Practice paper 1 (non-calculator)

1 $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ The term-to-term rule is: multiply by $\frac{1}{2}$, therefore there is a constant ratio between each number and the one before, so this sequence is geometric.

2 The triangles are identical.

3 $2^2 \times 5 \times 13$

4 $y^2 = 25 - x^2$, because it can be written in the form $x^2 + y^2 = r^2$

5 $3\frac{1}{3} \times 1\frac{3}{8} = \frac{10}{3} \times \frac{11}{8}$

$= \frac{55}{12}$

$= 4\frac{7}{12}$

6 $2^3 : 1^3 = 8 : 1$

7

n	1	2	3	4
$2n + 1$	3	5	7	9
$n^2 + 1$	2	2	10	17

5 is in both sequences.

Alternative method:

Let the particular value for n that is true for both sequences $= a$.

$2a + 1 = a^2 + 1$

$2a = a^2$

$a = 2$

$2a + 1 = 5$

$a^2 + 1 = 5$

So 5 is the value in both sequences.

8 $5^4 \times 5^{-1} \times 5 = 5^4 \times \frac{1}{5} \times 5 = 5^4 \times \frac{5}{5}$

$= 5^4$

9 Area = length × width

$= 30 \times 20$

$= 600\,\text{cm}^2 = 0.06\ \text{m}^2$

10 a area of triangle $= \frac{1}{2} \times 3x \times 2x = 3x^2$

area of rectangle $= 4xy$

As both areas are equal $3x^2 = 4xy$

$3x^2 - 4xy = 0$

$x(3x - 4y) = 0$

b Any pairs of values such that $3x - 4y = 0$, for example $x = 4$, $y = 3$ and $x = 8$, $y = 6$

11 P(green) $= \frac{x - 3}{x + x - 3 + 2x} = \frac{x - 3}{4x - 3}$

$\frac{x - 3}{4x - 3} = \frac{1}{5}$

$5(x - 3) = 4x - 3$

$5x - 15 = 4x - 3$

$x = 12$

total number of counters $= 4x - 3 = 4 \times 12 - 3 = 45$

number of blue counters $= 2x = 24$

P(blue) $= \frac{24}{45} = \frac{8}{15}$

12 $3.6 \times 10^4 = 36\,000$ and $3.6 \times 10^2 = 360$ hence $y = 3.6$

So $y \times 10^4 - y \times 10^2 = 3.6 \times 10^4 - 3.6 \times 10^2$

$= 36\,000 - 360$

$= 35\,640$

$= 3.564 \times 10^4$

13 a LHS $= x^2 + y^2 = 4^2 + 3^2 = 25$

LHS = RHS so point satisfies the equation, so the point (4, 3) lies on the circle.

b gradient of $OP = \frac{3}{4}$

gradient of tangent $= -\frac{4}{3}$

Equation of the tangent is $y - 3 = -\frac{4}{3}(x - 4)$

$3(y - 3) = -4(x - 4)$

$3y - 9 = -4x + 16$

$3y + 4x = 25$

or $y = -\frac{4}{3}x + \frac{25}{3}$

14 $\frac{8}{3\sqrt{2}} = \frac{8}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{8\sqrt{2}}{3 \times 2} = \frac{8\sqrt{2}}{6}$

$= \frac{4\sqrt{2}}{3}$

15 a $(2x^2y)^3 = 2^3 \times x^{2\times 3} \times y^{1\times 3} = 8x^6y^3$

b $2x^{-3} \times 3x^4 = 2 \times 3 \times x^{-3+4} = 6x$

c $\frac{15a^3b}{3a^3b^2} = 15 \div 3 \times a^{3-3} \times b^{1-2} = 5b^{-1} = \frac{5}{b}$

16 a True; x is always smaller than y, so $\frac{x}{y} < 1$.

b True; a value of $x = -1$ would give $x^3 = -1$, but the question says that x is a positive integer so x can never be -1.

c False; x is always smaller than y, so $x - y < 0$.

d False; since $y > x$, y^2 can never be equal to x^2.

17 Let $x = 0.1\dot{3}\dot{6} = 0.136363636\ldots$

$10x = 1.36363636\ldots$

$1000x = 136.363636\ldots$

$1000x - 10x = 136.363636\ldots - 1.36363636\ldots$

$990x = 135$

$x = \frac{135}{990} = \frac{3}{22}$

18 $\tan 30° = \frac{1}{\sqrt{3}}$ and $\sin 60° = \frac{\sqrt{3}}{2}$

$\tan 30° + \sin 60° = \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2}$

$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{2}$

$= \frac{2\sqrt{3}}{6} + \frac{3\sqrt{3}}{6}$

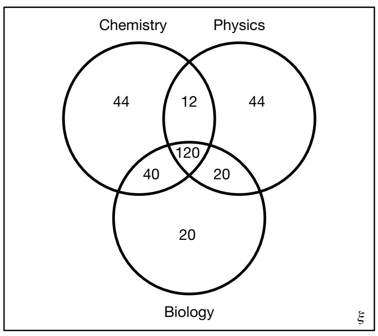
$= \frac{5\sqrt{3}}{6}$

19 $(x + 1)(x + 1)(x + 1) = (x + 1)(x^2 + 2x + 1)$

$= x^3 + 2x^2 + x + x^2 + 2x + 1$

$= x^3 + 3x^2 + 3x + 1$

20 a



b Let x = number who take only chemistry
(therefore the number who take only physics is also x)

$120 + 40 + 20 + 20 + 12 + x + x = 300$

$212 + 2x = 300$

$2x = 88$

$x = 44$

44 students take only chemistry (and 44 take only physics).

c total number who take biology = 200

number who take biology and chemistry = 160

P(biology student taking chemistry) = $\frac{160}{200} = \frac{4}{5}$

21 a angle $ACB = 55°$ (angle between tangent and chord equal to angle in the alternate segment)

b angle CAB = angle $ABC = \frac{180 - 55}{2} = 62.5°$ (base angles in isosceles triangle and angle sum in triangle)

angle $CBY = 180 - (62.5 + 55) = 62.5°$ (angles on a straight line add up to 180°)

22 a number of students in the class = $8 + 5 + 3 + 4 + 2 + 3 + 1 + 1 = 27$

total number of pets = $(1 \times 8) + (2 \times 5) + (3 \times 3) + (4 \times 4) + (5 \times 2) + (6 \times 3) + (7 \times 1) + (10 \times 1)$

$= 88$

mean number of pets = $\frac{88}{27} = 3.\dot{2}5\dot{9}$

= 3 (to the nearest integer)

Amy has 3 pets.

b There are 27 students so the median is the 14th value when the number of pets is put in order.

From the graph, the 14th value is 3.

Jacob has 3 pets.

23 a **i** $\overrightarrow{AM} = \overrightarrow{AO} + \frac{1}{2}\overrightarrow{OB}$

$= -\mathbf{a} + \frac{\mathbf{b}}{2}$

$= \frac{\mathbf{b}}{2} - \mathbf{a}$

ii $\overrightarrow{AR} = \frac{2}{3}\overrightarrow{AM}$

$= \frac{2}{3}\left(\frac{\mathbf{b}}{2} - \mathbf{a}\right)$

$= \frac{\mathbf{b}}{3} - \frac{2\mathbf{a}}{3}$

b $\overrightarrow{BP} = \overrightarrow{BO} + \overrightarrow{OP} = -\mathbf{b} + \frac{\mathbf{a}}{2} = \frac{\mathbf{a}}{2} - \mathbf{b} = \frac{1}{2}(\mathbf{a} - 2\mathbf{b})$

$\overrightarrow{BR} = \overrightarrow{BA} + \overrightarrow{AR} = \mathbf{a} - \mathbf{b} + \frac{\mathbf{b}}{3} - \frac{2\mathbf{a}}{3} = \frac{\mathbf{a}}{3} - \frac{2\mathbf{b}}{3}$

$= \frac{1}{3}(\mathbf{a} - 2\mathbf{b})$

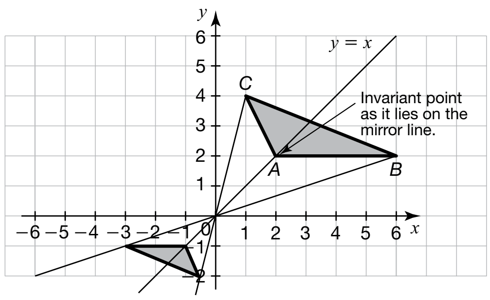
$\overrightarrow{BP}$ and $\overrightarrow{BR}$ have the same vector part ($\mathbf{a} - 2\mathbf{b}$) and are therefore parallel. As they both pass through point B, points P and R lie on the same straight line.

24 maximum attendance = 104 999

minimum attendance = 99 500

difference = 5499

25 a



b One invariant point (i.e. point A)

26 $6x + 5y = 35$ (1)

$x - 2y = 3$ (2)

Multiplying equation (1) by 2 and equation (2) by 5 gives:

$12x + 10y = 70$ (3)

$5x - 10y = 15$ (4)

(3) + (4) gives:

$17x = 85$, so $x = 5$

Substituting in (1): $30 + 5y = 35$, so $y = 1$

$x = 5$ and $y = 1$

27 $x^2 \leq 2x + 15$

$x^2 - 2x - 15 \leq 0$

Factorising $x^2 - 2x - 15 = 0$ gives

$(x - 5)(x + 3) = 0$: $x = 5$ or -3

As the coefficient of x^2 is positive, the graph of $y = x^2 - 2x - 15$ is U-shaped.

$x^2 - 2x - 15 \leq 0$ for the region below the x-axis (i.e. where $y < = 0$).

$-3 \leq x \leq 5$

28 a translation 3 units in the x-direction: turning point at (3, 6)

b reflection in x-axis: turning point at (0, −6)

29 gradient of $OA = \frac{4}{3}$

gradient of $BC = -\frac{3}{4}$

equation of BC is

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{3}{4}(x - 3)$

$4y - 16 = -3x + 9$

$3x + 4y - 25 = 0$

Practice paper 2 (calculator)

1 $2x^2 + \frac{3x^2}{x} + x = 2x^2 + 3x + x = 2x^2 + 4x$

2 $x - 3 = 0$ or $x + 4 = 0$, therefore $x = 3$ or -4

3 100

4 $3\mathrm{a} - 2\mathrm{b} = 3\begin{pmatrix}2\\1\end{pmatrix} - 2\begin{pmatrix}-1\\0\end{pmatrix} = \begin{pmatrix}3 \times 2 - 2 \times (-1)\\3 \times 1 - 2 \times 0\end{pmatrix} = \begin{pmatrix}8\\3\end{pmatrix}$

5 percentage increase = $\frac{\text{increase}}{\text{original value}} \times 100$

$= \frac{20\,000}{150\,000} \times 100$

= 13.3% (to 1 d.p.)

6 a 391.8954599

b $19.85^2 - \sqrt{98.67} \div 4.67 \approx 20^2 - \sqrt{100} \div 5$

$= 400 - 2$

$= 398$

7 Number of sides = $\frac{360}{60} = 6$, therefore the regular polygon is a hexagon.

8 The first digit could be from 1 to 9 (i.e. 9 options).

The second digit is fixed at 5 (i.e. 1 options).

The third digit could be any digit (i.e. 10 options).

The fourth digit would have to be even to make the whole number even so it would have to be 0, 2, 4, 6 or 8 (i.e. 5 options).

total number of possible numbers = $9 \times 1 \times 10 \times 5 = 450$

9 C

$y = \frac{k}{x}$. Since x is the denominator of the fraction, the larger x gets, the smaller y will be. Similarly, y will be big when x is as small as possible. Graph C shows this relationship.

10 Using $y = mx + c$, the equation of the straight line is $y = -\frac{2}{3}x + 2$

11 a $245 = 5 \times 49 = 5 \times 7 \times 7 = 5 \times 7^2$

b The only shared factors are 5 and 7, so:
highest common factor $= 5 \times 7 = 35$

12 $s = \frac{1}{2}at^2$

$= \frac{1}{2} \times 2.7 \times 10^8 \times (7.5 \times 10^{-3})^2$

$= 7593.75$

$= 7600$ to 2 s.f. (as the values used to calculate it were given to 2 s.f.)

13 $2^2 : 7^2 = 4 : 49$

14

Number of pets per household	Frequency	Midpoint	Frequency × midpoint
0–2	22	1	22
3–5	12	4	48
6–8	x	7	$7x$
9–11	3	10	30

total number of pets $= 22 + 48 + 7x + 30 = 100 + 7x$

total number of households $= 22 + 12 + x + 3 = 37 + x$

estimate of mean $= \frac{\text{total number of pets}}{\text{total number of households}} = \frac{100 + 7x}{37 + x}$

$\frac{100 + 7x}{37 + x} = 3.25$

$100 + 7x = 120.25 + 3.25x$

$3.75x = 20.25$

$x = 20.25 \div 3.75$

$x = 5.4$

5.4 is the correct answer for the figures given, but a frequency should be an integer so the question is not realistic. If the question gave midpoints of 1, 5, 6, 10, for example, the answer would be 3. Note that in the question, column 1 is also labelled as 'Number of households' when it should say 'Number of pets'.*

15 a Multiply the coefficient of x^2 by the number term: $10 \times 3 = 30$

Now look for a factor pair of 30 that adds to give the coefficient of :

$2 + 15 = 17$

$10x^2 + 17x + 3 = 10x^2 + 15x + 2x + 3$
$= 5x(2x + 3) + (2x + 3)$

Therefore, $10x^2 + 17x + 3 = (5x + 1)(2x + 3)$

b $(5x + 1)(2x + 3) = 0$

so $5x + 1 = 0$ giving $x = -\frac{1}{5}$

or $2x + 3$ giving $x = -\frac{3}{2}$

16 a volume $= 8 \times 6 \times 4 = 192\,\text{cm}^3$

b $1\,\text{m}^3 = 1\,000\,000\,\text{cm}^3$

volume in $\text{m}^3 = \frac{192}{1\,000\,000} = 1.92 \times 10^{-4}\,\text{m}^3$

c density in $\text{kg/m}^3 = \frac{\text{mass in kg}}{\text{volume in m}^3}$

mass in kg = density in $\frac{\text{kg}}{\text{m}^3}$ × volume in m^3

$= 8940 \times 1.92 \times 10^{-4}$

$= 1.72$ kg (to 2 d.p.)

d Using the answer from c, force in N = mass in kg × 9.81

$= 1.72 \times 9.81$

$= 16.87$ N (to 2 d.p.)

Or, more accurately, using the answer from c without rounding first:

$8940 \times 1.92 \times 10^{-4} \times 9.81$

$= 16.84$ N (to 2 d.p.)

e The greatest value for pressure will occur when the block is on its smallest area.

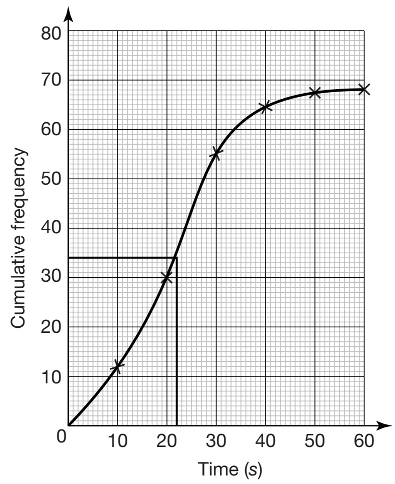
smallest area $= 6 \times 4 = 24\,\text{cm}^2 = \frac{24}{10\,000}\,\text{m}^2 = 0.0024\,\text{m}^2$

maximum pressure $= \frac{16.87}{0.0024} = 7030\,\text{N/m}^2$ (to 3 s.f.)

17 a

Time for call to be answered (t seconds)	Frequency	Cumulative frequency
$0 < t \le 10$	12	12
$10 < t \le 20$	18	30
$20 < t \le 30$	25	55
$30 < t \le 40$	10	65
$40 < t \le 50$	2	67
$50 < t \le 60$	1	68

b



c Reading from the graph above, the median time is 22 s, but any answer within the range of 21–22.5 is acceptable (depending on the curve drawn for part b).

18 a angle OAP = angle OBP = 90° (tangent and radius at right angle)

$OA = OB$ (radii of the circle)

Side OP is common to both triangles.

Both OAP and OBP are right-angled triangles and the two sides are the same, so by Pythagoras' theorem the third sides BP and AP must be equal.

b All the corresponding sides are equal in length, so the two triangles are congruent (SSS).

19 Let the length of the edge of the original cube $= x$ cm

volume of original cube $= x^3$

The three sides of the cuboid are $x + 3$, $x - 2$ and x.

volume of cuboid $= (x + 3)(x - 2)x$

difference in volumes $= (x + 3)(x - 2)x - x^3$

$= (x^2 + x - 6)x - x^3$

$= x^3 + x^2 - 6x - x^3$

$= x^2 - 6x$

*This information differs from that given in the Exam Practice book due to an error in our first edition. This has now been re-checked and corrected.

$x^2 - 6x = 55$

$x^2 - 6x - 55 = 0$

$(x - 11)(x + 5) = 0$

$x = 11$ or -5

The cube cannot have a negative length of side, so length of side of original cube = 11 cm.

20 $2ay + 2c = 3 - y$

$2ay + y = 3 - 2c$

$y(2a + 1) = 3 - 2c$

$y = \frac{3 - 2c}{2a + 1}$

21 3 + 5 = 8 parts

8 parts = 500 g so 1 part = 62.5 g

metal A: mass = 3 × 62.5 = 187.5 g

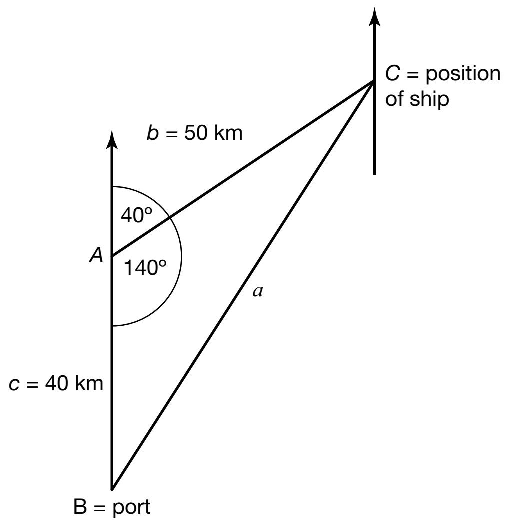
cost = $\frac{187.5}{1000}$ × 50 = £9.375

metal B: mass = 5 × 62.5 = 312.5 g

cost = $\frac{312.5}{1000}$ × 140 = £43.75

total cost = 9.375 + 43.75 = 53.125 = £53.13 (to nearest penny)

22 a



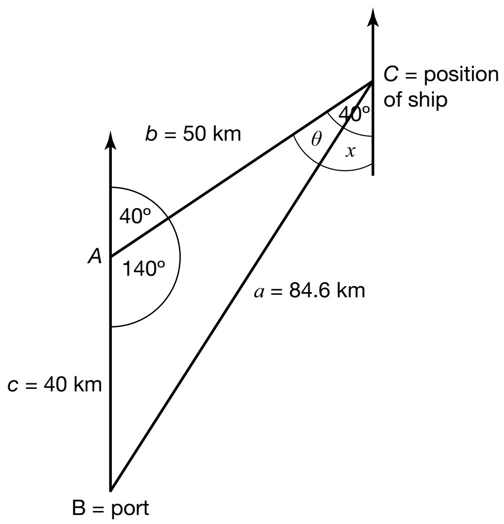
Using the cosine rule:

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 50^2 + 40^2 - 2 \times 50 \times 40 \times \cos 140°$

distance = 84.6 km (to 1 d.p.)

b



Using the sine rule:

$\frac{40}{\sin\theta} = \frac{84.6}{\sin 140°}$

$\sin\theta = \frac{40 \times \sin 140°}{84.6}$

$\theta = 17.7°$ (to 1 d.p.)

$x = 40 - \theta = 40 - 17.7 = 22.3°$

required angle = 180 + 22.3 = 202.3°

bearing = 202° to nearest degree.

23 a multiplier = $1 - \frac{\text{\% evaporation}}{100}$

$= 1 - \frac{2}{100} = 0.98$

volume at the end of 4 weeks = $A_0 \times (\text{multiplier})^n$

$= 30 \times (0.98)^4$

$= 27.7\,\text{m}^3$ (to 3 s.f.)

b Need to aim for a volume of 15 m³.

Try 14 weeks: volume = $30 \times (0.98)^{14} = 22.6$

Try 20 weeks: volume = $30 \times (0.98)^{20} = 20.0$

Try 24 weeks: volume = $30 \times (0.98)^{24} = 18.5$

Try 34 weeks: volume = $30 \times (0.98)^{34} = 15.1$

Try 35 weeks: volume = $30 \times (0.98)^{35} = 14.8$

Answer is 35 weeks.

24 a number of males = $\frac{4}{9} \times 36 = 16$

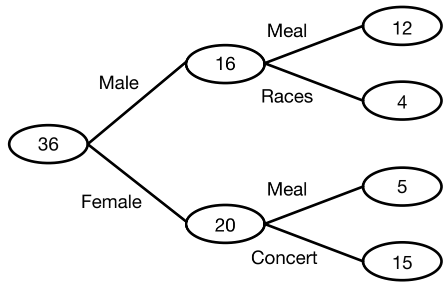
number of males + races = 25% of 16 = 4

number of males + meal = 16 − 4 = 12

number of females = 36 − 16 = 20

number of females + meal = $\frac{1}{4} \times 20 = 5$

number of females + concert = 20 − 5 = 15



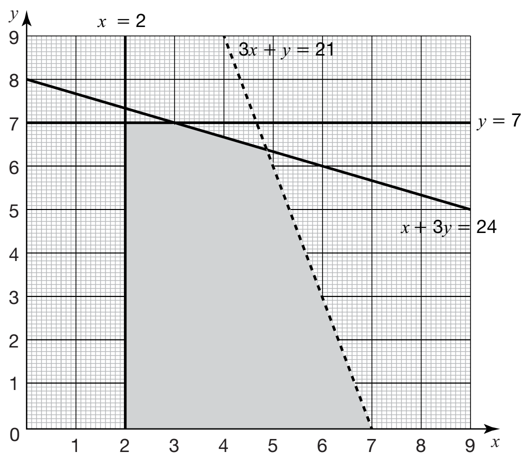
b P(meal) = $\frac{12 + 5}{36} = \frac{17}{36}$

25 $x + 3y = 24$: when $x = 0$, $y = 8$ and when $x = 6$, $y = 6$; required region below this line

$y + 3x = 21$: when $x = 5$, $y = 6$ and when $x = 7$, $y = 0$; required region below this line

$x \geq 2$: required region to the right of this line

$y \leq 7$: required region below this line



26 Using Pythagoras' theorem in triangle *ABC*:

$AC^2 = 2^2 + 3^2 = 13$

$AC = \sqrt{13}$ cm

Using Pythagoras' theorem in triangle *ACD*:

$AD^2 = (\sqrt{13})^2 + 3^2 = 13 + 9 = 22$

$AD = \sqrt{22}$ cm

27 **a** angle $PRQ = 60°$ (angles on the same arc are equal)

b angle $PSR = 90°$ (angle in a semicircle is 90°)

angle $PRS = 180 - (90 + 21) = 69°$ (angles in a triangle add up to 180°)