All Boards Foundation Mathematics Revision Guide

Full worked solutions

Number

Basic number techniques

1 **a** false **c** true **e** true

b true **d** true

2 −0.3, −1.5, −2.5, −4.2, −7.2

3 0.049, 0.124, 0.412, 0.442, 1.002

4 **a** $<$ **b** $<$ **c** $>$

Factors, multiples and primes

1 **a** 5 **b** 1, 12 **c** 1, 5, 45 **d** 1, 5

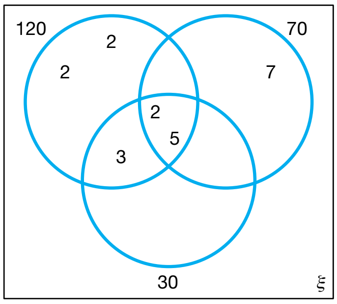
2 70 = 2 × 5 × 7

150 = 2 × 3 × 5 × 5

HCF = 10, LCM = 1050

3 $2 \times 3^2 \times 5$

4



a 2 × 5 = 10 **b** 2 × 2 × 2 × 3 × 5 × 7 = 840

5 12 and 18

Calculating with negative numbers

Stretch it! Multiplying three negative numbers together always gives a negative answer.

1 **a** −8 − 3 = −11 **d** 14 + 4 = 18

b 99 **e** 0

c −6 **f** 12 + 15 − 2 = 25

2 −8 and 9

3 32°C

Division and multiplication

Stretch it!

$$\begin{array}{cccccc l}
 & & & 6 & 2 & 1 & \\
 & & \times & 2 & 3 & 9 & \\ \hline
 & & 5 & 5 & 8 & 9 & \times 9 \\
 & & & {\scriptstyle 1} & & & \\
 & 1 & 8 & 6 & 3 & 0 & \times 30 \\
1 & 2 & 4 & 2 & 0 & 0 & \times 200 \\ \hline
1 & 4 & 8 & 4 & 1 & 9 & \\
 & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & &
\end{array}$$

148 419

1 **a**

$$\begin{array}{cccc}
 & 2 & 3 & 5 \\
 & \times & & 9 \\ \hline
2 & 1 & 1 & 5 \\ \hline
 & {\scriptstyle 3} & {\scriptstyle 4} &
\end{array}$$

2115

b

$$\begin{array}{ccccc l}
 & & 9 & 2 & 4 & \\
 & & \times & 6 & 1 & \\ \hline
 & & 9 & 2 & 4 & \times 1 \\
5 & 5 & 4 & 4 & 0 & \times 60 \\ \hline
 & {\scriptstyle 1} & {\scriptstyle 2} & & & \\
5 & 6 & 3 & 6 & 4 & \\ \hline
 & {\scriptstyle 1} & & & &
\end{array}$$

56 364

2 **a**

$$\begin{array}{r} 0\ \ 4\ \ 7 \\ 5\overline{)2\ \ {}^{2}3\ \ {}^{3}5} \end{array}$$

47

b

$$\begin{array}{r} 0\,5\,1\,6 \\ 8\overline{)4\,{}^{4}1\,{}^{1}2\,{}^{4}8} \end{array}$$

516

c

$$\begin{array}{rcccc}
 & 0 & 1 & 2 & 6 \\
17) & 2 & 1 & 4 & 6 \\
 & 1 & 7 & & \\ \hline
 & & 4 & 4 & \\
 & & 3 & 4 & \\ \hline
 & & 1 & 0 & 6 \\
 & & 1 & 0 & 2 \\ \hline
 & & & & 4
\end{array}$$

126 remainder 4, or 126 $\frac{4}{17}$

3

$$\begin{array}{r} 0\ \ 3\ \ 3 \\ 8\overline{)2^{2}\ 6^{2}\ 5^{1}} \end{array}$$

remainder 1

a 33 boxes

b 1 pencil

4

$$\begin{array}{r} 0\ 9\ 1.2\ 5 \\ 4\overline{)3\,{}^{3}6\ 5.{}^{1}0\,{}^{2}0} \end{array}$$

£91.25

5

$$\begin{array}{ccc}
 & 3 & 2 \\
 & \times & 9 \\ \hline
2 & 8 & 8 \\ \hline
 & {\scriptstyle 1} &
\end{array}$$

£288

6

$$\begin{array}{r} 3\ 0\ 7.6\ 6 \\ 3\overline{)9\ 2\,{}^{2}3.{}^{2}0\,{}^{2}0} \end{array}$$

$307.\dot{6} = 307\frac{2}{3}$

7

$$\begin{array}{ccccc l}
 & & 8 & 2 & 3 & \\
 & & \times & 3 & 5 & \\ \hline
 & 4 & 1 & 1 & 5 & \times 5 \\
 & & {\scriptstyle 1} & {\scriptstyle 1} & & \\
2 & 4 & 6 & 9 & 0 & \times 30 \\ \hline
2 & 8 & 8 & 0 & 5 & \\ \hline
 & & {\scriptstyle 1} & & &
\end{array}$$

28 805

8 **a** $239 + ? = 921$

Missing number $= 921 - 239$

$= 682$

b $? \times 87 = 1131$

Missing number $= 1131 \div 87$

$= 13$

c $23 \times ? + 8 = 123$

$23 \times ? = 123 - 8 = 115$

Missing number $= 115 \div 23$

$= 5$

9

$$\begin{array}{r} 0\ 3\ 7 \\ 12\overline{)4\ 5\ 0} \\ 3\ 6 \\ \hline 9\ 0 \\ 8\ 4 \\ \hline 6 \end{array}$$

37 remainder 6, so 37 boxes

10 He has not placed a zero in the ones column before multiplying through by 5. The × 50 line should have 5 digits: 36 300, so his final three rows of working should look like this:

$$\begin{array}{rl} 1\ 4\ 5\ 2\ 2 & \times\ 2 \\ 3\ 6\ 3\ 0\ 0 & \times\ 50 \\ \hline 3\ 7\ 7\ 5\ 2 & \end{array}$$

Calculating with decimals

Stretch it! $3.2 + 7.5 \times 2 = 3.2 + 15 = 18.2$

1 **a**

$$\begin{array}{r} 3\,.\,\overset{3}{\cancel{4}}\,\overset{1}{0} \\ -1\,.\,0\,7 \\ \hline 2\,.\,3\,3 \\ \hline \end{array}$$

2.33

b

$$\begin{array}{r} 1\ 9\cdot 3\ 0\ 0 \\ +\ \ 5\cdot 0\ 9\ 1 \\ \hline 2\ 4\cdot 3\ 9\ 1 \\ \hline {}_{1}\ \ \ \ \ \ \ \ \ \ \end{array}$$

24.391

c $5 \times 7 = 35$

$0.05 \times 0.7 = 0.035$

d

$$\begin{array}{rl} 3\ 2\ 1 & \\ \times\ 1\ \ 9 & \\ \hline 2\ 8\ 8\ 9 & \times\ 9 \\ {}^{1}\ \ \ \ & \\ 3\ 2\ 1\ 0 & \times\ 10 \\ \hline 6\ 0\ 9\ 9 & \\ \hline {}_{1}\ \ \ \ \ \ \ \ & \end{array}$$

$3.21 \times 1.9 = 6.099$

e

$$\begin{array}{r} 1.5\ 6\ 3 \\ 5\overline{)7.^{2}8\ ^{3}1\ ^{1}5} \end{array}$$

1.563

f $\frac{0.18}{0.3} = \frac{18}{30} = \frac{3}{5} = 0.6$

g $\frac{-0.3}{0.05} = \frac{-30}{5} = -6$

h $72 \times -7 = -504$

Therefore: 7.2×-0.7

$= -5.04$

2

$$\begin{array}{r} 6\ 4 \\ \times\ 2\ 4 \\ \hline 2\ 5\ 6 \\ 1\ 2\ \overset{1}{8}\ 0 \\ \hline 1\ 5\ 3\ 6 \\ \hline {}_{1}\ \ \ \ \ \ \end{array}$$

$24 \times 64 = 1536$

$24 \times £0.64 = £15.36$

$£20 - £15.36 = £4.64$

3

$$\begin{array}{r} 2\ 7.4\ 6 \\ 3\overline{)8\ ^{2}2.^{1}3\ ^{1}8} \end{array}$$

Erica $2 \times 27.46 = £54.92$

Freya $82.38 - 54.92 = £27.46$

Rounding and estimation

Stretch it! a 1.0 **b** 1.00 **c** 1.000

All the answers are 1

Stretch it! $6.5 \times 8.5 = 55.25\,\text{m}^2$ – an overestimate.

1 **a** 0.35 **c** 32.6

b 10 **d** 33 100

2 **a** $150 \leq x < 250$ **c** $3.15 \leq x < 3.25$

b $5.5 \leq x < 6.5$ **d** $5.055 \leq x < 5.065$

3 $\frac{30}{0.5 \times 6} = 10$

4 **a** 23 580 **c** 23 600 **e** 20 000

b 23 580 **d** 24 000

5 b is false since $18 \times 1 = 18$ so 18×0.9 cannot be 1.62.

c is false because if you divide by a number smaller than 1, the answer will be larger.

6 Night-time low tariff: 2.32 units × 1.622p = 3.76p

7.151 units × 2.315p = 16.55p

20.31p

One tariff: 2.320 + 7.151 = 9.471 units

9.471 units × 1.923p = 18.21p

Tarik should choose One tariff, since it is cheaper.

Converting between fractions, decimals and percentages

Stretch it! $0.\dot{1}, 0.\dot{2}, 0.\dot{3}, \ldots 0.\dot{4}, 0.\dot{5}$. The number of ninths is the same as the digit that recurs. The exception is $\frac{9}{9}$ which is the same as 1.

1 **a** $\frac{32}{100} = \frac{8}{25}$ **c** $\frac{33}{100}$

b $1\frac{24}{100} = 1\frac{6}{25}$ **d** $\frac{95}{100} = \frac{19}{20}$

2 **a**

$$\begin{array}{r} 0\,.\,4\,1\,6\,6\,6\ldots \\ 12\overline{)5^{5}.\,0^{2}0^{8}0^{8}0^{8}0} \end{array}$$

$0.41\dot{6}$

b

$$\begin{array}{r} 0\,.\,3\,7\,5 \\ 8\overline{)3\,.\,^{3}0^{6}0^{4}0\ 0} \end{array}$$

0.375

c 0.49

d 0.185

e

$$\begin{array}{r} 0\,.\,4\,2\,8\,5\,7\,1\,4\,2\ldots \\ 7\overline{)3\,.\,^{3}0^{2}0^{6}0^{4}0^{5}0^{1}0^{3}0^{2}0} \end{array}$$

$0.\dot{4}2857\dot{1}$

3 **a** $\frac{91}{100} = 91\%$

b $\frac{3}{10} = \frac{30}{100} = 30\%$

c $\frac{4}{5} = \frac{80}{100} = 80\%$

d $\frac{9}{15} = \frac{3}{5} = \frac{6}{10} = \frac{60}{100} = 60\%$

4 Divide 3 by 8:

$$\begin{array}{r} 0\,.\,3\,7\,5 \\ 8\overline{)3\,.\,^{3}0^{6}0^{4}0\ 0} \end{array} = 0.375 = 37.5\%$$

5 0.35 $\frac{2}{5} = \frac{4}{10} = 0.4$ 30% = 0.3

30%, 0.35, $\frac{2}{5}$

6 $\frac{15}{20} = \frac{75}{100} = 75\%$ – Amy

Rudi's mark was higher.

Ordering fractions, decimals and percentages

1 $\frac{1}{3} = \frac{8}{24}$ $\frac{3}{8} = \frac{9}{24}$ $\frac{7}{12} = \frac{14}{24}$

$\frac{7}{12}, \frac{3}{8}, \frac{1}{3}$

2 $-2.2, 7, \frac{1}{5} = 0.2, -\frac{1}{10} = -0.1, 15\% = 0.15, 1\% = 0.01, 0.1$

In order, this is: $-2.2, -\frac{1}{10}, 1\%, 0.1, 15\%, \frac{1}{5}, 7$.
The middle value is 0.1

3 Yes. If the numerator of a fraction is half the denominator then the fraction is equivalent to $\frac{1}{2}$.
If the numerator is smaller than this the fraction must be less than $\frac{1}{2}$.

Calculating with fractions

Stretch it! No – you could add the whole number parts, and then add the fraction parts, giving:

$1 + 2 = 3$

$\frac{3}{5} + \frac{1}{4} = \frac{17}{20}$

$= 3\frac{17}{20}$

1 **a** $2\frac{3}{8} - \frac{3}{4} = \frac{19}{8} - \frac{3}{4} = \frac{19}{8} - \frac{6}{8} = \frac{13}{8} = 1\frac{5}{8}$

b $\frac{\overset{3}{\cancel{15}}}{17} \times \frac{2}{\cancel{5}_1} = \frac{6}{17}$

c $\frac{1}{7} \times 3\frac{1}{3} = \frac{1}{7} \times \frac{10}{3} = \frac{10}{21}$

d $2\frac{2}{5} + 5\frac{3}{4} = 7 + \frac{2}{5} + \frac{3}{4} = 7 + \frac{8}{20} + \frac{15}{20}$

$= 7\frac{23}{20}$

$= 8\frac{3}{20}$

e $\frac{1}{5} \div 2\frac{1}{2} = \frac{1}{5} \div \frac{5}{2} = \frac{1}{5} \times \frac{2}{5} = \frac{2}{25}$

2 **a** $30 \div 5 = 6$

$6 \times 2 = 12$

b $40 \div 8 = 5$

$5 \times 7 = £35$

c $1818 \div 9 = 202$

$202 \times 4 = 808$ mm

3 $35 \div 7 = 5$

$3 \times 7 = 15$

$35 - 15 = 20$

4 The number must be a multiple of 5, and $\frac{2}{5}$ of it must be a multiple of 2.

$\frac{2}{5}$ of 45 = 18

$\frac{2}{5}$ of 40 = 16

$\frac{2}{5}$ of 35 = 14

$\frac{2}{5}$ of 30 = 12

$\frac{2}{5}$ of the number must be greater than 12, so the number is 35

Percentages

1 **a** $18 \div 100 = 0.18$ cm

$0.18 \times 10 = 1.8$ cm

b $1.20 \div 100 = £0.012$

$0.012 \times 25 = £0.30$

c 200 ml $\div 100 = 2$ ml

2 ml $\times 2 = 4$ ml

2 **a** $1.1 \times 30 = 33$

b $1.08 \times 500 = 540$

c $1.12 \times 91 = 101.92$, so £101.92

3 **a** $0.8 \times 600 = 480$

b $0.95 \times 140 = 133$

c $0.81 \times 18 = 14.58$, so £14.58

4 $1.09 \times 2800 = 3052$

5 $0.65 \times 22\,000 = £14\,300$

Order of operations

1 **a** 7

b $0.9 + 3.2 - \sqrt{36}$

$= 0.9 + 3.2 - 6$

$= -1.9$

c $(-1)^2 - 14$

$= 1 - 14$

$= -13$

2 30

3 $(8 - 3 + 5) \times 4$

Exact solutions

1 **a** π **b** 36π **c** $2\frac{1}{2}\pi$ or $\frac{5}{2}\pi$

2 **a** 7π **b** $\frac{5}{8}\pi$

3 Area $= \frac{2}{7} \times \frac{3}{4} = \frac{6}{28} = \frac{3}{14}$ cm^2

Perimeter $= \left(2 \times \frac{3}{4}\right) + \left(2 \times \frac{2}{7}\right) = \frac{3}{2} + \frac{4}{7} = \frac{21 + 8}{14} = \frac{29}{14}$

$= 2\frac{1}{14}$ cm

4 **a** $2 \times 9 \times \pi = 18\pi$ cm

b $12^2 \times \pi = 144\pi$ cm^2

5 Circumference $= 2 \times \pi \times 1 = 2\pi$ cm

Length of one side of square $= 2\pi \div 4 = \frac{1}{2}\pi$ cm

Indices and roots

1 **a** $\frac{1}{3}$ **b** $\frac{1}{0.4} = \frac{10}{4} = 2\frac{1}{2}$

c $\frac{1}{0.9} = \frac{10}{9} = 1\frac{1}{9}$

2 $3^2 = 9$ $1^3 = 1$ $\sqrt[3]{27} = 3$ $\sqrt[3]{8} = 2$ $\frac{1}{12} = 0.08\dot{3}$

In order, this gives $\frac{1}{12}, 1^3, \sqrt[3]{8}, \sqrt[3]{27}, 3.7, 3^2$

3 **a** -8 **b** 1 **c** 81 **d** 1

4 **a** $\frac{1}{4}$ **b** $\frac{1}{7^2} = \frac{1}{49}$

c $\frac{1}{1^4} = 1$ **d** $\frac{1}{3}$

5 $\frac{5^9}{5^5} = 5^4$

6 $(0.01 \times 798)^2 = 7.98^2$

$\approx 8^2$

$= 64$

Standard form

1 **a** 45 000 000 **b** 0.091

2 **a** 6.45×10^8 **b** 7.9×10^{-8}

3 $350\,000 - 4200 = 345\,800$

4 $3.2 \times 10^2 = 320$ $3.1 \times 10^{-2} = 0.031$
$3.09 \times 10 = 30.9$ $3 + (2.1 \times 10^2) = 213$

In order, this gives:
3.1×10^{-2} 3.09×10
$3 + (2.1 \times 10^2)$ 3.2×10^2

5 3×10^8 m/s

6 $200 \times 1.1 \times 10^{-4} = 2.2 \times 10^{-2} = 0.022$ m $= 2.2$ cm

Listing strategies

Stretch it!

red + small, red + medium, red + large,

green + small, green + medium, green + large,

blue + small, blue + medium, blue + large.

1 111,
112, 121, 211, 113, 131, 311,
222
221, 212, 122, 223, 232, 322
333
331 313 133 332 323 233
123 132 213 231 312 321

2 444 446 449
464 466 469
494 496 499

3 Small A, Small B, Small C, Small D
Medium A, Medium B, Medium C, Medium D
Large A, Large B, Large C, Large D.

Review it!

1 Total tickets sold = 34 592
Sold online = 21 298

$$\begin{array}{r} 3\;4\;\overset{4}{\cancel{5}}\;\overset{18}{\cancel{9}}\;{}^{1}2 \\ -2\;1\;2\;9\;8 \\ \hline 1\;3\;2\;9\;4 \\ \hline \end{array}$$

Sold at station = 13 294

2 7 and 6 (or 11 and 2, where both are prime and 2 is also a factor of 12)

3 $620 = 2 \times 2 \times 5 \times 31 = 2^2 \times 5 \times 31$

4 $18 = 2 \times 3 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
$40 = 2 \times 2 \times 2 \times 5$
HCF = 2

5 −11.5, −8.3, −3.5, −3.2, 1.4

6 a

$$\begin{array}{r} 3\;2\cdot9\;\;9 \\ +1\;8\cdot7\;\;4 \\ \hline 5\;1\cdot7\;\;3 \\ \hline {}_{1\;\;1\;\;\;1\;\;\;\;\;} \end{array}$$

£51.73

b

$$\begin{array}{r} 1\;8\;.\;3\;3 \\ 3\overline{)\,5\,{}^{2}4\;.\;9\;9} \end{array}$$

£18.33

7 a

$$\begin{array}{rl} 2\;\;3 & \\ \times\;1\;\;4 & \\ \hline 9\;\;2 & \times\;4 \\ 2\;\;\underset{}{\overset{1}{3}}\;\;0 & \times\;10 \\ \hline 3\;\;2\;\;2 & \\ \hline {}_{1}\;\;\;\;\;\;\;\;\; & \end{array}$$

$23 \times 0.14 = 3.22$

b

$$\begin{array}{rl} 1\;4\;9 & \\ \times\;2\;7 & \\ \hline 1\;0\;4\;3 & \times\;7 \\ 2\;\overset{3}{\underset{1}{9}}\;\overset{6}{8}\;0 & \times\;20 \\ \hline 4\;0^{1}2\;3 & \\ \hline {}_{1\;\;1\;\;\;\;\;\;} & \end{array}$$

4023

c $-3 \times 4 = -12$

There are two decimal places to put in, so $-0.3 \times 0.4 = -0.12$

d $-13.5 + 8.7 = -13.5 + 9 - 0.3$
$= -4.5 - 0.3$
$= -4.8$

e $\frac{-1.2}{0.3} = \frac{-12}{3} = -4$

8 $30 \times \sqrt{16} + 17 = 30 \times \pm 4 + 17$
$= \pm 120 + 17$
$= 137$ or -103

9 $81 \div 3 = 27$

10

$$\begin{array}{r} 0\;3\;1 \\ 11\overline{)\,3^{3}4^{1}5^{4}} \end{array} \quad \text{remainder 4}$$

$31\frac{4}{11}$

11 a

$$\begin{array}{r} 0\;.\;3\;7\;5 \\ 8\overline{)\,3\;.^{3}0^{6}0^{4}0\;0} \end{array}$$

0.375

b $0.7 \times 100 = 70\%$

12 a $70\% = \frac{70}{100} = \frac{7}{10}$

b $0.8 = \frac{8}{10} = \frac{4}{5}$

13 $\frac{1}{2} = \frac{2}{4}$ $\frac{1}{2}$ is larger
$\frac{2}{7} = \frac{8}{28}$ $\frac{1}{4} = \frac{7}{28}$ $\frac{2}{7}$ is larger
$\frac{3}{11} = \frac{12}{44}$ $\frac{1}{4} = \frac{11}{44}$ $\frac{3}{11}$ is larger
$\frac{2}{5} = \frac{8}{20}$ $\frac{1}{4} = \frac{5}{20}$ $\frac{2}{5}$ is larger

All of them.

14 a $\frac{3}{5} + \frac{1}{7} = \frac{21}{35} + \frac{5}{35} = \frac{26}{35}$

b $2\frac{1}{5} - \frac{7}{10} = \frac{11}{5} - \frac{7}{10} = \frac{22}{10} - \frac{7}{10} = \frac{15}{10} = 1\frac{1}{2}$

c $\frac{2}{3} \div \frac{4}{9} = \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}} = \frac{3}{2} = 1\frac{1}{2}$

15 $0.25 - 0.07 = 0.18 = \frac{18}{100} = \frac{108}{600}$
$\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6} = \frac{100}{600}$
0.25 − 0.07 is larger

16 $\frac{3}{\cancel{5}} \times \frac{\cancel{5}}{4} = \frac{3}{4}$

17 $\frac{45}{1000} = \frac{9}{200}$

18 $8.6 \div 100 = 0.086$
$0.086 \times 25 = £2.15$

19 a 9 **b** 5

20 a 3.4×10^9 **b** 3.04×10^{-7}

21 $37.55 \le x < 37.65$

22 a 51 **b** 12, 15, 21, 51, 25, 52

23 **a** $200 \times 9 \times 10 = 18000 = £180.00$

b Underestimate since all numbers were rounded down.

24 40% of $600 = 240$

$\frac{1}{5}$ of $600 = 120$

$600 - (240 + 120) = 240$

25 More than 33%, less than 50%, multiple of 5. 35%

26 No, since 2 is even and a prime number, and odd + odd + even = even.

27 $0.8 \times 349 = £279.20$

28 **a** 3.1 **b** 3.05

29 **a** 325 000 **b** 320 000

30 $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$

31 $26.25 + 18.23 + (4 \times 5.5) = £66.48$

$£66.48 \div 4 = £16.62$

32 $0.19 \times 18\,000 = 3420$

33 $102.3 \times 1.1 = 112.53$

The price in 2017 was £112.53

Algebra

Understanding expressions, equations, formulae and identities

1 **a** $3a + 6 = 10$ (It can be solved to find the value of a.)

b $C = \pi D$ (The value of C can be worked out if the value of D is known.)

c $3(a + 2)$ (It does not have an equals sign.)

d $3ab + 2ab = 5ab$ (Collecting the like terms on the left-hand side gives $5ab$ which is equal to the right-hand side.)

2 James is correct.

$4x - 2 = 2x$ can be solved to find the value of x so it is an equation.

Or, the two sides of $4x - 2 = 2x$ are not equal for all values of x so it cannot be an identity. For example, when $x = 2$:

(Left-hand side) $4x - 2 = 4 \times 2 - 2 = 6$

(Right-hand side) $2x = 2 \times 2 = 4$

$6 \neq 4$

Simplifying expressions

Stretch it!

The expressions must all contain algebra, so each part must include t.

There are four possible combinations that make $12t^3$: $12t \times t \times t$, $2t \times 6t \times t$, $2t \times 3t \times 2t$, $3t \times 4t \times t$.

1 **a** p^3

b $4 \times b \times c \times 7 = 4 \times 7 \times b \times c = 28bc$

c $4a \times 3b = 4 \times 3 \times a \times b = 12ab$

d $5x \times 4x = 5 \times 4 \times x \times x = 20x^2$

e $2g \times (-4g) = 2 \times (-4) \times g \times g = -8g^2$

f $2p \times 3q \times r = 2 \times 3 \times p \times q \times r = 6pqr$

2 **a** $10x \div 2 = \frac{10x}{2} = 5x$

b $\frac{14w}{-2} = -7w$

c $6p \div p = \frac{6p}{p} = 6$

d $8mn \div 2m = \frac{8mn}{2m} = 4n$

e $\frac{12xy}{3y} = 4x$

f $9abc \div bc = \frac{9abc}{bc} = 9a$

Collecting like terms

1 **a** $5f$

b $7b$

c $5mn$

d $4a + 6 - a - 5 = 4a - a + 6 - 5 = 3a + 1$

e $3d + 4e + d - 6e = 3d + d + 4e - 6e = 4d - 2e$

f $2x + 5y + 3x - 2y - 2 = 2x + 3x + 5y - 2y - 2$
$= 5x + 3y - 2$

g $3a - 2b + 4a + 7b = 3a + 4a - 2b + 7b =$
$7a + 5b$

h $2a - b - 5a - 3 = 2a - 5a - b - 3 = -3a - b - 3$

i $x^2 + x^2 = 2 \times x^2 = 2x^2$

j $2t^3 + 4 - t^3 - 4 = 2t^3 - t^3 + 4 - 4 = t^3$

k $2a + b^2$

l $(4 + 3)\sqrt{x} = 7\sqrt{x}$

m $(7 - 4)\sqrt{x} = 3\sqrt{x}$

n $(12 - 1 - 4)\sqrt{x} = 7\sqrt{x}$

Using indices

1 **a** $x^5 \times x^4 = x^{5+4} = x^9$

b $p \times p^4 = p^{1+4} = p^5$

c $2m^4 \times 3m^4 = 2 \times 3 \times m^4 \times m^4 = 6 \times m^{4+4} = 6m^8$

d $3m^4n \times 5m^2n^3$

$= 3 \times 5 \times m^4 \times m^2 \times n \times n^3$

$= 15 \times m^{4+2} \times n^{1+3} = 15m^6n^4$

e $u^{-2} \times u^5 = u^{-2+5} = u^3$

f $t^7 \times t^{-6} = t^{7+(-6)} = t$

2 **a** $x^4 \div x^2 = x^{4-2} = x^2$

b $\frac{y^7}{y^3} = y^{7-3} = y^4$

c $\frac{p^9}{p^8} = p^{9-8} = p$

d $8x^6 \div 4x^3 = \frac{8x^6}{4x^3}$

$(8 \div 4) \times (x^6 \div x^3) = 2 \times x^{6-3} = 2x^3$

e $m^3 \div m^5 = m^{3-5} = m^{-2} = \frac{1}{m^2}$

f $\frac{5x^8}{15x^4} = \frac{5}{15} \times \frac{x^8}{x^4} = \frac{1}{3} \times x^{8-4} = \frac{x^4}{3}$

g $3x^2 \div 9x = \frac{3x^2}{9x} = \frac{3}{9} \times \frac{x^2}{x} = \frac{1}{3} \times x^{2-1} = \frac{x}{3}$

3 **a** $(x^2)^3 = x^{2\times3} = x^6$

b $(y^4)^4 = y^{4\times4} = y^{16}$

c $(p^5)^2 = p^{5\times2} = p^{10}$

d $(4m^5)^2 = 4^2 \times (m^5)^2 \times 16 \times m^{5\times2} = 16m^{10}$

e $(x^2)^{-3} = x^{2\times(-3)} = x^{-6} = \frac{1}{x^6}$

f $(n^{-4})^{-2} = n^{-4\times(-2)} = n^8$

4 **a** $4x \times 3x^2 = 4 \times 3 \times x \times x^2 = 12 \times x^{1+2} = 12x^3$
b $\frac{5x^4}{x} = 5x^{4-1} = 5x^3$
c $\frac{1}{y^2}$
d $a^3b^2 \times a^2b = a^3 \times a^2 \times b^2 \times b = a^{3+2}b^{2+1} = a^5b^3$

5 $\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4} = x^{8-4} = x^4$

Expanding brackets

Stretch it!

a $a\sqrt{3} + a^2$ **or** $\sqrt{3}a + a^2$
b $b\sqrt{5} - b^2$ **or** $\sqrt{5}b - b^2$
c $c + d$

Stretch it!

1 2 + **4** = 6 and 2 × **4** = 8, so the numbers are 4 and 8.
$(x+2)(x+4) = x^2 + 6x + 8$
2 **a** $(x + 3)(2x + 2)$
$= 2x^2 + 6x + 2x + 6$
$= 2x^2 + 8x + 6$
b $(3x - 2)(x + 4)$
$= 3x^2 - 2x + 12x - 8$
$= 3x^2 + 10x - 8$
c $(2x + 3)(3x - 1)$
$= 6x^2 + 9x - 2x - 3$
$= 6x^2 + 7x - 3$
d $(x + 2y)(x - y)$
$= x^2 - xy + 2xy - 2y^2$
$= x^2 + xy - 2y^2$
e $(2x - y)(3x + y)$
$= 6x^2 + 2xy - 3xy - y^2$
$= 6x^2 - xy - y^2$

1 **a** $3a + 6$
b $4b - 16$
c $10c + 25$
d $6 - 2e$
e $4x + 4y + 8$
f $-2y - 4$
g $x^2 - 2x$
h $2a^2 + 10a$
i $3x + 6y$
j $-2a + 2b$
or $2b - 2a$
2 **a** $6a - (3a + 5)$
$= 6a - 3a - 5$
$= 3a - 5$
b $4x - 6 + 2(x + 5)$
$= 4x - 6 + 2x + 10$
$= 4x + 2x - 6 + 10$
$= 6x + 4$
3 **a** $2(2x + 3) + 4(x + 5) = 4x + 6 + 4x + 20 = 8x + 26$
b $3(3y + 1) + 2(4y - 3) = 9y + 3 + 8y - 6 = 17y - 3$
c $4(2m + 4) - 3(2m - 5) = 8m + 16 - 6m + 15 = 2m + 31$
4 **a** $(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$
b $(y - 3)(y + 4) = y^2 + 4y - 3y - 12 = y^2 + y - 12$
c $(a + 3)(a - 7) = a^2 - 7a + 3a - 21 = a^2 - 4a - 21$
d $(m - 1)(m - 6) = m^2 - 6m - m + 6 = m^2 - 7m + 6$
5 **a** $(x + 1)^2 = (x + 1)(x + 1)$
$= x^2 + x + x + 1 = x^2 + 2x + 1$
b $(x - 1)^2 = (x - 1)(x - 1)$
$= x^2 - x - x + 1 = x^2 - 2x + 1$
c $(m - 2)^2 = (m - 2)(m - 2)$
$= m^2 - 2m - 2m + 4 = m^2 - 4m + 4$
d $(y + 3)^2 = (y + 3)(y + 3)$
$= y^2 + 3y + 3y + 9 = y^2 + 6y + 9$

Factorising

Stretch it!

The width of the rectangle $= x + 1$, since $x^2 + 3x + 2 = (x + 2)(x + 1)$

Stretch it!

a $a^2 - 3 = a^2 - (\sqrt{3})^2$
$= (a + \sqrt{3})(a - \sqrt{3})$
b $b^2 - 5 = b^2 - (\sqrt{5})^2$
$= (b + \sqrt{5})(b - \sqrt{5})$

1 **a** $3(a + 3)$ **b** $5(b - 2)$
c $7(1 + 2c)$ **d** $d(d - 2)$
2 **a** $4(2a + 5)$ **b** $4(b - 3)$
c $9(2 + c)$ **d** $d(2d - 3)$
3 **a** $2(2x - 3y)$ **b** $m(a + b)$
c $4a(3a + 2)$ **d** $x(4x + 3y)$
e $n(2 - 9n)$ **f** $5x(1 + 2y)$
g $4p(q - 3)$ **h** $4y(x^2 - 2)$
4 $4(x - 3) + 3(2x + 6)$
$= 4x - 12 + 6x + 18$
$= 4x + 6x - 12 + 18$
$= 10x + 6$
$= 2(5x + 3)$
Compare $2(5x + 3)$ with $a(5x + b)$
$a = 2, b = 3$
5 **a** $(x + 1)(x + 7)$ **b** $(x - 1)(x + 5)$
c $(x + 2)(x - 4)$ **d** $(x - 2)(x - 3)$
e $(x - 3)(x - 3)$ **f** $(x + 3)(x + 4)$
g $(x - 2)(x + 5)$ **h** $(x + 4)(x - 5)$
6 **a** $x^2 - 16 = x^2 - 4^2 = (x + 4)(x - 4)$
b $x^2 - 36 = x^2 - 6^2 = (x + 6)(x - 6)$
c $x^2 - 81 = x^2 - 9^2 = (x + 9)(x - 9)$
d $y^2 - 100 = y^2 - 10^2 = (y + 10)(y - 10)$

Substituting into expressions

1 When $a = 3$ and $b = -2$,
$5a + 2b = 5 \times 3 + 2 \times (-2) = 15 + (-4) = 11$
2 **a** $2 - 2 \times (-4) = 2 - (-8) = 10$
b $3 \times 2 \times (-4) = -24$
c $4 \times (-4) - 3 \times 2 = -16 - 6 = -22$
d $2^2 + (-4)^2 = 4 + 16 = 20$
e $2 \times 2 + 4(2 - (-4)) = 2 \times 2 + 4 \times 6 = 4 + 24 = 28$
f $\frac{1}{2}(2 + (-4)) = \frac{1}{2} \times -2 = -1$

3 false
When $a = 3$: $3a^2 = 3 \times 3^2 = 3 \times 9 = 27$
4 When $p = \frac{1}{2}$ and $q = -4$,

a $10pq = 10 \times \frac{1}{2} \times (-4) = -20$
b $8p^2 = 8 \times \left(\frac{1}{2}\right)^2 = 8 \times \frac{1}{4} = 2$
c $\frac{q}{p} = \frac{-4}{\frac{1}{2}} = -4 \times 2 = -8$

d $2q^2 - 12p = 2 \times (-4)^2 - 12 \times \frac{1}{2}$

$= 2 \times 16 - 12 \times \frac{1}{2}$

$= 32 - 6$

$= 26$

5 When $d = 7$, $e = -3$ and $f = 10$,

$\frac{d(e-2)}{f} = \frac{7 \times (-3 - 2)}{10}$

$= \frac{7 \times (-5)}{10}$

$= \frac{-35}{10}$

$= -3.5$

Writing expressions

Stretch it!

a perimeter $= x + 2 + 2x + 4 + 5x - 2 = 8x + 4$

b perimeter of square = perimeter of triangle

$= 8x + 4$

$= 4(2x + 1)$

side of square $= \frac{4(2x+1)}{4} = 2x + 1$

1 **a** $4 - q$ **b** $n + m$ (or $m + n$)

c xy (or yx) **d** p^2

2 $x + y$

3 $\frac{y}{8}$

4 $100n + 75b$

5 Perimeter $= 3a + 2a + 4 + 4a - 2 = 9a + 2$

6 Area $= \frac{1}{2} \times 4 \times (2x + 5)$

$= 2 \times (2x + 5)$

$= 4x + 10$

Solving linear equations

1 **a** $5a = 35$

$a = \frac{35}{5}$

$a = 7$

b $b - 9 = 8$

$b = 8 + 9$

$b = 17$

c $\frac{c}{4} = 4$

$c = 4 \times 4$

$c = 16$

d $d + 4 = 2$

$d = 2 - 4$

$d = -2$

2 **a** $2x + 3 = 13$

$2x = 10$

$x = 5$

b $3y - 4 = 11$

$3y = 15$

$y = 5$

c $2p + 9 = 1$

$2p = -8$

$p = -4$

d $\frac{f}{3} - 7 = 4$

$\frac{f}{3} = 11$

$f = 33$

e $\frac{x+5}{2} = 8$

$x + 5 = 16$

$x = 11$

f $\frac{f-7}{3} = 4$

$f - 7 = 12$

$f = 19$

3 **a** $9 - m = 7$

$9 = 7 + m$

$m = 2$

b $10 - 3x = 1$

$10 = 1 + 3x$

$9 = 3x$

$3 = x$

c $7 - 2x = 2$

$7 = 2 + 2x$

$5 = 2x$

$x = \frac{5}{2}$

(**or** $x = 2.5$, **or** $x = 2\frac{1}{2}$)

d $5 = 1 - 2f$

$5 + 2f = 1$

$2f = -4$

$f = -2$

4 Hannah has not subtracted 4 from *both* sides.

Correct working:

$2x + 4 = 8$

$2x = 4$

$x = 2$

5 **a** $3(a + 2) = 15$

$3a + 6 = 15$

$3a = 9$

$a = 3$

b $4(b - 2) = 4$

$4b - 8 = 4$

$4b = 12$

$b = 3$

c $3(4c - 9) = 9$

$12c - 27 = 9$

$12c = 36$

$c = 3$

d $2(d + 3) + 4 = 2$

$2d + 6 + 4 = 2$

$2d + 10 = 2$

$2d = -8$

$d = -4$

e $4(2x + 3) - 2 = 6$

$8x + 12 - 2 = 6$

$8x + 10 = 6$

$8x = -4$

$x = -\frac{4}{8} = -\frac{1}{2}$

(**or** $x = -0.5$)

6 **a** $3m = m + 6$
$2m = 6$
$m = 3$

b $5t - 6 = 2t + 3$
$3t - 6 = 3$
$3t = 9$
$t = 3$

c $4x + 3 = 2x + 8$
$2x + 3 = 8$
$2x = 5$
$x = \frac{5}{2}$
(Or $x = 2.5$ or $x = 2\frac{1}{2}$)

d $3 - 2p = 6 - 3p$
$3 + p = 6$
$p = 3$

e $3y - 8 = 5y + 4$
$-8 = 2y + 4$
$-12 = 2y$
$-6 = y$

7 **a** $2(x + 5) = x + 6$
$2x + 10 = x + 6$
$x + 10 = 6$
$x = -4$

b $5(a - 1) = 4 - a$
$5a - 5 = 4 - a$
$6a - 5 = 4$
$6a = 9$
$a = \frac{9}{6} = \frac{3}{2}$ (or 1.5)

c $7b - 2 = 2(b + 4)$
$7b - 2 = 2b + 8$
$5b - 2 = 8$
$5b = 10$
$b = 2$

d $4(2y + 1) = 3(5y - 1)$
$8y + 4 = 15y - 3$
$4 = 7y - 3$
$7 = 7y$
$y = 1$

e $2x - 1 = 8 - 4x$
$6x - 1 = 8$
$6x = 9$
$x = \frac{9}{6} = \frac{3}{2}$
(Or $x = 1.5$ or $x = 1\frac{1}{2}$)

Writing linear equations

Stretch it!

$3(x + 3) = 5x - 12$
$3x + 9 = 5x - 12$
$12 + 9 = 5x - 3x$
$21 = 2x$
$x = \frac{21}{2} = 10.5$ cm

1 **a** Perimeter $= 4 \times (2s + 3) = 8s + 12$
(Or, Perimeter $= 2s + 3 + 2s + 3 + 2s + 3 + 2s + 3 = 8s + 12$)

b $8s + 12 = 84$
$8s = 72$
$s = 9$ cm

2 **a** Angles in a quadrilateral add up to 360° so:
$x + 20 + 2x - 15 + x + 65 + 2x - 10 = 360$
$6x + 60 = 360$
$6x = 300$
$x = 50$

b Largest angle: $x + 65 = 50 + 65 = 115°$
(Other angles: $x + 20 = 50 + 20 = 70°$;
$2x - 15 = 2 \times 50 - 15 = 85°$;
$2x - 10 = 2 \times 50 - 10 = 90°$)

3 Let a = Karen's age
Monica is 4 years younger: $a - 4$
$a + a - 4 = 64$
$2a - 4 = 64$
$2a = 68$
$a = 34$
Karen is 34 years old.
$a - 4 = 34 - 4 = 30$
Monica is 30 years old.

4 Let n = number.
$2n + 4 = 16 - n$
$3n + 4 = 16$
$3n = 12$
$n = 4$
The number is 4.

5 Let l = length of rectangle.
Width is 2 cm smaller: $l - 2$
Perimeter $= 2l + 2(l - 2)$
$= 2l + 2l - 4 = 4l - 4$
$4l - 4 = 36$
$4l = 40$
$l = 10$
Length is 10 cm.
$l - 2 = 10 - 2 = 8$
Width is 8 cm.

6 Base angles of an isosceles triangle are equal so:
$4a - 20 = 2a + 16$
$2a - 20 = 16$
$2a = 36$
$a = 18$
When $a = 18$: $4a - 20 = 4 \times 18 - 20 = 52$
So $2a + 16 = 52$
Angles in a triangle add up to 180° so:
$4b - 2a + 52 + 52 = 180$
$4b - 2a + 104 = 180$
$4b - 2a = 76$ (Substitute a = 18)
$4b - 2 \times 18 = 76$
$4b - 36 = 76$
$4b = 112$
$b = 28$

Linear inequalities

1 **a** $x = 3, 4, 5$

b $x = 2, 3, 4, 5$

c $x = 0, 1, 2, 3$

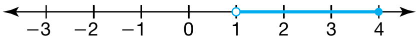
d $x = -3, -2, -1, 0, 1$

2 **a** $x < 3$ **b** $x \geq -2$

c $-1 \leq x \leq 5$

3 **a**

b



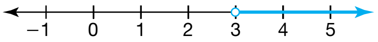
c

d

4 **a** $2x - 2 > 4$

$2x > 6$

$x > 3$



b $4x + 3 \leq 13$

$4x \leq 10$

$x \leq \frac{10}{4} = \frac{5}{2}$

(**or** $x \leq 2.5$ or $x \leq 2\frac{1}{2}$)

c $4x < 2x - 10$

$2x < -10$

$x < -5$

d $7x + 2 \geq 3x - 2$

$4x + 2 \geq -2$

$4x \geq -4$

$x \geq -1$

5 Olivia has not multiplied all the terms in the bracket by the term outside.

Correct working:

$3(x + 4) > 22$

$3x + 12 > 22$

$3x > 10$

$x > \frac{10}{3}$ (**or** $x > 3\frac{1}{3}$)

6 **a** $-12 < 4x \leq 8$

$-3 < x \leq 2$

$x = -2, -1, 0, 1, 2$

b $-8 \leq 2x < 14$

$-4 \leq x < 7$

$x = -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6$

c $-6 < 6x \leq 18$

$-1 < x \leq 3$

$x = 0, 1, 2, 3$

d $9 \leq 3n \leq 15$

$3 \leq n \leq 5$

$n = 3, 4, 5$

7 **a** $2x + 3 < 4x + 8$

$3 < 2x + 8$

$-5 < 2x$

$-\frac{5}{2} < x$ or $x > -\frac{5}{2}$ (**or** $x > -2.5$)

b Smallest integer value of x that satisfies the inequality is -2.

8 **a** $4 - x \leq 1$

$4 \leq x + 1$

$3 \leq x$ (**or** $x \geq 3$)

Alternative method:

$4 - x \leq 1$

$-x \leq -3$

$x \geq 3$

b $6 - 3x > 9$

$6 > 3x + 9$

$-3 > 3x$

$-1 > x$ (**or** $x < -1$)

Alternative method:

$6 - 3x > 9$

$-3x > 3$

$x < -1$

c $8 - 2x \geq 7$

$8 \geq 2x + 7$

$1 \geq 2x$

$\frac{1}{2} \geq x$ (**or** $x \leq \frac{1}{2}$)

Alternative method:

$8 - 2x \geq 7$

$-2x \geq -1$

$x \leq \frac{1}{2}$

d $-2 < -x \leq 3$

$2 > x \geq -3$

(**or** $-3 \leq x < 2$)

Formulae

Stretch it!

$4a = 5(2b^2 - a)$

$4a = 10b^2 - 5a$

$9a = 10b^2$

$\frac{9a}{10} = b^2$

$b = \sqrt{\frac{9a}{10}}$

1 Pay $= 8 \times 35 + 25 = 280 + 25 = 305$

£305

2 $P = 2(8 + 5.5) = 2 \times 13.5 = 27$

3 $F = \frac{9}{5} \times 45 + 32$

$= 81 + 32 = 113$

113 °F

4 $v = 10 + (-20) \times 5 = 10 + (-100) = -90$

5 $v^2 = 2.5^2 + 2 \times -9.8 \times 0.2$

$v^2 = 6.25 - 3.92$

$v^2 = 2.33$

$v = \sqrt{2.33} = 1.5$ (to 1 d.p.)

6 $C = 25d + 50$

7 **a** Distance in kilometres = $\frac{8}{5}$ × distance in miles

$k = \frac{8}{5}m$

b $k = \frac{8}{5} \times 200$

$= 320$ km

8 **a** $A = l \times l$ so

$A = l^2$

9 **a** $P = 2a + 2(a + 3) = 2a + 2a + 6 = 4a + 6$

(**or** $P = a + a + a + 3 + a + 3 = 4a + 6$)

b $P = 4 \times 6 + 6 = 24 + 6 = 30$

$P = 30$ cm

10 $-10 = \frac{D}{6.5}$

$D = -65$

11 **a** $v = u + at$

$v - u = at$

$\frac{v - u}{t} = a$

b $V = \frac{1}{3}Ah$

$3V = Ah$

$\frac{3V}{A} = h$

c $y = 3(x - 3)$

$y = 3x - 9$

$y + 9 = 3x$

$\frac{y + 9}{3} = x$

(**or** $x = \frac{y}{3} + 3$)

d $v^2 = u^2 + 2as$

$v^2 - u^2 = 2as$

$\frac{v^2 - u^2}{2a} = s$

e $s = \frac{1}{2}at^2$

$2s = at^2$

$\frac{2s}{a} = t^2$

$t = \sqrt{\frac{2s}{a}}$

f $T = \sqrt{\frac{2s}{g}}$

$T^2 = \frac{2s}{g}$

$gT^2 = 2s$

$g = \frac{2s}{T^2}$

g $4ax = 3 + a$

$4ax - a = 3$

$a(4x - 1) = 3$

$a = \frac{3}{4x - 1}$

Linear sequences

1 **a** **i** 2, 5, 8, 11, **14**, **17** (rule is ‘add 3’)

ii 23, 19, 15, 11, **7**, **3** (rule is ‘subtract 4’)

iii 3, 9, 15, 21, **27**, **33** (rule is ‘add 6’)

iv 4, 9, 14, 19, **24**, **29** (rule is ‘add 5’)

b **i** 2, 5, 8, 11, 14, 17, 20, 23, 26, **29**

(**or**, 10th term = $2 + (3 \times 9) = 29$)

ii 23, 19, 15, 11, 7, 3, −1, −5, −9, **−13**

(**or**, 10th term = $23 - (4 \times 9) = -13$)

iii 3, 9, 15, 21, 27, 33, 39, 45, 51, **57**

(**or**, 10th term = $3 + (6 \times 9) = 57$)

iv 4, 9, 14, 19, 24, 29, 34, 39, 44, **49**

(**or**, 10th term = $4 + (5 \times 9) = 49$)

2 **a** 1st term = $1 \times 4 - 2 = 2$

2nd term = $2 \times 4 - 2 = 6$

3rd term = $3 \times 4 - 2 = 10$

4th term = $4 \times 4 - 2 = 14$

b 20th term = $20 \times 4 - 2 = 78$

3 **a** Common difference is 7. Hence the term-to-term rule is add 7, so:

$-11 + 7 = -4$

$-4 + 7 = 3$

$3 + 7 = 10$

3, 10

b $15 - 2n < 0$

$-2n < -15$

$-n < -7.5$

$n > 7.5$

n is an integer.

8th term:

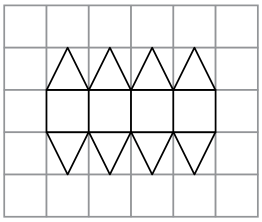
$15 - 2 \times 8 = -1$

Compare 7th term:

$15 - 2 \times 7 = 1$

So **8th term** is the first term with a negative value.

4 **a**



b Number of triangles: 2, 4, 6, 8, 10, 12, 14, 16

So there are 16 triangles in pattern number 8.

Or, $2 + 7 \times 2 = 16$ triangles

c No. The number of triangles forms an even number sequence and 35 is odd.

5 **a** 3, 7, 11, 15, 19

Common difference = +4

4 × term number = 4, 8, 12, 16, 20

−1 to get each term in the original sequence

So, nth term is $4n-1$

b If 99 is in the sequence then n will be an integer and:

$4n - 1 = 99$

$4n = 100$

$n = 25$

Yes, 99 is a term in the sequence because 25 is an integer.

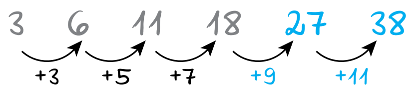
Non-linear sequences

1 1, 3, 5, 7, 9, … Arithmetic sequence (term-to-term rule is ‘add 2’)

1, 2, 4, 8, 16, … Geometric sequence (term-to-term rule is ‘multiply by 2’, or ‘double’)

1, 4, 5, 9, 14, … Fibonacci-type sequence (next term of sequence is found by adding the previous two terms together)

1, 4, 9, 16, 25, … Square-number sequence (sequence of square numbers: 1^2, 2^2, 3^2, 4^2,…)

2 

27, 38

3 **a** 4, 2, 1, $\mathbf{\frac{1}{2}}$, $\mathbf{\frac{1}{4}}$ rule is ‘divide by 2’

b 5, 0.5, 0.05, **0.005**, **0.0005** rule is ‘divide by 10’

c $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\mathbf{\frac{1}{16}}$, $\mathbf{\frac{1}{32}}$ rule is ‘divide by 2’

d $\frac{1}{9}$, $\frac{1}{3}$, 1, **3**, **9** rule is ‘multiply by 3’

e −0.1, −0.2, −0.4, **−0.8**, **−1.6** rule is ‘multiply by 2’

f 3, −6, 12, **−24**, **48** rule is ‘multiply by −2’

4 4th term = 6

5th term = 10

6th term = 6 + 10 = 16

7th term = 10 + 16 = 26

8th term = 16 + 26 = **42**

5 1st term = $1^2 + 5 = 6$

2nd term = $2^2 + 5 = 9$

3rd term = $3^2 + 5 = 14$

4th term = $4^2 + 5 = 21$

6, 9, 14, 21

6 When $n = 5$:

$3 \times 5^2 - 4 = 3 \times 25 - 4 = 71$

7 1st term = $1^2 + 2 \times 1 = 1 + 2 = 3$

2nd term = $2^2 + 2 \times 2 = 4 + 4 = 8$

3rd term = $3^2 + 2 \times 3 = 9 + 6 = 15$

3, 8, 15

8 **a** 1st term = a

2nd term = b

3rd term = $a + b$

4th term = $b + a + b = a + 2b$

5th term = $a + b + a + 2b = 2a + 3b$

b $b = 5$

$2a + 3b = 23$ (Substitute $b = 5$)

$2a + 3 \times 5 = 23$

$2a + 15 = 23$

$2a = 8$

$a = 4$

Show that…

Stretch it! If n is even, $n - 1$ is odd and $n + 1$ is odd. If you multiply two odd numbers the answer will always be odd. If m is odd, $m - 1$ is even and $m + 1$ is even. If you multiply two even numbers the answer will always be even.
or: $(n + 1)(n - 1) = n^2 - 1$ and $(m + 1)(m - 1) = m^2 - 1$
n^2 will be even × even = even, so $n^2 - 1$ will be odd.
m^2 will be odd × odd = odd, so $m^2 - 1$ will be even.

1 **a** 3 + 5 = 8 (or any other two primes except 2)

b Mo is not correct.

Let $\frac{n}{2} = x$. If x is even, then $x + 1$ is odd, and $2(x + 1)$ is even. Therefore, $\frac{n}{2}$ is not always even when n is even.

2 **a** LHS = $(x + 2)(x - 2) \equiv x^2 - 2x + 2x - 4 = x^2 - 4$

RHS = $x^2 - 4$

LHS $\equiv$ RHS

So $(x + 2)(x - 2) \equiv x^2 - 4$

b LHS = $(x - 3)^2 \equiv (x - 3)(x - 3)$

$= x^2 - 6x + 9$

RHS = $x^2 - 6x + 9$

LHS $\equiv$ RHS

So $(x - 3)^2 \equiv x^2 - 6x + 9$

c LHS = $(x + 1)^2 + 4 \equiv (x + 1)(x + 1) + 4$

$= x^2 + 2x + 1 + 4 = x^2 + 2x + 5$

RHS = $x^2 + 2x + 5$

LHS $\equiv$ RHS

So $(x + 1)^2 + 4 \equiv x^2 + 2x + 5$

d LHS = $6(a - 3) - 2(2a - 5) + 6$

$= 6a - 18 - 4a + 10 + 6 = 2a - 2$

RHS = $2(a - 1) = 2a - 2$

LHS $\equiv$ RHS

So $6(a - 3) - 2(2a - 5) + 6 \equiv 2(a - 1)$

e LHS = $4(x - 3) + 2(x + 5) = 4x - 12 + 2x + 10$

$= 6x - 2$

RHS = $3(2x - 1) + 1 = 6x - 3 + 1 = 6x - 2$

LHS $\equiv$ RHS

So $4(x - 3) + 2(x + 5) \equiv 3(2x - 1) + 1$

3 $4(ax - 2) + 5(3x + b) \equiv 23x - 3$

LHS = $4ax - 8 + 15x + 5b = (4a + 15)x + (5b - 8)$

Given that RHS $\equiv$ LHS,

$(4a + 15)x \equiv 23x$

so

$4a + 15 = 23$

$4a = 8$

$a = 2$

and $5b - 8 = -3$

$5b = 5$

$b = 1$

4 rod A = n

rod B = $n + 1$

rod C = $n + 2$

rod A + rod C = $n + n + 2$

$= 2n + 2$

$= 2(n + 1)$

This is 2 times the length of rod B.

Functions

1 **a** $y = 2 \times 2 - 4 = 0$

b $y = 1 \times 2 - 4 = -2$

c $y = -4 \times 2 - 4 = -12$

2 **a**

×1 → +2	
x	y
−2	**0**
0	**2**
2	4

b

$\times 2 \rightarrow -3$	
x	y
-2	$\mathbf{-7}$
0	-3
2	$\mathbf{1}$

3 **a** $y = 3x - 3$

b $10 \times 3 - 3 = 30 - 3 = 27$

c $y = 3x - 3$

$y + 3 = 3x$

$x = \frac{y + 3}{3}$

d If $x = y$ then $x = 3x - 3$

$2x = 3$, $x = \frac{3}{2}$ (or 1.5)

Substituting $x = 1.5$ into the function,

$y = 1.5 \times 3 - 3 = 1.5 = \frac{3}{2}$

So x and y can be equal.

Coordinates and midpoints

Stretch it!

Difference in x coordinates of *A* and *B* is equal to difference in x coordinates of *B* and *C*.

Difference $= 2 - (-1) = 3$

So x coordinate of *C* is:

$-1 - 3 = -4$

Difference in y coordinates of *A* and *B* is equal to difference in y coordinates of *B* and *C*.

Difference $= 4 - 3 = 1$

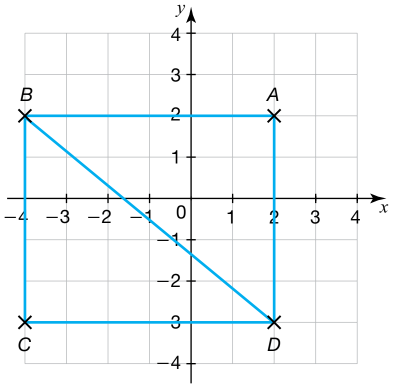
So y coordinate of *C* is:

$3 - 1 = 2$

C has coordinates $(-4, 2)$.

1 **a** $(2, 2)$

b, **c** and **d**



d *B* is $(-4, 2)$, *D* is $(2, -3)$

x coordinate of midpoint: $\frac{2 + (-4)}{2} = -1$

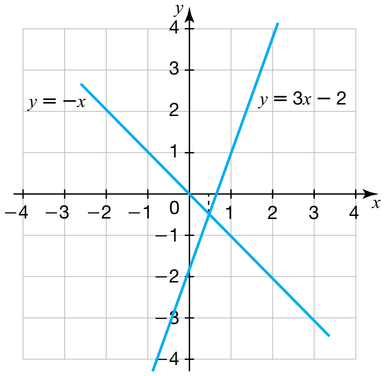
y coordinate of midpoint: $\frac{-3 + 2}{2} = -0.5$

Midpoint is $(-1, -0.5)$

Straight-line graphs

Stretch it!

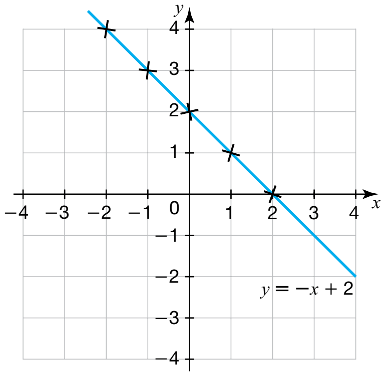
To solve the equation, you need to find where the graph of $y = 3x - 2$ intersects the graph of $y = -x$.



So the solution to $3x - 2 = -x$ is $x = 0.5$.

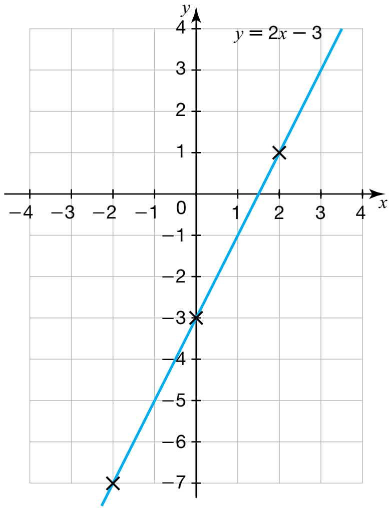
1

x	-2	-1	0	1	2
y	4	3	2	1	0

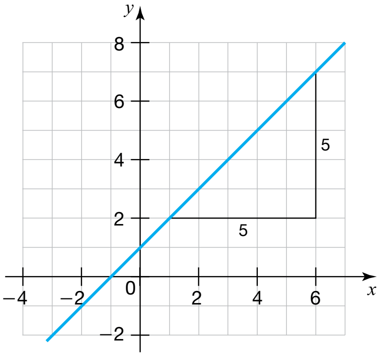


2

x	-2	0	2
y	-7	-3	1



3



Gradient, $m = \frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}} = \frac{5}{5} = 1$

y-intercept, $c = 1$

Using the general form of the equation of a line $y = mx + c$: the equation of the line is $y = x + 1$

4 Gradient, $m = 2$

$y = mx + c$ (General form of the equation of a line)

$y = 2x + c$

For point (1, −2), $x = 1$, $y = -2$:

$-2 = 2 \times 1 + c$

$-2 = 2 + c$

$-4 = c$

So the equation of the line is $y = 2x - 4$

5 Gradient $= \frac{\text{difference in } y \text{ coordinate}}{\text{difference in } x \text{ coordinate}} = \frac{4-2}{0-4} = -\frac{2}{4} = -\frac{1}{2}$

Gradient, $m = -\frac{1}{2}$

Given the point (0, 4):

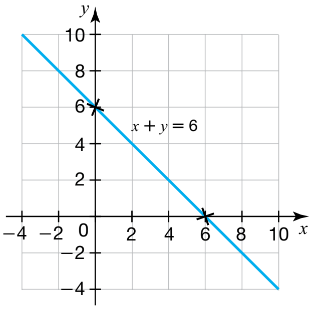
$4 = -\frac{1}{2}(0) + c$

$4 = c$ (y-intercept)

So the equation of the line is $y = -\frac{1}{2}x + 4$.

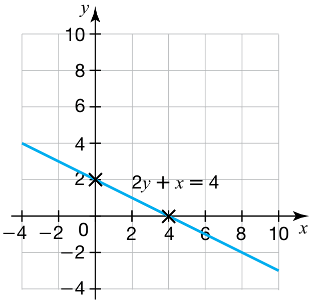
6 **a**

x	0	6
y	6	0



b

x	0	4
y	2	0



7 A: $y = 4x + 1$

B: $4x + 4y = 4$

$4y = -4x + 4$

$y = -x + 1$

C: $x - 2y = 2$

$-2y = -x + 2$

$y = \frac{x}{2} - 1$

D: $2y = 4 + 8x$

$y = 4x + 2$

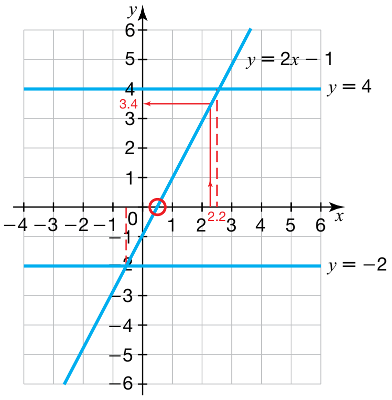
Lines A and D both have the same m value (4) so they are parallel.

8 $2y = x - 4$

$y = \frac{x}{2} - 2$

The y-intercept is (0, −2).

9



a $y = 3.4$ **b** $x = 0.5$

c $x = 2.5$ **d** $x = -0.5$

Solving simultaneous equations

1 $x + y = 16$ (1)

$x - y = 5$ (2)

(1) + (2):

$2x = 21$

$x = \frac{21}{2} = 10\frac{1}{2}$ (or 10.5)

Substitute $x = 10\frac{1}{2}$ into (2):

$10\frac{1}{2} - y = 5$

$y = 10\frac{1}{2} - 5$

$y = 5\frac{1}{2}$ (or 5.5)

Solution: $x = 10.5$, $y = 5.5$

2 **a** $2x + y = 4$ (1)

$3x - y = 1$ (2)

(1) + (2): $5x = 5$

$x = 1$

Substitute $x = 1$ in (1)

$2 \times 1 + y = 4$

$2 + y = 4$

$y = 2$

Solution: $x = 1$, $y = 2$

b $x - y = 5$ (1)

$2x + y = 4$ (2)

(1) + (2): $3x = 9$

$x = 3$

Substitute $x = 3$ into (1)

$3 - y = 5$

$3 = y + 5$

$y = -2$

Solution: $x = 3$, $y = -2$

c $2x + y = 8$ (1)

$x + y = 2$ (2)

(1) − (2): $x = 6$

Substitute $x = 6$ into (2)

$6 + y = 2$

$y = -4$

Solution: $x = 6$, $y = -4$

d $4x - y = 10$ (1)

$x + 2y = 7$ (2)

(1) × 2: $8x - 2y = 20$ (3)

(2) + (3): $9x = 27$

$x = 3$

Substitute $x = 3$ into (2):

$3 + 2y = 7$

$2y = 4$

$y = 2$

Solution: $x = 3$, $y = 2$

e $2x + y = 7$ (1)

$x - 4y = 8$ (2)

(1) × 4: $8x + 4y = 28$ (3)

(2) + (3): $9x = 36$

$x = 4$

Substitute $x = 4$ into (1):

$2 \times 4 + y = 7$

$8 + y = 7$

$y = -1$

Solution: $x = 4$, $y = -1$

f $2x + 3y = 7$ (1)

$3x - 2y = 4$ (2)

(1) × 2: $4x + 6y = 14$ (3)

(2) × 3: $9x - 6y = 12$ (4)

(3) + (4): $13x = 26$

$x = 2$

Substitute $x = 2$ into (1)

$2 \times 2 + 3y = 7$

$4 + 3y = 7$

$3y = 3$

$y = 1$

Solution: $x = 2$, $y = 1$

3 $x + y = 21$ (1)

$x - y = 7$ (2)

(1) + (2): $2x = 28$

$x = 14$

Substitute $x = 14$ into (1)

$14 + y = 21$

$y = 7$

The two numbers are 7 and 14.

4 Let b = burger and c = cola.

$3b + 2c = 505$ (1)

$3b + 4c = 725$ (2)

(2) − (1): $2c = 220$

$c = 110$

Substitute $c = 110$ into (1)

$3b + 2 \times 110 = 505$

$3b + 220 = 505$

$3b = 285$

$b = 95$

A burger costs 95p.

A cola costs £1.10.

5 $2a + 4c = 60$ (1)

$3a + 3c = 82.50$ (2)

(1) × 3 : $6a + 12c = 180$ (3)

(2) × 2 : $6a + 6c = 165$ (4)

(3) − (4) : $6c = 15$

$c = \frac{15}{6} = 2\frac{3}{6} = 2.5$

Substitute $c = 2.5$ into (1): $2a + 4 \times 2.5 = 60$

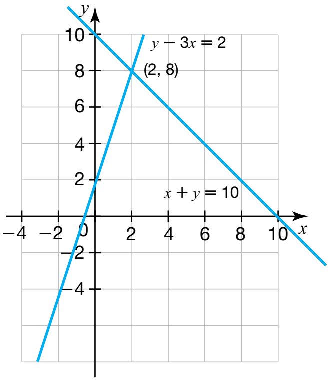
$2a + 10 = 60$

$2a = 50$

$a = 25$

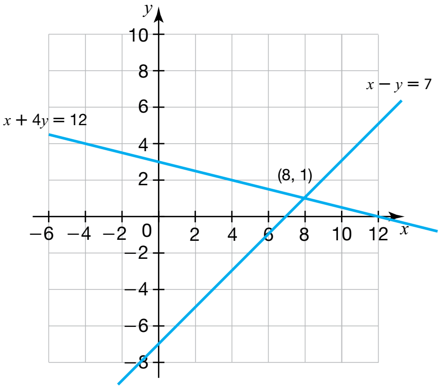
Adult ticket: £25 Child ticket: £2.50

6 **a**



$x = 2$, $y = 8$

b



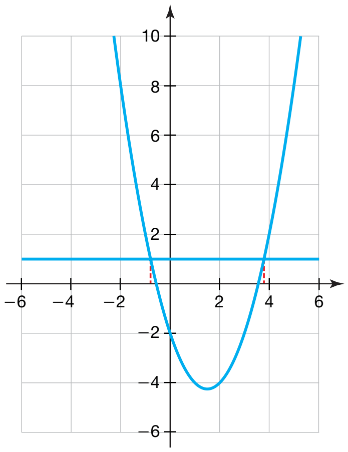
$x = 8$, $y = 1$

Quadratic graphs

Stretch it!

Rearrange $x^2 - 3x = 3$, to give $x^2 - 3x - 2 = 1$

You can solve this graphically by finding where the lines $y = x^2 - 3x - 2$ and $y = 1$ intersect.



So the solutions to the equation $x^2 - 3x = 3$ are $x = 3.8$ and $x = -0.8$. Acceptable readings from the graph would be in the range 3.6 to 3.9 and −0.6 to −0.9.

Stretch it!

a At points *A* and *B*, $y = 0$

Therefore, $(x - 2)(x - 4) = 0$

Either: $x - 2 = 0$

$x = 2$

or: $x - 4 = 0$

$x = 4$

Coordinates are *A*(2, 0) and *B*(4, 0)

b At point *C*, $x = 0$

$y = (0 - 2)(0 - 4)$

$= -2 \times -4 = 8$

Coordinates are *C*(0, 8)

c x coordinate of *D* is the midpoint of the x coordinates of *A* and *B*:

$\frac{2 + 4}{2} = 3$

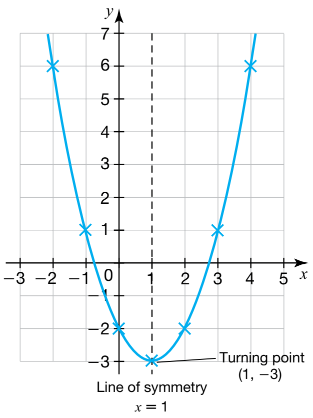
y coordinate $= (3 - 2)(3 - 4)$

$= 1 \times -1 = -1$

Coordinates are $D(3, -1)$

1 **a**

x	−2	−1	0	1	2	3	4
y	6	1	−2	−3	−2	1	6

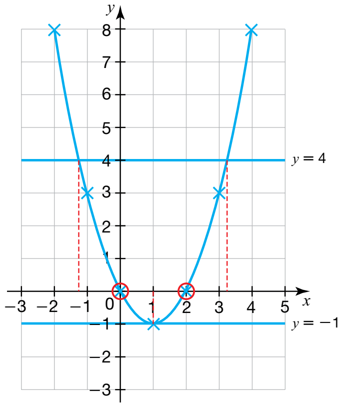


b **i** $x = 1$

ii (1, −3)

2

x	−2	−1	0	1	2	3	4
y	8	3	0	−1	0	3	8



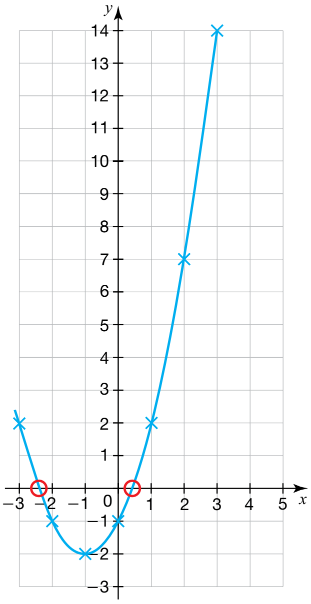
a $x = 0$ and $x = 2$

b $x \approx -1.2$ and $x \approx 3.2$

c $x = 1$

3 **a** and **b**

x	−3	−2	−1	0	1	2	3
y	2	−1	−2	−1	2	7	14



Read off the values of x where the graph cuts the line $y = 0$ (the x-axis).

$x \approx -2.4$ and $x \approx 0.4$

Solving quadratic equations

Stretch it!

$\frac{x^2}{2} = 8$

$x^2 = 16$

$x = \sqrt{16}$

So $x = 4$ or $x = -4$

$2x^2 = 50$

$x^2 = 25$

$x = \sqrt{25}$

So $x = 5$ or $x = -5$

Stretch it!

$x(x + 6) = 40$

$x^2 + 6x = 40$

$x^2 + 6x - 40 = 0$

$(x + 10)(x - 4) = 0$

Either $x + 10 = 0$ or $x - 4 = 0$

x must be positive as it is a length, therefore, $x = 4$ cm

1 **a** $x^2 - 4x = 0$

$x(x - 4) = 0$

Either $x = 0$ or $x - 4 = 0$

$x = 4$

So $x = 0$ or $x = 4$

b $x^2 + 7x = 0$

$x(x + 7) = 0$

Either $x = 0$ or $x + 7 = 0$

$x = -7$

So $x = 0$ or $x = -7$

c $x^2 - 16 = 0$ ($x^2 - 16 = x^2 - 4^2$, Factorise)

$(x + 4)(x - 4) = 0$

Either $x + 4 = 0$ or $x - 4 = 0$

$x = -4$ $\quad x = 4$

So $x = -4$ or $x = 4$

d $x^2 + 10x + 9 = 0$

$(x + 1)(x + 9) = 0$

Either $x + 1 = 0$ or $x + 9 = 0$

$x = -1$ $\quad x = -9$

So $x = -1$ or $x = -9$

e $x^2 + x - 12 = 0$

$(x - 3)(x + 4) = 0$

Either $x - 3 = 0$ or $x + 4 = 0$

$x = 3$ $\quad x = -4$

So $x = 3$ or $x = -4$

f $x^2 - 6x - 16 = 0$

$(x + 2)(x - 8) = 0$

Either $x + 2 = 0$ or $x - 8 = 0$

$x = -2$ $\quad x = 8$

So $x = -2$ or $x = 8$

2 **a** $y = x^2 - 49$ (Set $y = 0$)

$x^2 - 49 = 0$ ($x^2 - 49 = x^2 - 7^2$, Factorise)

$(x + 7)(x - 7) = 0$

Either $x + 7 = 0$ or $x - 7 = 0$

$x = -7$ $\quad x = 7$

So $x = -7$ or $x = 7$

b $y = x^2 - 3x$ (Set $y = 0$)

$x^2 - 3x = 0$

$x(x - 3) = 0$

Either $x = 0$ or $x - 3 = 0$

$x = 3$

So $x = 0$ or $x = 3$

c $y = x^2 + 7x + 6$ (Set $y = 0$)

$x^2 + 7x + 6 = 0$

$(x + 1)(x + 6) = 0$

Either $x + 1 = 0$ or $x + 6 = 0$

$x = -1$ $\quad x = -6$

So $x = -1$ or $x = -6$

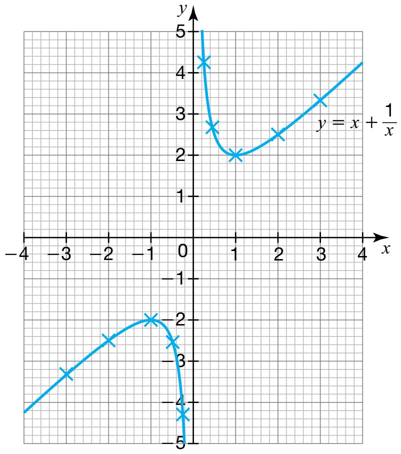
Cubic and reciprocal graphs

Stretch it!

a

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3
y	**$-3\frac{1}{3}$**	**$-2\frac{1}{2}$**	-2	**$-2\frac{1}{2}$**	$-4\frac{1}{4}$	**$4\frac{1}{4}$**	**$2\frac{1}{2}$**	**2**	**$2\frac{1}{2}$**	$3\frac{1}{3}$

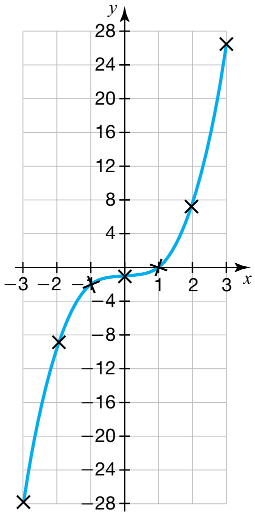
b



1 **a**

x	-3	-2	-1	0	1	2	3
y	-28	-9	-2	-1	0	7	26

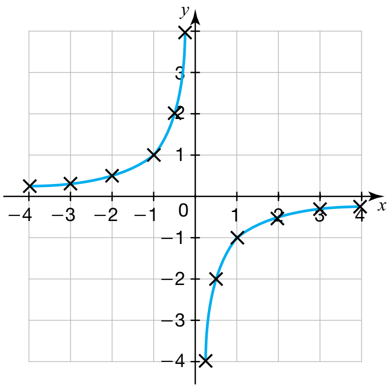
b



2 **a**

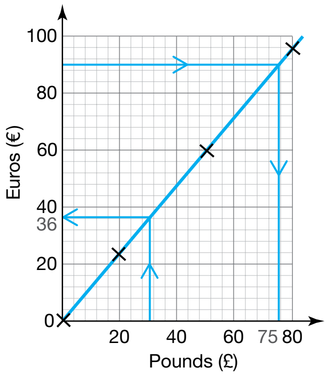
x	-4	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4
y	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	4	-4	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$

b



Drawing and interpreting real-life graphs

1 a



b The graph is a straight line with a positive gradient. As the number of pounds steadily increases, the corresponding number of euros steadily increases. This is direct proportion.

c See lines drawn on graph.

i €36 **ii** £75

d From the graph: £30 = €36

So £90 = €36 × 3 = €108

The ring is cheaper in France.

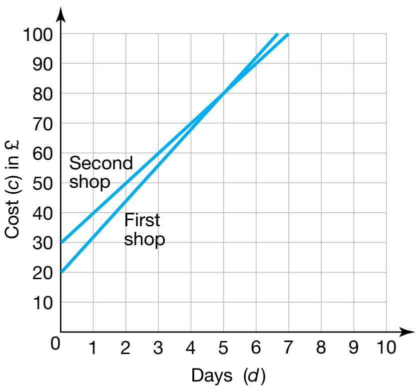
2 a Monthly charge = £10 (cost of 0 minutes from the graph)

b Gradient = $\frac{30}{240}$ = 0.125

Charge per minute of calls is 13p.

3 a and c

d	0	1	2	3	4
C	20	32	44	56	68



b This is the flat rate that you pay just for hiring the sander, before you pay for the number of days. It is the intercept with the vertical axis:

days (d) = 0

cost (C) = £20

c Using a graphical method: plot the second equation, $C = 10d + 30$, on the same axes. The line for the second shop has a lower gradient, and after the lines cross over (at $d = 5$), the second shop is cheaper. So you would use the second shop.

Alternatively, using an algebraic method:

Let $d = 6$ days (more than 5 days)
First shop: $C = 12 \times 6 + 20 = 92$
Second shop: $C = 10 \times 6 + 30 = 90$
To hire the sander for more than 5 days use the second shop as it is cheaper.

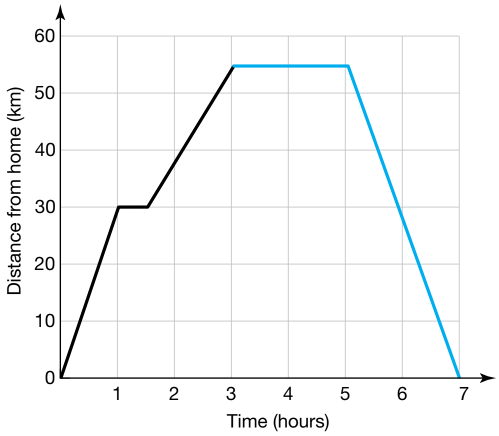
4 a 30 minutes (Horizontal line on graph)

b 55 km

c Speed before break = $\frac{\text{distance (km)}}{\text{time (hours)}} = \frac{30}{1} = 30$ km/hr

Speed after break = $\frac{\text{distance (km)}}{\text{time (hours)}} = \frac{25}{1.5} = 16.7$ km/hr

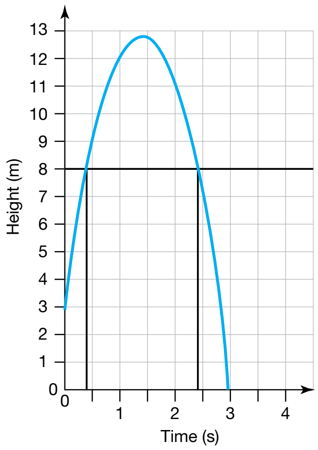
d



5 a Reading off maximum height value from graph: **12.8 m**.

b Reading from the graph, the ball is thrown at time = 0 seconds and returns to the ground at time = **3 seconds**.

c Draw a horizontal line on the graph at height = 8 m.



0.4 seconds and 2.4* seconds

d The ball is thrown from a height of 3 m above the ground.

*This answer differs from the one in the Revision and Exam Practice Book due to an error in our first edition. This answer has now been re-checked and corrected.

6 **a** 6 m/s

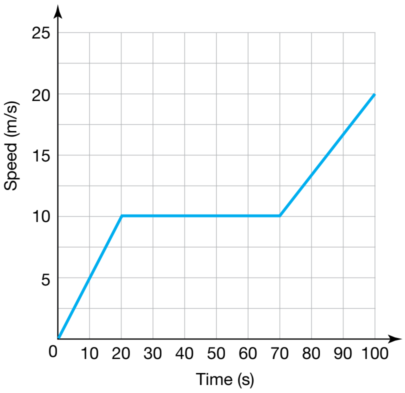
b 4 seconds

c 6 seconds

d acceleration $= \frac{\text{change in speed (m/s)}}{\text{time (s)}} = \frac{6}{4}$

$= 1.5$ m/s^2

7 **a**



b Acceleration $= \frac{\text{change in speed (m/s)}}{\text{time (s)}} = \frac{20-10}{30} = \frac{10}{30}$

$= 0.\dot{3}$ m/s^2

8 **a** The maximum depth of water in the bath before the person got in was 35 cm.

b Between C and D, the person was taking their bath.

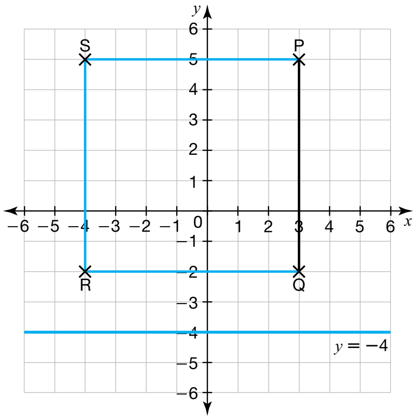
c Between D and E, the person got out of the bath.

d Running water into the bath was quicker. The slope of the line between O and A (filling the bath) is steeper than the slope of the line between E and F (emptying the bath).

Review it!

1 **a** P(3, 5)

b, c and e



d Q(3, −2), S(−4, 5)

x-coordinate: $-4 + 3 = -1$

$-1 \div 2 = -0.5$

y-coordinate: $5 + (-2) = 3$

$3 \div 2 = 1.5$

Midpoint is (−0.5, 1.5)

2 **a** $2x + 8 = 4$

$2x = -4$

$x = -2$

b When $x = 2$ and $y = -4$

A: $\frac{y}{x} = \frac{-4}{2} = -2$

B: $x - y = 2 - (-4) = 6$

C: $xy = 2 \times -4 = -8$

Expression C has the smallest value.

c Millie is correct.

When $x = 4$, $3x^2 = 3 \times 4^2 = 3 \times 16 = 48$

(George has worked out $(3x)^2$ instead.)

3 **a** $7a - (3a + 4) = 7a - 3a - 4 = 4a - 4$

b $4(2x + 3)$

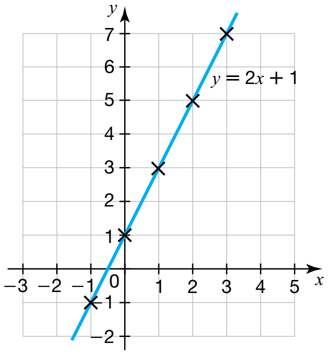
c $m^4 \times m = m^{4+1} = m^5$

d $\frac{x^8}{x^3} = x^{8-3} = x^5$

4 **a**

x	−1	0	1	2	3
y	−1	1	3	5	7

b



c Compare $y = 2x + 1$ with $y = mx + c$ (general form of the equation of a line):

Gradient, $m = 2$

5 $4x + 4 = x + 13$

$3x + 4 = 13$

$3x = 9$

$x = 3$

6 **a** 2 is included, and so are all values lower than 2.

$x \leq 2$

b

c 1 can be included, but 4 cannot.

$x = 1, 2, 3$

d $4x + 2 \leq 2x + 5$

$2x + 2 \leq 5$

$2x \leq 3$

$x \leq \frac{3}{2}$

(or $x \leq 1\frac{1}{2}$)

7 **a** $6x$ or $x + 65$

b $6x = x + 65$

$5x = 65$

$x = 13$

Luke is 13 years old.

8 **a** The term-to-term rule is ‘add 6’.

$27 + 6 = 33$

b No. The nth term is $6n - 3$.

$6n = 2 \times 3n$ = always even

Because 3 (odd) is always taken away from $6n$, every term in the sequence will be odd. As 44 is even it is not in the sequence.

c When $n = 5$:

$2n^2 - 3 = 2 \times 5^2 - 3 = 2 \times 25 - 3 = 47$

9 $(x + 3)(x + 4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

10 Smallest value of $a - b$ is where a is as small as possible and b is as large as possible.

$a > 30$ so its smallest value is 31

$b < 20$ so its largest value is 19

Using $a = 31$ and $b = 19$:

$a - b = 31 - 19 = 12$

11 The opposite sides of a rectangle are equal in length so:

$5x - 8 = 2x + 4$

$3x - 8 = 4$

$3x = 12$

$x = 4$

$14 - 2y = 4y + 2$

$14 = 6y + 2$

$12 = 6y$

$y = 2$

12 a $4x^2 + 6x$

$= x(4x + 6)$

$= 2x\ (2x + 3)$

b $x^2 - 100$

$= (x + 10)(x - 10)$

c $x^2 + 9x + 18 = 0$

$(x + 3)(x + 6) = 0$

Either $x + 3 = 0$ or $x + 6 = 0$

$x = -3$ or $x = -6$

13 $3(ax - 4) + 2(4x + b) \equiv 14x - 6$

$3ax - 12 + 8x + 2b \equiv 14x - 6$

$3ax + 8x - 12 + 2b \equiv 14x - 6$

$3ax + 8x \equiv 14x$

$3a + 8 = 14$

$3a = 6$

$a = 2$

$-12 + 2\mathrm{b} \equiv -6$

$2b = 6$

$b = 3$

14 a $12m$

b $3p \times 4p = 3 \times 4 \times p \times p = 12p^2$

c $12x \div 2 = \frac{12x}{2} = 6x$

15 a $5(w - 4) = 35$

$5w - 20 = 35$

$5w = 55$

$w = 11$

b When $a = 7$ and $b = -2$,

$5a + 7b = 5 \times 7 + 7 \times (-2)$

$= 35 + (-14)$

$= 21$

c $5a + 8b$

16 x coordinate of N = x coordinate of L + 6

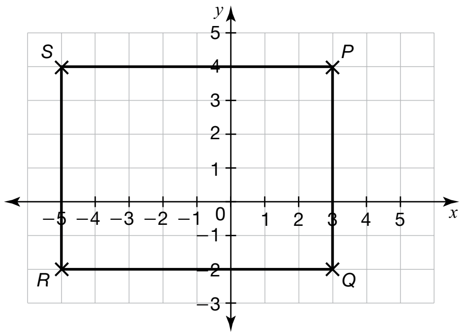
$= 2 + 6 = 8$

y coordinate of N = y coordinate of L − 6

$= 3 - 6 = -3$

Coordinates are $N(8, -3)$

17 a



P and Q are vertically above each other, because they share an x coordinate (3).

R and Q are horizontally aligned, because they share a y coordinate (−2).

As the fourth vertex, S must share an x coordinate with R (−5) and a y coordinate with P (4).

S is the point (−5, 4).

b Length is x coordinate of P or Q minus x coordinate of R or S:

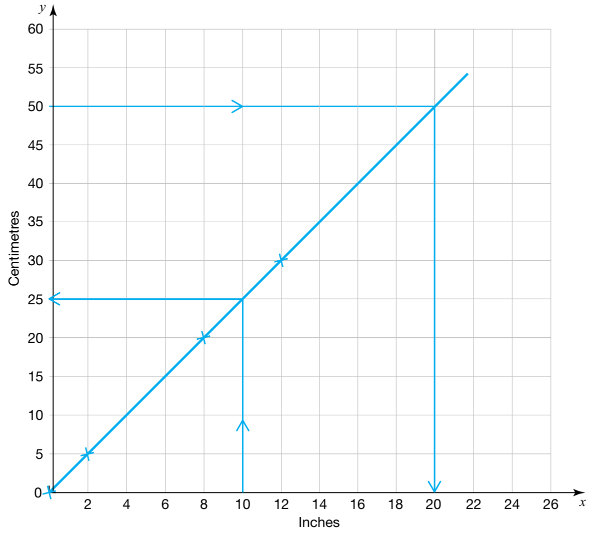
$= 3 - -5 = 8$

Width is y coordinate of S or P minus x coordinate of R or Q:

$= 4 - -2 = 6$

Length is 8 units and width is 6 units.

18 a



b i From the graph: 10 inches = 25 cm

ii From the graph: 50 cm = 20 inches

So 50 cm = 10 × 2 = 20 inches

c From the graph: 10 inches = 25 cm

So 60 inches = 25 × 6 = 150 cm

Cost of beading = 150 × 2 = 300p

Cost = £3.00

19 $4 \times 15 = 80 - 8k$

$60 = 80 - 8k$

$8k = 80 - 60$

$8k = 20$

$k = \frac{20}{8} = 2\frac{4}{8} = 2\frac{1}{2}$

20 **a** $T = 12.50x + 10$

b $72.50 = 12.50x + 10$

$62.50 = 12.50x$

$5 = x$

Suzanne hired the costume for 5 days.

21 Equation of a line: $y = mx + c$

y-intercept $= c = 5$

gradient $= m = \frac{5-0}{0-2} = -\frac{5}{2}$

$y = -2.5x + 5$ **or** $y = -\frac{5}{2}x + 5$

22 **a** $4x + 2 \le 8$

$4x \le 6$

$x \le \frac{6}{4}$

$x \le \frac{3}{2}$

(**or** $x \le 1\frac{1}{2}$ or 1.5)

b $3x - 4 < 17$

$3x < 21$

$x < 7$

$4x + 2 \ge 22$

$4x \ge 20$

$x \ge 5$

If $x < 7$ and $x \ge 5$ then $x = 6$ and $x = 5$ satisfy both.

23 Ollie has squared each term inside the brackets rather than squaring the whole bracket.

Correct working:

$(x + 4)^2 = (x + 4)(x + 4) = x^2 + 4x + 4x + 16 = x^2 + 8x + 16$

24 $P = \frac{Q}{4} + R$

$P - R = \frac{Q}{4}$

$4(P - R) = Q$

25 **a** $m(m + 8)$

b $(x + 3)(x + 4)$

26 **a** 2, 5, 8, 11, 14

Common difference = +3

3 × term number: 3, 6, 9, 12, 15

− 1 to get each term in the original sequence

So nth term $= 3n - 1$

b $2n - 3 = 112$

$2n = 115$

$n = 57.5$

No, Kadena is incorrect.

112 cannot be term in the sequence because 57.5 is not an integer.

27 **a** $4(x + 5) - 3(2x - 1) = 4x + 20 - 6x + 3 = -2x + 23$

b $4a^3b^2 \times 5a^2b = 4 \times 5 \times a^3 \times a^2 \times b^2 \times b$

$= 20 \times a^{3+2} \times b^{2+1} = 20a^5b^3$

28 Perimeter $= 3x - 2 + 2x + 1 + 3x + 5 + 2x = 10x + 4$

$10x + 4 = 49$

$10x = 45$

$x = 4.5$

29 A: output $= 6x - 4$

B: output $= 3x + 2$

$6x - 4 = 4(3x + 2)$

$6x - 4 = 12x + 8$

$-4 = 6x + 8$

$-12 = 6x$

$-2 = x$

Input $= -2$

30 1st term: $4 + 2a$

2nd term: $4 + 4a$

3rd term: $4 + 6a$

4th term: $4 + 8a$

5th term: $4 + 10a$

$4 + 10a = 64$

$10a = 60$

$a = 6$

31 **a** 1st term: a

2nd term: b

3rd term: $a + b$

4th term: $b + a + b = a + 2b$

5th term: $a + b + a + 2b = 2a + 3b$

6th term: $a + 2b + 2a + 3b = 3a + 5b$

7th term: $2a + 3b + 3a + 5b = 5a + 8b$

b $a + b = 5$ (1)

$5a + 8b = 34$ (2)

(1) × 5: $5a + 5b = 25$ (3)

(2) − (3): $3b = 9$

$b = 3$

Substitute $b = 3$ into (1):

$a + 3 = 5$

$a = 2$

32 Roots 2 and −4 are x-intercepts where the curve cuts the x-axis.

These give factors:

$(x - 2)$ and $(x - -4)$

Equation is $(x - 2)(x + 4) = 0$

Equation C.

Ratio, proportion and rates of change

Units of measure

1 **a** 3000 m

b 75 mins

c 13 000 cm^2

d 3.52 litres

e 7200 seconds

f 14 kg

2 $4.5 - 0.325 = 4.175$ kg or $4500 - 325 = 4175$ g

3 $5 \div 2.2 = 2.\dot{2}\dot{7}$ kg

Ratio

Stretch it! 31 + 25 = 56, fraction male = $\frac{31}{56}$

1 **a** 1:4 **b** 1:3:4 **c** 4:5

2 35:5 = 7:1

3 375 ÷ 250 = 1.5.
Allow 1 part cement for 1.5 parts sand.

4 **a** number in evening = 7 parts
number in afternoon = 1 part
7:1

b total parts = 7 + 1 = 8
1 part = 800 ÷ 8 = 100
1 part sold in afternoon
So **100** tickets were sold in the afternoon.

5 **a** There are 5 parts to the ratio. Ratio is:
3 : (5 − 3)
= 3 : 2

b 1 part = 200 ÷ 5 = 40
40 × 3 = 120
120 cats

6 9 = 3 × 3 so multiply the other lengths by 3.
4 × 3 = 12
5 × 3 = 15
12 cm and 15 cm

7 To work out the number of students per teacher (s), you multiply the number of teachers (t) by 20, so: $s = 20t$.

8 **a** There are 5 parts to the ratio.
1 part = 1.5 ÷ 5 = 0.3 kg
2 parts of sugar needed:
2 × 0.3 = 0.6
0.6 kg or 600 g

b 5 − 2 = 3 parts = 60 g more flour
60 ÷ 3 = 20
2 × 20 = 40
40 g of sugar

Scale diagrams and maps

Stretch it! 50 miles on ground = 50 ÷ x or $\frac{50}{x}$ miles on map

1 mile = 1610 m = 161 000 cm
50 miles on ground = $\frac{50}{x}$ × 161 000 cm* on map

1 A, B, F

2 **a** 3 × 12 = 36 km
b 15 ÷ 12 = 1.25 cm

3 12 × 1000 = 12 000 cm = 120 m

4 **a** 2 cm: 2 × 50 000 = 100 000 cm = 1 km
(Any answer within the range of 1 km – 1.1 km is acceptable.)
b 250°

Fractions, percentages and proportion

1 $\frac{20}{3500} = \frac{1}{175}$

2 2 + 3 + 8 = 13 hours
24 − 13 = 11 hours
$\frac{11}{24}$ of the day remaining

3 **a** $\frac{15}{20} = \frac{3}{4}$
b $1 - \frac{3}{4} = \frac{1}{4} = 25\%$

4 1 + 2 + 7 = 10, $\frac{1}{10}$ = 10%

5 School A: 125 : 145 = 25 : 29
School B: 100 : 120 = 5 : 6
No since the ratios are not equivalent.

6 150 ÷ 100 = 1.5
1.5 × 22 g = 33 g
33 g ÷ 8 = 4.125 g

Direct proportion

Stretch it!

For two values to be in direct proportion, when one is 0 the other must be 0. Here, when distance is 0 miles, the fee is £2.

1 A and E

2 **a** **i** 20 meringues = 2 eggs, divide both by 2 to give:
10 meringues = 1 egg
3 eggs: 3 × 10 = 30 meringues

ii 20 meringues = 120 g of sugar, divide both by 2 to give: 10 meringues = 60 g of sugar. Multiply both by 10 to give 100 meringues

b 20 meringues = 2 eggs, divide both by 2 to give 10 meringues = 1 egg, multiply both by 7 to give 70 meringues = 7 eggs

3 675 ÷ 4.5 = 150 minutes = 2 hours 30 minutes

4 A, D

Inverse proportion

1 D

2 At 60 miles it takes 15 minutes.
$60 \times \frac{2}{3} = 40$
$15 \div \frac{2}{3} = 22.5$ mins

3 2 × 3 = 6 decorators
$5 \div 3 = 1\frac{2}{3}$ of a day

4 **a** 2
b The age of the chicken and the number of eggs it lays are in inverse proportion, this means that as the age of the chicken increases, the number of eggs it lays decreases.

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*This answer differs from the one in the Revision and Exam Practice Book due to an error in our first edition. This answer has now been re-checked and corrected.

Working with percentages

Stretch it! £128

Stretch it! Let percentage rate $= x$

$$(1 + \tfrac{x}{100})^5 \times £100 = £110$$

$$(1 + \tfrac{x}{100})^5 = \frac{110}{100}$$

$$1 + \frac{x}{100} = \sqrt[5]{\frac{110}{100}}$$

$$1 + \frac{x}{100} = 1.02$$

$$\frac{x}{100} = 0.02$$

$$x = 2$$

Percentage interest is 2%

1 **a** $1.03 \times 50 = £51.50$

b $2.48 \times 400 = 992$

c $0.195 \times 64 = 12.48$

2 $45 - 40 = 5$, $\frac{5}{40} \times 100 = 12.5\%$

3 $24 \div 115 = 0.209$, $0.209 \times 100 = 20.9°C$

4 $15\,000 \times 1.20^3 = 25\,920$

5 20% is $\frac{1}{5}$ of the price.

$30 \times 5 = £150$

6 $(200 \div 225) \times 100 = 88.9\%$ (to 1 d.p.)

The number of employees in Year 2 is 88.9% of the number in Year 1.

Compound units

Stretch it! $\frac{100}{x}$ mph

1 $29.50 \div 0.18 = 164$ or $2950 \div 18 = 164$ units

2 Time $= \frac{80}{120} = \frac{2}{3}$ hour $= 40$ minutes

3 Density $= \frac{0.72}{3} = 0.24\ \text{g/cm}^3$

4 Pressure $= \frac{12}{2} = 6\ \text{N/m}^2$

5 3 m/s = 3 × 60 m/minute = 3 × 60 × 60 m/hour
= 10800 m/hour = 10.8 km/hour

6 0.6 litres per second = 0.6 × 60 litres per minute
= 0.6 × 60 × 60 litres per hour
= 2160 litres per hour.

2160 ÷ 4.55 = 475 gallons

475 gallons per hour (to the nearest whole number)

7 Bolt: 100 m in 9.58 seconds = 10.4 m/s

Cheetah: 120 km/h = 120000 m/hour
= 120000 ÷ 60 m/min
= 2000 m/min
= 2000 ÷ 60 m/sec = 33.3 m/s

The cheetah is faster.

Review it!

1 **a** $3.2 \times 1000 = 3200$ m

b $9 \times 60 = 540$ seconds

c $0.4 \times 1000 = 400$ ml

2 $4600 \div 1000 = 4.6$ km

3 $2.5 \times 60 = 150$ minutes

4 $1.1 \times 0.32 = 0.352\ \text{m}^2$ or $110 \times 32 = 3520\ \text{cm}^2$

5 $3 \times 10000 = 30\,000\ \text{cm}^2$

6 $\frac{5}{12}$

7 26 : 18 = 13 : 9

8 $100 - 85 = 15$, $15 \div 3 = 5$ minutes

9 density $= \frac{345}{0.15} = 2300\ \text{kg/m}^3$

10 $10 - 8 = 2$ km, $\frac{2}{8} \times 100 = 25\%$

11 $25 - 13 = 12$, $\frac{12}{25}$ **or** 48%

12 $15 + 5 + 3 = 23$ mins

$\frac{23}{90}$

13 $20 \div \left(\frac{4}{5}\right) = 25$ hours = 1 day and 1 hour

14 **a** $50 \div 5 = 10$, Josie: $1 \times 10 = 10$ marbles, Charlie: $4 \times 10 = 40$ marbles, Charlie has 30 more.

b $C = 4J$

15 $\frac{100}{360} \times 100 = 28\%$

16 $0.8 \times 1200 = £960$

$0.9 \times 960 = £864$

17 1 cm: 50 000 cm

50 000 cm = 0.5 km

3 km ÷ 0.5 = 6

6 cm

18 $1.02^3 \times 1500 = £1591.81$

19 $32\,000 \div 4 = 8000$ people

20 $393 \div 125 = 3.144$ hours = 3 hours 9 minutes

21 $2.50 + 1.90 + (2 \times 5.30) = £15$

$1.05 \times £15 = £15.75$

22 $37 + 15 + 4 + 19 = 75$

$\frac{15}{75} \times 100 = 20\%$

23 $0.045 \times 3000 = £135$

$3000 + (5 \times 135) = £3675$

24 $30 \div 3 = 10$

boys = 2 × 10 = 20

Girls = 1 × 10 = 10

Boys = 20 − 2 = 18

Girls = 10 + 3 = 13

18 : 13

25 Men to women is 7 : 6 = 35 : 30

Ratio of women to children is 15 : 2 = 30 : 4

Ratio of men to women to children is 35 : 30 : 4

35 + 30 + 4 = 69

3450 ÷ 69 = 50

35 × 50 = 1750 men

26 No – for two things to be in direct proportion when one is zero the other must be zero; the graph does not go through the origin so this is not the case.

27 Neither, since the time taken to cook increases as the weight increases it is not in indirect proportion. It is not in direct proportion since a graph to illustrate the relationship would not go through the origin.

28 speed $= \frac{\text{distance}}{\text{time}} = \frac{0.05}{17} = \frac{1}{340}$ hours $= \frac{3}{17}$ mins
= 11 seconds

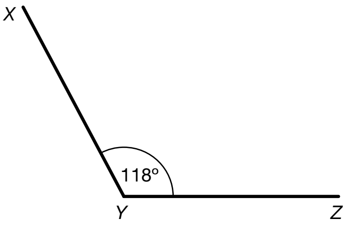
29 She is incorrect since the ratio of females to males must be the same for them to have equivalent proportions: 35 : 60 is not equivalent to 12 : 37.

Geometry and measures

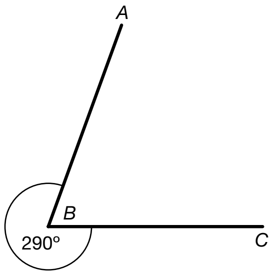
Measuring and drawing angles

1 **a** 43° **b** Acute

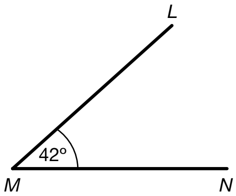
2 **a**



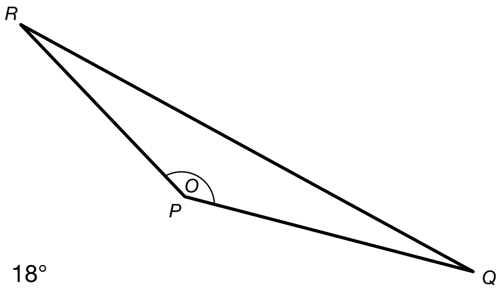
b



c



3 **a**



b 18°

Using the properties of angles

Stretch it!

Angles of triangle are in the ratio 1 : 2 : 3

Total number of parts = 6

1 part = $\frac{180}{6}$ = 30°

Angles in the triangle are 30°, 60° and 90°. It is a right-angled triangle.

1 Angles around a point add up to 360° so:

$a + 112 + 88 + 106 = 360$

$a + 306 = 360$

$a = 54°$

2 **a** **i** $a = (180 - 40) \div 2 = 70°$

ii Base angles of an isosceles triangle are equal.

b Exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices so:

$b = 70 + 40$

$b = 110°$

Or, angles on a straight line add up to 180° so:

$b = 180 - 70 = 110°$

3 Angles around a point add up to 360° so:

$5x + 9x + 108 = 360$

$14x + 108 = 360$

$14x = 252$

$x = 18°$

4 **a** **i** $x = 180 - 126 = 54°$

ii Angles on a straight line add up to 180°.

b Angles in a quadrilateral add up to 360° so:

$y + 135 + 54 + 88 = 360$

$y + 277 = 360$

$y = 83°$

5 **a** Angles on a straight line add up to 180° so:

$x = 180 - 84$

$x = 96°$

b **i** $y = 96°$

ii Use the fact that corresponding angles are equal, then the fact that vertically opposite angles are equal.
Or, use the fact that alternate angles are equal, then use angles on a straight line add up to 180°.

6 **a** Base angles of an isosceles triangle are equal so $a = 58°$.

b Angles in a triangle add up to 180° so:

$b = 180 - 58 - 58$

$b = 64°$

c Alternate angles are equal so $c = 58°$ (since angle a = angle c).

Or, since opposite angles of a parallelogram are equal:

$b + c = 122$

$64 + c = 122$

$c = 58°$

7 Angle *BAD* = 62° (Opposite angles of a parallelogram are equal)

Angle *ADE* = 62° (Alternate angles are equal)

$x = 180 - 62 - 62$ (Base angles of an isosceles triangle are equal)

$x = 56°$

8 Angle *ACB* = 36° (Base angles of an isosceles triangle are equal)

Angle *ABC* = 180 − 36 − 36 (Angles in a triangle add up to 180°)

Angle *ABC* = 108°

$x = 108°$ (Alternate angles are equal)

Using the properties of polygons

Stretch it!

1 The angle sum of a triangle is 180°.

Sum of interior angles of a hexagon = 4 × 180° = 720°.

2

Polygon	Number of sides (n)	Number of triangles formed	Sum of interior angles
Triangle	3	1	180°
Quadrilateral	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°
Heptagon	7	5	900°
Octagon	8	6	1080°
Decagon	10	8	1440°

3 $n - 2$

4 $180 \times (n - 2)$

Stretch it! Exterior angle of a regular hexagon = 360 ÷ 6 = 60°

Interior angle = 180 − 60 = 120°

Three hexagons meet at a point, so 120 + 120 + 120 = 360°

Similarly, interior angle of an octagon = 180 − (360 ÷ 8)
= 135°

Interior angle of a square = 90°, so 135 + 135 + 90
= 360°.

Regular pentagons have an interior angle of 108°. This does not divide equally into 360°, so these shapes will not fit together at a point in this way.

1 Regular decagon has 10 equal sides.

Exterior angle = 360° ÷ 10 = 36°

2 **a** Number of sides = 360° ÷ 15° = 24

b Angles on a straight line add up to 180° so:

Interior angle + exterior angle = 180

Interior angle + 15 = 180

Interior angle = 165°

Sum of interior angles = 24 × 165 = 3960°

3 Sum of interior angles of regular pentagon = 180° × (5 − 2)

= 180° × 3 = 540°

One interior angle of regular pentagon = 540° ÷ 5 = 108°

If this is a regular pentagon, *AB* = *AE* and triangle *ABE* is isosceles.

In triangle *ABE*:

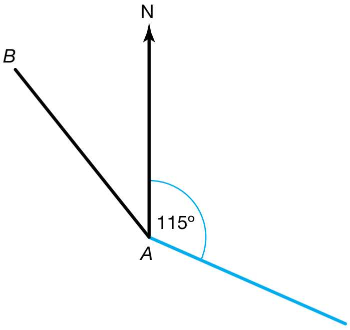
angle *ABE* = angle *AEB* = (180° − 108°) ÷ 2 = 36°

so angle *CBE* = 108° − 36° = 72°

Using bearings

1 **a** 360 − 45 (acute angle) = 315°

b



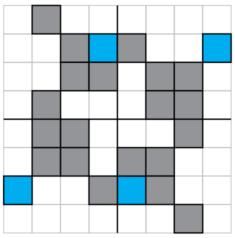
2 Bearing of P from Q = 180° + 164° = 344°

3 Kirsty is correct.

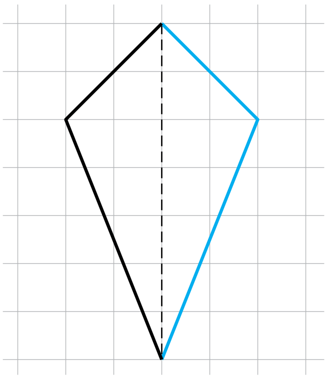
The bearing is 314° (360° − 46°) as it must be measured clockwise from North.

Properties of 2D shapes

Stretch it!

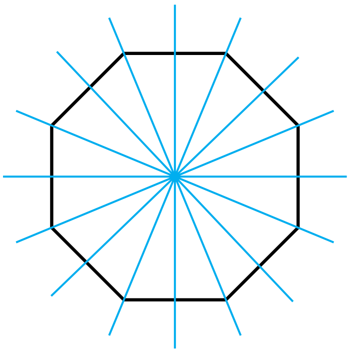


1 **a**



b kite

2 **a** 8 possible lines of symmetry:



b 8

3 **a** A rectangle has rotational symmetry of order **2.**

b A **rhombus** has all sides equal and rotational symmetry of order 2.

c A kite has **1** line of symmetry and **no** rotational symmetry.

d The diagonals of a **square** and a **rhombus** bisect each other at 90°.

Congruent shapes

1 Any accurate copy of shape A, in any orientation.

2 **a** Corresponding angles are equal so $x = 120°$

b Corresponding sides are the same length so $y = 12$ cm

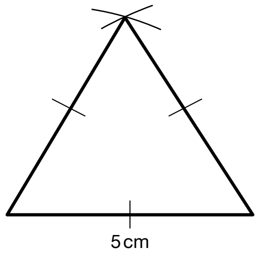
3 **a** SSS (each triangle has equal sides: 3 cm, 3 cm, 2.5 cm)

b ASA (two angles, 70° and 60°, and the included side, 8 cm, are equal)

Constructions

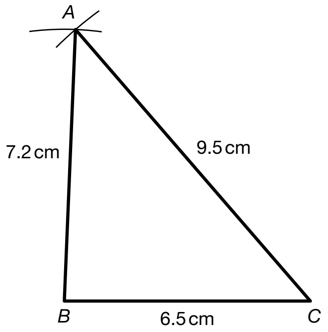
Stretch it!

A triangle with sides of 5cm with constructions lines indicating the use of compasses



Angle size 60°

1



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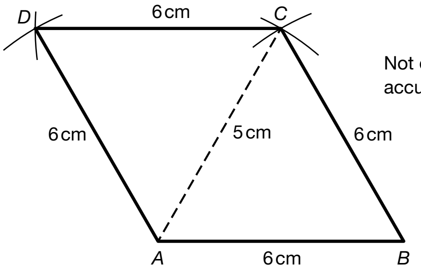
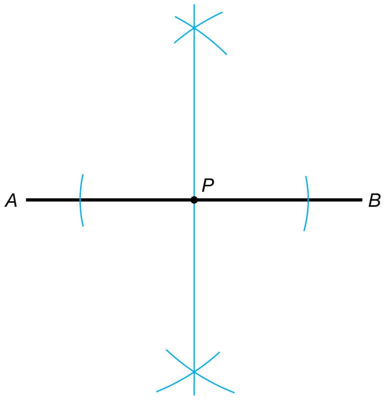


Not drawn accurately

3



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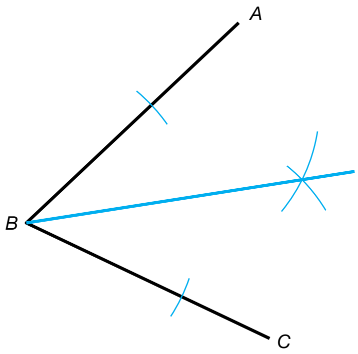

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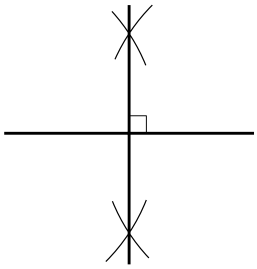


Drawing circles and parts of circles

1 a A **chord** is a straight line that does not pass through the centre of a circle but touches the circumference at each end.

b A **tangent** is a straight line that touches the outside of a circle at one point only.

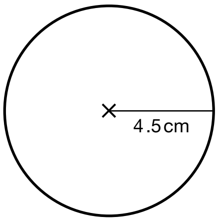
c A **diameter** is a straight line through the centre of a circle that touches the circumference at each end.

d An **arc** is part of the circumference of a circle.

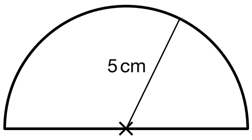
e A **radius** is a straight line from the centre of a circle that is half the length of the diameter.

f The part of a circle that has a chord and an arc as its boundary is called a **segment.**

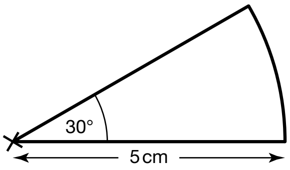
2 a



b

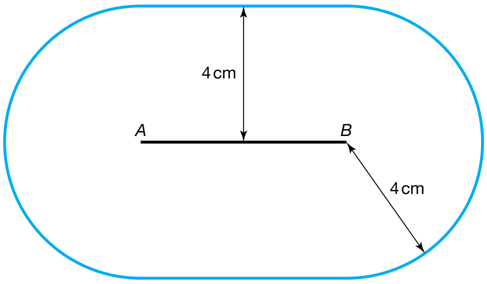


3

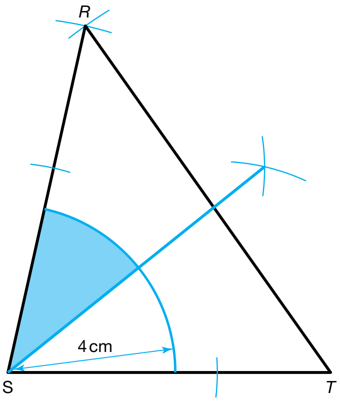


Loci

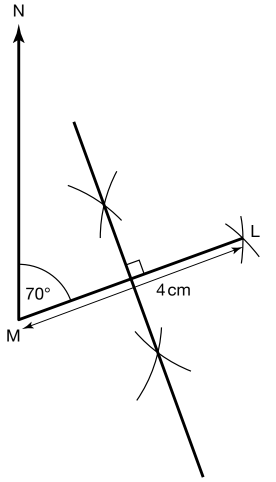
1



2



3 **a** and **b**



Perimeter

1 $4 \times 7.2 = 28.8$ cm

2 $7 + 9 + 9 + 5 + 5 \ + 7 = 42$ cm

3 Curved edge $= 2\pi r \div 2 = (2 \times \pi \times 4) \div 2 = 4\pi$

Perimeter $= 4\pi + 2 \times r = 4\pi + 8$ cm

So $k = 4$ and $b = 8$

4 Perimeter $= (\pi \times 30) + 100 + 100 = 200 + 30\pi$ m

5 Perimeter $= \left(\frac{1}{2} \times \pi \times 32\right) + 32 + 32 = 16\pi + 64$ cm

Ribbon $= 16\pi + 64 + 5 = 16\pi + 69$ cm $= 119.3$ cm

120 cm must be bought

$12 \times$ £0.15 = £1.80

Area

Stretch it! Area of a semicircle $= \frac{\pi r^2}{2}$, area of a quarter circle $= \frac{\pi r^2}{4}$

1 **a** $4.5 \times 2 = 9.0\text{ cm}^2$

b $3 \times 1.5 = 4.5\text{ cm}^2$

c $\frac{(5+9)}{2} \times 4 = 28.0\text{ cm}^2$

d $\frac{1}{2} \times 2 \times 5 = 5.0\text{ cm}^2$

e $\pi \times 4.5^2 = 63.6\text{ cm}^2$

2 Length of side $= 12 \div 4 = 3$ cm

Area $= 3^2 = 9\text{ cm}^2$

3 Shaded triangles would fit together to form one triangle with base $10 - 6 = 4$.

So area of shaded triangles $= \frac{1}{2} \times 4 \times 7 = 14\text{ cm}^2$

Area of trapezium $= \frac{(6+10)}{2} \times 7 = 56\text{ cm}^2$

Fraction of the shape that is shaded $= \frac{14}{56} = \frac{1}{4}$

4 Area of whole shape $= 6 \times 8 = 48\text{ cm}^2$

Fraction shaded $= \frac{6}{16} = \frac{3}{8}$

Area shaded $= \left(\frac{3}{8}\right) \times 48 = 18\text{ cm}^2$

5 Area of square $= 46 \times 46 = 2116\text{ cm}^2$

Each circle has radius $= 11.5$ cm

Area of four circles $= 4 \times \pi \times 11.5^2 = 1661.9\text{ cm}^2$

Shaded area $= 2116 - 1661.9 = 454.1\text{ cm}^2$

Sectors

1 Area $= \frac{1}{2} \times \pi \times 5^2 = 39.3\text{ cm}^2$

Perimeter $= \frac{1}{2} \times \pi \times 10 + 10 = 25.7$ cm

2 Area $= \frac{3}{4} \times \pi \times 4^2 = 12\pi\text{ cm}^2$

3 Area $= \frac{1}{2} \times \pi \times 3^2 = 14.1\text{ m}^2$

$14.1 \div 2 = 7.05$, so 8 bags needed.

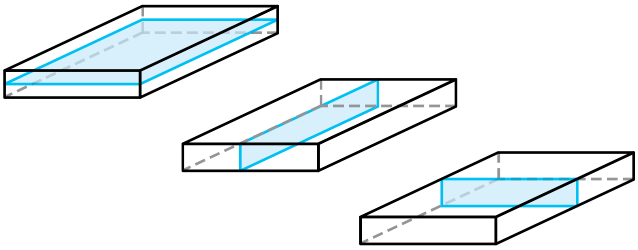
$8 \times 14.99 =$ £119.92

3D shapes

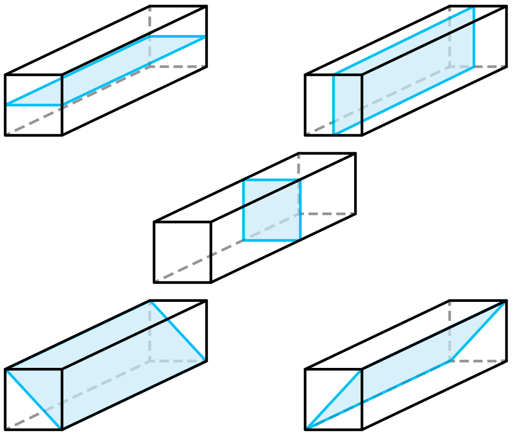
Stretch it!

3D shape	Faces	Edges	Vertices
Cube	6	12	8
Cuboid	6	12	8
Square-based pyramid	5	8	5
Tetrahedron	4	6	4
Triangular prism	5	9	6
Hexagonal prism	8	18	12

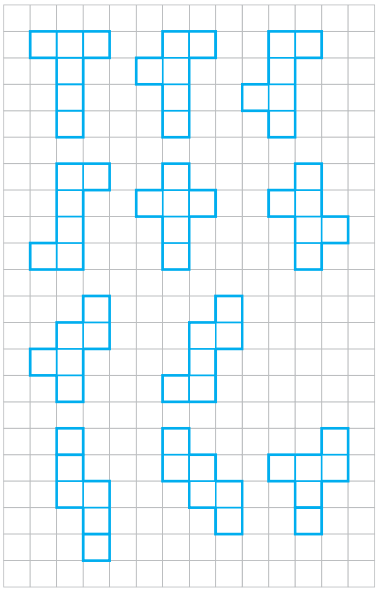
Stretch it! There are three planes of symmetry for the first cuboid:



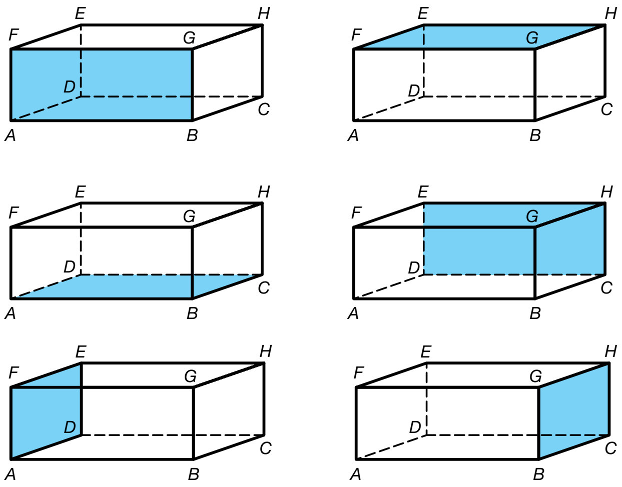
There are 5 planes of symmetry for the second cuboid: the same 3 planes as the first cuboid, plus two more planes along the diagonals of the square faces.



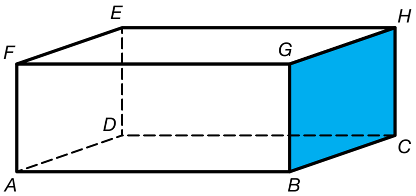
Stretch it!



1 **a** 6 possible rectangular faces:

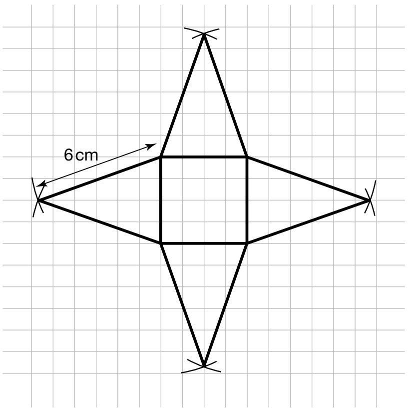


b

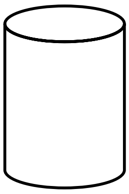


c Kelli has not counted the hidden edges.

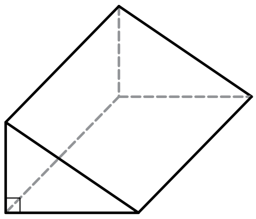
2 Draw a square in the middle with sides of 4 units (1 unit represents 1 cm). Set your compasses to 6 units and draw pairs of intersecting arcs from the corners of the square. These are the apices (top points) of the triangular sides. Draw lines for the sides of the triangles.



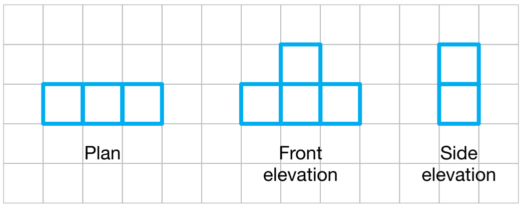
3 **a**



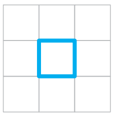
b



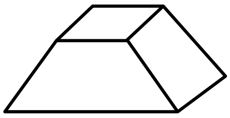
4



5 **a**



b



Volume

1 $\frac{4}{3} \times \pi \times 4.5^3 = 381.7 = 382\,\text{cm}^3$ (to 3 s.f.)

2 $\pi r^2 h + \frac{1}{3}\pi r^2 h = \pi \times 0.5^2 \times 2 + \frac{1}{3} \times \pi \times 0.5^2 \times 1.5$

$= 0.625\,\pi = 1.96\,\text{m}^3$

3 $\frac{1}{3} \times \pi \times 6^2 \times 22 = \frac{1}{3} \times 792 \times \pi = 264\pi\,\text{cm}^3$

$k = 264$

4 Volume of water $= 18 \times 7 \times 7 = 882\,\text{cm}^3$

$882 = 7 \times 20 \times h$

$882 = 140 \times h$

$h = 6.3\,\text{cm}$

Surface area

1 $6 \times (5 \times 5) = 150\,\text{cm}^2$

2 $4\pi r^2 = 4 \times \pi \times 3^2 = 36\pi\,\text{cm}^2$

3 $18 - 4 = 14\,\text{cm}^2$

4 Sloping surface $= \pi \times 14 \times 45 = 630\pi\,\text{cm}^2$

Base $= \pi \times 14^2 = 196\pi\,\text{cm}^2$

Total surface area $= 196\pi + 630\pi = 826\pi$

Percentage yellow $= \frac{630}{826} \times 100 = 76.3\%$

Using Pythagoras' theorem

1 Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$AC^2 = AB^2 + BC^2$

$15^2 = 11^2 + BC^2$

$BC^2 = 15^2 - 11^2 = 104$

$BC = \sqrt{104}$

$BC = 10.2\,\text{cm}$ (to 3 s.f.)

2 $c^2 = a^2 + b^2$

$c^2 = 2.4^2 + 5.5^2$

$c^2 = 36.01$

$c = \sqrt{36.01}$

$c = 6.0008...$

The ladder is 6 m long, to the nearest metre.

3 Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$XZ^2 = XY^2 + YZ^2$

$15^2 = XY^2 + 9^2$

$XY^2 = 15^2 - 9^2 = 144$

$XY = \sqrt{144}$

$XY = 12$ cm

Area $= \frac{1}{2}bh = \frac{1}{2} \times 9 \times 12$

Area $= 54$ cm^2

4 If the triangle is right-angled, $PQ^2 = PR^2 + RQ^2$

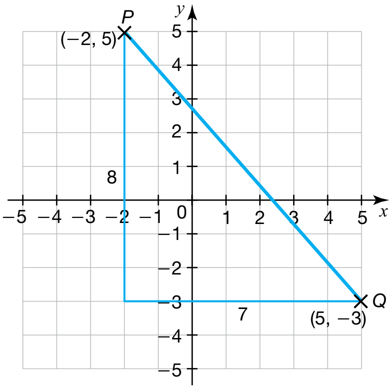
$PQ^2 = 13^2 = 169$

$PR^2 + RQ^2 = 9^2 + 7^2 = 81 + 49 = 130$

$PQ^2 \neq PR^2 + RQ^2$

Claudia is not correct.

5 $P(-2, 5)$, $Q(5, -3)$

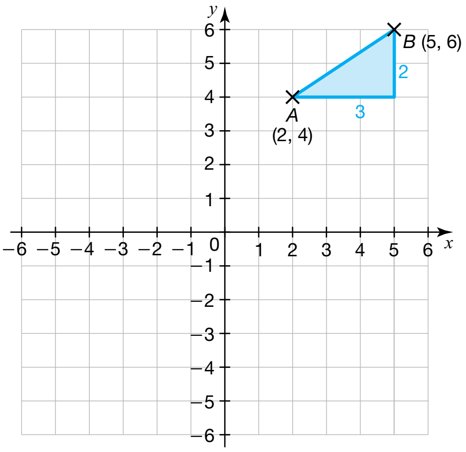


$PQ^2 = 8^2 + 7^2 = 113$

$PQ = \sqrt{113}$

$PQ = 10.63$ (to 2 d.p.)

6



$A(2, 4)$, $B(5, 6)$

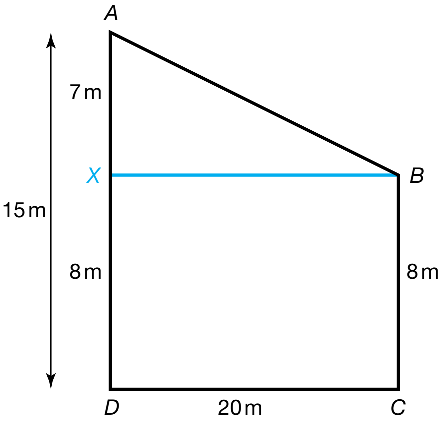
$AB^2 = (6 - 4)^2 + (5 - 2)^2$

$= 2^2 + 3^2$

$= 13$

$AB = \sqrt{13}$

7



Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$AB^2 = AX^2 + BX^2$

$AB^2 = 7^2 + 20^2 = 449$

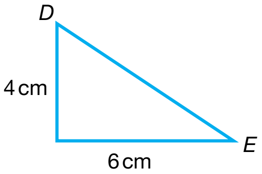
$AB = \sqrt{449}$

$AB = 21.2$ (to 3 s.f.)

Perimeter of field $ABCD = 15 + 20 + 8 + 21.2$
$= 64.2 \approx 65$m

Cost of fencing $= 65 \times$ £14 $=$ £910

8



$DE^2 = 6^2 + 4^2 = 52$

$DE = \sqrt{52}$

$= 7.2$ cm (1 d.p.)

Trigonometry

Stretch it!

Opposite could have been 1 m, hypotenuse could have been 2 m. They could be any lengths that keep opposite and hypotenuse in the ratio 1 : 2.

1 **a** 0.4 **b** 0.6 **c** 1.0

d 26.6 **e** 48.6 **f** 54.7

2 Cos 72° $= \frac{MN}{15}$ $MN = 15 \cos 72° = 4.6$ cm

3 Tan $ABC = \frac{6}{7}$

$ABC = \tan^{-1}\left(\frac{6}{7}\right)$

$ABC = 40.6°$

4 Let x be the depth of water.

$\sin 15° = \frac{x}{10}$

$x = 10 \sin 15°$

$x = 2.6$ m

Exact trigonometric values

1 **a** 0.5 **b** 0 **c** 0

d $\frac{1}{\sqrt{2}}$ **e** $\sqrt{3}$

2 $\tan 45° = 1 = \frac{\text{opposite}}{\text{adjacent}} = \frac{4}{AC}$

Therefore $AC = 4\,\text{cm}$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{4}{BC}$

$BC = 4\sqrt{2}$

Therefore $BC = 4\sqrt{2}$ cm

3 Since: $\tan 30° = \frac{1}{\sqrt{3}}$ one angle must be 30° and therefore the other is 60°

4 $\sin 30° = \frac{1}{2}$ therefore $ABC = 30°$

5 $\cos 30° = \frac{\sqrt{3}}{2} = 0.866$ (3 d.p.) $\tan 45° = 1$

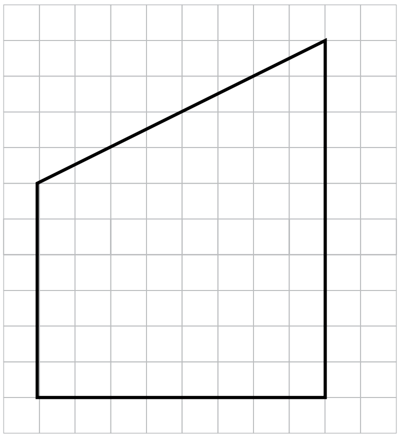
Smallest to largest $= 0.5, \frac{3}{4}$, cos 30°, tan 45°

Transformations

Stretch it!

Yes. Reflection in the x-axis followed by reflection in the y-axis (or vice versa) will always produce a rotation of 180°.

1



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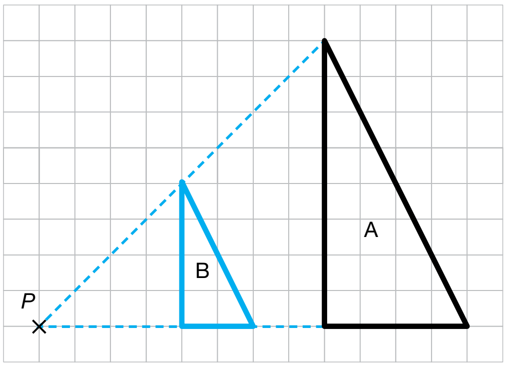
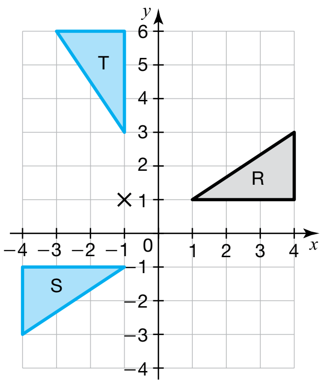


3 Translation by vector $\begin{pmatrix}-4\\-2\end{pmatrix}$

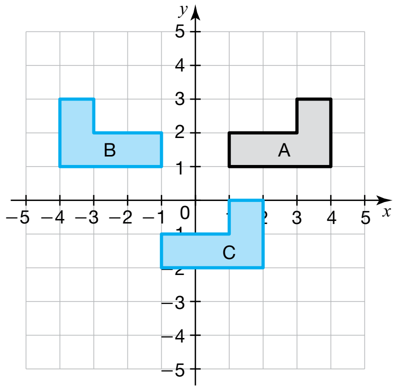
4 **a** and **b**



5 Reflection in the y-axis

6 Enlargement by scale factor $\frac{1}{2}$, centre (3, 3)

7 **a** and **b**



c Rotation of 90° clockwise about (0, 0)

Similar shapes

Stretch it!

Perimeter of $ABC = 3 + 6 + 5 = 14\,\text{cm}$

Perimeter of $DEF = 6 + 12 + 10 = 28\,\text{cm}$

The perimeter of a shape enlarged by scale factor 2 will also be enlarged by scale factor 2.

In general, all lengths on an enlarged shape, including the perimeter, are enlarged by the same scale factor.

Stretch it!

Angle BAC = angle CDE (alternate angles are equal)

Angle ABC = angle CED (alternate angles are equal)

Angle BCA = angle DCE (vertically opposite angles are equal)

All three pairs of angles are equal so triangles ABC and EDC are similar.

1 **a** Angle $DFE = 30°$ (Corresponding angles are the same)

b Scale factor of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{12}{3} = 4$

Length of $EF = 4\,\text{cm} \times 4 = 16\,\text{cm}$

c Length of $AB = 8\,\text{cm} \div 4 = 2\,\text{cm}$

2 **a** Angle $MLO = 80°$ (Corresponding angles are the same: angle MLO = angle QPS)

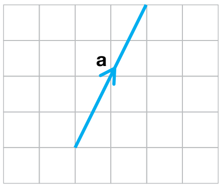
b Scale factor of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{9}{3} = 3$

Length of $QR = 4.4\,\text{cm} \times 3 = 13.2\,\text{cm}$

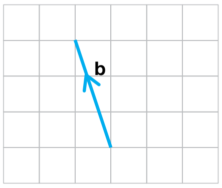
c Length of $LO = 12\,\text{cm} \div 3 = 4\,\text{cm}$

Vectors

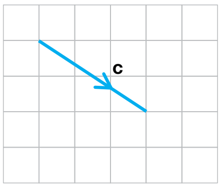
1 **a**



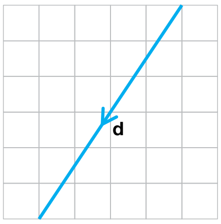
b



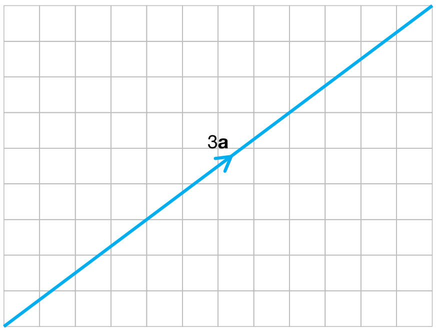
c



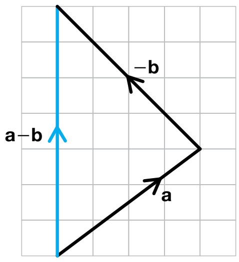
d



2 **a**



b



c $\mathbf{a} = \begin{pmatrix}4\\3\end{pmatrix}$

So $-2\mathbf{a} = -2 \times \begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}-8\\-6\end{pmatrix}$

3 **i** $\mathbf{c} = 2\mathbf{a}$

ii $\mathbf{d} = \mathbf{a} - \mathbf{b}$

iii $\mathbf{e} = \mathbf{a} + \mathbf{b}$

4 **a** $\mathbf{a} + \mathbf{b} = \begin{pmatrix}2\\6\end{pmatrix} + \begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}-3\\9\end{pmatrix}$

b $\mathbf{a} + 2\mathbf{b} = \begin{pmatrix}2\\6\end{pmatrix} + 2 \times \begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix} + \begin{pmatrix}-10\\6\end{pmatrix} = \begin{pmatrix}-8\\12\end{pmatrix}$

c $\mathbf{a} - \mathbf{b} = \begin{pmatrix}2\\6\end{pmatrix} - \begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}7\\3\end{pmatrix}$

d $\mathbf{b} - 2\mathbf{a} = \begin{pmatrix}-5\\3\end{pmatrix} - 2 \times \begin{pmatrix}2\\6\end{pmatrix} = \begin{pmatrix}-5\\3\end{pmatrix} - \begin{pmatrix}4\\12\end{pmatrix} = \begin{pmatrix}-9\\-9\end{pmatrix}$

5 $\mathbf{a} = \begin{pmatrix}2\\3\end{pmatrix}$

$\mathbf{b} = \begin{pmatrix}4\\-6\end{pmatrix} = 2\begin{pmatrix}2\\-3\end{pmatrix}$

$\mathbf{c} = \begin{pmatrix}-2\\-3\end{pmatrix} = -\begin{pmatrix}2\\3\end{pmatrix} = -\mathbf{a}$

$\mathbf{d} = \begin{pmatrix}20\\30\end{pmatrix} = 10\begin{pmatrix}2\\3\end{pmatrix} = 10\mathbf{a}$

a, c and **d** are parallel.

6 **a** $\overrightarrow{PQ} = 4\mathbf{a}$ ($\overrightarrow{PQ}$ and $\overrightarrow{SR}$ are parallel and the same length)

b $\overrightarrow{QR} = -3\mathbf{b}$ ($\overrightarrow{QR}$ and $\overrightarrow{PS}$ are parallel and the same length; $\overrightarrow{PS}$ has opposite direction to $\overrightarrow{SP}$)

c $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

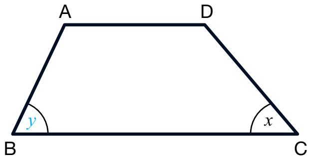
$= 4\mathbf{a} - 3\mathbf{b}$

d $\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS}$

$= -4\mathbf{a} - 3\mathbf{b}$

Review it

1 **a**



b $BC = 3.8$ cm

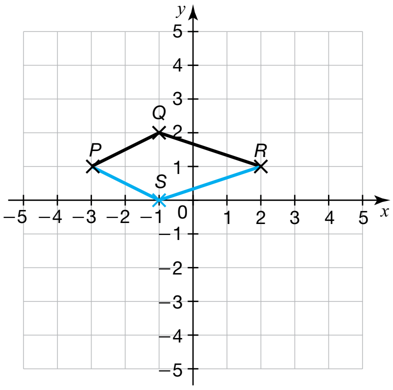
c $x = 50°$

2 **a** 5 faces

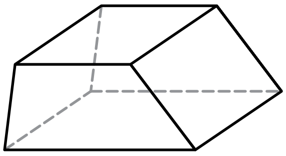
b 6 vertices

3 **a** (−3, 1)

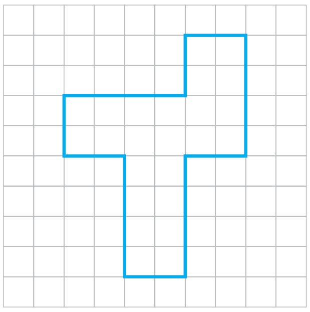
b



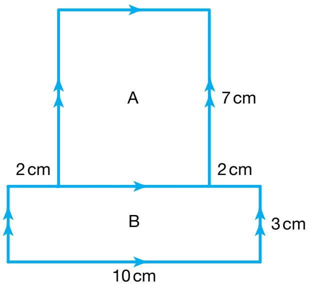
4



5 Any accurate copy of the shape



6 Area A = 7 × (10 − 2 − 2) = 7 × 6 = 42 cm^2



Area B = 10 × 3 = 30 cm^2

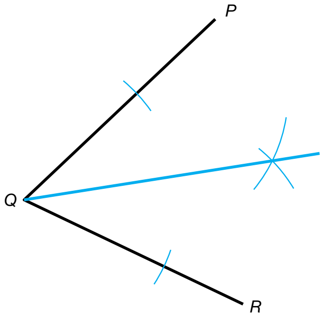
Total area = 42 + 30 = 72 cm^2

7 Area of parallelogram = 3 × 12 = 36 cm^2

Length of side of square = $\sqrt{36}$ = 6 cm

Perimeter of square = 4 × 6 = 24 cm

8



9 Rotation of 180° about (1, 0)

10 Sum of interior angles = $180(n - 2)$

$n = 8$, therefore sum of interior angles = 180(8 − 2)

= 180 × 6

= 1080

$x = \frac{1080}{8} = 135$

$x = 135°$

11 Angle *CFE* = 112° (corresponding angles are equal)

Angle *CFG* = 180 − 112 = 68° (angles on a straight line add up to 180°)

Angle *GCF* = angle *CFG* (base angles of an isosceles triangle are equal)

$x = (180 - 68 - 68) = 44°$ (angles in a triangle add up to 180°)

12 Shaded area = $(10 \times 12) - ((\frac{1}{2} \times 12 \times 3) + (\frac{1}{2} \times 8 \times 7) + (\frac{1}{2} \times 10 \times 4))$

= 120 − (18 + 28 + 20)

= 54 cm^2

Proportion = $\frac{54}{120} = \frac{9}{20}$ = 45%

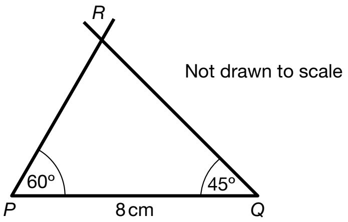
13 If triangle *ABC* is right-angled, $c^2 = a^2 + b^2$

$c^2 = 8^2 = 64$

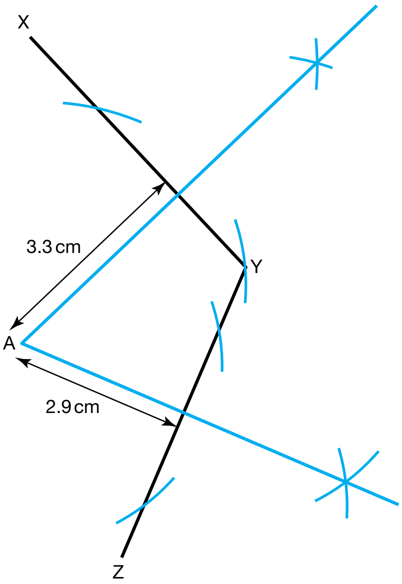
$a^2 + b^2 = 4^2 + 6^2 = 16 + 36 = 52$

$c^2 \neq a^2 + b^2$ so triangle *ABC* is not right-angled.

14



15 a



Scale is 1 : 200

b Distance from *A* to *YZ* = 2.9 cm

2.9 × 200 = 580 cm = 5.8 m

Distance from *A* to *YX* = 3.3 cm

3.3 × 200 = 660 cm = 6.6 m

Difference in distance = 6.6 − 5.8 = 0.8 m

16 a $\cos 45° = \frac{1}{\sqrt{2}}$

b Ratio of adjacent to hypotenuse is 1:2

Therefore *AB* = 3 cm

17 a $\mathbf{a} - \mathbf{b} = \begin{pmatrix} 4 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -8 \end{pmatrix}$

b $\mathbf{a} + 2\mathbf{b} = \begin{pmatrix} 4 \\ -5 \end{pmatrix} + 2 \times \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -5 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

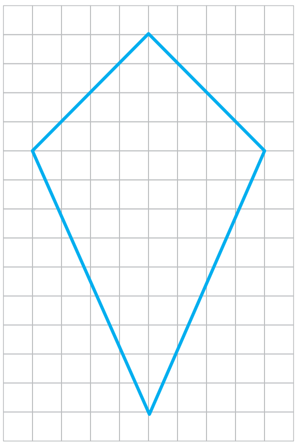
$= \begin{pmatrix} 8 \\ 1 \end{pmatrix}$

c $\mathbf{b} - 2\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2 \times \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 8 \\ -10 \end{pmatrix}$

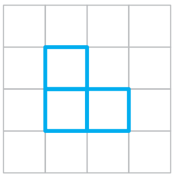
$= \begin{pmatrix} -6 \\ 13 \end{pmatrix}$

18 Any correct answer will have two pairs of equal adjacent sides, two equal angles, and one line of symmetry.



19 Three lines of symmetry and all sides the same length mean it must be an **equilateral triangle**.

20



21 a **i** 35°

ii Triangle *WYZ* is isosceles, and base angles of an isosceles triangle are equal.

b Angles in a triangle add up to 180° so:

$b = 180 - 35 - 35$

$= 110°$

c Triangle *XYZ* is isosceles, and base angles of an isosceles triangle are equal so:

$c = (180 - 70) \div 2 = 55°$

22 Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$AC^2 = AB^2 + BC^2$

$14^2 = 6^2 + \text{BC}^2$

$BC^2 = 14^2 - 6^2 = 160$

$BC = \sqrt{160}$

$BC = 12.6\,\text{cm}$ (to 1 d.p.)

23 Interior angle of a square = 90°

Sum of interior angles of an octagon (with $n = 8$) $= 180 \times (n - 2) = 180 \times (8 - 2) = 1080°$

Interior angle of a regular octagon = 1080° ÷ 8 = 135°

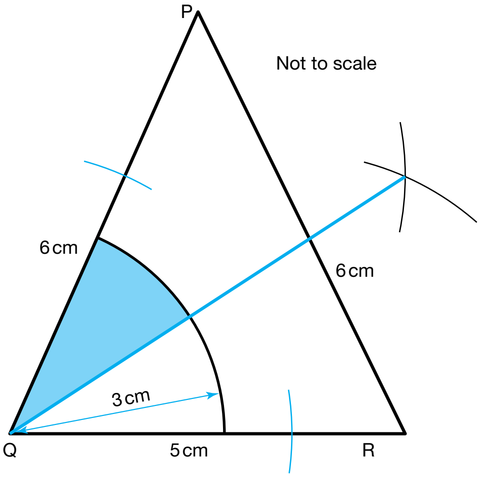
(Or, exterior angle of a regular octagon = 360° ÷ 8 = 45°. Then interior angle = 180° − 45° = 135°)

Angles around a point add up to 360° so:

$x = 360 - 90 - 135$

$x = 135°$

24



25 Divide the trapezium into a rectangle and a triangle. Draw a line *DX* parallel to *AB*, with *X* on the line *BC*. *BX* = 5 cm, *CX* = 7 cm.
Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$DC^2 = CX^2 + DX^2$

$DC^2 = 7^2 + 4^2 = 65$

$DC = \sqrt{65}$

$DC = 8.06$ (to 2 d.p.)

Perimeter of *ABCD* = 4 + 5 + 8.06 + 12 = 29.06 cm

26 Length of arc $= \frac{1}{4} \times 2 \times \pi \times 4 = 2\pi$

Perimeter $= 4 + (2 \times 9) + 4 + 2\pi = 32.3\,\text{cm}$

27 Area of square $= 6 \times 6 = 36\,\text{cm}^2$

Area of circle $= \pi \times 3^2 = 9\pi\,\text{cm}^2$

Shaded area $= 36 - 9\pi = 7.7\text{cm}^2$ (1 d.p.)

28 Volume of cylinder $= \pi \times 3^2 \times 15 = 135\pi\,\text{cm}^3$

2 litres = 2000 ml = 2000 cm^3

$2000 \div 135\pi = 4.7$

Glass can be completely filled 4 times.

29 Scale factor of enlargment $= \frac{\text{enlarged length}}{\text{original length}} = \frac{11}{5} = 2.2$

Length $x = 6\,\text{cm} \times 2.2 = 13.2\,\text{cm}$

30 Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$PR^2 = PQ^2 + RQ^2$

$PR^2 = 10^2 + 6^2 = 136$

$PR^2 = PS^2 + SR^2$

$136 = 11^2 + x^2$

$x^2 = 136 - 11^2 = 136 - 121 = 15$

$x = \sqrt{15}$

$x = 3.87$ cm (to 3 s.f.)

31 a Curved surface area $= \pi \times 6 \times 10 = 60\pi\ \text{cm}^2$

Base area $= \pi \times 6^2 = 36\pi\ \text{cm}^2$

Total surface area $= 60\pi + 36\pi = 96\pi = 300\,\text{cm}^2$ to 2 s.f.

b Volume $= \frac{1}{3} \times \pi \times 6^2 \times 8 = 96\pi = 300\,\text{cm}^3$

32 $\tan x = \frac{8}{6}$

$x = \tan^{-1}\left(\frac{8}{6}\right)$

$x = 53.1°$

33 Translation by vector $\begin{pmatrix}-7\\-6\end{pmatrix}$

34 $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= 2\mathbf{a} + 3\mathbf{b} + 3\mathbf{a} - \mathbf{b}$

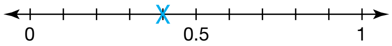
$= 5\mathbf{a} + 2\mathbf{b}$

Probability

Basic probability

Stretch it! No – each time the probability of getting an even number is $\frac{1}{2}$. You would expect to get even numbers approximately 50 times but cannot guarantee it.

1 $\frac{4}{10}$



2 a Total number of sweets = 12 + 3 + 10 = 25

$\frac{3}{25}$

b $\frac{(3+10)}{25} = \frac{13}{25}$

3 Pair **a**, because when you flip a coin, you can't get both a head and a tail at the same time. (Prime numbers on a dice are 2, 3, 5 and odd numbers are 1, 3, 5, so events **b** are **not** mutually exclusive because 3 is in both groups.)

4 Pair **b**, because the first sweet chosen is replaced, so the possible outcomes of the second choice remain the same. (If the first sweet chosen is eaten, the possible outcomes of the second choice are altered, and so events **a** are **not** independent.)

5 $P(6) = 1 - (0.1 + 0.15 + 0.1 + 0.02 + 0.2)$
$= 1 - 0.57 = 0.43$

6 P(green or red) $= 1 - 0.4 = 0.6$

P(green) $= 2 \times$ P(red)

P(red) $= \frac{0.6}{3} = 0.2$

P(green) $= 2 \times 0.2 = 0.4$

Two-way tables and sample space diagrams

1

	Chicken	Beef	Vegetarian
Fruit	12	6	4
Cake	5	3	8
Total	17	9	12

a 12 (this is worked out by using the numbers in the 'Total' row, which must add up to 38)

b As shown in the table.

2 a

		Dice 1					
		1	2	3	4	5	6
Dice 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b **i** $\frac{2}{36} = \frac{1}{18}$

ii $\frac{3}{36} = \frac{1}{12}$

iii 0

3 To score 6, the player must pick two cards showing 3. To score 2, the player must pick two cards showing 1. Since the probability of getting 3 and 3 is more than 0, and the probability of getting 1 and 1 is more than 0, there must be at least 2 of each of those numbers. So the cards must be 1, 1, 3, 3.

Sets and Venn diagrams

Stretch it!

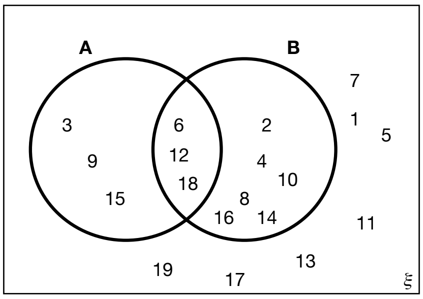
M {−1, 0, 1, 2}

N {2, 3}

2 is in both sets

Stretch it! None

1 a



b $A \cap B$ = {multiples of 6 less than 20} because these numbers are multiples of both 2 and 3.

2 a $C \cap T$ is the set of students who travel by car **and** train

$C' \cap B$ is the set of students who do **not** travel by car **and** travel by bus.

b **i** $P(C) = \frac{(14 + 11 + 11 + 2)}{(14 + 11 + 11 + 2 + 17 + 19 + 26)} = \frac{38}{100} = \frac{19}{50}$

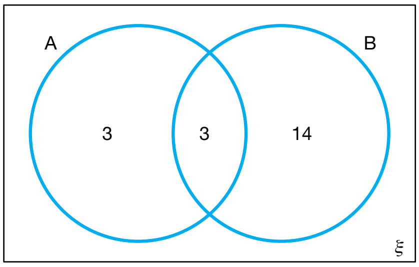
ii $P(B \cup T) = \frac{(19 + 11 + 2 + 0 + 11 + 17)}{100} = \frac{60}{100} = \frac{3}{5}$

iii $P(B' \cap T) = \frac{(11 + 17)}{100} = \frac{28}{100} = \frac{7}{25}$

3 $P(A \cap B) = \frac{3}{20}$ so there must be 3 elements in the intersection.

$P(A) = \frac{3}{10} = \frac{6}{20}$ so there must be a total of 6 elements in A.

The total number of elements must sum to 20.



4 a 3, 4, 5, 6

b 1, 2

c 1, 2, 3

Frequency trees and tree diagrams

1 a Apple = 123

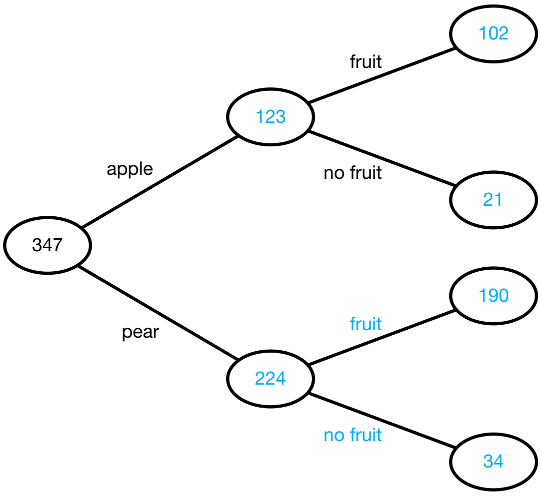
Pear = 347 − 123 = 224

Apple fruiting = 102

Apple not fruiting = 123 − 102 = 21

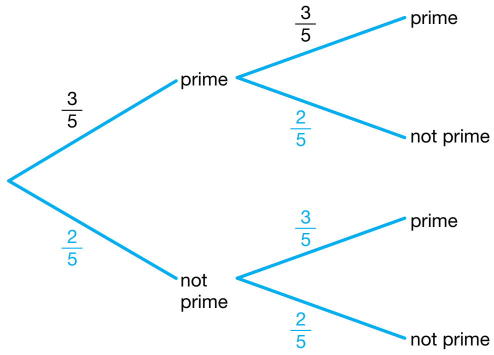
Pear not fruiting = 34

Pear fruiting = 224 − 34 = 190



b $\frac{190}{347}$

2 a



b $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

c $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

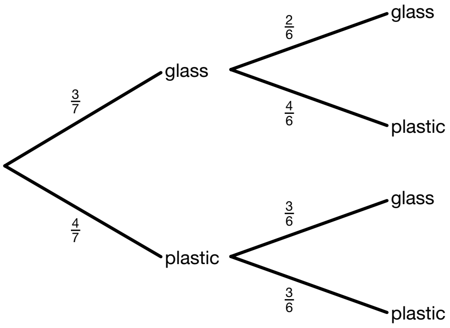
$\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

$\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

P(at least one prime) = 1 − P(no primes)

$= 1 - \frac{2}{5} \times \frac{2}{5} = 1 - \frac{4}{25} = \frac{21}{25}$

3 a



b P(two glass marbles) $= \frac{3}{7} \times \frac{2}{6} = \frac{6}{42}$

P(glass then plastic) $= \frac{3}{7} \times \frac{4}{6} = \frac{12}{42}$

P(plastic then glass) $= \frac{4}{7} \times \frac{3}{6} = \frac{12}{42}$

P(at least one glass) = 1 − P(both plastic)

$= 1 - \frac{4}{7} \times \frac{3}{6} = 1 - \frac{12}{42}$

$= 1 - \frac{2}{7} = \frac{5}{7}$

Expected outcomes and experimental probability

Stretch it! The dice has not been rolled enough times to decide if it is biased. More tests need to be carried out.

1 $0.45 \times 300 = 135$

2 Red $= \frac{2}{10} = \frac{1}{5}$

$\frac{1}{5} \times 100 = 20$ red sweets

3 $\frac{1}{2} \times 100 = 50$ primes

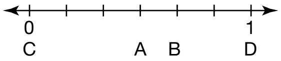
4 a Charlie – he has carried out the most tests.

b $\frac{(112 + 10 + 28)}{(112 + 10 + 28 + 74 + 7 + 19)} \times 10 = 6$

Review it!

1 $0.12 \times 250 = 30$

2



3 $1 - 0.3 = 0.7$

4 B, C

5 a $\frac{3}{5}$ **b** $\left(\frac{1}{5}\right) \times 25 = 5$

6

	Pizza	Pasta	Risotto	Total
Cake	12	6	1	19
Ice Cream	10	11	10	31
Total	22	17	11	50

7 $0.2 + 5x + 0.2 + x = 1$

$6x + 0.4 = 1$

$x = 0.1$

P(white) $= 5x + 0.2 = 0.7$

8 a No, he has not tested his dice enough times.

b P(2) $= \frac{9}{(12 + 9 + 16 + 7 + 6 + 0)} = \frac{9}{50}$

$\frac{9}{50} \times 100 = 18$

9 P(R, R) $= 0.1 \times 0.5 = 0.05$

P(R, G) $= 0.1 \times 0.5 = 0.05$

P(G, R) $= 0.9 \times 0.5 = 0.45$

$0.05 + 0.05 + 0.45 = 0.55$

Or P(at least one red) = 1 − P(green, green)

$= 1 - (0.9 \times 0.5)$

$= 1 - 0.45$

$= 0.55$

10 a

		Dice					
		1	2	3	4	5	6
Coin	Heads	2	4	6	8	10	12
	Tails	3	4	5	6	7	8

b i $\frac{2}{12} = \frac{1}{6}$

ii $\frac{2}{12} = \frac{1}{6}$

11 A{2,3,4}

B{−2,−1, 0, 1, 2, 3}

A∩B = {2, 3}

12 a i 6 **ii** 1 **iii** 5

b $\frac{4}{8} = \frac{1}{2}$

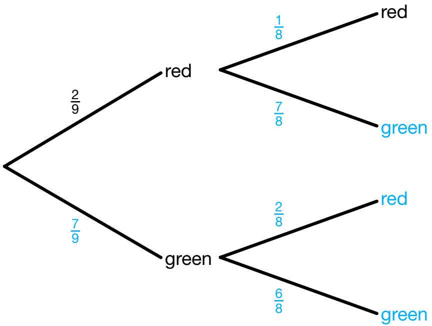
13 a i P(S) $= \frac{9 + 7}{14 + 7 + 9} = \frac{16}{30} = \frac{8}{15}$

ii P(S ∩ P) $= \frac{7}{14 + 7 + 9} = \frac{7}{30}$

iii P(P) $= \frac{14 + 7}{14 + 7 + 9} = \frac{21}{30} = \frac{7}{10}$

b No. If you use P(S) + P(P), you will be counting P(S ∩ P) twice.

14 a



b $\frac{7}{9} \times \frac{6}{8} = \frac{42}{72} = \frac{7}{12}$

15 a Possible fractions: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{2}{4}, \frac{3}{4}$

Less than $\frac{1}{2}$ are $\frac{1}{3}$ and $\frac{1}{4}$.

P(less than $\frac{1}{2}$) $= \frac{2}{6} = \frac{1}{3}$

$\frac{1}{3} \times 30 = 10$

b $\frac{1}{3}$ is only a theoretical probability and therefore will not necessarily be accurate in real life.

16 45% of 300 = 135

135 boys and 165 girls.

$\frac{2}{3}$ of 135 = 90

$\frac{4}{5}$ of 165 = 132

Total playing sport = 222

Probability $= \frac{222}{300} = \frac{37}{50} = 0.74$

17 P(hooking a winning duck) $= \frac{5}{20} = 0.25$

If 100 people play, expected number of winners = 0.25 × 100 = 25 people.

The game makes £1 × 100 people = £100.

The money paid out in prizes = 25 winners × £2 = £50

Profit = £100 − £50 = £50

18 a Milo will have the better estimate as he has surveyed a greater number of people.

b Number of left-handed students = 5 + 4 + 7 + 7
= 23

Number of right-handed students = 23 + 18 + 51 + 60
= 152

P(left-handed) $= \frac{23}{23 + 152} = \frac{23}{175}$

$\frac{23}{175} \times 2000 = 262.8$

You would expect to find 263 left-handed students in a school with 2000 students.

Statistics

Data and sampling

Stretch it! A random sample could be taken; you could allocate a number to each pupil and randomly generate the numbers to survey. Any method is acceptable as long as each person in the school has an equally likely chance of being chosen. Alternatively a stratified sample could be taken.

1 Primary source: Recording the data by measuring it yourself.

Secondary source: Any sensible source, e.g. the Meteorological Office, local paper etc.

2 Qualitative data.

3 It is cheaper and quicker than surveying the whole population.

4 a The people working for an animal charity are more likely to be opposed to wearing real fur; every member of the population does not have an equal chance of being chosen.

b Surveying people in the street, a random telephone survey, any sensible method that ensures that any member of the population has an equal chance of being chosen.

5 a $\frac{400}{1600 + 400} = \frac{400}{2000} = \frac{1}{5}$

b $\frac{1}{5} \times 50 = 10$ bottles

6 a $\frac{3}{200} \times 800\,000 = 12\,000$

b The sample is relatively small. The sample is not a random sample as it is taken on one day in a year.

7 a Two of the following: the groups overlap; they are unequal in width; there is no group for anyone with a journey of more than 60 minutes; most people will be in the middle group.

b How long do you spend travelling to school in the morning?

$0 < t \leq 10$

$10 < t \leq 20$

$20 < t \leq 30$

$30 < t \leq 40$

$t > 40$

Frequency tables

1

Number of people on the bus	Frequency
0–9	4
10–19	12
20–29	3
30–39	1

2 a

Number of courgettes	Frequency
0	1
1	0
2	1
3	1
4	9
5	3
6	0

b (0 × 1) + (1 × 0) + (2 × 1) + (3 × 1) + (4 × 9) + (5 × 3) = 56

3 a There are gaps between his groups – times that fall between groups cannot be recorded, e.g. 15.5 hours.

His groups do not have the same width.

b Although one or more of the data values may fall in the $30 \leq h \leq 40$ group, this doesn't mean that those people trained for 40 hours. They could have trained for any length of time between 30 and 40 hours.

Bar charts and pictograms

1 **a** $15 + 4 + 1 = 20$

b $4 + 1 = 5$

$\frac{5}{20} \times 100 = 25\%$

2 **a** $11 - 7 = 4$

b Total number of people surveyed $= 18 + 18 + 12 + 3 = 51$

Total number of boys $= 11 + 6 + 3 = 20$

$\frac{20}{51} \times 100 = 39.2\%$

c Proportion of boys who played two sports $= \frac{6}{18} = \frac{1}{3}$

Proportion of boys who played three sports $= \frac{3}{12}$ $= \frac{1}{4}$

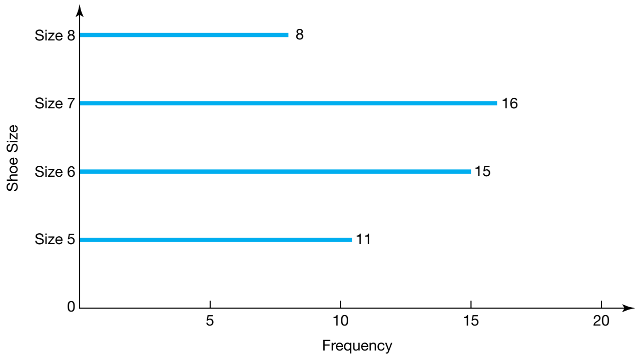
$\frac{1}{3} > \frac{1}{4}$ so the proportion who played two sports is larger.

3 $50 - (11 + 15) = 24$

$24 \div 3 = 8$

Therefore: $2 \times 8 = 16$ size 7 shoes

$1 \times 8 = 8$ size 8 shoes



4 **a** $90 - 20 = 70$

b Total number of bikes $= 50 + 50 + 20 + 90 = 210$

$\frac{50}{210} = \frac{5}{21}$

Pie charts

Stretch it!

Round appropriately – but check the angles sum to 360°

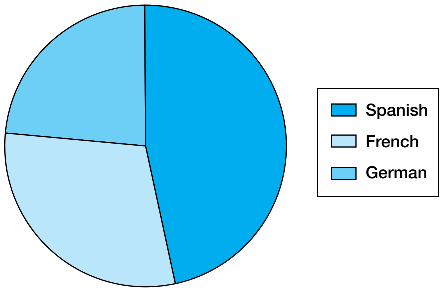
1 $27 + 42 + 21 = 90$

$360° \div 90 = 4°$

French $= 27 \times 4 = 108°$

Spanish $= 42 \times 4 = 168°$

German $= 21 \times 4 = 84°$



2 **a** $\frac{1.5}{360} \times 240 = 1$ student earned more than £40 000.

b $\frac{288 + 63}{360} \times 100 = 97.5\%$ (**or** $\frac{39}{40}$) of students earned less than £30 000.

3 **a** $18 + 10 = 28$

b The bar chart, since the frequency is easy to read from the bar chart.

Stem and leaf diagrams

1 **a** 7

b $4.3 - 4.1 = 0.2$ kg (200 g)

2 **Age of people using a dentist**

2	0 0 0 0 1 1 1
3	2 5 5 7
4	1 2 2 6

Key
2 | 0 means 20 years old

The leaves were not in ascending order, the spaces between leaves were not regular, and there was no key.

3 **a** Stem and leaf diagram – you can see the smallest number of passengers was 3; however, on the bar chart you only know it is between 0 and 9.

b Both since the shape of the data is preserved in both.

Measures of central tendency: mode

1 The other three must be 12.2.

2 $1 < t \leq 2$

3 There are three equal 7 'leaves' on the 1 stem. So: 17

4 Max is correct, the modal number of pets is the group with the highest frequency, therefore 2 pets is the mode.

Measures of central tendency: median

1 Ordering the data gives; 2.9, 3.1, 4.3, 6.5, 8.7, 9.2

Median $= \frac{4.3 + 6.5}{2} = 5.4$

2 $29 + 28 + 30 + 3 + 10 = 100$

$\frac{(100 + 1)}{2} = 50.5$ – median term is between the 50th and 51st terms.

Both these lie in the $2 \leq b < 4$ class.

3 **a** Group A $= \frac{82 + 85}{2} = 83.5$

Group B $= \frac{75 + 79}{2} = 77$

b Group A has a higher median, so they did better on the test.

Measures of central tendency: mean

Stretch it! **a** mode **b** mean/median **c** mean/median

1 **a** Total frequency $= 12 + 3 + 5 = 20$

Mean $= \frac{(2 \times 12) + (6 \times 3) + (10 \times 5)}{20} = 4.6$

b You are using the midpoint of the groups as an estimate of the actual value for each group.

2 $\frac{(5 \times 9) + 6}{5 + 1} = 8.5$

3 No – they could be any pair of numbers which sum to 10.

Range

1 $9.5 - 0.7 = 8.8$

2 **a** Girls = $18 - 15 = 3$

b Boys = $18 - 16 = 2$

3 Range for Athlete A = $15.2 - 13.0 = 2.2$

Range for Athlete B = $15.2 - 14.3 = 0.9$

Athlete A has the greatest range.

4 $45\% - 10 = 35\%$ or $45\% + 30 = 75\%$

Comparing data using measures of central tendency and range

1 **a** **i** Mean = $\frac{(32 + 29 + 18 + 41 + 362 + 19)}{6} = \frac{501}{6}$

= 83.5 minutes

ii Ordered data: 18, 19, 29, 32, 41, 362

Median = $\frac{(29+32)}{2}$ = 30.5 minutes

b The extreme value (362 mins) affects the mean but not the median.

2 All the data is used to find the mean.

3 Either as long as suitably justified:

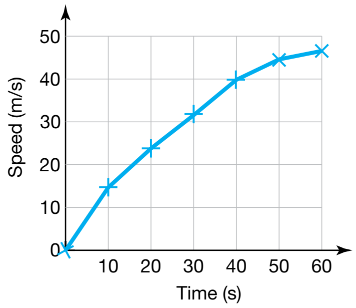
Car A – although the mean time is higher, it is more consistent in performance since the range is smaller.

Car B – the acceleration is quicker on average.

4 **a** and **b** The mode or median since the mean will not be a whole number and therefore not meaningful.

Time series graphs

1



2 **a** 67°C

b Approx. 27°C

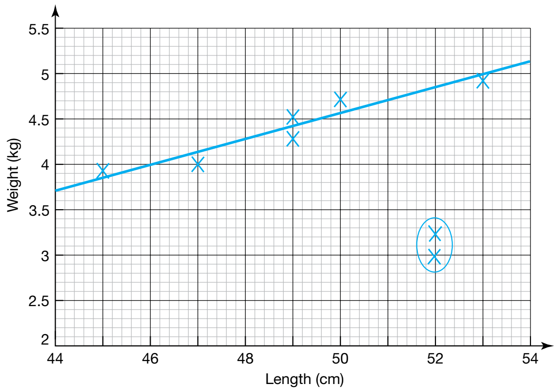
c No, since it is extrapolation (beyond the limits of the data).

3 **a** 17 000 **b** **i** April **ii** August

c The number of tourists peaks in April and again in December. The low seasons are February/March and July/August/September/October*.

Scatter graphs

1 **a** and **b**

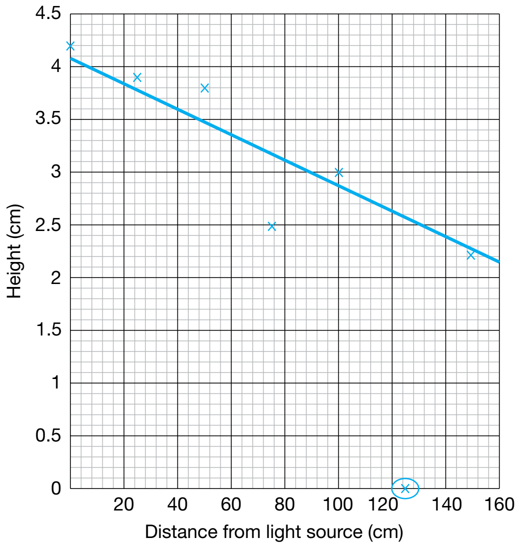


c Positive

d This will vary according to the line of best fit: approximately 4.7 kg. A range of 4.6kg to 4.8kg would be acceptable.

e This is beyond the limits of the data and therefore extrapolation.

2 **a**, **b** and **c**



c The seeds failed to germinate or the seedling died.

d The further the seedling is from the light source the shorter its height.

3 No, although the two things correlate one does not cause another. There may be many reasons why the crime rate is high in the area, perhaps there is poverty and inequality causing social tension.

Graphical misrepresentation

1 The organisation has only shown a small section of the data. This is not enough to understand the overall trend.

2 Chart C correctly shows the information. Chart A has an incorrect vertical axis, suggesting there are more women than there actually are. Chart B has unequal bar widths, also exaggerating the number of women.

Review it!

1 The sample is too small and he only asked his friends. His data is therefore not representative of the population of TV viewers.

*This answer differs from the one in the Revision and Exam Practice Book due to an error in our first edition. This answer has now been re-checked and corrected.

2 **a** Margherita

b Total frequency = 11 + 2 + 6 + 1 = 20

$\frac{1}{20} = \frac{5}{100} = 5\%$

c 360° ÷ 20 = 18°

Pepperoni = 1 × 18° = 18°

(or 5% of 360° = 18°)

3 **a** $\frac{90}{360} = \frac{1}{4}$

b $45° = \frac{1}{8}$ of 360°

Therefore $\frac{1}{8}$ of the pie chart represents 60 cars.

The whole pie chart = 8 × 60 = 480 cars

c $\left(\frac{105}{360}\right) \times 480 = 140$ cars

4 **a** The number of people doing their grocery shopping online is increasing.

b Any sensible answer, approximately 75%

c No – it is outside the limits of the data therefore extrapolation.

5 **a** Outside: **i** Mode = 21 and 31

ii Median = $\frac{28 + 29}{2} = 28.5$

iii Range = 41 – 20 = 21

Greenhouse: **i** Mode = 47

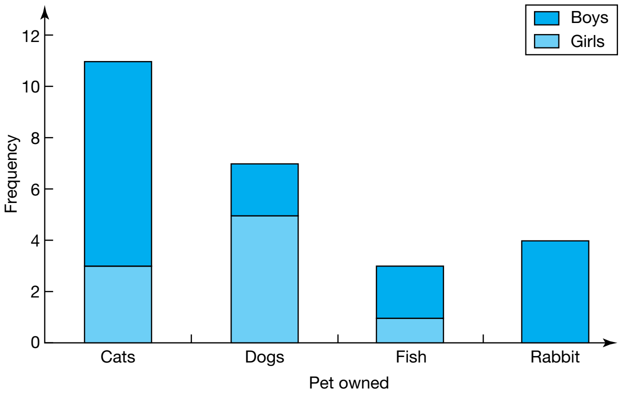
ii Median = $\frac{47 + 47}{2} = 47$

iii Range = 51 – 37 = 14

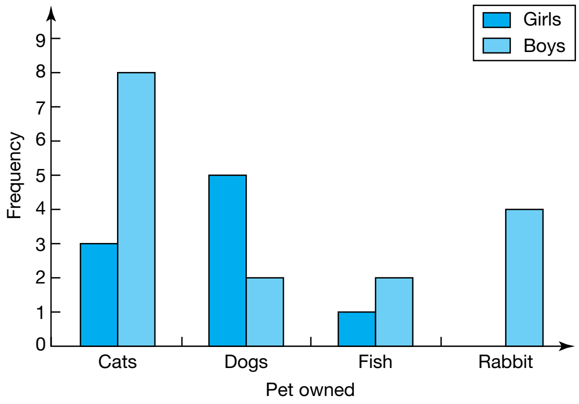
b The seedlings are taller in the greenhouse since both mode and median is larger, the range of data is smaller in the greenhouse so the height the seedlings reach is more consistent.

c Range = 51 – 20 = 31

6 **a** Comparative bar chart or compound bar chart:



Or:



b Total number of students = (3 + 5 + 1 + 0 + 8 + 2 + 2 + 4) = 25

Number of cats = 3 + 8 = 11

$\frac{11}{25}$

7 **a** Total frequency = 17 + 2 + 32 + 23 + 9 = 83

Median value = $\frac{(83 + 1)}{2}$ = 42nd term

42nd term is in group 40–59

Median class = 40–59

b The youngest person is between 0 and 19, the youngest may be any age in this range and the oldest is between 80 and 99 therefore any age in this range.

8 **a** 7

b Size 5

c Mean = $\frac{(3 \times 2) + (4 \times 1) + (5 \times 7) + (6 \times 5) + (7 \times 3)}{2 + 1 + 7 + 5 + 3} = 5.\dot{3}$

d Mode – the mean is not an actual shoe size.

9 **a** Time for 800 m (seconds)

11	2 2 5 8 9
12	0 1 9
13	1 2

Key: 11|2 = 112 seconds

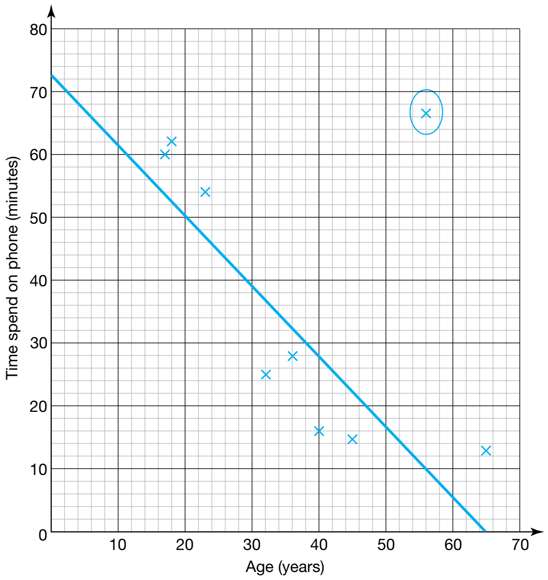
b $\frac{5}{10} = \frac{1}{2}$*

10 a $\frac{50}{150} = \frac{1}{3}$

b 60 – 40 = 20

c Biology

11 **a** and **c**



b Negative

d Approximately 40 minutes: it depends on line of best fit.

e This is outside the limits of the data and therefore extrapolation.

f As the age of the customer increases the time spent on the phone decreases.

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*This answer differs from the one in the Revision and Exam Practice Book due to an error in our first edition. This answer has now been re-checked and corrected.

12 a $\dfrac{(65 \times 3) + (75 \times 5) + (85 \times 2)}{3 + 5 + 2} = 74\,\text{kg}$

b The midpoint of the class is used as the age of each of the patients rather than the actual age.

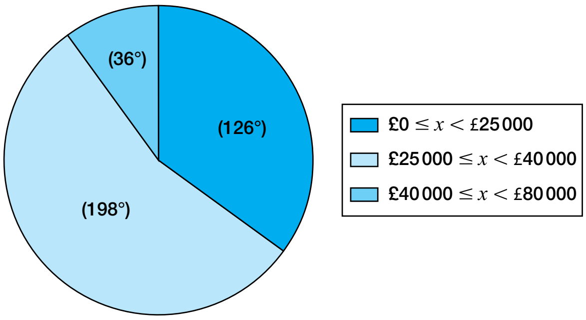
13 Male $< 50 = \left(\dfrac{12\,201}{36\,579}\right) \times 120 = 40$

Female $< 50 = \left(\dfrac{10\,678}{36\,579}\right) \times 120 = 35$

Male $\geq 50 = \left(\dfrac{5699}{36\,579}\right) \times 120 = 19$

Female $\geq 50 = \left(\dfrac{8001}{36\,579}\right) \times 120 = 26$

14 Annual income for surveyed population



15 Mean $= \dfrac{(10.3 \times 10) + 9.5}{11} = 10.2$ (1 d.p.)

16 Mean is 3.8 so the sum of the scores is $3.8 \times 5 = 19$

Mode is 3 so she must roll at least two 3s.

Range is 4.

If the range is 4 then the lowest and highest must be either 1 and 5 or 2 and 6.

The numbers are: 2, 3, 3, 5 and 6

Foundation Mathematics Exam Practice Book for all Exam Boards Full worked solutions

Number

Basic number techniques

1 Negative numbers are smaller than zero

−12 is further left on the number line than −8, −1 is larger than −8 (and −12) so it appears next.

Then comes 0, then 2.

So the order is:

−12, −8, −1, 0, 2

2 First look at the place value for 10ths: 0.32 and 0.3 have the higher number of 10ths.

Now compare their 100ths. 0.32 has 2 100ths but 0.3 doesn't have any, so it's smaller.

Similarly, 0.23 and 0.203 both have 2 10ths, but 0.23 is bigger than 0.203 because it has 3 100ths while 0.203 only has 3 1000ths.

So the order is:

0.32, 0.3, 0.23, 0.203

3 **a** $-4 < 0.4$ (the negative number is lower)

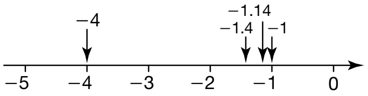
b $4.200 < 4.3$ (the higher number has more 10ths)

c $-0.404 > -0.44$ (because they are both negative, the one with more 100ths is lower)

d $0.33 < 0.4$ (the larger number has more 10ths)

4 The positive number (1.4) is the highest.

Think where the negative numbers fit on a number line:



Order is:

−4, −1.4, −1.14, −1, 1.4

Factors, multiples and primes

1 $18 = 2 \times 3 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

Find the factors that they both share (2 and 3) and multiply together:

$2 \times 3 = 6$, so HCF is 6.

2 Numbers between 15 and 25:

16, 17, 18, 19, 20, 21, 22, 23, 24

Note that this does not include 15 and 25 because the question said 'between 15 and 25' not 'from 15 to 25'.

$16 = 2 \times 2 \times 2 \times 2$

$18 = 2 \times 3 \times 3$

$20 = 2 \times 2 \times 5$

$21 = 3 \times 7$

$22 = 2 \times 11$

$24 = 2 \times 2 \times 2 \times 3$

17, 19, and 23 have no factors except for 1 and the number itself.

17, 19 and 23.

3 $60 = 20 \times 3$

$= 2 \times 2 \times 5 \times 3$

$= 2^2 \times 3 \times 5$

4 Drummer 1 hits her drum at: 6 12 18 24 30 36 42 48 54 60 seconds

Drummer 2 hits his drum at: 8 16 24 32 40 48 56 seconds

They hit their drums at the same time twice (two times), after 24 seconds and after 48 seconds.

Calculating with negative numbers

1 **a** $-7 + -3 = -7 - 3 = -10$

b $-7 - -3 = -7 + 3 = -4$

c $8 + -5 - -2 = 8 - 5 + 2 = 5$

d $-4 - -6 + -1 = -4 + 6 - 1 = 1$

2 **a** −18

b $-12 \div -3 = 12 \div 3 = 4$

c $-4 \times -2 \times 5 = 4 \times 2 \times 5 = 40$

d $(-24 \div 3) \times 2 = -8 \times 2 = -16$

3 $2 - 4 = -2$

$-2 - 5 = -7$

−7°C

4 Let a = number of correct answers, b = number of incorrect answers

$3a - 2b = -5$ (1)

There are five questions, so $a + b = 5$ and $b = 5 - a$ (2)

Substituting this for b in (1): $3a - 2(5 - a) = -5$

$3a - 10 + 2a = -5$

$5a = 5$

$a = 1$

Substituting this in (2):

$b = 5 - 1$

$b = 4$

Sally got 1 correct answer and 4 incorrect answers in the test.

Division and multiplication

1 **a**

$$\begin{array}{r} 3\,5\,7 \\ \times\quad 6 \\ \hline 2\,1\,4\,2 \end{array}$$

2142

b

$$\begin{array}{rl} 2\,6\,1 & \\ \times\ 4\,3 & \\ \hline 7\,8\,3 & (=261 \times 3) \\ 1\,0\,4\,4\,0 & (=261 \times 40) \\ \hline 1\,1\,2\,2\,3 & \end{array}$$

11223

c

$$\begin{array}{r} 0\,9\,2 \\ 6\overline{)\,5\,{}^{5}5\,{}^{1}2} \end{array}$$

92

d

$$\begin{array}{r} 0\,5\,2 \\ 13\overline{)\,6\,7\,6} \\ -6\,5 \\ \hline 0\,2\,6 \end{array}$$

52

2 **a**

$$\begin{array}{rl} 0\,1\,2 & \text{remainder 12} \\ 24\overline{)\,3\,0\,{}^{6}0} & \end{array}$$

So 12 boxes are filled.

b $24 \times 12 = 20 \times 12 + 4 \times 12 = 240 + 48 = 288$

$300 - 288 = 12$

There are 12 books left over.

3 $12\,500 - 440 = 12\,060$

$$\begin{array}{r} 0\,0\,3\,3\,5 \\ 36\overline{)\,1\,{}^{1}\cancel{2}\,{}^{1}0\,6\,0} \\ -1\,0\,8 \\ \hline 1\,{}^{1}\cancel{2}\,{}^{1}6 \\ -1\,0\,8 \\ \hline 0\,1\,8\,0 \end{array}$$

Each repayment is £335.

4 **a** $36 \times (52 - 6)$

$= 36 \times 46$

$= 1656$ hours

b She will work $\frac{36 \times 2}{3} = 24$ hours per week, with $\frac{6 \times 2}{3} = 4$ weeks' holiday.

Hours worked:

$24 \times (52 - 4)$

$= 24 \times 48$

$= 1152$ hours

Calculating with decimals

1 First note the combined number of decimal places in both numbers (2).

Remove the decimal points to do the calculation:

$$\begin{array}{rl} 92 & \\ \times\ 83 & \\ \hline 276 & (= 92 \times 3) \\ 7360 & (= 92 \times 80) \\ 1 & \\ \hline 7636 & \\ \hline 1 & \end{array}$$

Now you've got the digits right put the decimal point back, counting in from the right 2 places, to give a number with 2 decimal places:

76.36

2

$$\begin{array}{r} 22.50 \\ +\ 19.99 \\ \hline 42.49 \\ \hline \end{array}$$

$$\begin{array}{r} \overset{4}{\cancel{5}}\overset{1}{\cancel{0}}.\overset{9}{\cancel{0}}\overset{1}{0} \\ -\ 42.49 \\ \hline 07.51 \\ \hline \end{array}$$

She should get £7.51 change.

3

$$\begin{array}{r} 0\,3\,8.2\,9 \\ 6\overline{)\,2^{2}2^{4}9^{1}.7^{5}4} \end{array}$$

38.29

4 Kirsty raises $\frac{172.50}{5 + 1}$

Kirsty raises £28.75 × 5 = 28.75 × 10 ÷ 2 = 287.5 ÷ 2 = 143.7

$$\begin{array}{r} 0\,2\,8.7\,5 \\ 6\overline{)\,1^{1}7^{5}2^{4}.5^{3}0} \end{array}$$

Kirsty raises

£143.75

$$\begin{array}{r} 1\overset{6}{\cancel{7}}\overset{11}{\cancel{2}}.\overset{14}{\cancel{5}}\overset{1}{0} \\ -\ 143.75 \\ \hline 028.75 \\ \hline \end{array}$$

Flo raises £28.75

Rounding and estimation

1 Digit after second decimal place is 8, so round previous digit (9) up, to 10, and round the 7 up to 8. You must still include a zero in the second decimal place, to show the required level of accuracy.

0.80 (to 2 d.p.)

2 4.09 could have been rounded up or down.

Lower bound: 4.085, because 5 rounds up, giving 9.

Upper bound: 4.095, because everything between 4.09 and this number rounds down, giving 9. You will need to use a < sign, because 4.095 is not included (it would round to 4.10).

$4.085 \leq x < 4.095$

3 $\frac{9.74 \times 4.02}{7.88} \approx \frac{10 \times 4}{8} = 5$

4 **a** $40 \times 500 = 20\,000$

$20\,000 - 12\,500 = £7500$

b Overestimate, because the concert ticket price and number of tickets sold were rounded up, and so the amount of income was estimated more than it really is.

Converting between fractions, decimals and percentages

1 **a** There are $0 \times 10^{\text{ths}}$, $7 \times 100^{\text{ths}}$ and $1 \times 1000^{\text{ths}}$ $= 071 \times 1000^{\text{ths}}$ so: $\boldsymbol{\frac{71}{1000}}$

b $63 \div 100 = 0.63$

c $0.4 \times 100 = 40\%$

d $32\% = \frac{32}{100} = \frac{8}{25}$

2 **a** $5 \div 16 = 0.3125$

b To convert a number to a percentage, multiply its decimal value by 100.

$0.3125 \times 100 = 31.25\%$

3 $\frac{5}{8} = 0.625$, $60\% = 0.6$, so 0.65 is the largest.

Ordering fractions, decimals and percentages

1 **a** $\frac{1}{2} = \frac{5}{10} = 0.5$, so

$\frac{1}{2} < 0.6$

b $\frac{3}{4} = 3 \div 4 = 0.75$, so

$\frac{3}{4} > 0.7$

c $\frac{-3}{10} = -0.3$, so

$\frac{-3}{10} < 0.2$

2 **a** LCM of 12, 15 and 20 is 60

$\frac{5}{12} = \frac{25}{60}$

$\frac{7}{15} = \frac{28}{60}$

$\frac{9}{20} = \frac{27}{60}$

So order from lowest to highest is $\frac{5}{12}, \frac{9}{20}, \frac{7}{15}$

b $45\% = \frac{45}{100} = 0.45$

$\frac{1}{25} = \frac{4}{100} = 0.04$

$0.04 < 0.4 < 0.45$

So order is:

$\frac{1}{25}$, 0.4, 45%

3 Shop C is cheapest $\left(\frac{2}{5} = 40\%\right)$, then Shop A $\left(\frac{1}{3} = 33.3...\%\right)$, and Shop B offers the least discount at 30%.

4 $\frac{5}{9} = 0.\dot{5}$

$38.5\% = 0.385$

$\frac{3}{10} = 0.3$

So the order is $\frac{5}{9}$, 38.5%, 0.38, $\frac{3}{10}$

Calculating with fractions

1 $\frac{1}{5} + \frac{4}{9} = \frac{9}{45} + \frac{20}{45} = \frac{9 + 20}{45} = \frac{29}{45}$

2 $2\frac{3}{4} - 2\frac{2}{3} = \frac{11}{4} - \frac{8}{3} = \frac{33}{12} - \frac{32}{12} = \frac{1}{12}$

3 $1\frac{5}{6} \times \frac{2}{7} = \frac{11}{6} \times \frac{2}{7} = \frac{22}{42} = \frac{11}{21}$

4 $6 \div \frac{3}{5} = 6 \times \frac{5}{3} = \frac{30}{3} = 10$

Jo can make 10 necklaces.

Percentages

1 $\frac{40}{100} \times 25 = 10$

2 $16 \times 0.85 = £13.60$

3 $12450 \times 1.14 = 14193$

4 $40 \times 7 \times 3 = £840$

$840 \times 1.2 = £1008$

Order of operations

1 $3^2 - 6 \div (2 + 1) = 9 - \frac{6}{3} = 9 - 2 = 7$

2 $2^3 + 3x\sqrt{25} = 8 + (3 \times 5) = 8 + 15 = 23$

3 $(1.7 - 0.12)^2 + \sqrt[3]{4.096} = 4.0964$

Exact solutions

1 Area of triangle $= \frac{1}{2} \times$ base $\times$ vertical height $= 0.5 \times 0.76 \times 0.35 = 0.133\,\text{cm}^2$

2 $\left(1\frac{1}{3}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9} = 1\frac{7}{9}\,\text{m}^2$

3 $\sqrt{2} \times \sqrt{6} = \sqrt{12} = 2\sqrt{3}\,\text{cm}^2$

4 Area of a circle $= \pi r^2$

The fraction of the circle shown $= \frac{3}{4}$

The area of the circle shown $= \frac{3}{4} \times \pi r^2$

The radius = 2 cm

So area of shape shown $= \frac{3}{4} \times \pi \times 2^2 = 3\pi\,\text{cm}^2$

Indices and roots

1 **a** $7 \times 7 \times 7 \times 7 = 7^4$

b $\frac{1}{5 \times 5 \times 5} = \frac{1}{5^3} = 5^{-3}$

2 **a** $2^4 = 2 \times 2 \times 2 \times 2 = 16$

b Reciprocal means make it the denominator of a fraction with 1 as the numerator:

$\frac{1}{100}$

3 $2^3 = 2 \times 2 \times 2 \times 2 = 8$

$3^{-2} = \frac{1}{9}$

$\sqrt[3]{27} = 3$

$4^0 = 1$

$\sqrt{25} = 5$ or -5

Assuming the square root of 25 is positive, the answer is:

$3^{-2}, 4^0, \sqrt[3]{27}, \sqrt{25}, 2^3$

If it were negative, the answer would be:

$\sqrt{25}, 3^{-2}, 4^0, \sqrt[3]{27}, 2^3$

4 $\frac{9^5}{9^3 \times 9^2} = \frac{9^5}{9^5} = 1$

Standard form

1 2750

2 1.5×10^8

3 Move the decimal point three places to the right to give 6.42×10^{-3}

4 $(1.4 \times 10^{-5}) \times 20 = (2.8 \times 10^{-5}) \times 2 \times 10 = 2.8 \times 10^{-4}$ km

Listing strategies

1 259, 295, 529, 592, 925, 952

2 **a**

		4-sided spinner			
		0	**1**	**2**	**3**
3-sided spinner	**1**	1	2	3	4
	2	2	3	4	5
	3	3	4	5	6

b 4

3

		Dice					
		1	2	3	4	5	6
Coin	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

4 spj; spi; sfj; sfi ; bpj; bpi; bfj; bfi

Algebra

Understanding expressions, equations, formulae and identities

1 **a** identity **b** equation **c** expression

2 **a** Equation, because it has an equals sign and can be solved.

b Formula, because it has letter terms, an equals sign and the values of the letters can vary.

c Expression, because it has letter terms and no equals sign.

d Formula, because it has letter terms, an equals sign and the values of the letters can vary.

3 **a** Any of: $2x + 10$ or $10x + 2$ or $x + 210$ or $x + 102$

b Any of: $2x = 10$ or $10x = 2$

Simplifying expressions

1 $8x$

2 **a** $6a \times 8a = (6 \times 8) \times (a \times a) = 48 \times a^2 = 48a^2$

b $2p \times 3p \times 5p = (2 \times 3 \times 5) \times (p \times p \times p)$

$= 30 \times p^3 = 30p^3$

3 $35yz \div 7z = (35 \div 7) \times (yz \div z) = 5 \times y = 5y$

4 $\frac{32uv}{4v} = \frac{32}{4} \times \frac{uv}{v} = 8 \times u = 8u$

Collecting like terms

1 **a** $7m + 6n - 4m - 2n = (7 - 4)m + (6 - 2)n = 3m + 4n$

b $9q - 5r - 12q + 3r = (9 - 12)q + (3 - 5)r = -3q - 2r$

2 **a** $a + b \times b$

Use BIDMAS: **M**ultiplication before **A**ddition.

$a + b^2$

b $6c - 4d - 7c + 5d = (6 - 7)c + (5 - 4)d = -c + d$

3 **a** $9p^3 + p - 4p^3 = (9 - 4)p^3 + p = 5p^3 + p$

b $12 - 5x^2 + 3x - 2x^2 = 12 - (5 + 2)x^2 + 3x$

$= -7x^2 + 3x + 12$

4 $3\sqrt{5} - f - 8\sqrt{5} + 2f = (3 - 8)\sqrt{5} + (2 - 1)f = -5\sqrt{5} + f$

Using indices

1 **a** $p^3 \times p = p^{(3 + 1)} = p^4$

b $4y^2 \times 3y^3 = (4 \times 3) \times y^{(2 + 3)} = 12 \times y^5 = 12y^5$

c $2a^4b \times 5ab^2 = (2 \times 5) \times a^{(4 + 1)} \times b^{(1 + 2)}$

$= 10 \times a^5 \times b^3 = 10a^5b^3$

2 **a** $q^{-2} \times q^{-4} = q^{(-2-4)} = q^{-6}$

b $(u^{-3})^2 = u^{((-3) \times 2)} = u^{-6}$

c $x^{-1} \times x = x^{-1} \times x^1 = x^{(-1+1)} = x^0 = 1$

3 **a** $b^4 \div b^3 = b^{(4-3)} = b^1 = b$

b $\frac{f^5}{f^2} = f^{(5-2)} = f^3$

c $\frac{xy^3}{x^2y} = x^{(1 - 2)} \times y^{(3 - 1)} = x^{-1} \times y^2 = \frac{1}{x} \times y^2 = \frac{y^2}{x}$

4 Let the first box $= x$ and the second box $= y$

$(xm^3)^y = x^y m^{3y} = 8m^9$

comparing terms, $3y = 9$

$y = 3$

Substitute in the y value: $(xm^3)^3 = 8m^9$

$x^3 = 8$

$x = \sqrt[3]{8} = 2$

Therefore, the completed expression is $(2m^3)^3$

Expanding brackets

1 **a** $4(m + 3) = (4 \times m) + (4 \times 3) = 4m + 12$

b $2(p - 1) = (2 \times p) + (2 \times -1) = 2p - 2$

c $10(3x - 5) = (10 \times 3)x + (10 \times -5) = 30x - 50$

2 **a** $3(m + 2) + 5(m + 1) = 3m + 6 + 5m + 5 = 8m + 11$

b $6(x - 1) - 2(x - 4) = 6x - 6 - 2x + 8 = 4x + 2$

3 **a** $(y + 3)(y + 7) = y^2 + 7y + 3y + 21 = y^2 + 10y + 21$
b $(b + 2)(b - 4) = b^2 - 4b + 2b - 8 = b^2 - 2b - 8$
c $(x - 4)(x - 6) = x^2 - 6x - 4x + 24 = x^2 - 10x + 24$

4 **a** $(q + 1)^2 = (q + 1)(q + 1) = q^2 + q + q + 1 = q^2 + 2q + 1$
b $(z + 2)^2 = (z + 2)(z + 2)$
$= z^2 + 2 \times z + z \times 2 + 2 \times 2$
$= z^2 + 2z + 2z + 4$
$= z^2 + 4z + 4$
c $(c - 3)^2 = (c - 3)(c - 3) = c^2 - 3c - 3c + 9 = c^2 - 6c + 9$

Factorising

1 Divide the expression by the highest common factor (HCF) of both terms to find the bracket, and then place the HCF outside of the bracket to give the full factorisation.
a $(4x + 8) \div 4 = x + 2$
factorisation: $4(x + 2)$
b $(3d - 15) \div 3 = d - 5$
factorisation: $3(d - 5)$
c $(8y - 12) \div 4 = 2y - 3$
factorisation: $4(2y - 3)$

2 Divide the expression by the common term to find the bracket, and then place the common term outside of the bracket to give the full factorisation.
a $(q^2 + q) \div q = q + 1$
factorisation: $q(q + 1)$
b $(a^2 + 6a) \div a = a + 6$
factorisation: $a(a + 6)$
c $(10z^2 + 15z) \div 5z = (2z + 3)$
factorisation: $5z(2z + 3)$

3 Find which factors of the number term add together to give the coefficient of the x term.
a $12 = 3 \times 4$
$7 = 3 + 4$
factorisation: $(x + 3)(x + 4)$
b $-16 = (-2) \times 8$
$6 = -2 + 8$
factorisation: $(x - 2)(x + 8)$
c $24 = (-6) \times (-4)$
$-10 = (-6) + (-4)$
factorisation: $(a - 6)(a - 4)$

4 **a** Write $y^2 - 4$ in the form of $a^2 - b^2$:
$y^2 - 2^2$
Using the formula for the difference of two squares, the factorisation is
$(y + 2)(y - 2)$
b Write $x^2 - 9$ in the form of $a^2 - b^2$:
$x^2 - 3^2$
Using the formula for the difference of two squares, the factorisation is
$(x + 3)(x - 3)$
c Write $p^2 - 100$ in the form of $a^2 - b^2$:
$p^2 - 10^2$
Using the formula for the difference of two squares, the factorisation is
$(p + 10)(p - 10)$

Substituting into expressions

1 $4x + 5y = 4 \times 3 + 5 \times (-2) = 12 - 10 = 2$

2 $s = ut + \frac{1}{2}at^2$
$= 12 \times 2 + \frac{1}{2} \times 10 \times 2^2$
$= 12 \times 2 + \frac{1}{2} \times 40$
$= 24 + 20$
$s = 44$

3 **a** $12mn = 12 \times 6 \times \left(-\frac{1}{2}\right)$
$= 12 \times -3$
$= -36$
b $\frac{m}{n} = \frac{6}{-\frac{1}{2}}$ or $6 \div -\frac{1}{2}$
$= 6 \times \frac{-2}{1}$
$= -12$

4 **a** $f = 3c - 2(c - d)$
$= 3 \times 7 - 2 \times (7 - (-5))$
$= 21 - 2 \times (12)$
$= 21 - 24$
$f = -3$
b $f = -c(d^2 - 3c)$
$= -7 \times ((-5)^2 - 3 \times 7)$
$= -7 \times (25 - 21)$
$= -7 \times 4$
$f = -28$
c $f^2 = 7c - 3d$
$= 7 \times 7 - 3 \times (-5)$
$= 49 + 15$
$= 64$
$f = \sqrt{64}$
$f = \pm 8$

Writing expressions

1 **a** $n + 3$ **b** $(n \times 2) - 9 = 2n - 9$

2 **a** $x + y$ **b** $5 \times x = 5x$
c $(12 \times x) + (11 \times y) = 12x + 11y$

3 $2 \times 9p + 2(5p + 2) = 18p + 10p + 4 = 28p + 4$

4 The area of the rectangle is given by height × length, which is $s \times (5s + 1) = s(5s + 1)$.

Solving linear equations

1 **a** $x = 12 - 5$
$x = 7$
b $x = 10 + 3$
$x = 13$
c $x = \frac{20}{4}$
$x = 5$
d $x = 6 \times 3$
$x = 18$

2 **a** $2x + 3 = 15$
$2x = 12$
$x = 6$
b $3x - 5 = 16$
$3x = 21$
$x = 7$
c $\frac{x}{5} + 3 = 8$
$\frac{x}{5} = 5$
$x = 25$
d $7 - 2x = 1$
$7 = 2x + 1$
$6 = 2x$
$x = 3$

3 **a** $3(x + 9) = 30$
$3x + 27 = 30$
$3x = 3$
$x = 1$
b $5(p - 2) = 10$
$5p - 10 = 10$
$5p = 20$
$p = 4$

c $2(10 - 3m) = 8$
$20 - 6m = 8$
$12 = 6m$
$m = 2$

d $4(8 - 2q) = 8(4 - q) = 0$
So $4 - q = 0$
$q = 4$

4 **a** $4x - 6 = x + 9$
$3x - 6 = 9$
$3x = 15$
$x = 5$

b $2y + 5 = 4y - 3$
$5 = 2y - 3$
$8 = 2y$
$y = 4$

c $4(2x + 3) = 11x + 3$
$8x + 12 = 11x + 3$
$9 = 3x$
$x = 3$

d $3(n + 4) = 2(2n + 3)$
$3n + 12 = 4n + 6$
$12 = n + 6$
$n = 6$

Writing linear equations

1 Sum of the angles in a triangle are 180°
$(2x + 3) + 81 + (3x - 4) = 180$
$5x + 80 = 180$
$5x = 100$
$x = 20$

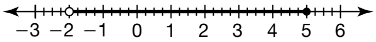
2 Let Jamie's age = x years. Sophie's age = $\frac{x}{2}$
$x + \frac{x}{2} = 18$
$\frac{3x}{2} = 18$
$3x = 36$
$x = 12$
Jamie is 12 years old

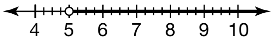
3 Let width = x so length = $x + 3$
Perimeter = $2x + 2(x + 3)$
$46 = 4x + 6$
$x = 10$
Length = 10 cm and width = 13 cm
Area = 10 × 13 = 130 cm^2

4 Opposite angles are equal so $3x + 10 = 5x - 20$
$30 = 2x$ giving $x = 15$
Also $3x + 10 + 7x + 5y = 180$
$10x + 10 + 5y = 180$
Now $x = 15$ so $150 + 10 + 5y = 180$
Solving this gives $y = 4$

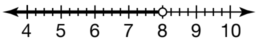
Linear inequalities

1 **a** The signs show that −2 is not included, but 5 is: −1, 0, 1, 2, 3, 4, 5

b


2 **a** $4x > 20$
$x > 5$


b $3x - 8 \leq 13$
$3x \leq 21$
$x \leq 7$


c $2(x - 3) < 10$
$2x - 6 < 10$
$2x < 16$
$x < 8$


3 **a** $2 \leq 3x + 5$
$-3 \leq 3x$
$-1 \leq x$
$3x + 5 < 11$
$3x < 6$
$x < 2$
Hence $-1 \leq x < 2$

b $-4 > 5x + 6$
$-10 < 5x$
$-2 < x$
$5x + 6 \leq 6$
$5x \leq 0$
$x \leq 0$
Hence $-2 < x \leq 0$

4 $n + n + 3 < 15$
$2n + 3 < 15$
$2n < 12$
$n < 6$
Possible integer values of n = 1, 2, 3, 4, 5

Formulae

1 **a** $t = (40 \times 2) + 20 = 100$ minutes = 1 hour 40 minutes

b $t = (40 \times 1.5) + 20 = 80$ minutes = 1 hour 20 minutes
The chicken should be put in the oven 1 hour and 20 minutes earlier than 1.30 pm, which is a time of 12.10 pm.

2 **a** $C = l + kn$

b $C = 90 + 6.5 \times 3$
$C = £109.50$

3 **a** $p = \frac{qs}{3}$
$3p = qs$
$q = \frac{3p}{s}$

b $p = \frac{q}{r} + t$
$p - t = \frac{q}{r}$
$q = rp - rt$ or $r(p - t)$

c $p = 3(q + r)$
$\frac{p}{3} = q + r$
$q = \frac{p}{3} - r = \frac{p - 3r}{3}$

d $p = \sqrt{2q}$
$p^2 = 2q$
$q = \frac{p^2}{2}$

Linear sequences

1 **a** The term in position 1 is 1 × 5 + 1 = 6
The term in position 2 is 2 × 5 + 1 = 11
The term in position 3 is 3 × 5 + 1 = 16
The term in position 4 is 4 × 5 + 1 = 21

b The term in position 50 is 50 × 5 + 1 = 251

2 **a** Each pattern has 2 more dots than the last, so pattern 7 will have 8 more dots than pattern 3. Pattern 7 will have 19 dots.

b No, Rachel is not correct, because the number of triangles is not the pattern number multiplied by 2. Instead, it is the pattern number plus 2, so there will be 6 triangles in pattern 4.

3 **a** Common difference = 11, so $11n$ is in the sequence.

When $n = 1$:

$11n = 11$, but the 1st term is 3.

$3 = 11n - 8$

So the expression for the sequence is $11n - 8$

b Assume 100 is in the sequence. Then:

$11n - 8 = 100$

$11n = 108$

$n = 108 \div 11 = 9$ remainder 9

But n must be a whole number, and it is not; so 100 is not in this sequence.

Non-linear sequences

1 **a** Rule is multiply by 2.

$8 \times 2 = 16$

$16 \times 2 = 32$

So terms are 16, 32.

b Rule is divide by 10.

$1 \div 10 = 0.1$

$0.1 \div 10 = 0.01$

So terms are 0.1, 0.01

c Rule is multiply by −2.

$-12 \times -2 = 24$

$24 \times -2 = -48$

So terms are 24, −48.

d They involve multiplying and dividing, not adding and subtracting, so they are geometric.

2 **a** $1 = 1 \times 1$, $4 = 2 \times 2$, $9 = 3 \times 3$, $16 = 4 \times 4$…

$1^2, 2^2, 3^2, 4^2$…

square numbers

b $1 = 1 \times 1 \times 1$, $8 = 2 \times 2 \times 2$, $27 = 3 \times 3 \times 3$, $64 = 4 \times 4 \times 4$…

$1^3, 2^3, 3^3, 4^3$…

cube numbers

3 **a** Count the dots in each triangle:

1, 3, 6, 10

b Add another row (of 5 dots) under the 4th triangle:

$10 + 5 = 15$

Now add another row again (6 this time):

$15 + 6 = 21$

15, 21

4 **a** Next term = 6 + 9 =15

b 5th term = 6 + 9 = 15

6th term = 9 + 15 = 24

7th term = 15 + 24 = 39

8th term = 24 + 39 = 63

9th term = 39 + 63 = 102

The 9th term is the first term in the sequence over 100

5 **a**

Day	Mon	Tue	Wed	Thu	Fri
Number of ladybirds	2	8 (= 2 × 4)	32 (= 8 × 4)	32 × 4 = 128	128 × 4 = 512

The gardener is correct. There will be more than 500 ladybirds.

b Saturday, because 512 × 4 = 2048.

6 **a** First term: $\frac{1}{2} \times 1^2 = \frac{1}{2}$

Second term: $\frac{1}{2} \times 2^2 = 2$

Third term: $\frac{1}{2} \times 3^2 = \frac{9}{2} = 4\frac{1}{2}$

b If 32 is in the sequence, then:

$\frac{1}{2}n^2 = 32$

$n^2 = 64$

$n = 8$

This gives n as a whole number, 8, so 32 is the 8th term in the sequence.

Show that...

1 LHS = $2x + 1$; RHS = $2x + 1$; LHS = RHS. Therefore, $2\left(x + \frac{1}{2}\right) \equiv x + x + 1$

2 LHS = $x^2 - 25 + 9 = x^2 - 16$; RHS = $x^2 - 16$

3 Let the three consecutive numbers be n, $n + 1$ and $n + 2$.

$n + n + 1 + n + 2 = 3n + 3 = 3(n+1)$. Therefore, the sum of three consecutive numbers is a multiple of 3.

4 **a** Width of pond = $x - y + x + x + x - y = 4x - 2y$
Length of pond = $4x$

Perimeter = $4x - 2y + 4x - 2y + 4x + 4x = 16x - 4y$

b Yes Sanjit is correct, because $16x - 4y = 4(4x - y)$, showing that when x and y are whole numbers, the perimeter is always a multiple of 4.

Functions

1 **a** when $x = 3$, $y = 3 \times 4 - 1 = 11$

b when $y = 23$, $4x - 1 = 23$, therefore $x = (23 + 1) \div 4 = 6$

c To get y you multiply x by 4 and subtract 1, so $y = 4x - 1$

2

x	Operations	y
−2	(−2) × 2 + 3	−1
0	0 × 2 + 3	3
3	(9 − 3) ÷ 2	9

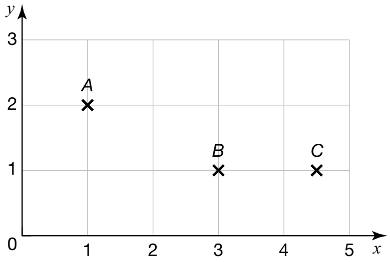
3

x	Operations	y
−2	(−2) ÷ 2 + 1	0
1	(1) ÷ 2 + 1	$1\frac{1}{2}$
8	(5 − 1) × 2	5

Coordinates and midpoints

1 **a** 1 along the x-axis and 2 up the y-axis: (1, 2)

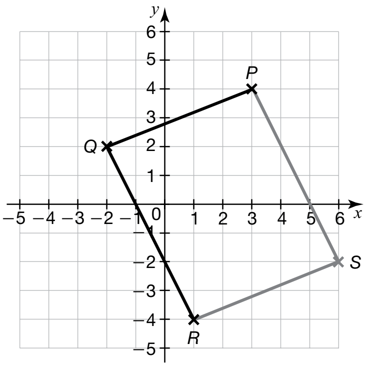
b



c $4\frac{1}{2}$ along the x-axis and 1 up the y-axis: $(4\frac{1}{2}, 1)$

2 **a** 1 along the x-axis and −4 ‘up’ the y-axis: (1, −4)

b



S = (6, −2) to make a parallelogram

3 **a** x coordinate $= \frac{4 + (-2)}{2} = 1$

y coordinate $= \frac{5 + 1}{2} = 3$

Midpoint of *XY* is (1, 3).

b Midpoint of *XZ* $= \left(\frac{4 + 4}{2}, \frac{5 + (-4)}{2}\right) = (4, \frac{1}{2})$

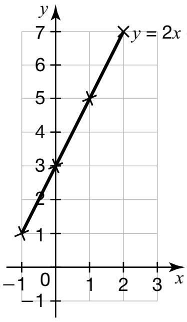
c Midpoint of *YZ* $= \left(\frac{(-2) + 4}{2}, \frac{1 + (-4)}{2}\right) = (1, -1\frac{1}{2})$

Straight-line graphs

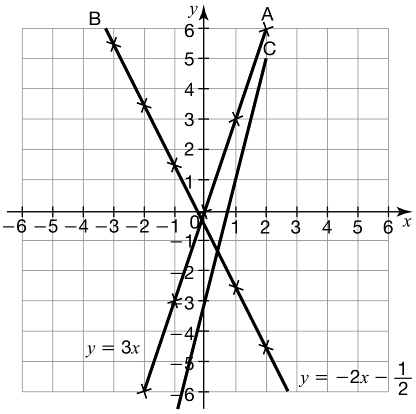
1 **a** For $y = 2x + 3$

x	−1	0	1	2
Operations	2 × (−1) + 3	2 × (0) + 3	2 × (1) + 3	2 × (2) + 3
y	**1**	**3**	**5**	**7**

b



2 **a** and **b**



a For $y = 3x$

x	−2	−1	0	1	2
Operations	3 × (−2)	3 × (−1)	3 × (0)	3 × (1)	3 × (2)
y	**−6**	**−3**	**0**	**3**	**6**

b Rearrange the equation to give $y = -2x - \frac{1}{2}$

x	Operations	y
−3	$-2 \times (-3) - \frac{1}{2}$	$5\frac{1}{2}$
−2	$-2 \times (-2) - \frac{1}{2}$	$3\frac{1}{2}$
−1	$-2 \times (-1) - \frac{1}{2}$	$1\frac{1}{2}$
0	$-2 \times (0) - \frac{1}{2}$	$-\frac{1}{2}$
1	$-2 \times (1) - \frac{1}{2}$	$-2\frac{1}{2}$
2	$-2 \times (2) - \frac{1}{2}$	$-4\frac{1}{2}$

c Line C goes through points (0, −3), (1, 1) and (2, 5)

The y intercept is −3.

The gradient is $\frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}} = \frac{5 - 1}{2 - 1} = 4$

The equation of line C is $y = 4x - 3$.

3 **a** B and C, because they have the same gradient of 2.

b A and B, because they both have a y-intercept at (0, 1).

4 The gradient is $\frac{2 - (-6)}{3 - (-1)} = \frac{8}{4} = 2$

Using point (3, 2) and gradient m = 2:

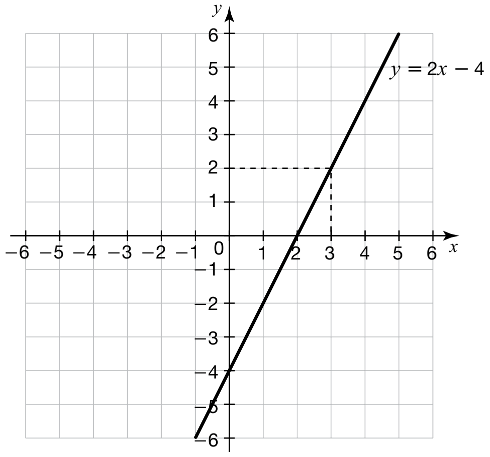
$y = 2x + c$

$2 = 2 \times 3 + c$

$c = -4$

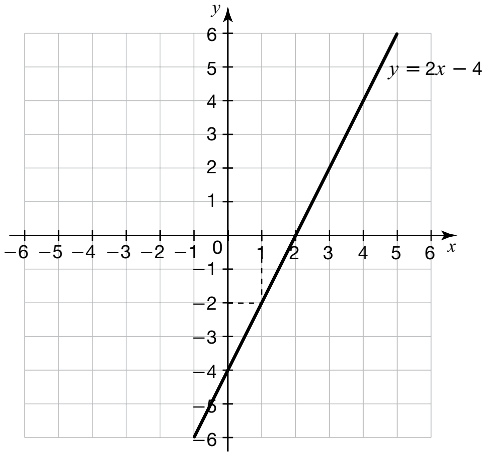
Equation of the line is $y = 2x - 4$

5 **a**



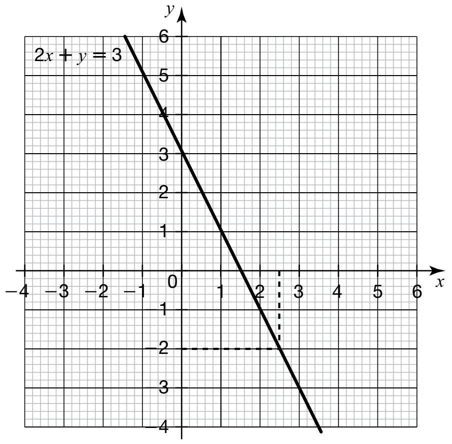
Draw a vertical line up from $x = 3$ to the graph, and then a horizontal line to the y-axis to read off the result: $y = 2$.

b



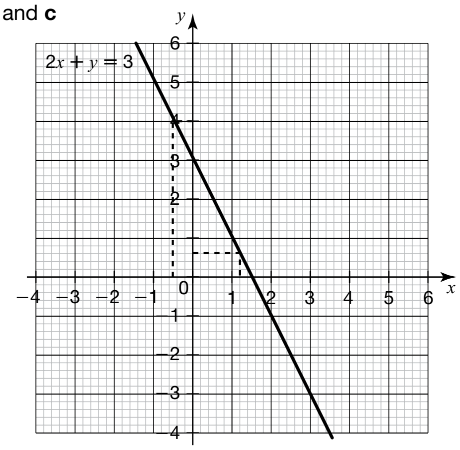
Draw a horizontal line across from $y = -2$ to the graph, and then a vertical line up to the x-axis to read off the result: $x = 1$.

6 **a**



Draw a horizontal line across from $y = -2$ to the graph, and then a vertical line up to the x-axis to read off the result: $x = 2.5$.

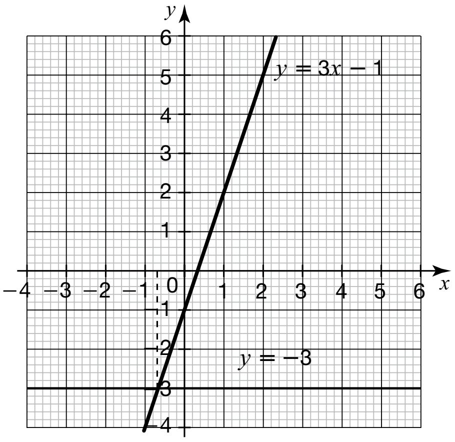
b and **c**



b Draw a vertical line up from $x = -0.5$ to the graph, and then a vertical line across to the y-axis to read off the result: $y = 4$.

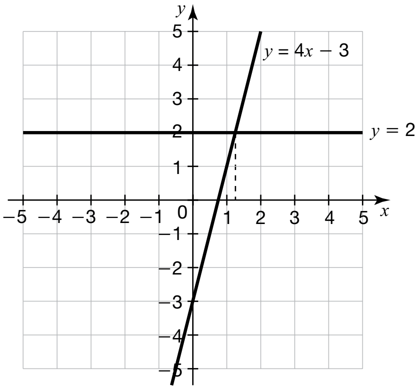
c Reading up from $x = 1.2$ and then across to y axis gives $y = 0.6$. Any value from 0.6 to 0.75 is acceptable.

7 **a** and **b**



b Where the graphs cross, draw a vertical line up to the x-axis. It meets the axis two thirds of the way between $x = -1$ and $x = 0$, so the solution is approximately $x = -0.67$. Any value from −0.6 to −0.7 is acceptable.

8



Compare the equations $y = 4x - 3$ and $4x - 3 = 2$. y has been replaced with 2, so add line $y = 2$ to the graph.

The intersection point of the two graphs gives the solution to the equation $4x - 3 = 2$

$x = 1.25$. Any answer between 1.2 and 1.3 is acceptable.

Solving simultaneous equations

1 **a** Substituting $y = 2x$ into the first equation gives

$3x + 2x = 15$

$5x = 15$

$x = 3$

When $x = 3$, $y = 2 \times 3 = 6$

b $3x = 12$

$x = 4$

Substituting $x = 4$ into the first equation gives

$8 + y = 9$

$y = 1$

c $5x = 10$

$x = 2$

Substituting $x = 2$ into the first equation gives

$6 + y = 4$

$y = -2$

2 **a** $2x + 2y = 14$ (1)

$3x + y = 11$ (2)

(2) × 2 $6x + 2y = 22$

(3) − (1) $4x = 8$

$x = 2$

Substitute into (1) $4 + 2y = 14$

$2y = 10$

$y = 5$

Solution is $x = 2$, $y = 5$

b $4x - 2y = 2$ (1)

$2x - 3y = 7$ (2)

(2) × 2 $4x - 6y = 14$ (3)

(3) − (1) $-4y = 12$

$y = -3$

Substitute into (1) $4x + 6 = 2$

$4x = -4$

$x = -1$

Solution is $x = -1$, $y = -3$

c $2x + 3y = 20$ (1)

$3x + 2y = 15$ (2)

(1) × 2 $4x + 6y = 40$ (3)

(2) × 3 $9x + 6y = 45$ (4)

(4) − (3) $5x = 5$

$x = 1$

Substituting $x = 1$ into equation (1) $2 + 3y = 20$

$3y = 18$

$y = 6$

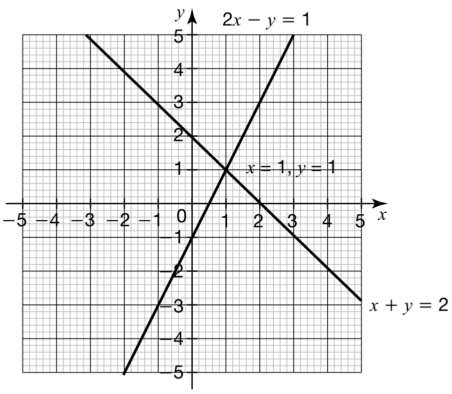
Solution is $x = 1$, $y = 6$

3 **a** $x + y = 2$ $2x - y = 1$

x	0	2
y	2	0

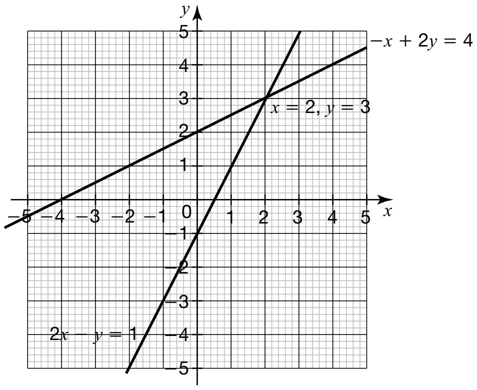
x	0	$\frac{1}{2}$
y	−1	0

b



From the intersection of the two lines, $x = 1$, $y = 1$.

c



From the intersection of the two lines, $x = 2$, $y = 3$.

Quadratic graphs

1 **a** C and D – they are straight lines so they are linear.

b E – it is quadratic, with a positive multiplier for x^2, and is symmetrical about the origin.

c A – it is quadratic, with a negative multiplier for x^2, and is symmetrical about the origin.

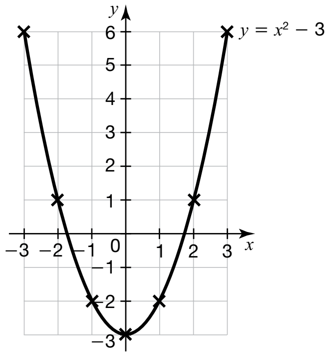
d D – the x coordinates are all different, but all the y coordinates on this line are 1.

e B – it is the same as E except that it has been moved 1 unit up the y-axis.

2 **a**

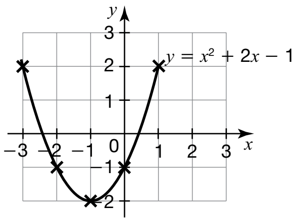
x	−3	−2	−1	0	1	2	3
y	6	1	−2	−3	−2	1	6

b



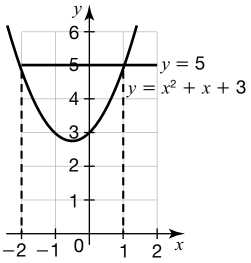
c $x = 0$ **d** (0, −3)

3 **a**



b $x = -1$ **c** (−1, −2)

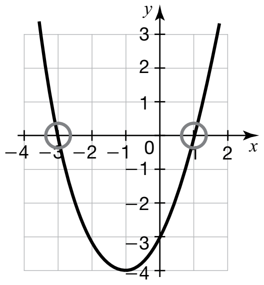
4 **a**



b $x = -2$ or $x = 1$

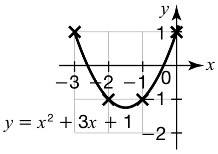
5 The roots are where the curve cuts the x-axis.

$x = -3$ or $x = 1$

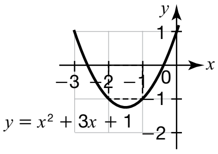


6 **a**

x	−3	−2	−1	0
$x^2 + 3x + 1$	$(-3)^2 + 3\times(-3) + 1$	$(-2)^2 + 3\times(-2) + 1$	$(-1)^2 + 3\times(-1) + 1$	$(0)^2 + 3\times(0) + 1$
y	**1**	**−1**	**−1**	**1**



b Compare the equations $y = x^2 + 3x + 1$ and $x^2 + 3x + 1 = -1$. y has been replaced with −1, so the solutions to the equation $x^2 + 3x + 1 = -1$ are where $y = -1$.



$x = -2$ and $x = -1$

c Compare the equations $y = x^2 + 3x + 1$ and $x^2 + 3x + 1 = 0$. y has been replaced with 0, so the solutions to the equation $x^2 + 3x + 1 = 0$ are where the graph crosses the x-axis.

$x = -2.6$ and $x = -0.38$ (any answer close to −0.4 is acceptable)

Solving quadratic equations

1 **a** $x(x + 6) = 0$

$x = 0$ or -6

b $y(y - 11) = 0$

$y = 0$ or 11

c $3d(d - 3) = 0$

$d = 0$ or 3

2 **a** $(x + 4)(x - 4) = 0$

$x = 4$ or -4

b $(a + 9)(a - 9) = 0$

$a = 9$ or -9

c $(z - 10)(z + 10) = 0$

$z = 10$ or -10

3 **a** $(x + 3)(x + 2) = 0$

$x = -2$ or -3

b $(x + 5)(x - 2) = 0$

$x = -5$ or 2

c $(x - 7)(x - 2) = 0$

$x = 2$ or 7

4 **a** $0 = x(x - 3)$

$x = 0$ or 3

b $0 = (x - 5)(x + 5)$

$x = 5$ or -5

c $0 = (x + 6)(x - 3)$

$x = -6$ or 3

Cubic and reciprocal graphs

1 **a** A, C and D – they are not continuous curves with two turning points (s-shaped curves).

b B – it has two turning points and has rotational symmetry about the origin.

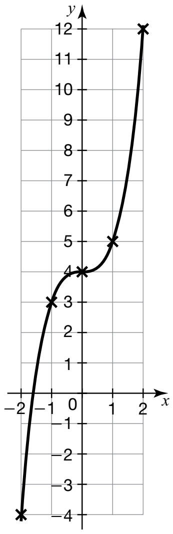
c E – it has two turning points and is a reflection of B, raised up one unit on the y-axis.

d D – this is the form for a reciprocal graph.

2 **a**

x	−2	−1	0	1	2
y	−4	3	4	5	12

b

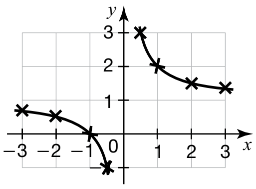


3 **a** cubic **b** (0, −8) **c** (2, 0)

4 **a**

x	−3	−2	−1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	3
y	$\frac{2}{3}$	$\frac{1}{2}$	0	−1	3	2	$1\frac{1}{2}$	$1\frac{1}{3}$

b



Drawing and interpreting real-life graphs

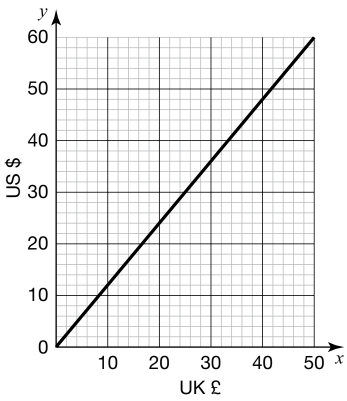
1 **a** The initial charge is the value at 0 miles, where the graph cuts the y-axis (the y-intercept): $3.

b The charge per mile is given by the gradient of the graph.

$$\text{Gradient} = \frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}} = \frac{23-3}{8-0} = \frac{20}{8} = 2.5$$

Charge per mile is $2.50

c



d From the conversion graph, £15 = $18.

From the graph of the cost of a taxi in New York, $18 allows you to travel 6 miles.

2 **a** You can read this from the highest point of the graph: 16 m.

b Read from the highest point down to the value on the horizontal axis: 4 seconds.

c This is when the ball reaches 0 height for the second time: 8 seconds.

d Read from 12 on the vertical axis across to the curve: 2 seconds and 6 seconds.

3 **a** This is the highest value on the vertical axis: 10 m/s.

b The cyclist is travelling at a constant speed of 10 m/s.

c The cyclist is decelerating, so it will be a negative value.

Acceleration $= \frac{-10}{20} = -\frac{1}{2}$ m/s^2

d The speed returns to 0: the cyclist stops.

4 **a** Assuming that water pours into the container at a constant rate,

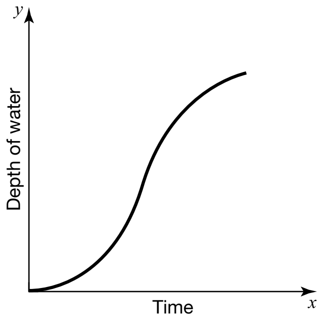
A: depth goes up increasingly slowly as the container widens, so 2.

B: depth rises steadily and fairly slowly in a broad container of consistent diameter, so 4.

C: depth rises quickly at first, then more slowly as the container widens, then more quickly as it gets narrow towards the top, so 3.

D: depth rises steadily and quickly in a narrow container of consistent diameter, so 1.

b



Ratio, proportion and rates of change

Units of measure

1 **a** 4 (m) × 100 = 400 cm

b 500 (g) ÷ 1000 = 5 kg

c 1.5 (l) × 1000 = 1500 ml

d 8250 (m) ÷ 1000 = 8.25 km

2 6 (litres) × 1000 = 6000 ml

6000 − 3500 = 2500

Sally had **2500 ml** of lemonade left.

3 **a** Luke: 240 seconds; Adam: 3 × 60 + 47 = 227 seconds. Adam arrived first.

or Luke: 240 ÷ 60 = 4 minutes; Adam: 3 minutes 47 seconds. Adam arrived first.

b 4 minutes − 3 minutes 47 seconds = 13 seconds. Adam waited **13 seconds** for Luke to arrive at school.

4 Ben = 1.25 m = 3.2 + 0.8 feet = 4 feet. Tom is taller.

or Tom = 4.8 feet = 3.2 + 1.6 feet = 1 + 0.5 metres = 1.5 metres. Tom is taller.

Ratio

1 12 stories and 8 colouring

Ratio of story: colouring = 12 : 8 = 3 : 2

2 **a** density of tin : density of copper = 2 : 1

b 10 g is 1 part

So 90 g is $\frac{90}{10}$ = 9 parts

9

c weight of tin : weight of copper = 1 : 9.

density of tin: density of copper = 2 : 1

$\text{density} = \frac{\text{weight}}{\text{volume}}$ and so $\text{volume} = \frac{\text{weight}}{\text{density}}$

So ratio of volume of tin to copper in the bronze

$= \frac{1}{2} : \frac{9}{1}$

Multiply both parts by 2:

= 1 : 18

3 Total parts = 1 + 2 + 3 = 6

1 part = $\frac{60}{6}$ = 10

Amount given to charity = 3 × 10 = £30

4 Ratio of blue to yellow required is 3 : 7.

There are 3 + 7 = 10 parts. He needs to make 5 litres.

10 parts = 5000 ml

1 part = 500 ml

Phil needs 3 × 500 ml = 1500 ml = 1.5 litres of blue paint. He has 2 litres of blue paint.

Phil needs 7 × 500 ml = 3500 ml = 3.5 litres of yellow paint. He has 3 litres of yellow paint.

Phil has enough blue paint, but does not have enough yellow paint.

Scale diagrams and maps

1 1 cm on the map is **10 000 cm** in real life.

This means 1 cm on the map is **100 m** in real life.

2 1 cm on the map represents 50 m in real life.

3 × 50 m = 150 m, so the bus stop is **3 cm** from the village shop on the map.

3 Measure the distance between the trees on the diagram = 5 cm

1 cm on the diagram represents 4 m in real life.

5 × 4 = 20

The trees are **20 m** apart.

4 A scale of 1 : 400 means 1 cm on the model represents 4 m (= 400 cm) in real life.

96 ÷ 4 = 24

The scale mode is **24 cm** tall.

Fractions, percentages and proportion

1 1 + 3 = 4 parts so Bess receives $\frac{3}{4}$

2 a 1 : 3 : 6

b 1 + 3 + 6 = 10 items in the basket

Fruit = $\frac{3}{10}$

c Tins = $\frac{6}{10}$ = 60%

3 $\frac{50}{4000} = \frac{1}{80}$

4 Total parts = 3 + 8 + 14 = 25

$\frac{8}{25} = \frac{32}{100}$ = 32%

Direct proportion

1 a One ticket costs £80 ÷ 5 = £16

b Nine tickets cost 9 × £16 = £144

2 a Read up from 6 packs on the horizontal axis, to the line, then across to the vertical axis to find the cost: £1.20

b There are 10 pencils in a pack, so 1 pencil is 0.1 of a pack. Reading off the graph using this value, the price is 2p.

c It is a straight-line graph; the graph passes through the origin (0, 0).

3 a Sally needs to make 28 ÷ 4 = 7 lots of the recipe.

She will need 1 × 7 = 7 teaspoons of turmeric, 2 × 7 = 14 teaspoons of chilli powder and $2\frac{1}{2} \times 7 = 17\frac{1}{2}$ teaspoons of cumin.

b Sally has 75 g of chilli powder. That is 75 ÷ 3 = 25 teaspoons.

Sally needs 14 teaspoons to make the curry for her class. She does have enough.

Inverse proportion

1 a Start from 5 on the x-axis, read up to the graph, then left to the scale on the y-axis.

5 winners will each get **£400**.

b Start from 200 on the y-axis, read right to the graph, then down to the scale on the x-axis. 10 winners each get £200, so there are **9** other winners.

c Multiply the number of winners by the amount each one gets to find the total prize money. e.g. 5 × £400 = **£2000**.

2 a The total time needed to decorate the room is 3 × 2 = 6 hours.

b 6 hours ÷ 12 people = 0.5 hours = **30 minutes** (or 0.5 hours).

3 The printer can print 240 ÷ 4 = 60 pages per minute.

600 ÷ 60 = 10, so it would take **10 minutes** to print the larger document.

4 $y = 3x$ means that as x increases, y increases (3 times as quickly).

$y = \frac{3}{x}$ means that as x increases, 3 is divided by a bigger number, so y gets smaller.

$y = x - 3$ means that as x increases, y increases but y is always 3 smaller than x.

$y = \frac{x}{3} = \frac{1}{3} \times x$ means that as x increases, y increases ($\frac{1}{3}$ as quickly).

$y = x + 3$ means that as x increases, y increases but y is always 3 bigger than x.

So the answer is $y = \frac{3}{x}$

Working with percentages

1 125 − 75 = 50

$\frac{50}{125} \times 100$ = 40%

2 a Amount of increase = 24 − 15 = £9 million

$\frac{9}{15} \times 100$ = **60%** increase in sales for Company X

b 125% = £35 million

35 ÷ 125 × 100 = **£28 million** sales in 2006.

3 a $\frac{2}{100} \times 265$ = 5.30 and 3 × 5.30 = 15.90

15.90 + 265 = £280.90

b 265 × $(1.02)^3$ = £281.22

Compound units

1 $\frac{3\text{ km}}{\text{minute}} = \frac{3000\text{ m}}{\text{minute}} = \frac{3000\text{ m}}{60\text{ seconds}}$ = 50 m/s

2 20 ÷ 5 = 4 minutes to fill the tank.

3 Pressure = $\frac{300}{0.05}$ = 6000 Newtons/m^2 (**or** 6000 N/m^2)

4 On Saturday Sami drove 4 × 50 = 200 miles; on Sunday Sami drove 356 ÷ 8 × 5 = 222.5 miles. Sami drove further on Sunday.

Geometry and measures

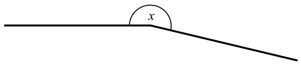
Measuring and drawing angles

1 a 123°

b Use a protractor to measure angle (angle *ABC*): 42°

c 331°

2



3 a 100°, 120°, 140°, 160°

b First angle + second angle = 87°. This means both angles are less than 87°, and so they both must be acute.

Using the properties of angles

1 a x = 360 − 111 − 102 − 94

= 53° (angles around a point sum to 360°)

b x = 180 − 49

= 131° (alternate angles are equal and angles on a straight line add up to 180°)

c Angle *ACB* = 52° (Angles in a triangle add up to 180°)

x = 128° (Angles on a straight line add up to 180°)

d Angle *ADC* = 86° (Angles in a quadrilateral add up to 360°)
$x = 94°$ (Angles on a straight line add up to 180°)

2 Angles in a triangle add up to 180°, so apex of Meg's teepee = 180°− 2θ = apex of Jonah's teepee.

Since Jonah's teepee is symmetrical, both its other angles are equal. So each angle is $\frac{2\theta}{2} = \theta$.

So the outlines of both teepees have all three angles the same and are congruent.

3 **a** Angle *BED* = 39° (Alternate angles are equal)
Angle *BDE* = 39° (Base angles in an isosceles triangle are equal)
$x = 102°$ (Angles in a triangle add up to 180°)

b Angle *DCF* = 98° (Vertically opposite angles are equal)
$x = 98°$ (Corresponding angles are equal)

4 Angle *CFG* = 62° (Co-interior angles add up to 180°)
$x = 66°$ (Angles on a straight line add up to 180°)

5 $x + 40 + 3x + 5x - 40 = 180°$
$9x = 180°$
$x = 20°$
Angle *BAC* = $x + 40 = 20 + 40 = 60°$
Angle *ACB* = $3x = 3 \times 20 = 60°$
Angle *ABC* = $5x - 40 = 5 \times 20 - 40 = 60°$
Triangle *ABC* has equal angles of 60°. Therefore, it is an equilateral triangle.

6 angle *ABC* = angle ***XAB*** (alternate angles)
angle *ACB* = angle ***YAC*** (alternate angles)
angle *BAC* + angle ***XAB*** + angle ***YAC*** = 180° (angles on a straight line add up to 180°)
So angle *BAC* + angle *ABC* + angle *ACB* = 180°

Using the properties of polygons

1 **a** 180° × (6 − 2) = 720°
b 720° ÷ 6 = 120°
c 180° − 120° = 60° or 360° ÷ 6 = 60°

2 **a** It is an octagon because it has eight sides.
b All angles are equal; all sides are equal.
c 180° × (8 − 2) = 1080°
1080° ÷ 8 = 135°
or 180° − (360° ÷ 8) = 135°

3 exterior angle = 180° − 144° = 36°
number of sides = 360 ÷ 36 = 10
Therefore it is a decagon.

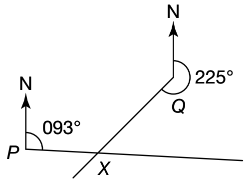
Using bearings

1 **a** This is the angle measured clockwise from North at A: 065°.
b 180 − 138 = 42° This is the acute angle at *C*.
Bearing of *B* from *C* = 360 − 42° = 318°
c 180 − 65 = 115° This is the angle between the north line at *B* and *AB*, measured anticlockwise.
Bearing of *A* from *B* = 360 − 115 = 245°

2 To find a reciprocal bearing, subtract 180 from the original bearing (or add 180 to it).
The bearing of *O* from *X* = 276 − 180 = 096°

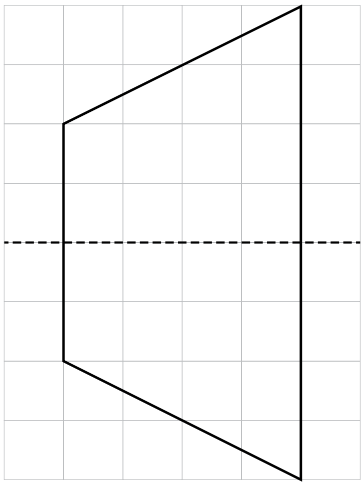
3 **a** Draw a North line at *P*, then join *P* to *Q* and measure the angle between the North line and this line: 060° (any value 058° to 062° accepted).

b



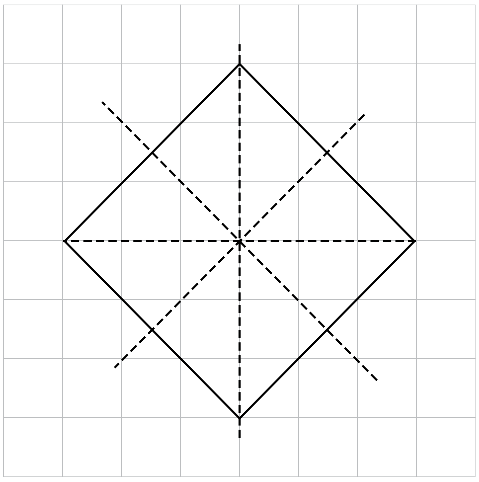
Properties of 2D shapes

1 **a**



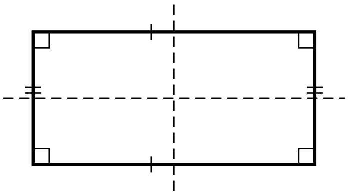
b trapezium
c one pair of parallel sides

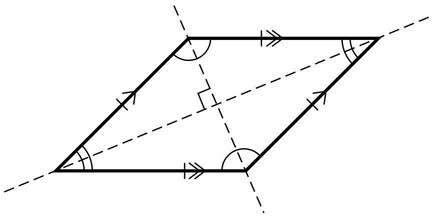
2 **a**



b 4
c square
d Two from: all sides equal in length; all angles are 90°; diagonals are equal; diagonals bisect each other at 90°.

3 **a** rectangle, rhombus
b





Congruent shapes

1 D and F are exactly the same as A – they are the same size and shape (it doesn't matter that they are rotated). E is similar to A, not congruent – it is smaller.

2 If the triangles are congruent, all three angles must be the same in both. You know that two of the angles are 35° and 82°, so $x = 180 - 35 - 82 = 63°$.

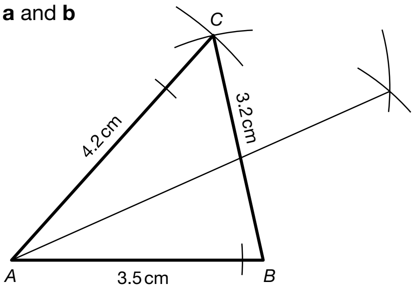
3 **a** Identify what values match: SAS (side, angle, side – two sides and the angle between them).

b Identify what values match: ASA (angle, side angle – two angles and a corresponding side).

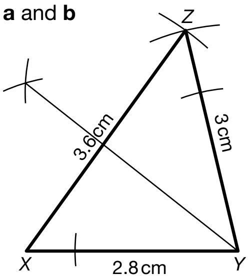
4 No, they are not congruent. They have the same angles, but the sides may not be the same size (one triangle could be an enlargement of the other).

Constructions

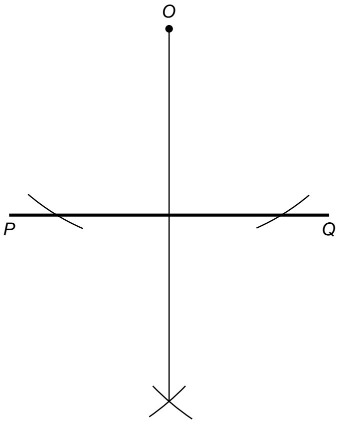
1 **a** and **b**



2 **a** and **b**



3 **a**



b Distance on the diagram = 2.5 cm

2.5 × 100 = 250 cm in real life = 2.5 m

Drawing circles and parts of circles

1 **a–d**

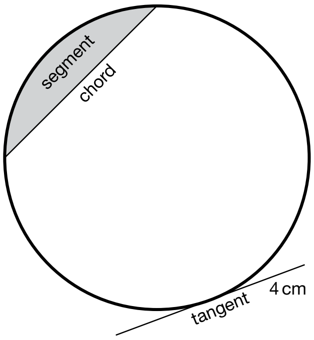


Diagram not to scale

2 **a** and **b**

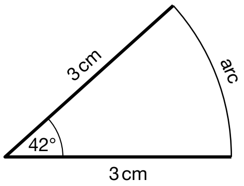
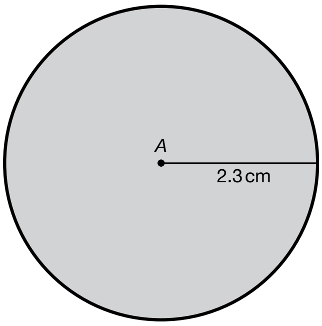


Diagram not to scale

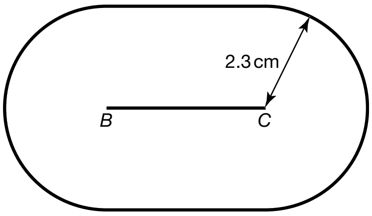
3 No, Donald is not correct. A segment of a circle is the area enclosed by a chord and an arc; a sector of a circle is the area enclosed by two radii and the arc between them.

Loci

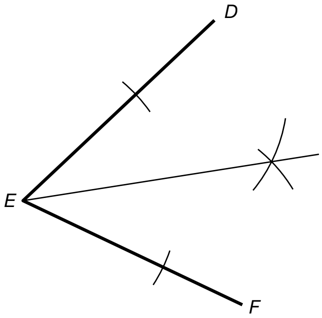
1 **a**



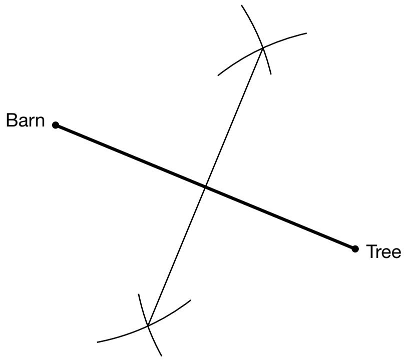
b

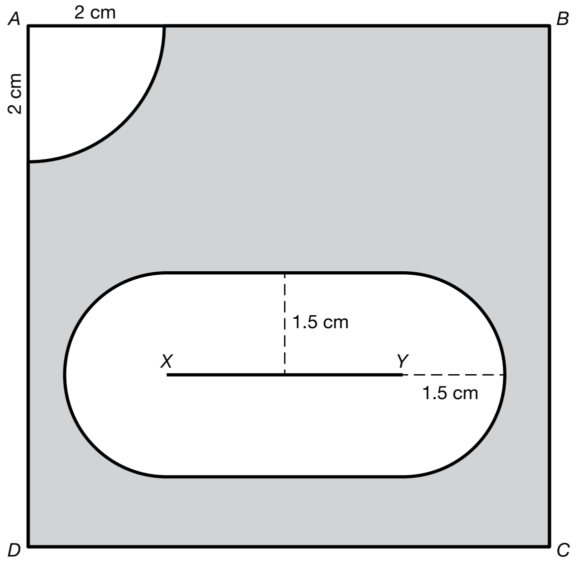


c



2



3


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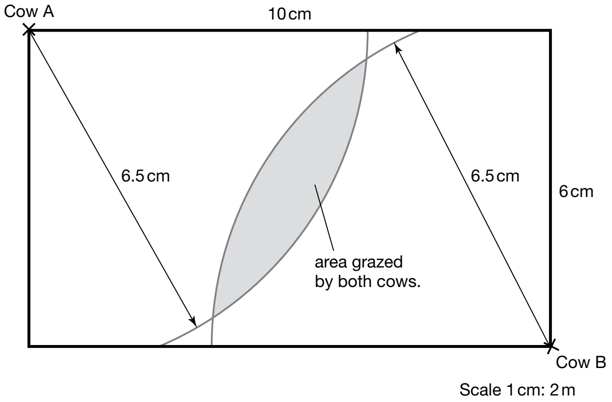


Perimeter

1 A hexagon has 6 sides so perimeter = 6 × 9 = 54 cm

2 Missing vertical length = 20 − 5 − 5 − 4 = 6 mm

Missing horizontal lengths are all equal = 25 − 13 = 12 mm each

Perimeter = 20 + 25 + 6 + 12 + 4 + 12 + 5 + 12 + 5 + 13
= 114 mm

114 ÷ 10 = 11.4 cm

3 Perimeter of cushion = $\frac{1}{2} \times 2 \times \pi \times 24 + 30 + 48 + 30$
= 183 cm (to nearest cm)
= 1.83 m. So no, Greta does not have enough lace.

Area

1 **a** Area = 12 × 6 = 72 cm^2

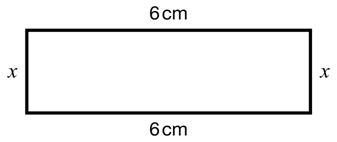
b Area = $\frac{1}{2} \times (3 + 8) \times 4 = 22$ cm^2

c Area of rectangle = 2 × 10 = 20 cm^2

Area of trapezium = $\frac{1}{2}(a + b)h = \frac{1}{2}(2.5 + 10)10$
= 62.5 cm^2

Area of shape = 20 + 62.5 = 82.5 cm^2

2 First draw a diagram. Two sides are equal, and are 6 cm. The two other sides are equal, and are x cm.



$x + x + 6 + 6 = 16$

$2x + 12 = 16$

$x = \frac{16 - 12}{2} = 2.$

Area = 6 × 2 = 12 cm^2

3 Radius of circle = 3.5 cm

Area of circle = $\pi r^2 = \pi \times 3.5^2 = 38.48$ cm^2

Area of square = 49 cm^2

Area of shaded part = $\frac{49 - 38.48}{4} = 2.63$ cm^2

Sectors

1 **a** $\frac{120}{360} = \frac{1}{3}$

b Area = $\frac{1}{3} \times \pi \times 3^2 = 3\pi$ cm^2

2 **a** Area = $\frac{1}{4} \times \pi \times 2.8^2 = 6.2$ cm^2 (1 d.p.)

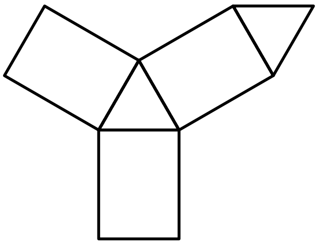
b Perimeter = $2.8 + 2.8 + \frac{1}{4} \times 2 \times \pi \times 2.8$
= 10.0 cm (1 d.p.)

3 **a** Area = $\frac{40}{360} \times \pi \times 5^2 = 8.73$ cm^2 (2 d.p.)

b Arc $AB = \frac{40}{360} \times 2 \times \pi \times 5 = 3.49$ cm (2 d.p.)

3D shapes

1 **a**

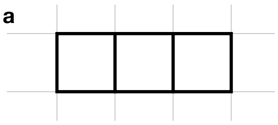


b triangular prism

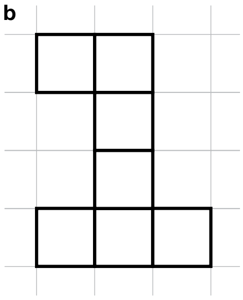
c

Number of faces	Number of edges	Number of vertices
5	9	6

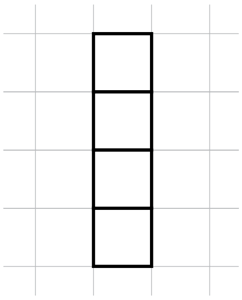
2 **a**



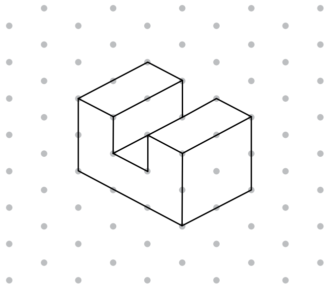
b



c



3 **a**



3 **b** The front elevation shows 5 cubes and the side shows that the shape is 2 cubes deep. 5 × 2 = 10, so 10 cubes make up the shape.

Volume

1 The front elevation shows 5 cubes and the side shows that the shape is 4 cubes deep.Volume $= 5 \times 4 = 20\text{ cm}^3$

2 **a** Volume = area of cross-section × length $= \pi \times 5^2 \times 12$

$= 942\text{ cm}^3$ (to 3 s.f.)

b Volume $= \frac{1}{3} \times$ area of cross-section × length

$= \frac{1}{3} \times \pi \times 7^2 \times 15 = 770\text{ cm}^3$ (to nearest cm)

3 Volume $= \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3} \times \pi \times 8^3 = 1072.33\text{ cm}^3$ (to 2 d.p.)

4 Volume of tank $= 40 \times 40 \times 60 = 96\,000\text{ cm}^3$

Volume of water in tank, 80% full $= 0.8 \times 96\,000$

$= 76\,800\text{ cm}^3$

Height of water in pond (1st fill) $= 76\,800 \div (80 \times 60) = 16\text{ cm}$

Height of water in pond (2nd fill) $= 16 \times 2 = 32\text{ cm}$

Height of water in pond (3rd fill) $= 16 \times 3 = 48\text{ cm}$

Three tanks of water are needed to fill the pond.

Alternative method: divide volume of pond by volume of water in tank.

$\frac{80 \times 60 \times 48}{76\,800} = 3$

Surface area

1 **a** 6 faces

b Surface area $= 60 + 60 + 5 + 5 + 3 + 3 = 136\text{ cm}^2$

2 surface area = area of triangular side × 4 + area of square base

$= \left(\frac{1}{2} \times 6 \times 5\right) \times 4 + 6^2$

$= (15 \times 4) + 36$

Surface area $= 96\text{ cm}^2$

3 **a** Surface area of a sphere $= 4\pi r^2 = 4 \times \pi \times 14^2$

$= 2463.01\text{ cm}^2$ (to 2 d.p.)

b Surface area of a cone $= \pi rl + \pi r^2 = \pi \times 6 \times 10 + \pi \times 6^2$

$= 301.59\text{ cm}^2$ (to 2 d.p.)

4 Area of cylinder $= 2\pi rh = 2\pi \times 6 \times 1.5 = 56.55\text{ cm}^2$

Area of circular base $= \pi r^2 = \pi \times 6^2 = 113.10\text{ cm}^2$

Area of curved surface area of cone $= \pi rl = \pi \times 6 \times 11.5$

$= 216.77\text{ cm}^2$

Total surface area $= 56.55 + 113.10 + 216.77 = 386.42\text{ cm}^2$ (to 2 d.p.)

Using Pythagoras' theorem

1 $x^2 = 3^2 + 4^2$

$= 9 + 16$

$= 25$

$x = \sqrt{25}$

$= 5\text{ cm}$

$15^2 = y^2 + 12^2$

$225 = y^2 + 144$

$81 = y^2$

$y = \sqrt{81}$

$= 9\text{ cm}$

2 $6^2 = 4.5^2 + w^2$

$36 = 20.25 + w^2$

$w = \sqrt{15.75}$

$= 3.97\text{ cm}$

Area $= l \times w = 4.5 \times 3.97 = 17.9\text{ cm}^2$

3 $AB^2 = 2^2 + 4^2$

$= 4 + 16$

$= 20$

$AB = \sqrt{20}$

$= \sqrt{4 \times 5}$

$= 2\sqrt{5}$ units

4 Square of diagonal of doorway $= 70^2 + 190^2 = 41\,000$

Diagonal of doorway $= \sqrt{41\,000}$

$= 202.48\text{ cm} = 2.0248\text{ m} = 2.02\text{ m}$ (2 d.p.)

Yes, the artwork will fit through the diagonal of the doorway.

Trigonometry

1 $\tan x = \frac{8}{13}$

$x = 31.6°$

2 $\cos 42 = \frac{17}{AC}$

$AC = \frac{17}{\cos 42} = 22.9\text{ cm}$

3 $\sin 49 = \frac{h}{6}$

$h = 6 \sin 49$

$= 4.53\text{ m}$

Exact trigonometric values

1 **a** $\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{1} = 1$

b $x = \tan^{-1}(1) = 45°$

2 $\cos 30 = \frac{\sqrt{3}}{PR}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{PR}$

$PR = 2\text{ cm}$

3 $\frac{YZ}{20} = \sin 30$

$YZ = 20 \sin 30$

$= 20 \times \frac{1}{2}$

$= 10\text{ cm}$

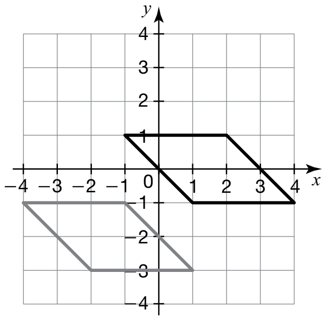
4 $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$

So the second angle in the triangle must be 45°.

The third angle will be $180 - 90 - 45 = 45°$.

Transformations

1



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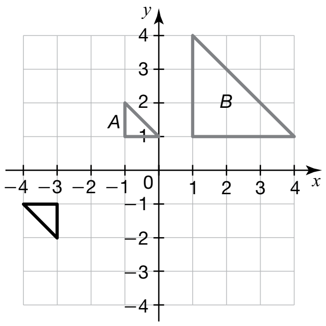
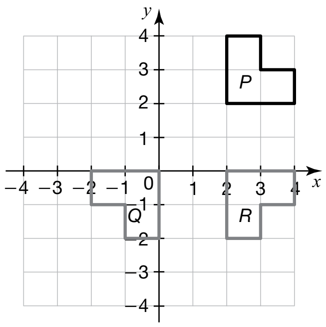


3 Rotation 90° clockwise about (1, −1); or rotation 270° anticlockwise about (1, −1).

4 **a** and **b**



c Reflection in $y = 1$

Similar shapes

1 **a** *YZW*

b Scale of enlargement = $\frac{\text{enlarged length}}{\text{original length}} = \frac{6.3}{2.1} = 3$

c $WZ = 4 \times 3 = 12$ cm

2 **a** 37.5°

b Scale of enlargement = $\frac{\text{enlarged length}}{\text{original length}} = \frac{5}{2.5} = 2$

Length of $AB = 8 \div 2 = 4$ cm

c They are isosceles, because they have two equal sides and two equal angles. Note that the diagrams are not drawn to scale, as is common practice in maths questions – you have to go by the numbers.

d Length of BC = length of $AC = 2.5$ cm

3 **a** Scale of enlargement = $\frac{\text{enlarged length}}{\text{original length}} = \frac{4}{6} = \frac{2}{3}$

b Length of $RT = 4.5 \times \frac{2}{3} = 3$ cm

4 **a** Scale of enlargement = $\frac{\text{enlarged length}}{\text{original length}} = \frac{2.2}{4.4} = \frac{1}{2}$

b Length of CE = length of $CD \div \frac{1}{2} = 2.8 \times 2 = 5.6$ cm

c Angle $ACE = 180 - 70 - 64 = 46°$

Vectors

1 $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ $\mathbf{d} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$

2 **a**

b $-\mathbf{p} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$

c $2\mathbf{p} = \begin{pmatrix} -12 \\ 2 \end{pmatrix}$

d $2\mathbf{p} + \mathbf{p} = \begin{pmatrix} -12 \\ 2 \end{pmatrix} + \begin{pmatrix} -6 \\ 1 \end{pmatrix} = \begin{pmatrix} -18 \\ 3 \end{pmatrix}$

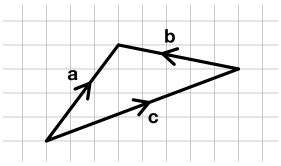
$3\mathbf{p} = 3 \times \begin{pmatrix} -6 \\ 1 \end{pmatrix} = \begin{pmatrix} -18 \\ 3 \end{pmatrix}$

Therefore, $2\mathbf{p} + \mathbf{p} = 3\mathbf{p}$

3 **a** $\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -5 \\ 1 \end{pmatrix} = \begin{pmatrix} 3-5 \\ 4+1 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

b $\mathbf{c} = \mathbf{a} - \mathbf{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -5 \\ 1 \end{pmatrix} = \begin{pmatrix} 3-(-5) \\ 4-1 \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

c



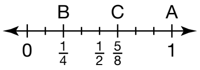
Probability

Basic probability

1 Probability = $\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}} = \frac{1}{10}$ (**or** 0.1 **or** 10%)

2 P(not rain) = 1 – P(rain) = 1 – 0.6 = 0.4

3 **a b c**



a P(a number from 1 to 8) = $\frac{8}{8} = 1$

b P(a multiple of 3) = $\frac{2}{8} = \frac{1}{4}$

c P(a number greater than 3) = $\frac{5}{8}$

4
$$\begin{aligned} 2p - 0.1 + 2p + 0.1 + p &= 1 \\ 5p &= 1 \\ p &= 0.2 \end{aligned}$$

Outcome	Red	Blue	Green
Probability	$2p - 0.1$ $= 2 \times 0.2 - 0.1$ $= 0.3$	$2p + 0.1$ $= 2 \times 0.2 + 0.1$ $= 0.5$	$p = 0.2$

Blue is most likely.

Two-way tables and sample space diagrams

1 Work out the missing values one by one, for example in the order shown from first to seventh. (There is more than one order you can do it in.)

	Single	Double	King	Totals
Oak	2	Fourth: 42 – 12 – 14 = 16	Fifth: 30 – 16 – 2 = 12	30
Pine	First: 54 – 14 – 17 = 23	14	17	54
Walnut	1	12	Sixth: 32 – 12 – 17 = 3	Seventh: 1 + 12 + 3 = 16
Totals	Second: 2 + 23 + 1 = 26	Third: 100 – 26 – 32 = 42	32	100

2 **a**

		Spinner			
		1	2	3	4
Coin	**Heads**	1, H	2, H	3, H	4, H
	Tails	1, T	2, T	3, T	4, T

b P(1, T) = $\frac{1}{8}$

c P(2, H) + P(3, H) + P(4, H) = $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

3 **a**

	Study sciences	Do not study sciences	Totals
Boys	$\frac{1}{5} \times 120 = 24$	45 – 24 = 21	$\frac{3}{8} \times 120 = 45$
Girls	75 – 40 = 35	$\frac{1}{3} \times 120 = 40$	120 – 45 = 75
Totals	24 + 35 = 59	21 + 40 = 61	120

b 21 boys do not study science, so probability = $\frac{21}{120}$ **or** $\frac{7}{40}$

c There are 75 girls and 35 of them study science, so probability = $\frac{35}{75} = \frac{7}{15}$

Sets and Venn diagrams

1 **a** ξ = {21, 22, 23, 24, 25, 26, 27, 28, 29}

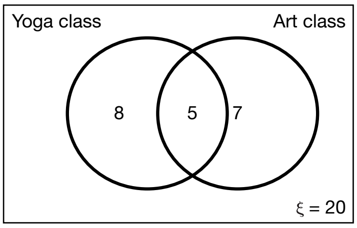
b A = {21, 24, 27}

c B = {24, 28}

d A ∪ B ={21, 24, 27, 28} – A 'union' B means **all** the values in A and **all** the values in B

e A ∩ B ={24} – A 'intersect' B means only those value that are in **both** A and B.

2 **a**



b There are 7 adults who only go to art class, so probability = $\frac{7}{20}$

c 8 adults only go to yoga class and 7 adults only go to art class.

Probability = $\frac{8+7}{20} = \frac{3}{4}$

3 **a** –3 is not included, but 2 is:

–2, –1, 0, 1, 2

b Upper and lower bounds are not integers. Values are: 8, 9

c Upper and lower bounds are not integers. Values are: 2, 3, 4, 5, 6

4 **a** Total number of respondents = ξ

$= 25 + 11 + 2 + 12 + 0 + 7 + 3 = 60$

P(supermarket only) $= \frac{25}{60} = \frac{5}{12}$

b $\text{P(F)} = \frac{12 + 2 + 7 + 3}{60} = \frac{24}{60} = \frac{2}{5}$

c $S = \{25 + 11 + 2 + 12\} = \{50\}$

$S' = \{60 - 50\} = \{10\}$

$\xi = \{50 + 0 + 3 + 7\} = \{60\}$

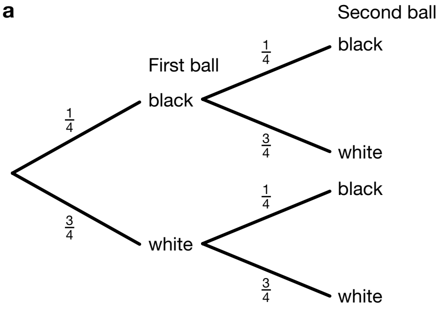
$\text{P}(S') = \frac{10}{60}$

$= \frac{1}{6}$

(This is the probability that the person never shops at a supermarket.)

Frequency trees and tree diagrams

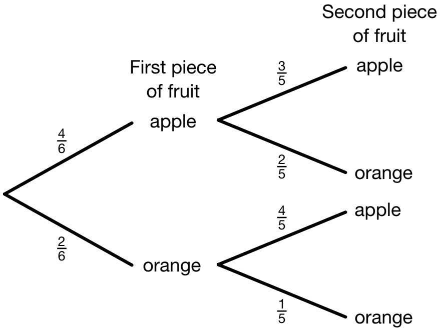
1 **a**



b $\text{P(W, B)} = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$

c $\text{P(B, B)} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

2 **a**



b From tree diagram:

$\text{P(A, O)} = \frac{4}{6} \times \frac{2}{5}$

$= \frac{8}{30}$

$= \frac{4}{15}$

c From tree diagram:

$\text{P(O, O)} = \frac{2}{6} \times \frac{1}{5}$

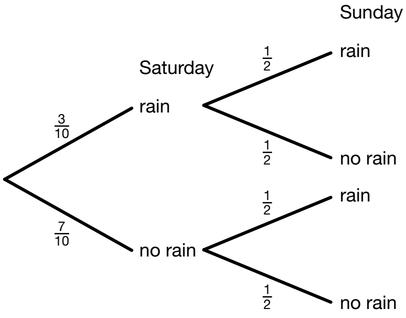
$= \frac{2}{30}$

$= \frac{1}{15}$

d $\text{P(O, O)} + \text{P(A, A)} = \left(\frac{2}{6} \times \frac{1}{5}\right) + \left(\frac{4}{6} \times \frac{3}{5}\right) = \frac{2}{30} + \frac{12}{30} = \frac{14}{30}$

$= \frac{7}{15}$

3 **a**



b $\text{P(R, R)} = \frac{3}{10} \times \frac{1}{2} = \frac{3}{20}$

c $\text{P}(R, R') + \text{P}(R', R) = \left(\frac{3}{10} \times \frac{1}{2}\right) + \left(\frac{7}{10} \times \frac{1}{2}\right)$

$= \frac{3}{20} + \frac{7}{20} = \frac{10}{20} = \frac{1}{2}$

The probability of one day having rain and one day having no rain is 50%.

Expected outcomes and experimental probability

1 **a** The spinner was spun 12 + 13 + 10 + 15 = 50 times.

b Estimated probability of blue $= \frac{13}{50}$

c Estimated probability of yellow $= \frac{15}{50} = \frac{3}{10}$

d You would expect $\frac{10}{15} \times 100 =$ **20** green outcomes from 100 spins.

2 0.75 × 20 = **15** students would be expected to pass the exam.

3 **a** Total number of customers = 26 + 20 + 6 + 5 + 3 = 60

Estimated probability that someone will buy stamps

$= \frac{20}{60} = \frac{1}{3}$

b $\frac{1}{3} \times 450 =$ **150** customers buy stamps each day

c $\frac{6}{60} \times 450 =$ **45** customers buy foreign currency each day

d 450 × 6 = **2700** customers each week

$\frac{5}{60} \times 2700 =$ **225** customers use the post office for banking each week

Statistics

Data and sampling

1 65, because that is 10% of 650 (the entire population).

2 $\frac{50}{25\,000} \times 100 = 0.2\%$. The sample is not big enough.

People in the town centre may not be the only ones using buses. For example, some people may take buses to the local train station, school or hospital.

3 **a** $\frac{9}{45} \times 400 = 80$ people like carrot cake

Sam needs to make 80 cakes.

b Assumptions

Assumed that these are individual carrot cakes. If instead each cake is a large one divided into 8 slices, then only 80 ÷ 8 = 10 carrot cakes would be needed.

Assumed the sample is representative of the population; this could affect the answer because not all the 400 people who have accepted the invitation may turn up.

Frequency tables

1 **a** 1 + 11 + 9 + 6 + 1 + 2 = 30 tables

b (1 × 1) + (2 × 11) + (3 × 9) + (4 × 6) + (5 × 1) + (6 × 2) = 91 people

2

Number of electronic devices	Tally	Frequency
0–1	|||	3
2–3	𝍸 𝍸	10
4–5	𝍸	5
6–7	𝍸	5
8–9	|	1

3 **a** Continuous

b

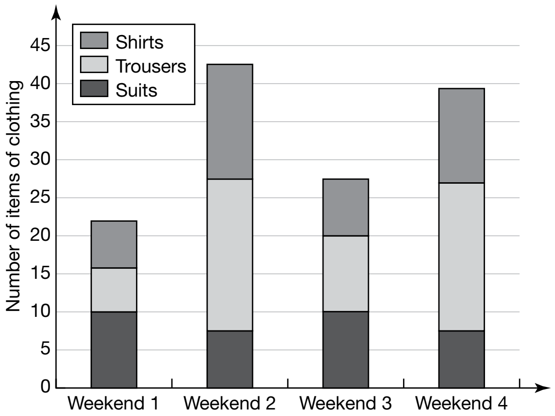
Mass, m (kg)	Tally	Frequency
$50 \leq m < 60$	|||	3
$60 \leq m < 70$	𝍸	5
$70 \leq m < 80$	||||	4
$80 \leq m < 90$	𝍸	5
$90 \leq m < 100$	|||	3

Bar charts and pictograms

1 **a** 9 − 4 = **5** more boys than girls prefer squash

b 15 + 6 + 9 + 7 + 4 = **41** girls were surveyed

2



3 **a** 3 + 3 + 3 + 3 + 2 = 14 hours of sunshine

b Yorkshire gets 9 hours, Inverness-shire gets 7 hours. Yorkshire gets 2 more hours of sunshine each day than Inverness-shire.

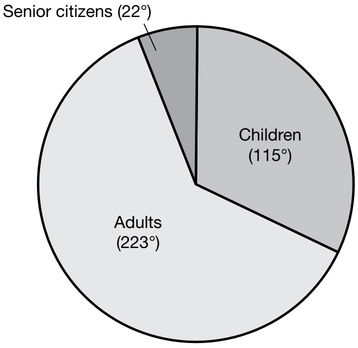
Pie charts

1 **a** $\frac{150}{360} \times 300 =$ **125** people last saw an action movie

b Romance = 90°, so $\frac{3}{4} \times 300 =$ **225** people did not see a romance movie.

2

	Children	**Adults**	**Senior citizens**
Calculation	$\frac{80}{250} \times 360$ $= 115.2$	$\frac{155}{250} \times 360$ $= 223.2$	$\frac{15}{250} \times 360$ $= 21.6$
Angle	115°	223°	22°



3 Angle for 8:30−9pm = 72°

$\frac{360}{72} \times 12 =$ **60** people were surveyed*

Stem and leaf diagrams

1 **a** 71 cm − this is the highest stem value combined with the highest leaf value

b 20 − count the number of leaf values

2 **a** longest − shortest = 4.26 − 2.03 = 2.23 m

b 3 girls jumped more than 3.5 m (3.65 m, 3.74 m and 4.06 m)

3 Write all the amounts in order, and convert them to pounds.

Year 9s	0.50	0.80	0.95	1.30	1.75	2.00	2.65	3.05
Year 10s	0.65	0.75	1.50	1.85	2.70	3.10		

Year 9s		Year 10s
95 80 50	0	65 75
75 30	1	50 85
65 00	2	70
05	3	10

Key
Year 9s Year 10s
50|0 means £0.50 0|65 means £0.65

Measures of central tendency: mode

1 There are two modes: 3 minutes and 4 minutes, since each appears twice. An alternative correct answer is to say that there is no mode.

2 This is the one with the highest frequency: $12 < a \leq 13$.

3 This is the class with the biggest slice of the pie chart: £10−£20.

4 Look for where there are most repeated digits to the right of the vertical line: 5, 5, 5.

Use key to work out these numbers: 25, 25, 25.

So **25 kg** is the modal weight.

5 The mode is 108, so this number must have the highest frequency.

2 of the numbers are 54, so 3 of the numbers must be 108.

The numbers are 54, 54, 108, 108, 108, 120.

The ‘other 3 numbers’ are all 108.

Measures of central tendency: median

1 Ages in order: 11 11 12 13 13 13 13 14 15 15 16 16 17 17 18 18

Median = $14\frac{1}{2}$ years old

2 Total frequency = 5 + 12 + 17 + 10 + 6 = 50

Median = $\frac{50 + 1}{2}$ = 25.5th person

Median class = $12 < a \leq 13$

3 The median is the middle value.

Counting the digits in each row:

2 + 5 + 14 + 6 + 4 + 2 + 5 + 2 = 40

The median is between the 20th and 21st values.

2 + 5 + 14 = 21, so the 20th and 21st values are the last two values in the 3rd row:

28 and 29 (using the key)

median = $\frac{28 + 29}{2}$

= 28.5 or $28\frac{1}{2}$

Measures of central tendency: mean

1 Mean age = $\frac{6 + 7 + 11 + 13 + 18}{5}$ = 11 years old

2 Number of bedrooms = (1 × 4) + (2 × 7) + (3 × 13) + (4 × 17) = 125

Number of houses = 4 + 7 + 13 + 17 = 41

Mean number of bedrooms = 125 ÷ 41 = 3.05 ≈ 3 bedrooms

3 Number of holidays = (0 × 4) + (1 × 21) + (2 × 9) + (3 × 2) = 45

Number of employees = 4 + 21 + 9 + 2 = 36

Mean number of holidays = 45 ÷ 36 = 1.25 ≈ 1 holiday

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*This answer is missing from the first edition of the Revision and Exam Practice Book.

4

Age of patients, a	Midpoint	Frequency	Midpoint × frequency
$0 < a \leq 10$	5	3	15
$10 < a \leq 20$	15	18	270
$20 < a \leq 30$	25	6	150
$30 < a \leq 40$	35	11	385
$40 < a \leq 50$	45	10	450
$50 < a \leq 60$	55	19	1045
$60 < a \leq 70$	65	16	1040
$70 < a \leq 80$	75	17	1275
		Total = 100	Total = 4630

Mean age of patients = $\frac{4630}{100} = 46.3 \approx 46$ years old

5 Let mean of first three measurements = m.

$3m = 3 \times 75.6 = 226.8$

There is now an extra measurement (4 in total).

New mean is:

$\frac{226.8 + 75.2}{4} = 75.5$ cm

Range

1 **a** Range of boys' ages = 4 − 2 = 2 years

b Range of boys' ages = 4 − 1 = 3 years

2 **a** Range in temperatures for Resort A = 25 − 16 = 9°C

b Range in temperatures for Resort B = 28 − 13 = 15°C

3 **a** Business A: Range = 45 816 − 23 561 = £22 255

Mean profit = $\frac{23\,561 + 30\,485 + 392\,10 + 45\,816}{4}$ = £34 768

b Business B: Range = 63 248 − 17 894 = £45 354

Mean profit = $\frac{32\,820 + 40\,328 + 17\,894 + 63\,248}{4}$ = £38 572.50

c **Either** Business A because its range in profit is lower and the profit is increasing each year, and so it shows a more consistent performance.

or Business B because its mean profit is higher, and its most recent profit (in Year 4) is £17 432 more than Business A.

Comparing data using measures of central tendency and range

1 **a** Mean time for bus journey = $\frac{32 + 30 + 39 + 32 + 43 + 31}{6}$ = 34.5 minutes

b Range for bus journey = 43 − 30 = 13 minutes

c Mean time for train journey = $\frac{16 + 24 + 18 + 26 + 70 + 17}{6}$ = 28.5 minutes

d Range for train journey = 70 − 16 = 54 minutes

e **Either** The bus is better because although it takes longer (on average), the range is lower, and so you can predict the time it takes for the journey.

or The train is better because it is quicker than the bus (on average), although the range suggests it may be less reliable.

2 **a** Mean = 13. This does not represent the age of the people using the playground. In fact, those using the playground are small children (under 10) and their parents (over 25).

b There are five modes (3, 4, 5, 7, 8), and so the mode does not represent the age of the people using the playground.

c Ages in order: 3 3 4 4 5 5 7 7 8 8 26 30 33 39

Median position = $\frac{14 + 1}{2}$ = 7.5th value

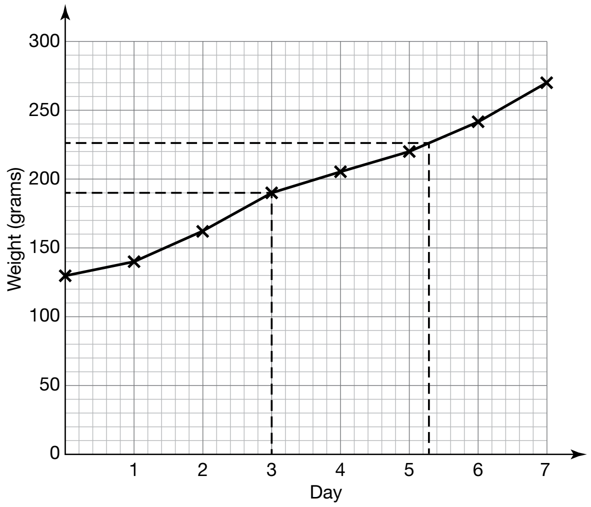
Median age = 7 years old

3 Mode = 0; Median = 0; Mean = 2 days. Mode or median are the best averages to use, because the mean is skewed by the student who is absent due to sickness for 24 days.

Time series graphs

1 **a** and **b**

Weight (grams) per day of a male kitten



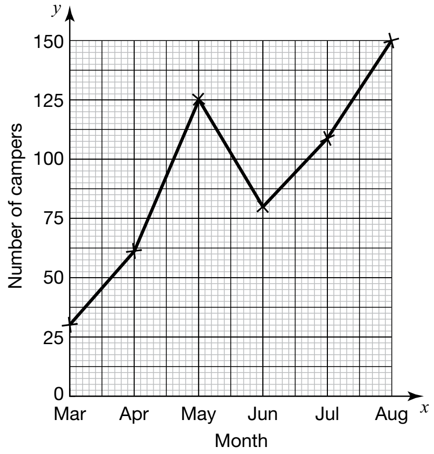
a Start at Day 3, read up to the graph line then left to the weight scale: 190 g

b According to the graph the kitten was just over 5 days old: 5 days (to the nearest day)

c Mean weight gained = $\frac{\text{total weight gained}}{\text{number of days}} = \frac{270 - 130}{7}$

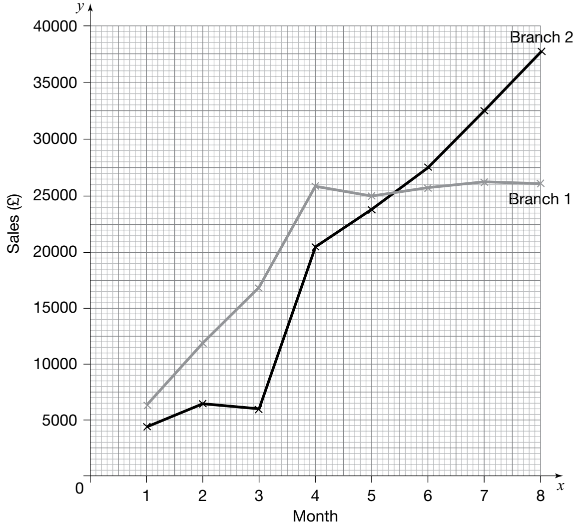
= 20 g per day

2 **a*** Number of campers on a campsite from March to August



b There is an increase in campers in May. This may be due to May bank holidays, or May half-term, or perhaps there was some very sunny weather.

3 **a** Sales figures for two different branches of a DIY store



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*This answer differs from the one in the Revision and Exam Practice Book due to an error in our first edition. This answer has now been re-checked and corrected.

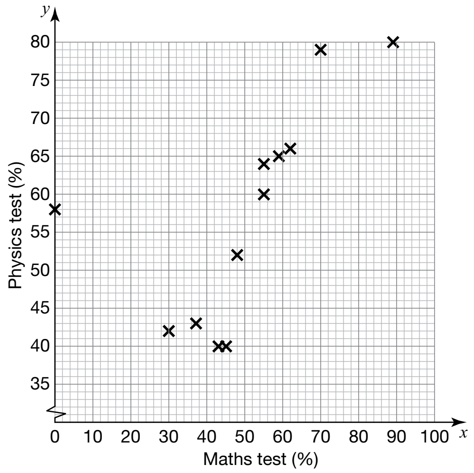
b Branch 1 had a steady increase in sales for the first four months. Then sales levelled off to stay at around £25 000. Branch 2 had a slow start to its sales in the first three months. Then perhaps it had a promotion, because sales increased a lot in month 4. Sales have been increasing ever since.

Scatter graphs

1 **a** Positive correlation. This means as the temperature rises, more pairs of flip flops are sold.

b Negative correlation. This means as the temperature rises, fewer wellington boots are sold.

2 **a**

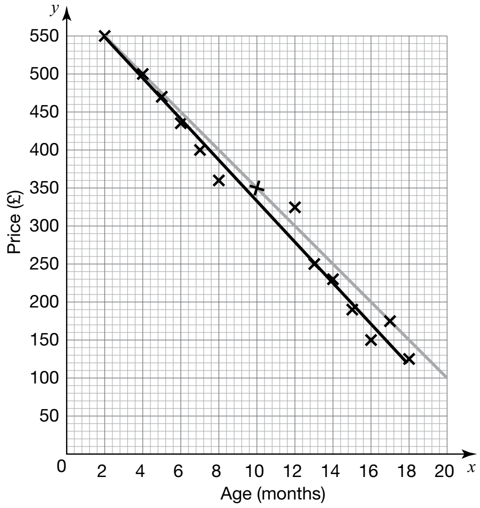


b The scatter diagram shows a positive correlation between students' maths and physics test percentages. Therefore, the students who got a low percentage in the maths test got the lower percentages in the physics test; the students who got a high percentage in the maths test got the higher percentages in the physics test.

c The outlier is the point marked at (0, 58).

d The student was absent for the maths test.

3 **a**



The black line shows the line of best fit.

The grey line shows the line where a laptop loses £150 every 6 months.

The shop owner is not correct. The line of best fit shows on average a laptop loses approximately £159/£160 every 6 months.

b The line of best fit cannot make a prediction outside the available data. The data only goes as far as 18 months.

Graphical misrepresentation

1 The pictogram suggests the weather is mostly sunny.

In the key, each symbol represents a different number of days. Also, it is not good practice to use a mixture of different symbols.

If drawn accurately, the pictogram would use symbols that all have the same value, showing that the weather is mostly cloudy (15 days), it rains on 9 days, and it is sunny for 7 days.

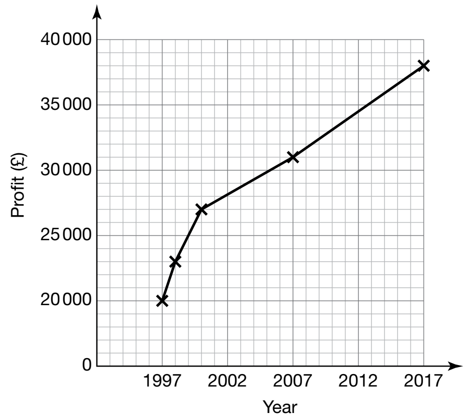
2 The bar chart suggests most people say yes to the supermarket, but actually, 40% of people said no, and just over 50% said yes.

On the vertical axis, the scale is only labelled from 40 to 50 percent. If the axes were less misleadingly labelled, the chart would show that the responses were reasonably close.

3 **a** No, his claim is not accurate.

The graph suggests that the growth in profits has been accelerating, but the years are not equally spread.

b



c Profits are increasing, but not as quickly as they did in the 1990s.

Practice papers

Non-calculator

1 It is in the 100 000s column, so 700 000.

2 10% = 70 ÷ 10 = 7
30% = 3 × 7 = 21

3 No, Sandeep is not correct:
$\frac{2}{5} = \frac{4}{10} = 0.4$
But 4% = 0.04

4 35 = 5 × 7
5 and 7

5 Distance on diagram between lamp post A and lamp post B = 6.1 cm
6.1 × 20 = 122 m*

6 E appears twice (2 times) out of 7 times.
P(E) = $\frac{2}{7}$

7 Yes. Fun run + music festival = £9689 + £9689 + £6370
= £25 748

8 Total number of parts = 5 + 7 = 12
Fraction that are fiction = $\frac{5}{12}$

9 Prize A = 8 × 4 = 32 tickets
Prize B = 2 + 4 + 8 + 16 = 30 tickets
Prize A gives more tickets.

10 **a** There are 2 triangles in Pattern 1, and 4 more are added for each pattern.
Number of triangles = $4p - 2$
In Pattern 8, there are 4 × 8 − 2 = 30 triangles

b No, Harry is incorrect. The number of triangles is not the pattern number multiplied by 4. Rather, it is add 4 triangles each time.

11 **a** £3.50

b £5.00 − appears the most times

12 call out = 55
fee for hours worked = 2 × 40 = 80
total before VAT = 55 + 80 = 135
VAT at 20% = 135 ÷ 10 × 2 = 13.5 × 2 = 27
total bill = 135 + 27 = 162
£162

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*This answer differs from the one in the Revision and Exam Practice Book due to an error in our first edition. This answer has now been re-checked and corrected.

13 Bus 2A will stop at: 10:00, 10:15, 10:30, 10:45, 11:00

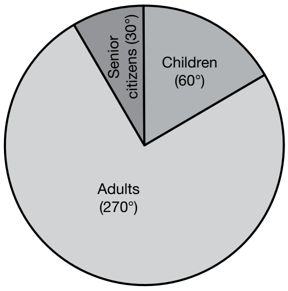
Bus 2B will stop at: 10:00, 10:12, 10:24, 10:36, 10:48, 11:00

They next arrive at the bus stop together at 11 am.

14 Total number of patients = 10 + 45 + 5 = 60

Angle per patient = 360 ÷ 60 = 6°

	Children	**Adults**	**Senior citizens**
Number	10	45	5
Angle	$10 \times 6 = 60°$	$45 \times 6 = 270°$	$5 \times 6 = 30°$



15 Cost of T shirts = 8 × 12 = £96 (for 48 shirts)

Saturday: $\frac{3}{8} \times 48 = 18$ $18 \times 5 = £90$

Sunday: 30 shirts for £3 each 30 × 3 = £90

Profit = 90 + 90 − 96 = £84

16 **a** $\frac{5}{8} - \frac{7}{12} = \frac{15}{24} - \frac{14}{24} = \frac{1}{24}$

b $2\frac{2}{3} \div \frac{4}{9} = \frac{8}{3} \times \frac{9}{4} = 2 \times 3 = 6$

17 **a** $13 - 4x$

b $(x + 3)(x - 4) = x^2 - 4x + 3x - 12 = x^2 - x - 12$

18 £21 = 25%

100% is original price.

4 × 25% = 100%

4 × 21 = 84

£84

19 **a** 3 along and 4 down (or 3 in the x direction and −4 in the y direction)

(3, −4)

b Coordinates of A are −3 in the x direction and 0 in the y direction: (−3, 0).

Coordinates of C are (3, −4).

Coordinates of midpoint are:

the average (mean) of both x coordinates and the average (mean) of both y coordinates:

$\left(\frac{3 + -3}{2}, \frac{-4 + 0}{2}\right)$

$= \left(\frac{0}{2}, \frac{-4}{2}\right)$

$= (0, -2)$

Check on the diagram to see if this looks sensible – it does.

20 Notice there are 2 + 1 = 3 decimal places in total.

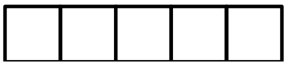
Ignore the decimal points while you work out the numbers:

$$\begin{array}{r} 635 \\ \underline{12} \\ 1270 \\ 6350 \\ \underline{7620} \end{array}$$

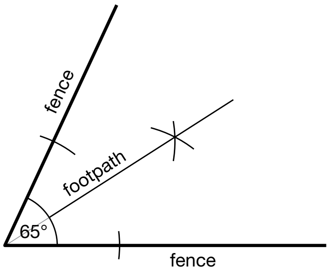
Now put the 3 decimal places back in, to give your answer the correct place values:

7.620 = 7.62

21 Plan:



22



23 $\frac{8.02 \times 3.76}{15.98} \approx \frac{8 \times 4}{16} \approx 2$

24 **a** $K = \frac{1}{2} \times 11 \times 3^2$

$K = 49.5$

b Rearrange the formula:

$K = \frac{1}{2}mv^2$

$v^2 = \frac{2K}{m}$

$v = \sqrt{\frac{2K}{m}}$

Substitute values for K and m to find v

$v = \sqrt{\frac{2 \times 180}{10}}$

$v = 6$ or -6

25 $x = 180 - (90 + 36) = 54°$

26 **a** The first graph shows only a small part of the vertical scale to exaggerate the increase, she is using graph A.

Note that although the scale on graph B gives a truer overall impression of the sales figures, the points are not plotted quite accurately to match the data in graph A – all but the first one are a little too high. The answers to **b** and **c** below are based on graph A.

b actual increase = 11 000 − 10 000 = 1000

£1000

c percentage increase = $\frac{1000 \times 100}{10000} = \frac{1000}{100} = 10\%$

27 $5x + 3 = 6x - 7$ (as base angles of an isosceles triangle are equal)

$10 = x$

Hence base angles are 53°

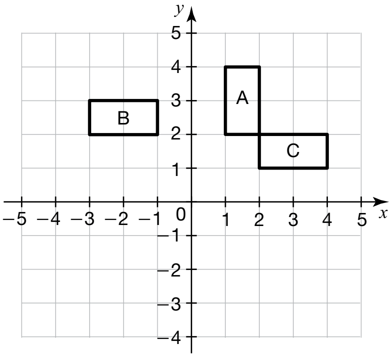
Other angle = 180 − (53 + 53) = 74°

So $74° = 7y - 10$

Solving gives $y = 12°$

Hence $x = 10°$ and $y = 12°$

28 **a** and **b**



c Reflection in $x = y$, or rotation of 90° anticlockwise about the point (1.5, 1.5)

29 $3\mathbf{a} = 3 \times \begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}-3\\6\end{pmatrix}$

(Notice it's not the same as multiplying a fraction – you multiply both parts of a vector because they are like coordinates.)

$\mathbf{b} - 3\mathbf{a} = \begin{pmatrix}3\\8\end{pmatrix} - \begin{pmatrix}-3\\6\end{pmatrix}$

$= \begin{pmatrix}3+3\\8-6\end{pmatrix}$

$= \begin{pmatrix}6\\2\end{pmatrix}$

Calculator

1 Enter 3 ÷ 8 into your calculator:

0.375

2 Enter 450 × 1.3 (or 450 × 130%) into your calculator:

£585

3 46.25 ÷ 5 = 9.25

Rate is £9.25 per hour.

9.25 × 37 = 342.25

£342.25

4 255 × 1.16 = 295.8 so the price in France is €295.80. The games console is cheaper in the UK.

5 On your calculator: $3\sqrt{13.824} + (4.5 - 0.38)^2 = 19.4$ (to 3 s.f.)

6 Using Pythagoras' theorem:

$YZ^2 = XY^2 + XZ^2$

$5.7^2 = XY^2 + 4.2^2$

$XY^2 = 5.7^2 - 4.2^2$

$= 32.49 - 17.64$

$= 14.85$

$XY = \sqrt{14.85}$

$= 3.9$ cm (1 d.p.)

7 a Friday

b On Thursday Jeff received 18 work emails and 2 personal emails.

$\frac{2}{20} \times 100 = 10\%$ of his emails on Thursday were personal.

c Total number of emails received = 8 + 16 + 4 + 15 + 5 + 16 + 2 + 18 + 11 + 10 = 105

Total number of work emails received = 16 + 15 + 16 + 18 + 10 = 75

$\frac{75}{105} = \frac{5}{7}$ of his emails that week were about work.

8 Factors of 15 are: 1, 3, 5 and 15.

Multiples of 8 are: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80…

Let answer = a.

$75 < a < 85$

So a could be 76, 77, 78, 79, 80, 81, 82, 83, 84.

Of these, 80 is a multiple of 8.

80 × 1 = 80, and 1 is a factor of 15.

So one pair is 1 and 80.

Which possible solutions for a have 3 or 5 as factors?

78, 81 and 84 have 3 as factors: 26 × 3 = 78, 27 × 3 = 81 and 28 × 3 = 84.

But 26, 27 and 28 are not multiples of 8.

80 has 5 as a factor: 5 × 16 = 80, and 16 is a multiple of 8.

So another pair is 5 and 16.

Pairs are 5, 16 and 1, 80.

9 −1, 0, 1, 2, 3, 4 – the range is less than 5 so this is not included

10 Area to be painted = 16 × 1.8 × 1.8 = 51.84 m^2

2 tins are enough for 2 × 20 = 40 m^2 and 3 tins are enough for 3 × 20 = 60 m^2.

$40 < 51.84 < 60$

So Geraldine needs 3 tins, at a price of 3 × 22.50 = £67.50.

11 $6 \times 9 \times 7 + \frac{1}{2} \times 6 \times 8 \times 7 = 546$ cm^3

12 84 km/h $= \frac{84 \times 1000}{60 \times 60}$ m/s $= 23.\dot{3}$ m/s

13 Felicity gets 5 parts, Ian gets 3.

£22 = 2 parts (difference between Felicity's share and Ian's)

1 part = £11

11 × 5 = 55

Felicity gets £55.

14 $PQ^2 = 2^2 + 4^2$

$PQ^2 = 20$

$PQ = \sqrt{20}$

$= \sqrt{4 \times 5}$

$= 2\sqrt{5}$

15 $\frac{2}{3}$ are the 42 women so $\frac{1}{3}$ is 21 women and

$\frac{3}{3} = 21 \times 3 = 63$

So there are 63 men and women and this represents 50% of the total.

Hence there are 126 people in the audience.

16 58 million = $58 \times 10^6 = 5.8 \times 10^7$ km

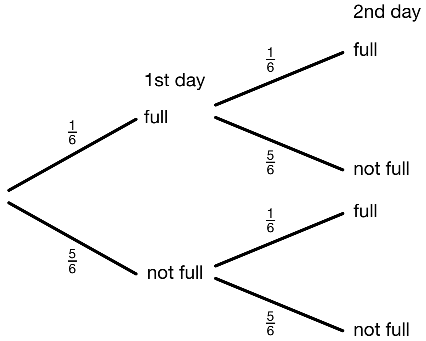
17 Let number by Seren = x

Number by Tom = $x + 3$

Number by Rohan = $x + 3 + 2 = x + 5$

Total number = $x + x + 3 + x + 5 = 3x + 8$

18 a



b P(full and not full) + P(not full and full) = $\left(\frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{1}{6}\right) = \frac{10}{36} = \frac{5}{18}$

19 $\tan x = \frac{8}{5}$

$x = \tan^{-1} \frac{8}{5}$

$x = 58°$

$\cos 59° = \frac{y}{11}$

$y = 11 \cos 59°$

$= 5.7$ cm (to 1 d.p.)

20 Mean for Team A $= \frac{(0 \times 3) + (1 \times 4) + (2 \times 4) + (3 \times 2) + (4 \times 1) + (5 \times 2)}{16}$

$= \frac{32}{16} = 2$

Mean for Team B $= \frac{(0 \times 3) + (1 \times 4) + (2 \times 8) + (3 \times 4) + (4 \times 1)}{20}$

$= \frac{36}{20} = 1.8$

Team A scored more goals on average.

21 a Option A is best value if you plan to visit the gym once per week, as in one month 4 visits cost £20 (or 5 visits cost £25). This is less than the monthly fee of £30.

b The graphs intersect at 6, so the cost is the same for 6 visits.

It is cheaper to pay the monthly fee if you visit the gym more than 6 times.

22 Area of semicircle $= \frac{1}{2}\pi r^2$

Area of large semicircle $= \frac{1}{2} \times \pi \times 7.5^2 = \frac{225\pi}{8}$

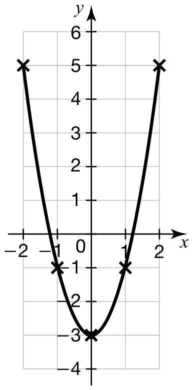
Area of small semicircles $= 3 \times \left(\frac{1}{2} \times \pi \times 2.5^2\right) = 3 \times \frac{25\pi}{8} = \frac{75\pi}{8}$

Area of lawn $= \frac{225\pi}{8} - \frac{75\pi}{8} = \frac{150\pi}{8} = 58.9$ m^2 (to 3 s.f.)

23 **a**

x	**Operation**	y
–2	$2 \times (-2)^2 - 3 =$	5
–1	$2 \times (-1)^2 - 3 =$	-1
0	$2 \times (0)^2 - 3 =$	-3
1	$2 \times (1)^2 - 3 =$	-1
2	$2 \times (2)^2 - 3 =$	5

b



24 **a** Draw a line of best fit to show that, yes, the car dealer is correct.

b Start at 55 000 miles and read up to your line of best fit, then read across to the price scale. The car dealer should charge £3500 to £4000. (Value varies with students' own graphs.)

25 $(n + 5)(n - 3) = n^2 + 2n - 15$