{original length}} = \frac{12}{3} = 4$

Length of EF = 4 cm × 4 = 16 cm

**c** Length of AB = 8 cm ÷ 4 = 2 cm

**2 a** Angle MLO = 80° (Corresponding angles are the same: angle MLO = angle QPS)

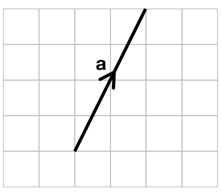
**b** Scale factor of enlargement = $\frac{\text{enlarged length}}{\text{original length}} = \frac{9}{3} = 3$

Length of QR = 4.4 cm × 3 = 13.2 cm

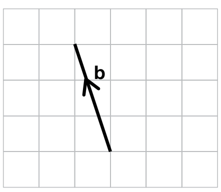
**c** Length of LO = 12 cm ÷ 3 = 4 cm

## Vectors

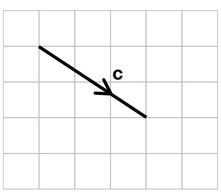
**1 a**



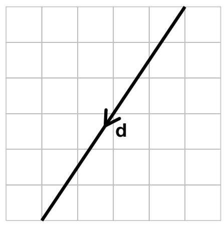

**b**



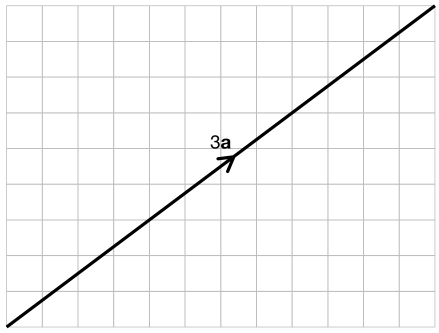

c



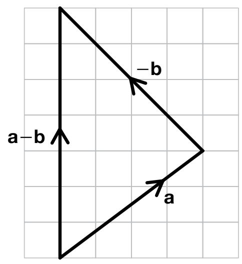

**d**




**2 a**




**b***




**c** $\mathbf{a} = \begin{pmatrix}4\\3\end{pmatrix}$

So $-2\mathbf{a} = -2 \times \begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}-8\\-6\end{pmatrix}$

**3 a** $\mathbf{a} + \mathbf{b} = \begin{pmatrix}2\\6\end{pmatrix} + \begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}-3\\9\end{pmatrix}$

**b** $\mathbf{a} + 2\mathbf{b} = \begin{pmatrix}2\\6\end{pmatrix} + 2 \times \begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix} + \begin{pmatrix}-10\\6\end{pmatrix} = \begin{pmatrix}-8\\12\end{pmatrix}$

**c** $\mathbf{a} - \mathbf{b} = \begin{pmatrix}2\\6\end{pmatrix} - \begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}7\\3\end{pmatrix}$

**d** $\mathbf{b} - 2\mathbf{a} = \begin{pmatrix}-5\\3\end{pmatrix} - 2 \times \begin{pmatrix}2\\6\end{pmatrix} = \begin{pmatrix}-5\\3\end{pmatrix} - \begin{pmatrix}4\\12\end{pmatrix} = \begin{pmatrix}-9\\-9\end{pmatrix}$

**4** No, only Jordan is correct.

$\mathbf{a} = \begin{pmatrix}2\\3\end{pmatrix}$, $\mathbf{d} = \begin{pmatrix}20\\30\end{pmatrix}$

Vectors **a** and **d** are parallel since $10\mathbf{a} = 10 \times \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}20\\30\end{pmatrix}$

So Jordan is correct.

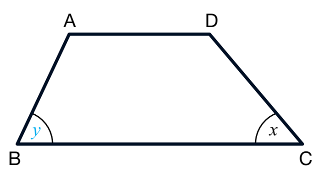
$\mathbf{a} = \begin{pmatrix}2\\3\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}4\\-6\end{pmatrix}$

Vectors a and b are not parallel as $-2\mathbf{a} = \begin{pmatrix}-4\\-6\end{pmatrix}$

So Jenna is not correct.

## Review it

**1 a**



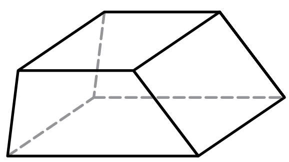

**b** BC = 3.8 cm

**c** $x = 50°$

**2 a** 5 faces

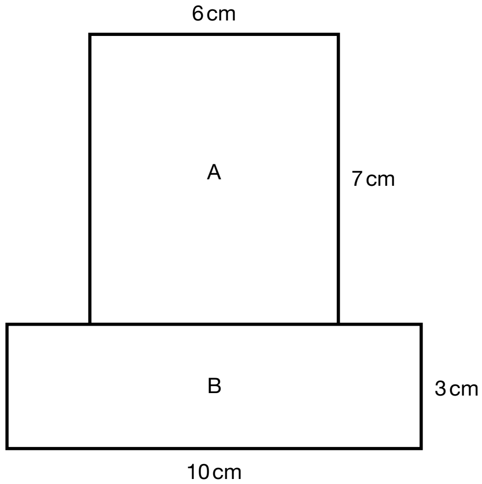
**b** 6 vertices

**3**



*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

**4** Area A $= 7 \times 6 = 42\text{ cm}^2$




Area B $= 10 \times 3 = 30\text{ cm}^2$

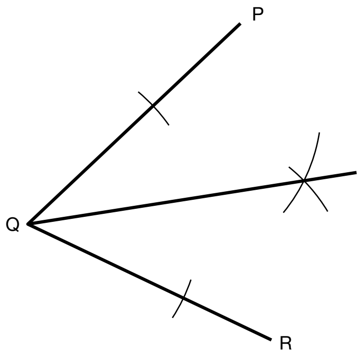
Total area $= 42 + 30 = 72\text{ cm}^2$

**5** Area of parallelogram $= 3 \times 12 = 36\text{ cm}^2$

Length of side of square $= \sqrt{36} = 6\text{ cm}$

Perimeter of square $= 4 \times 6 = 24\text{ cm}$

**6**




**7** Rotation of 180° about (1, 0)

**8** Angle CFE = 112° (Corresponding angles are equal)

Angle CFG = 180 − 112 = 68° (Angles on a straight line add up to 180°)

Angle GCF = angle CFG (Base angles of an isosceles triangle are equal)

$x = (180 - 68 - 68) = 44°$ (Angles in a triangle add up to 180°)

**9** Shaded Area $= (10 \times 12) - ((\frac{1}{2} \times 12 \times 3) + (\frac{1}{2} \times 8 \times 7) + (\frac{1}{2} \times 10 \times 4))$

$= 120 - (18 + 28 + 20)$

$= 54\text{ cm}^2$

Proportion $= \frac{54}{120} = \frac{9}{20} = 45\%$

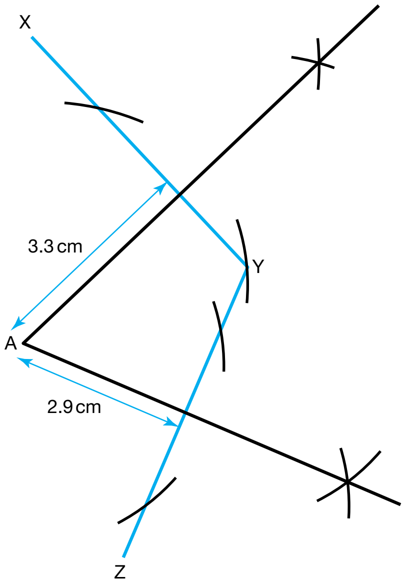
**10** If triangle ABC is right-angled, $c^2 = a^2 + b^2$

$c^2 = 8^2 = 64$

$a^2 + b^2 = 4^2 + 6^2 = 16 + 36 = 52$

$c^2 \neq a^2 + b^2$ so triangle ABC is not right-angled.

**11 a**




Scale is 1 : 200

**b** Distance from A to YZ = 2.9 cm

$2.9 \times 200 = 5800\text{ cm} = 5.8\text{ m}$

Distance from A to YX = 3.3 cm

$3.3 \times 200 = 6600\text{ cm} = 6.6\text{ m}$

Difference in distance = 6.6 − 5.8 = 0.8* m

**12 a** $\cos 45° = \frac{1}{\sqrt{2}}$

**b** Ratio of adjacent to hypotenuse is 1:2

Therefore AB = 3 cm

**13 a** $\mathbf{a} + 2\mathbf{b} = \begin{pmatrix} 4 \\ -5 \end{pmatrix} + 2 \times \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -5 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

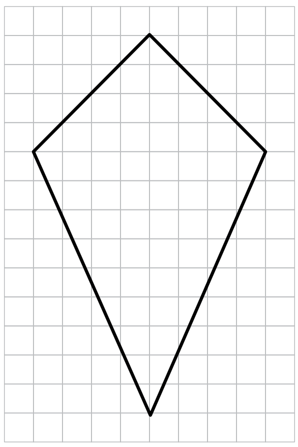
$= \begin{pmatrix} 8 \\ 1 \end{pmatrix}$

**b** $\mathbf{b} - 2\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2 \times \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

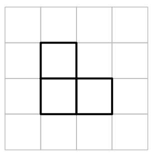
$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 8 \\ -10 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 13 \end{pmatrix}$

**14**



**15**



*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

**16 a** **i** 35°

**ii** Triangle WYZ is isosceles, and base angles of an isosceles triangle are equal.

**b** Angles in a triangle add up to 180° so:

$b = 180 - 35 - 35$

$= 110°$

**c** Triangle XYZ is isosceles, and base angles of an isosceles triangle are equal so:

$c = (180 - 70) \div 2 = 55°$

**17** Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$AC^2 = AB^2 + BC^2$

$14^2 = 6^2 + BC^2$

$BC^2 = 14^2 - 6^2 = 160$

$BC = \sqrt{160}$

BC = 12.6 cm (to 1 d.p.)

**18** Interior angle of a square = 90°

Sum of interior angles of an octagon (with $n = 8$)
$= 180 \times (n - 2) = 180 \times (8 - 2) = 1080°$

Interior angle of a regular octagon = 1080° ÷ 8 = 135°

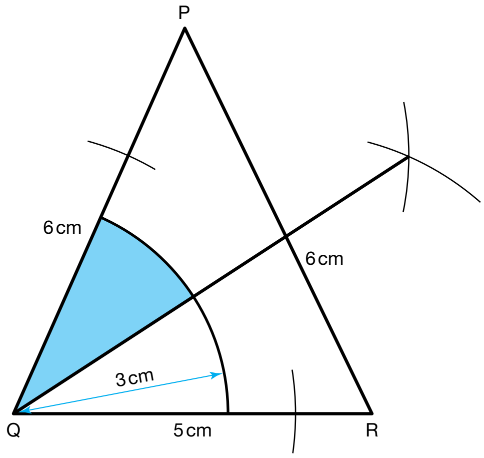
(Or, exterior angle of a regular octagon = 360° ÷ 8 = 45°. Then interior angle = 180° − 45° = 135°)

Angles around a point add up to 360° so:

$x = 360 - 90 - 135$

$x = 135°$

**19**




**20** Divide the trapezium into a rectangle and a triangle. Draw a line DX parallel to AB, with X on the line BC. BX = 5 cm, CX = 7 cm.
Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$DC^2 = CX^2 + DX^2$

$DC^2 = 7^2 + 4^2 = 65$

$DC = \sqrt{65}$

DC = 8.06 (to 2 d.p.)

Perimeter of ABCD = 4 + 5 + 8.06 + 12 = 29.06 cm

**21** Length of arc $= \frac{1}{4} \times 2 \times \pi \times 4 = 2\pi$

Perimeter $= 4 + (2 \times 9) + 4 + 2\pi = 32.3$ cm

**22** Area of square = 6 × 6 = 36 cm²

Area of circle $= \pi \times 3^2 = 9\pi\,\text{cm}^2$

Shaded area $= 36 - 9\pi = 7.7\,\text{cm}^2$

**23** Volume of cylinder $= \pi \times 3^2 \times 15 = 135\pi\,\text{cm}^3$

2 litres = 2000 ml = 2000 cm³

$2000 \div 135\pi = 4.7$

Glass can be completely filled 4 times.

**24** Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$PR^2 = PQ^2 + RQ^2$

$PR^2 = 10^2 + 6^2 = 136$

$PR^2 = PS^2 + SR^2$

$136 = 11^2 + x^2$

$x^2 = 136 - 11^2 = 136 - 121 = 15$

$x = \sqrt{15}$

$x = 3.87$ cm (to 3 s.f.)

**25 a** Curved surface area $= \pi \times 6 \times 10 = 60\pi\,\text{cm}^2$

Base area $= \pi \times 6^2 = 36\pi\,\text{cm}^2$

Total surface area $= 60\pi + 36\pi = 96\pi = 300\,\text{cm}^2$ to 2 s.f.

**b** Volume $= \frac{1}{3} \times \pi \times 6^2 \times 8 = 96\pi = 300\,\text{cm}^3$

**26** $\tan x = \frac{8}{6}$

$x = \tan^{-1}\left(\frac{8}{6}\right)$

$x = 53.1°$

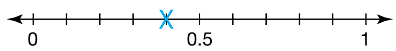
**27** Translation by vector $\begin{pmatrix} -7 \\ -6 \end{pmatrix}$

# Probability

## Basic probability

**Stretch it!** No – each time the probability of getting an even number is $\frac{1}{2}$. You would expect to get even numbers approximately 50 times but cannot guarantee it.

**1** $\frac{4}{10}$




**2 a** Total number of sweets = 12 + 3 + 10 = 25

$\frac{3}{25}$

**b** $\frac{(3+10)}{25} = \frac{13}{25}$

**3** Pair **a**, because when you flip a coin, you can't get both a head and a tail at the same time. (Prime numbers on a dice are 2, 3, 5 and odd numbers are 1, 3, 5, so events **b** are **not** mutually exclusive because 3 is in both groups.)

**4** Pair **b**, because the first sweet chosen is replaced, so the possible outcomes of the second choice remain the same. (If the first sweet chosen is eaten, the possible outcomes of the second choice are altered, and so events **a** are **not** independent.)

**5** P(6) = 1 − (0.1 + 0.15 + 0.1 + 0.02 + 0.2)
= 1 − 0.57 = 0.43

**6** P(green or red) = 1 − 0.4 = 0.6

P(green) = 2 × P(red)

P(red) $= \frac{0.6}{3} = 0.2$

P(green) = 2 × 0.2 = 0.4

## Two-way tables and sample space diagrams

**1**

| | Chicken | Beef | Vegetarian |
|---|---|---|---|
| Fruit | 12 | 6 | 4 |
| Cake | 5 | 3 | 8 |
| Total | 17 | 9 | 12 |

**a** 12 (this is worked out by using the numbers in the 'Total' row, which must add up to 38)

**b** As shown in the table.

**2 a**

| | | Dice 1 | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Dice 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

**b i** $\frac{2}{36} = \frac{1}{18}$

**ii** $\frac{3}{36} = \frac{1}{12}$

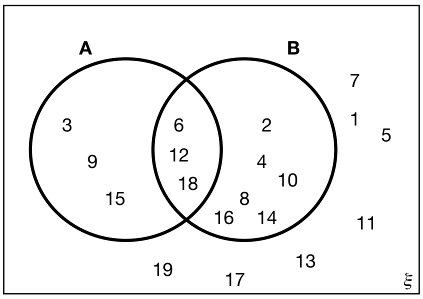
**iii** 0

**3** To score 6, the player must pick two cards showing 3. To score 2, the player must pick two cards showing 1. Since the probability of getting 3 and 3 is more than 0, and the probability of getting 1 and 1 is more than 0, there must be at least 2 of each of those numbers. So the cards must be 1, 1, 3, 3.

## Sets and Venn diagrams

**Stretch it!** None

**1 a**




**b** A ∩ B = {multiples of 6 less than 20} because these numbers are multiples of both 2 and 3.

**2 a** C ∩ T is the set of students who travel by car AND train

C' ∩ B is the set of students who do NOT travel by car AND travel by bus.*

**b i** $P(C) = \frac{(14 + 11 + 11 + 2)}{(14 + 11 + 11 + 2 + 17 + 19 + 26)} = \frac{38}{100} = \frac{19}{50}$

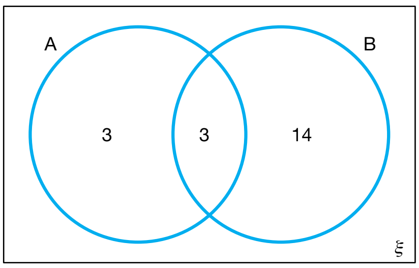
**ii** $P(B \cup T) = \frac{(19 + 11 + 2 + 0 + 11 + 17)}{100} = \frac{60}{100} = \frac{3}{5}$

**iii** $P(B' \cap T) = \frac{(11 + 17)}{100} = \frac{28}{100} = \frac{7}{25}$

**3** $P(A \cap B) = \frac{3}{20}$ so there must be 3 elements in the intersection.

$P(A) = \frac{3}{10} = \frac{6}{20}$ so there must be a total of 6 elements in A.

The total number of elements must sum to 20.




## Frequency trees and tree diagrams

**1 a** Apple = 123

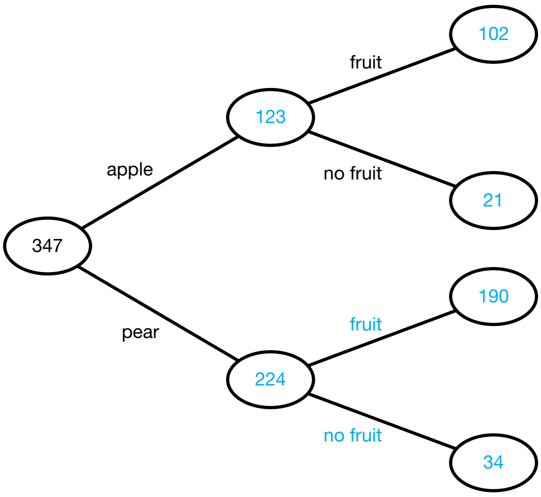
Pear = 347 − 123 = 224

Apple fruiting = 102

Apple not fruiting = 123 − 102 = 21

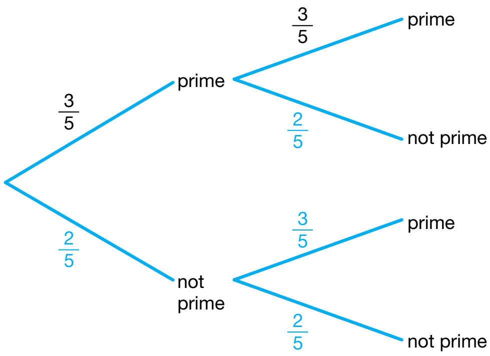
Pear not fruiting = 34

Pear fruiting = 224 − 34 = 190




**b** $\frac{190}{347}$

**2 a**




**b** $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

**c** $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

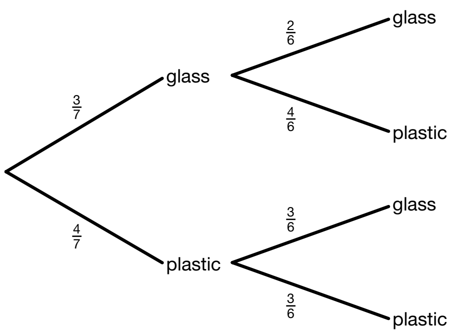
$\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

$\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

P(at least one prime) = 1 − P(no primes)

$= 1 - \frac{2}{5} \times \frac{2}{5} = 1 - \frac{4}{25} = \frac{21}{25}$

**3 a**




**b** P(two glass marbles) $= \frac{3}{7} \times \frac{2}{6} = \frac{6}{42}$

P(glass then plastic) $= \frac{3}{7} \times \frac{4}{6} = \frac{12}{42}$

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

P(plastic then glass) $= \frac{4}{7} \times \frac{3}{6} = \frac{12}{42}$

P(at least one glass) $= 1 -$ P(both plastic)

$= 1 - \frac{4}{7} \times \frac{3}{6} = 1 - \frac{12}{42}$

$= 1 - \frac{2}{7} = \frac{5}{7}$

## Expected outcomes and experimental probability

**Stretch it!** The dice has not been rolled enough times to decide if it is biased. More tests need to be carried out.

**1** $0.45 \times 300 = 135$

**2** Red $= \frac{2}{10} = \frac{1}{5}$

$\frac{1}{5} \times 100 = 20$ red sweets

**3** $\frac{1}{2} \times 100 = 50$ primes

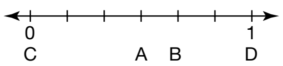
**4** **a** Charlie – he has carried out the most tests.

**b** $\frac{(112 + 10 + 28)}{(112 + 10 + 28 + 74 + 7 + 19)} \times 10 = 6$

## Review it!

**1** $0.12 \times 250 = 30$

**2**




**3** $1 - 0.3 = 0.7$

**4** B, C

**5** **a** $\frac{3}{5}$

**b** $(\frac{1}{5}) \times 25 = 5$

**6**

| | Pizza | Pasta | Risotto | Total |
|---|---|---|---|---|
| Cake | 12 | 6 | 1 | 19 |
| Ice Cream | 10 | 11 | 10 | 31 |
| Total | 22 | 17 | 11 | 50 |

**7** $0.2 + 5x + 0.2 + x = 1$

$6x + 0.4 = 1$

$x = 0.1$

P(white) $= 5x + 0.2 = 0.7$

**8** **a** No, he has not tested his dice enough times.

**b** P(2) $= \frac{9}{(12 + 9 + 16 + 7 + 6 + 0)} = \frac{9}{50}$

$\frac{9}{50} \times 100 = 18$

**9** P(R, R) $= 0.1 \times 0.5 = 0.05$

P(R, G) $= 0.1 \times 0.5 = 0.05$

P(G, R) $= 0.9 \times 0.5 = 0.45$

$0.05 + 0.05 + 0.45 = 0.55$

**Or** P(at least one red) $= 1 -$ P(green, green)

$= 1 - (0.9 \times 0.5)$

$= 1 - 0.45$

$= 0.55$

**10** **a**

| | | Dice | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Coin | Heads | 2 | 4 | 6 | 8 | 10 | 12 |
| | Tails | 3 | 4 | 5 | 6 | 7 | 8 |

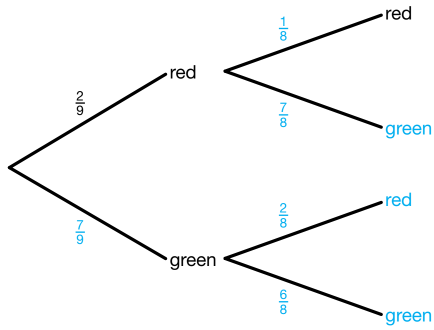
**b** **i** $\frac{2}{12} = \frac{1}{6}$

**ii** $\frac{2}{12} = \frac{1}{6}$

**11** **a** **i** 6 **ii** 1 **iii** 5

**b** $\frac{4}{8} = \frac{1}{2}$

**12** **a**




**b** $\frac{7}{9} \times \frac{6}{8} = \frac{42}{72} = \frac{7}{12}$

**13** **a** Possible fractions: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{2}{4}, \frac{3}{4}$

Less than $\frac{1}{2}$ are $\frac{1}{3}$ and $\frac{1}{4}$.

P(less than $\frac{1}{2}$) $= \frac{2}{6} = \frac{1}{3}$

$\frac{1}{3} \times 30 = 10$

**b** $\frac{1}{3}$ is only a theoretical probability and therefore will not necessarily be accurate in real life.

**14** 45% of $300 = 135$

135 boys and 165 girls.

$\frac{2}{3}$ of $135 = 90$

$\frac{4}{5}$ of $165 = 132$

Total playing sport $= 222$

Probability $= \frac{222}{300} = \frac{37}{50} = 0.74$

**15** P(hooking a winning duck) $= \frac{5}{20} = 0.25$

If 100 people play, expected number of winners $= 0.25 \times 100 = 25$ people.

The game makes £1 × 100 people = £100.

The money paid out in prizes = 25 winners × £2 = £50

Profit = £100 – £50 = £50

**16** **a** Milo will have the better estimate as he has surveyed a greater number of people.

**b** Number of left-handed students $= 5 + 4 + 7 + 7$

$= 23$

Number of right-handed students $= 23 + 18 + 51 + 60$

$= 152$

P(left-handed) $= \frac{23}{23 + 152} = \frac{23}{175}$

$\frac{23}{175} \times 2000 = 262.8$

You would expect to find 263 left-handed students in a school with 2000 students.

# Statistics

## Data and sampling

**Stretch it!** A random sample could be taken; you could allocate a number to each pupil and randomly generate the numbers to survey. Any method is acceptable as long as each person in the school has an equally likely chance of being chosen. Alternatively a stratified sample could be taken.

1 Primary source: Recording the data by measuring it yourself.

Secondary source: Any sensible source, e.g. the Meteorological Office, local paper etc.

2 Qualitative data.

3 It is cheaper and quicker than surveying the whole population.

4 a The people working for an animal charity are more likely to be opposed to wearing real fur; every member of the population does not have an equal chance of being chosen.

b Surveying people in the street, a random telephone survey, any sensible method that ensures that any member of the population has an equal chance of being chosen.

5 a $\frac{400}{2000} = \frac{1}{5}$

b $\frac{1}{5} \times 50 = 10$ bottles

6 a $\frac{3}{200} \times 800\,000 = 12\,000$

b The sample is relatively small, the sample is not a random sample as it is taken on one day in a year.

## Frequency tables

1

| Number of people on the bus | Frequency |
|---|---|
| 0*–9 | 4 |
| 10–19 | 12 |
| 20–29 | 3 |
| 30–39 | 1 |

2 a

| Number of courgettes | Frequency |
|---|---|
| 0 | 1 |
| 1 | 0 |
| 2 | 1 |
| 3 | 1 |
| 4 | 9 |
| 5 | 3 |
| 6 | 0 |

b $(0 \times 1) + (1 \times 0) + (2 \times 1) + (3 \times 1) + (4 \times 9) + (5 \times 3) = 56$

3 There are gaps between his groups – where would he record someone who spent 15.5 hours training?

His groups do not have the same width.

## Bar charts and pictograms

1 a $15 + 4 + 1 = 20$

b $4 + 1 = 5$

$\frac{5}{20} \times 100 = 25\%$

2 a $11 - 7 = 4$

b Total number of people surveyed $= 18 + 18 + 12 + 3 = 51$

Total number of boys $= 11 + 6 + 3 = 20$

$\frac{20}{51} \times 100 = 39.2\%$

c Proportion of boys who played two sports $= \frac{6}{18} = \frac{1}{3}$

Proportion of boys who played three sports $= \frac{3}{12} = \frac{1}{4}$

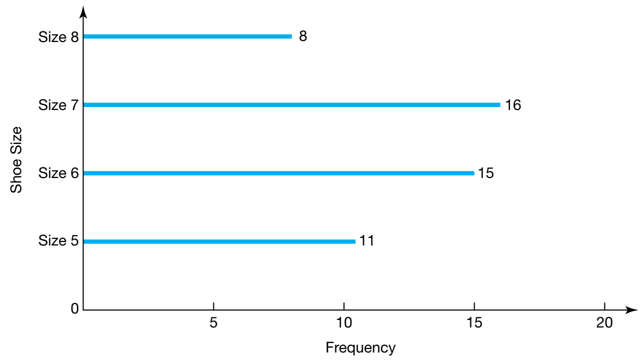
$\frac{1}{3} > \frac{1}{4}$ so Jasmine is incorrect, and you can't prove that what she says is true.*

3 $50 - (11 + 15) = 24$

$24 \div 3 = 8$

Therefore: $2 \times 8 = 16$ size 7 shoes

$1 \times 8 = 8$ size 8 shoes




4 a $90 - 20 = 70$

b Total number of bikes $= 50 + 50 + 20 + 90 = 210$

$\frac{50}{210} = \frac{5}{21}$

## Pie charts

**Stretch it!**

Round appropriately – but check the angles sum to 360°

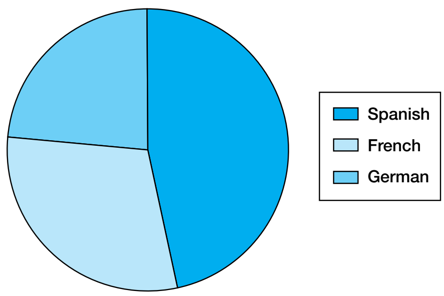
1 $27 + 42 + 21 = 90$

$360° \div 90 = 4°$

French $= 27 \times 4 = 108°$

Spanish $= 42 \times 4 = 168°$

German $= 21 \times 4 = 84°$




2 a $\frac{1.5}{360} \times 240 = 1$ student earned more than £40 000.

b $\frac{288 + 63}{360} \times 100 = 97.5\%$ of students earned less than £30 000.

3 a $18 + 10 = 28$

b The bar chart, since the frequency is easy to read from the bar chart.

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

## Stem and leaf diagrams

**1** **a** 7

**b** 4.3 − 4.1 = 0.2 kg

**2** Age of people using a dentist

| | |
|---|---|
| 2 | 0 0 0 0 1 1 1 |
| 3 | 2 5 5 7 |
| 4 | 1 2 2 6 |

The leaves were not in ascending order, the spaces between leaves were not regular.

**3** **a** Stem and leaf diagram – you can see the smallest number of passengers was 3; however, on the bar chart you only know it is between 0 and 9.

**b** Both since the shape of the data is preserved in both.

## Measures of central tendency: mode

**1** The other three must be 12.2.

**2** $1 < t \leq 2$

**3** 17

## Measures of central tendency: median

**1** Ordering the data gives; 2.9, 3.1, 4.3, 6.5, 8.7, 9.2

Median $= \frac{4.3 + 6.5}{2} = 5.4$

**2** $29 + 28 + 30 + 3 + 10 = 100$

$\frac{(100 + 1)}{2} = 50.5$ – median term is between the 50th and 51st terms.

Both these lie in the $2 \leq b < 4$ class.

**3** **a** Group A $= \frac{(82 + 85)}{2} = 83.5$

Group B $= \frac{(75+79)}{2} = 77$

**b** Group A has a higher median, so they did better on the test.

## Measures of central tendency: mean

**Stretch it!** **a** mode **b** mean/median **c** mean/median

**1** **a** Total frequency = 12 + 3 + 5 = 20

Mean $= \frac{(2 \times 12) + (6 \times 3) + (10 \times 5)}{20} = 4.6$

**b** You are using the midpoint of the groups as an estimate of the actual value for each group.

**2** $\frac{(5 \times 9) + 6}{5 + 1} = 8.5$

**3** No – they could be any pair of numbers which sum to 10.

## Range

**1** 9.5 − 0.7 = 8.8

**2** Girls = 18 − 15 = 3

Boys = 18 − 16 = 2

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

**3** Range for Athlete A = 15.2 − 13.0 = 2.2

Range for Athlete B = 15.2 − 14.3 = 0.9

Athlete A has the greatest range.

**4** 45% − 10 = 35% or 45% + 30 = 75%

## Comparing data using measures of central tendency and range

**1** **a** **i** Mean $= \frac{(32 + 29 + 18 + 41 + 362 + 19)}{6} = \frac{501}{6}$

$= 83.5$ minutes

**ii** Ordered data: 18, 19, 29, 32, 41, 362

Median $= \frac{(29+32)}{2} = 30.5$ minutes

**b** The extreme value (362 mins) affects the mean but not the median.

**2** All the data is used to find the mean.

**3** Either as long as suitably justified:

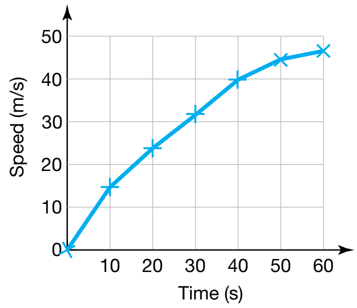
Car A – although the mean time is higher, it is more consistent in performance since the range is smaller.

Car B – the acceleration is quicker on average.

**4** **a** and **b** The mode or median since the mean will not be a whole number and therefore not meaningful.

## Time series graphs

**1**




**2** **a** 67°C

**b** Approx. 27°C

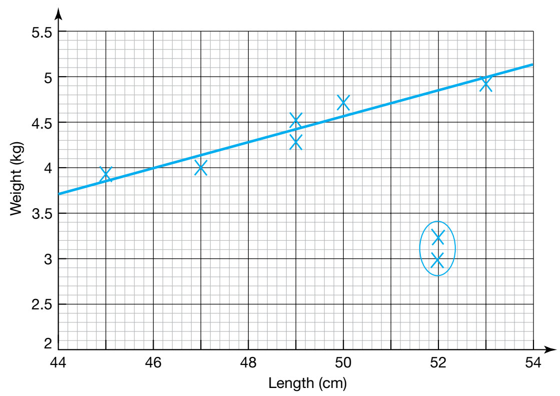
**c** No, since it is extrapolation (beyond the limits of the data).

**3** **a** 17 000 **b** **i** April **ii** August

**c** The number of tourists peaks in April and again in December. The low seasons are February/March and July/August/September/October.

## Scatter graphs

**1** **a** and **b***

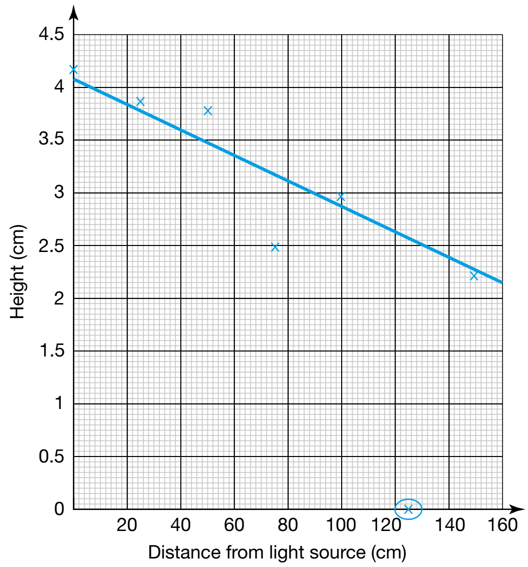



**c** Positive

**d** This will vary according to the line of best fit: approximately 4.7 kg. A range of 4.6kg to 4.8kg would be acceptable.

**e** This is beyond the limits of the data and therefore extrapolation.

**2** **a** and **b**




**c** The seeds failed to germinate or the seedling died.

**d** The further the seedling is from the light source the shorter its height.

**3** No, although the two things correlate one does not cause another. There may be many reasons why the crime rate is high in the area, perhaps there is poverty and inequality causing social tension.

## Review it!

**1** The sample is too small and he only asked his friends. His data is therefore not representative of the population of TV viewers.

**2** **a** Margherita

**b** Total frequency = 11 + 2 + 6 + 1 = 20

$\frac{1}{20} = \frac{5}{100} = 5\%$

**c** 360° ÷ 20 = 18°

Pepperoni = 1 × 18° = 18°

(or 5% of 360° = 18°)

**3** **a** $\frac{90}{360} = \frac{1}{4}$

**b** $45° = \frac{1}{8}$ of 360°

Therefore $\frac{1}{8}$ of the pie chart represents 60 cars.

The whole pie chart = 8 × 60 = 480 cars

**c** $\left(\frac{105}{360}\right) \times 480 = 140$ cars

**4** **a** The number of people doing their grocery shopping online is increasing.

**b** Any sensible answer, approximately 75%

**c** No – it is outside the limits of the data therefore extrapolation.

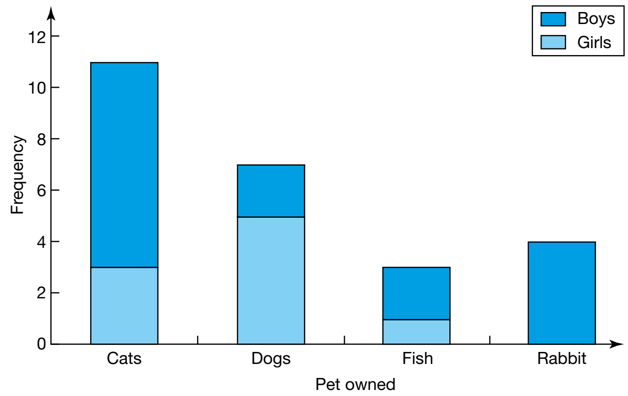
**5** **a** Outside: **i** Mode = 21 and 31 **ii** Median = $\frac{(28 + 29)}{2}$ = 28.5 **iii** Range = 41 – 20 = 21

Greenhouse: **i** Mode = 47 **ii** Median = $\frac{(47 + 47)}{2}$ = 47 **iii** Range = 51 – 37 = 14

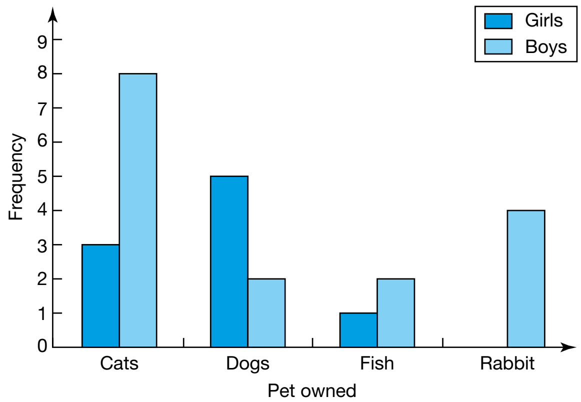
**b** The seedlings are taller in the greenhouse since both mode and median is larger, the range of data is smaller in the greenhouse so the height the seedlings reach is more consistent.

**c** Range = 51 – 20 = 31

**6** **a** Comparative bar chart or compound bar chart:




**Or**:




**b** Total number of students = (3 + 5 + 1 + 0 + 8 + 2 + 2 + 4) = 25

Number of cats = 3 + 8 = 11

$\frac{11}{25}$

**7** **a** Total frequency = 17 + 2 + 32 + 23 + 9 = 83

Median value = $\frac{(83 + 1)}{2}$ = 42nd term

42nd term is in group 40–59

Median class = 40–59

**b** The youngest person is between 0 and 19, the youngest may be any age in this range and the oldest is between 80 and 99 therefore any age in this range.

**8** **a** 7

**b** Size 5

**c** Mean = $\frac{(3 \times 2) + (4 \times 1) + (5 \times 7) + (6 \times 5) + (7 \times 3)}{2 + 1 + 7 + 5 + 3} = 5.\dot{3}$

**d** Mode – the mean is not an actual shoe size.

**9** **a** Time for 800 m (seconds)

| | |
|---|---|
| 11 | 2 2 5 8 9 |
| 12 | 0 1 9 |
| 13 | 1 2 |

Key: 11|2 = 112 seconds

**b** $\frac{1}{2}$ *

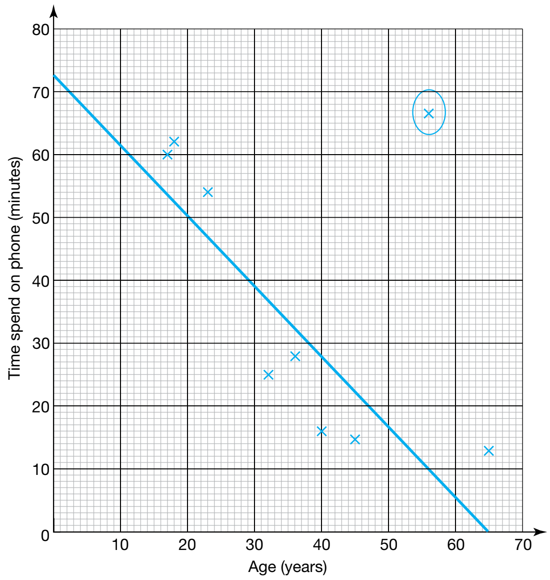
*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

**10 a** $\frac{50}{150} = \frac{1}{3}$

**b** $60 - 40 = 20$

**c** Biology

**11 a*** and **c**




**b** Negative

**d** Approximately 40 minutes: it depends on line of best fit.

**e** This is outside the limits of the data and therefore extrapolation.

**f** As the age of the customer increases the time spent on the phone decreases.

**12 a** $\frac{(65 \times 3) + (75 \times 5) + (85 \times 2)}{3 + 5 + 2} = 74\,\text{kg}$

**b** The midpoint of the class is used as the age of each of the patients rather than the actual age.

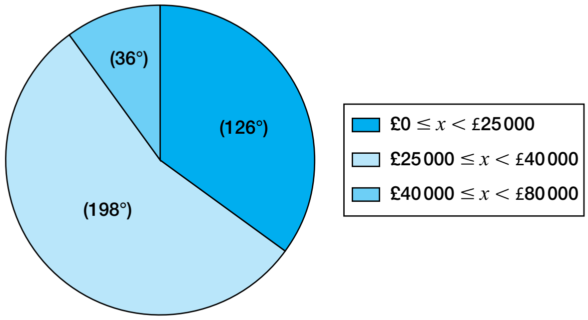
**13** Male $< 50 = \left(\frac{12\,201}{36579}\right) \times 120 = 40$

Female $< 50 = \left(\frac{10678}{36579}\right) \times 120 = 35$

Male $\geq 50 = \left(\frac{5699}{36579}\right) \times 120 = 19$

Female $\geq 50 = \left(\frac{8001}{36579}\right) \times 120 = 26$

**14** Annual income for surveyed population




**15** Mean $= \frac{(10.3 \times 10) + 9.5}{11} = 10.2$ (1 d.p.)

**16** Mean is 3.8 so the sum of the scores is $3.8 \times 5 = 19$

Mode is 3 so she must roll at least two 3s.

Range is 4.

If the range is 4 then the lowest and highest must be either 1 and 5 or 2 and 6.

The numbers are: 2, 3, 3, 5 and 6



*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.