Foundation Mathematics Exam Practice Book for all Exam Boards Full worked solutions

Number

Basic number techniques

1 Negative numbers are smaller than zero

-12 is further left on the number line than -8, -1 is larger than -8 (and -12) so it appears next.

Then comes 0, then 2.

So the order is:

$-12, -8, -1, 0, 2$

2 First look at the place value for 10ths: 0.32 and 0.3 have the higher number of 10ths.

Now compare their 100ths. 0.32 has 2 100ths but 0.3 doesn't have any, so it's smaller.

Similarly, 0.23 and 0.203 both have 2 10ths, but 0.23 is bigger than 0.203 because it has 3 100ths while 0.203 only has 3 1000ths.

So the order is:

0.32, 0.3, 0.23, 0.203

3 **a** $-4 < 0.4$ (the negative number is lower)

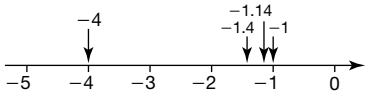
b $4.200 < 4.3$ (the higher number has more 10ths)

c $-0.404 > -0.44$ (because they are both negative, the one with more 100ths is lower)

d $0.33 < 0.4$ (the larger number has more 10ths)

4 The positive number (1.4) is the highest.

Think where the negative numbers fit on a number line:



Order is:

$-4, -1.4, -1.14, -1, 1.4$

Factors, multiples and primes

1 $18 = 2 \times 3 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

Find the factors that they both share (2 and 3) and multiply together:

$2 \times 3 = 6$, so HCF is 6.

2 Numbers between 15 and 25:

16, 17, 18, 19, 20, 21, 22, 23, 24

Note that this does not include 15 and 25 because the question said 'between 15 and 25' not 'from 15 to 25'.

$16 = 2 \times 2 \times 2 \times 2$

$18 = 2 \times 3 \times 3$

$20 = 2 \times 2 \times 5$

$21 = 3 \times 7$

$22 = 2 \times 11$

$24 = 2 \times 2 \times 2 \times 3$

17, 19, and 23 have no factors except for 1 and the number itself.

17, 19 and 23.

3 $60 = 20 \times 3$

$= 2 \times 2 \times 5 \times 3$

$= 2^2 \times 3 \times 5$

4 Drummer 1 hits her drum at: 6 12 18 24 30 36 42 48 54 60 seconds

Drummer 2 hits his drum at: 8 16 24 32 40 48 56 seconds

They hit their drums at the same time twice (two times), after 24 seconds and after 48 seconds.

Calculating with negative numbers

1 **a** $-7 + -3 = -7 - 3 = -10$

b $-7 - -3 = -7 + 3 = -4$

c $8 + -5 - -2 = 8 - 5 + 2 = 5$

d $-4 - -6 + -1 = -4 + 6 - 1 = 1$

2 **a** -18

b $-12 \div -3 = 12 \div 3 = 4$

c $-4 \times -2 \times 5 = 4 \times 2 \times 5 = 40$

d $(-24 \div 3) \times 2 = -8 \times 2 = -16$

3 $2 - 4 = -2$

$-2 - 5 = -7$

-7°C

4 Let a = number of correct answers, b = number of incorrect answers

$3a - 2b = -5$ (1)

There are five questions, so $a + b = 5$ and $b = 5 - a$ (2)

Substituting this for b in (1): $3a - 2(5 - a) = -5$

$3a - 10 + 2a = -5$

$5a = 5$

$a = 1$

Substituting this in (2):

$b = 5 - 1$

$b = 4$

Sally got 1 correct answer and 4 incorrect answers in the test.

Division and multiplication

1 **a**

$$\begin{array}{r} 357 \\ \times \quad 6 \\ \hline 2142 \end{array}$$

2142

b

$$\begin{array}{r} 261 \\ \times \quad 43 \\ \hline 783 \ (=261 \times 3) \\ 10440 \ (=261 \times 40) \\ \hline 11223 \end{array}$$

11 223

c

$$6\overline{)5^{5}5^{1}2} = 92$$

92

d

$$\begin{array}{r} 052 \\ 13\overline{)676} \\ -65 \\ \hline 026 \end{array}$$

52

2 **a**

$$24\overline{)300} = 12 \text{ remainder } 12$$

So 12 boxes are filled.

b $24 \times 12 = 20 \times 12 + 4 \times 12 = 240 + 48 = 288$

$300 - 288 = 12$

There are 12 books left over.

3 $12500 - 440 = 12060$

$$\begin{array}{r} 0033\,5 \\ 36\overline{)1^{1}2^{1}060} \\ -108 \\ \hline 1^{1}2^{1}6 \\ -108 \\ \hline 0180 \end{array}$$

Each repayment is £335.

4 **a** $36 \times (52 - 6)$
$= 36 \times 46$
$= 1656$ hours

b She will work $\frac{36 \times 2}{3} = 24$ hours per week, with $\frac{6 \times 2}{3} = 4$ weeks' holiday.
Hours worked:
$24 \times (52 - 4)$
$= 24 \times 48$
$= 1152$ hours

Calculating with decimals

1 First note the combined number of decimal places in both numbers (2).
Remove the decimal points to do the calculation:

$$\begin{array}{r} 92 \\ \times\ 83 \\ \hline 276\ (=92\times 3) \\ 7360\ (=92\times 80) \\ \hline 7636 \\ {\scriptstyle 1} \end{array}$$

Now you've got the digits right put the decimal point back, counting in from the right 2 places, to give a number with 2 decimal places:
76.36

2
$$\begin{array}{r} 22.50 \\ +\ 19.99 \\ \hline 42.49 \end{array}$$

$$\begin{array}{r} {\scriptstyle 4\ 1\ 9\ 1} \\ 50.00 \\ -\ 42.49 \\ \hline 07.51 \end{array}$$

She should get £7.51 change.

3 $6\overline{)2^{2}2^{4}9.1^{1}7^{5}4}$ $= 38.29$

4 Kirsty raises $\frac{172.50}{5+1}$

Kirsty raises £28.75 × 5 = 28.75 × 10 ÷ 2 = 287.5 ÷ 2 = 143.7

$6\overline{)1^{1}7^{5}2.5^{3}0}$ $= 28.75$

Kirsty raises
£143.75

$$\begin{array}{r} {\scriptstyle 6\ 11\ 14\ 1} \\ 172.50 \\ -\ 143.75 \\ \hline 028.75 \end{array}$$

Flo raises £28.75

Rounding and estimation

1 Digit after second decimal place is 8, so round previous digit (9) up, to 10, and round the 7 up to 8. You must still include a zero in the second decimal place, to show the required level of accuracy.
0.80 (to 2 d.p.)

2 4.09 could have been rounded up or down.
Lower bound: 4.085, because 5 rounds up, giving 9.
Upper bound: 4.095, because everything between 4.09 and this number rounds down, giving 9. You will need to use a < sign, because 4.095 is not included (it would round to 4.10).
$4.085 \leq x < 4.095$

3 $\frac{9.74 \times 4.02}{7.88} \approx \frac{10 \times 4}{8} = 5$

4 **a** $40 \times 500 = 20000$
$20000 - 12500 = £7500$

b Overestimate, because the concert ticket price and number of tickets sold were rounded up, and so the amount of income was estimated more than it really is.

Converting between fractions, decimals and percentages

1 **a** There are 0×10^{ths}, 7×100^{ths} and 1×1000^{ths}
$= 071 \times 1000^{ths}$ so: $\frac{71}{1000}$

b $63 \div 100 = 0.63$

c $0.4 \times 100 = 40\%$

d $32\% = \frac{32}{100} = \frac{8}{25}$

2 **a** $5 \div 16 = 0.3125$

b To convert a number to a percentage, multiply its decimal value by 100.
$0.3125 \times 100 = 31.25\%$

3 $\frac{5}{8} = 0.625$, $60\% = 0.6$, so 0.65 is the largest.

Ordering fractions, decimals and percentages

1 **a** $\frac{1}{2} = \frac{5}{10} = 0.5$, so
$\frac{1}{2} < 0.6$

b $\frac{3}{4} = 3 \div 4 = 0.75$, so
$\frac{3}{4} > 0.7$

c $\frac{-3}{10} = -0.3$, so
$\frac{-3}{10} < 0.2$

2 **a** LCM of 12, 15 and 20 is 60
$\frac{5}{12} = \frac{25}{60}$
$\frac{7}{15} = \frac{28}{60}$
$\frac{9}{20} = \frac{27}{60}$
So order from lowest to highest is $\frac{5}{12}$, $\frac{9}{20}$, $\frac{7}{15}$

b $45\% = \frac{45}{100} = 0.45$
$\frac{1}{25} = \frac{4}{100} = 0.04$
$0.04 < 0.4 < 0.45$
So order is:
$\frac{1}{25}$, 0.4, 45%

3 Shop C is cheapest $\left(\frac{2}{5} = 40\%\right)$, then Shop A $\left(\frac{1}{3} = 33.3...\%\right)$, and Shop B offers the least discount at 30%.

4 $\frac{5}{9} = 0.\dot{5}$
$38.5\% = 0.385$
$\frac{3}{10} = 0.3$
So the order is $\frac{5}{9}$, 38.5%, 0.38, $\frac{3}{10}$

Calculating with fractions

1 $\frac{1}{5} + \frac{4}{9} = \frac{9}{45} + \frac{20}{45} = \frac{9+20}{45} = \frac{29}{45}$

2 $2\frac{3}{4} - 2\frac{2}{3} = \frac{11}{4} - \frac{8}{3} = \frac{33}{12} - \frac{32}{12} = \frac{1}{12}$

3 $1\frac{5}{6} \times \frac{2}{7} = \frac{11}{6} \times \frac{2}{7} = \frac{22}{42} = \frac{11}{21}$

4 $6 \div \frac{3}{5} = 6 \times \frac{5}{3} = \frac{30}{3} = 10$
Jo can make 10 necklaces.

Percentages

1 $\frac{40}{100} \times 25 = 10$

2 $16 \times 0.85 = £13.60$

3 $12450 \times 1.14 = 14193$

4 $40 \times 7 \times 3 = £840$

$840 \times 1.2 = £1008$

Order of operations

1 $3^2 - 6 \div (2 + 1) = 9 - \frac{6}{3} = 9 - 2 = 7$

2 $2^3 + 3x\sqrt{25} = 8 + (3 \times 5) = 8 + 15 = 23$

3 $(1.7 - 0.12)^2 + \sqrt[3]{4.096} = 4.0964$

Exact solutions

1 Area of triangle $= \frac{1}{2} \times$ base $\times$ vertical height $= 0.5 \times 0.76 \times 0.35 = 0.133\,\text{cm}^2$

2 $\left(1\frac{1}{3}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9} = 1\frac{7}{9}\,\text{m}^2$

3 $\sqrt{2} \times \sqrt{6} = \sqrt{12} = 2\sqrt{3}\,\text{cm}^2$

4 Area of a circle $= \pi r^2$

The fraction of the circle shown $= \frac{3}{4}$

The area of the circle shown $= \frac{3}{4} \times \pi r^2$

The radius $= 2$ cm

So area of shape shown $= \frac{3}{4} \times \pi \times 2^2 = 3\pi\,\text{cm}^2$

Indices and roots

1 **a** $7 \times 7 \times 7 \times 7 = 7^4$

b $\frac{1}{5 \times 5 \times 5} = \frac{1}{5^3} = 5^{-3}$

2 **a** $2^4 = 2 \times 2 \times 2 \times 2 = 16$

b Reciprocal means make it the denominator of a fraction with 1 as the numerator:

$\frac{1}{100}$

3 $2^3 = 2 \times 2 \times 2 \times 2 = 8$

$3^{-2} = \frac{1}{9}$

$\sqrt[3]{27} = 3$

$4^0 = 1$

$\sqrt{25} = 5$ or -5

Assuming the square root of 25 is positive, the answer is:

$3^{-2}, 4^0, \sqrt[3]{27}, \sqrt{25}, 2^3$

If it were negative, the answer would be:

$\sqrt{25}, 3^{-2}, 4^0, \sqrt[3]{27}, 2^3$

4 $\frac{9^5}{9^3 \times 9^2} = \frac{9^5}{9^5} = 1$

Standard form

1 2750

2 1.5×10^8

3 Move the decimal point three places to the right to give 6.42×10^{-3}

4 $(1.4 \times 10^{-5}) \times 20 = (2.8 \times 10^{-5}) \times 2 \times 10 = 2.8 \times 10^{-4}$ km

Listing strategies

1 259, 295, 529, 592, 925, 952

2 **a**

		4-sided spinner			
		0	**1**	**2**	**3**
3-sided spinner	**1**	1	2	3	4
	2	2	3	4	5
	3	3	4	5	6

b 4

3

		Dice					
		1	2	3	4	5	6
Coin	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

4 spj; spi; sfj; sfi ; bpj; bpi; bfj; bfi

Algebra

Understanding expressions, equations, formulae and identities

1 **a** identity **b** equation **c** expression

2 **a** Equation, because it has an equals sign and can be solved.

b Formula, because it has letter terms, an equals sign and the values of the letters can vary.

c Expression, because it has letter terms and no equals sign.

d Formula, because it has letter terms, an equals sign and the values of the letters can vary.

3 **a** Any of: $2x + 10$ or $10x + 2$ or $x + 210$ or $x + 102$

b Any of: $2x = 10$ or $10x = 2$

Simplifying expressions

1 $8x$

2 **a** $6a \times 8a = (6 \times 8) \times (a \times a) = 48 \times a^2 = 48a^2$

b $2p \times 3p \times 5p = (2 \times 3 \times 5) \times (p \times p \times p)$
$= 30 \times p^3 = 30p^3$

3 $35yz \div 7z = (35 \div 7) \times (yz \div z) = 5 \times y = 5y$

4 $\frac{32uv}{4v} = \frac{32}{4} \times \frac{uv}{v} = 8 \times u = 8u$

Collecting like terms

1 **a** $7m + 6n - 4m - 2n = (7 - 4)m + (6 - 2)n = 3m + 4n$

b $9q - 5r - 12q + 3r = (9 - 12)q + (3 - 5)r = -3q - 2r$

2 **a** $a + b \times b$

Use BIDMAS: **M**ultiplication before **A**ddition.

$a + b^2$

b $6c - 4d - 7c + 5d = (6 - 7)c + (5 - 4)d = -c + d$

3 **a** $9p^3 + p - 4p^3 = (9 - 4)p^3 + p = 5p^3 + p$

b $12 - 5x^2 + 3x - 2x^2 = 12 - (5 + 2)\,x^2 + 3x$
$= -7x^2 + 3x + 12$

4 $3\sqrt{5} - f - 8\sqrt{5} + 2f = (3 - 8)\sqrt{5} + (2 - 1)f = -5\sqrt{5} + f$

Using indices

1 **a** $p^3 \times p = p^{(3+1)} = p^4$

b $4y^2 \times 3y^3 = (4 \times 3) \times y^{(2+3)} = 12 \times y^5 = 12y^5$

c $2a^4b \times 5ab^2 = (2 \times 5) \times a^{(4+1)} \times b^{(1+2)}$
$= 10 \times a^5 \times b^3 = 10a^5b^3$

2 **a** $q^{-2} \times q^{-4} = q^{(-2-4)} = q^{-6}$

b $(u^{-3})^2 = u^{((-3)\times 2)} = u^{-6}$

c $x^{-1} \times x = x^{-1} \times x^1 = x^{(-1+1)} = x^0 = 1$

3 **a** $b^4 \div b^3 = b^{(4-3)} = b^1 = b$

b $\frac{f^5}{f^2} = f^{(5-2)} = f^3$

c $\frac{xy^3}{x^2y} = x^{(1-2)} \times y^{(3-1)} = x^{-1} \times y^2 = \frac{1}{x} \times y^2 = \frac{y^2}{x}$

4 Let the first box $= x$ and the second box $= y$

$(xm^3)^y = x^y m^{3y} = 8m^9$

comparing terms, $3y = 9$

$y = 3$

Substitute in the y value: $(xm^3)^3 = 8m^9$

$x^3 = 8$

$x = \sqrt[3]{8} = 2$

Therefore, the completed expression is $(2m^3)^3$

Expanding brackets

1 **a** $4(m + 3) = (4 \times m) + (4 \times 3) = 4m + 12$

b $2(p - 1) = (2 \times p) + (2 \times -1) = 2p - 2$

c $10(3x - 5) = (10 \times 3)x + (10 \times -5) = 30x - 50$

2 **a** $3(m + 2) + 5(m + 1) = 3m + 6 + 5m + 5 = 8m + 11$

b $6(x - 1) - 2(x - 4) = 6x - 6 - 2x + 8 = 4x + 2$

3 **a** $(y + 3)(y + 7) = y^2 + 7y + 3y + 21 = y^2 + 10y + 21$

b $(b + 2)(b - 4) = b^2 - 4b + 2b - 8 = b^2 - 2b - 8$

c $(x - 4)(x - 6) = x^2 - 6x - 4x + 24 = x^2 - 10x + 24$

4 **a** $(q + 1)^2 = (q + 1)(q + 1) = q^2 + q + q + 1 = q^2 + 2q + 1$

b $(z + 2)^2 = (z + 2)(z + 2)$

$= z^2 + 2 \times z + z \times 2 + 2 \times 2$

$= z^2 + 2z + 2z + 4$

$= z^2 + 4z + 4$

c $(c - 3)^2 = (c - 3)(c - 3) = c^2 - 3c - 3c + 9 = c^2 - 6c + 9$

Factorising

1 Divide the expression by the highest common factor (HCF) of both terms to find the bracket, and then place the HCF outside of the bracket to give the full factorisation.

a $(4x + 8) \div 4 = x + 2$

factorisation: $4(x + 2)$

b $(3d - 15) \div 3 = d - 5$

factorisation: $3(d - 5)$

c $(8y - 12) \div 4 = 2y - 3$

factorisation: $4(2y - 3)$

2 Divide the expression by the common term to find the bracket, and then place the common term outside of the bracket to give the full factorisation.

a $(q^2 + q) \div q = q + 1$

factorisation: $q(q + 1)$

b $(a^2 + 6a) \div a = a + 6$

factorisation: $a(a + 6)$

c $(10z^2 + 15z) \div 5z = (2z + 3)$

factorisation: $5z(2z + 3)$

3 Find which factors of the number term add together to give the coefficient of the x term.

a $12 = 3 \times 4$

$7 = 3 + 4$

factorisation: $(x + 3)(x + 4)$

b $-16 = (-2) \times 8$

$6 = -2 + 8$

factorisation: $(x - 2)(x + 8)$

c $24 = (-6) \times (-4)$

$-10 = (-6) + (-4)$

factorisation: $(a - 6)(a - 4)$

4 **a** Write $y^2 - 4$ in the form of $a^2 - b^2$:

$y^2 - 2^2$

Using the formula for the difference of two squares, the factorisation is

$(y + 2)(y - 2)$

b Write $x^2 - 9$ in the form of $a^2 - b^2$:

$x^2 - 3^2$

Using the formula for the difference of two squares, the factorisation is

$(x + 3)(x - 3)$

c Write $p^2 - 100$ in the form of $a^2 - b^2$:

$p^2 - 10^2$

Using the formula for the difference of two squares, the factorisation is

$(p + 10)(p - 10)$

Substituting into expressions

1 $4x + 5y = 4 \times 3 + 5 \times (-2) = 12 - 10 = 2$

2 $s = ut + \frac{1}{2}at^2$

$= 12 \times 2 + \frac{1}{2} \times 10 \times 2^2$

$= 12 \times 2 + \frac{1}{2} \times 40$

$= 24 + 20$

$s = 44$

3 **a** $12mn = 12 \times 6 \times \left(-\frac{1}{2}\right)$

$= 12 \times -3$

$= -36$

b $\frac{m}{n} = \frac{6}{-\frac{1}{2}}$ or $6 \div -\frac{1}{2}$

$= 6 \times \frac{-2}{1}$

$= -12$

4 **a** $f = 3c - 2(c - d)$

$= 3 \times 7 - 2 \times (7 - (-5))$

$= 21 - 2 \times (12)$

$= 21 - 24$

$f = -3$

b $f = -c(d^2 - 3c)$

$= -7 \times ((-5)^2 - 3 \times 7)$

$= -7 \times (25 - 21)$

$= -7 \times 4$

$f = -28$

c $f^2 = 7c - 3d$

$= 7 \times 7 - 3 \times (-5)$

$= 49 + 15$

$= 64$

$f = \sqrt{64}$

$f = \pm 8$

Writing expressions

1 **a** $n + 3$ **b** $(n \times 2) - 9 = 2n - 9$

2 **a** $x + y$ **b** $5 \times x = 5x$

c $(12 \times x) + (11 \times y) = 12x + 11y$

3 $2 \times 9p + 2(5p + 2) = 18p + 10p + 4 = 28p + 4$

4 The area of the rectangle is given by height × length, which is $s \times (5s + 1) = s(5s + 1)$.

Solving linear equations

1 **a** $x = 12 - 5$

$x = 7$

b $x = 10 + 3$

$x = 13$

c $x = \frac{20}{4}$

$x = 5$

d $x = 6 \times 3$

$x = 18$

2 **a** $2x + 3 = 15$

$2x = 12$

$x = 6$

b $3x - 5 = 16$

$3x = 21$

$x = 7$

c $\frac{x}{5} + 3 = 8$

$\frac{x}{5} = 5$

$x = 25$

d $7 - 2x = 1$

$7 = 2x + 1$

$6 = 2x$

$x = 3$

3 **a** $3(x + 9) = 30$

$3x + 27 = 30$

$3x = 3$

$x = 1$

b $5(p - 2) = 10$

$5p - 10 = 10$

$5p = 20$

$p = 4$

c $2(10 - 3m) = 8$
$20 - 6m = 8$
$12 = 6m$
$m = 2$

d $4(8 - 2q) = 8(4 - q) = 0$
So $4 - q = 0$
$q = 4$

4 a $4x - 6 = x + 9$
$3x - 6 = 9$
$3x = 15$
$x = 5$

b $2y + 5 = 4y - 3$
$5 = 2y - 3$
$8 = 2y$
$y = 4$

c $4(2x + 3) = 11x + 3$
$8x + 12 = 11x + 3$
$9 = 3x$
$x = 3$

d $3(n + 4) = 2(2n + 3)$
$3n + 12 = 4n + 6$
$12 = n + 6$
$n = 6$

Writing linear equations

1 Sum of the angles in a triangle are 180°
$(2x + 3) + 81 + (3x - 4) = 180$
$5x + 80 = 180$
$5x = 100$
$x = 20$

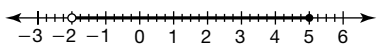
2 Let Jamie's age $= x$ years. Sophie's age $= \frac{x}{2}$
$x + \frac{x}{2} = 18$
$\frac{3x}{2} = 18$
$3x = 36$
$x = 12$
Jamie is 12 years old

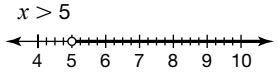
3 Let width $= x$ so length $= x + 3$
Perimeter $= 2x + 2(x + 3)$
$46 = 4x + 6$
$x = 10$
Length $= 10$ cm and width $= 13$ cm
Area $= 10 \times 13 = 130\,\text{cm}^2$

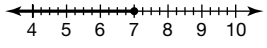
4 Opposite angles are equal so $3x + 10 = 5x - 20$
$30 = 2x$ giving $x = 15$
Also $3x + 10 + 7x + 5y = 180$
$10x + 10 + 5y = 180$
Now $x = 15$ so $150 + 10 + 5y = 180$
Solving this gives $y = 4$

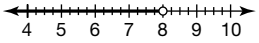
Linear inequalities

1 a The signs show that -2 is not included, but 5 is:
$-1, 0, 1, 2, 3, 4, 5$

b


2 a $4x > 20$
$x > 5$


b $3x - 8 \leq 13$
$3x \leq 21$
$x \leq 7$


c $2(x - 3) < 10$
$2x - 6 < 10$
$2x < 16$
$x < 8$


3 a $2 \leq 3x + 5$
$-3 \leq 3x$
$-1 \leq x$
$3x + 5 < 11$
$3x < 6$
$x < 2$
Hence $-1 \leq x < 2$

b $-4 > 5x + 6$
$-10 < 5x$
$-2 < x$
$5x + 6 \leq 6$
$5x \leq 0$
$x \leq 0$
Hence $-2 < x \leq 0$

4 $n + n + 3 < 15$
$2n + 3 < 15$
$2n < 12$
$n < 6$
Possible integer values of $n = 1, 2, 3, 4, 5$

Formulae

1 a $t = (40 \times 2) + 20 = 100$ minutes $= 1$ hour 40 minutes

b $t = (40 \times 1.5) + 20 = 80$ minutes $= 1$ hour 20 minutes
The chicken should be put in the oven 1 hour and 20 minutes earlier than 1.30 pm, which is a time of 12.10 pm.

2 a $C = l + kn$

b $C = 90 + 6.5 \times 3$
$C = £109.50$

3 a $p = \frac{qs}{3}$
$3p = qs$
$q = \frac{3p}{s}$

b $p = \frac{q}{r} + t$
$p - t = \frac{q}{r}$
$q = rp - rt$ or $r(p - t)$

c $p = 3(q + r)$
$\frac{p}{3} = q + r$
$q = \frac{p}{3} - r = \frac{p - 3r}{3}$

d $p = \sqrt{2q}$
$p^2 = 2q$
$q = \frac{p^2}{2}$

Linear sequences

1 a The term in position 1 is $1 \times 5 + 1 = 6$
The term in position 2 is $2 \times 5 + 1 = 11$
The term in position 3 is $3 \times 5 + 1 = 16$
The term in position 4 is $4 \times 5 + 1 = 21$

b The term in position 50 is $50 \times 5 + 1 = 251$

2 a Each pattern has 2 more dots than the last, so pattern 7 will have 8 more dots than pattern 3. Pattern 7 will have 19 dots.

b No, Rachel is not correct, because the number of triangles is not the pattern number multiplied by 2. Instead, it is the pattern number plus 2, so there will be 6 triangles in pattern 4.

3 **a** Common difference = 11, so $11n$ is in the sequence.
When $n = 1$:
$11n = 11$, but the 1st term is 3.
$3 = 11n - 8$
So the expression for the sequence is $11n - 8$

b Assume 100 is in the sequence. Then:
$11n - 8 = 100$
$11n = 108$
$n = 108 \div 11 = 9$ remainder 9
But n must be a whole number, and it is not; so 100 is not in this sequence.

Non-linear sequences

1 **a** Rule is multiply by 2.
$8 \times 2 = 16$
$16 \times 2 = 32$
So terms are 16, 32.

b Rule is divide by 10.
$1 \div 10 = 0.1$
$0.1 \div 10 = 0.01$
So terms are 0.1, 0.01

c Rule is multiply by -2.
$-12 \times -2 = 24$
$24 \times -2 = -48$
So terms are 24, -48.

d They involve multiplying and dividing, not adding and subtracting, so they are geometric.

2 **a** $1 = 1 \times 1$, $4 = 2 \times 2$, $9 = 3 \times 3$, $16 = 4 \times 4$…
$1^2, 2^2, 3^2, 4^2$…
square numbers

b $1 = 1 \times 1 \times 1$, $8 = 2 \times 2 \times 2$, $27 = 3 \times 3 \times 3$, $64 = 4 \times 4 \times 4$…
$1^3, 2^3, 3^3, 4^3$…
cube numbers

3 **a** Count the dots in each triangle:
1, 3, 6, 10

b Add another row (of 5 dots) under the 4th triangle:
$10 + 5 = 15$
Now add another row again (6 this time):
$15 + 6 = 21$
15, 21

4 **a** Next term = $6 + 9 = 15$

b 5th term = $6 + 9 = 15$
6th term = $9 + 15 = 24$
7th term = $15 + 24 = 39$
8th term = $24 + 39 = 63$
9th term = $39 + 63 = 102$
The 9th term is the first term in the sequence over 100

5 **a**

Day	Mon	Tue	Wed	Thu	Fri
Number of ladybirds	2	8 (= 2 × 4)	32 (= 8 × 4)	32 × 4 = 128	128 × 4 = 512

The gardener is correct. There will be more than 500 ladybirds.

b Saturday, because $512 \times 4 = 2048$.

6 **a** First term: $\frac{1}{2} \times 1^2 = \frac{1}{2}$
Second term: $\frac{1}{2} \times 2^2 = 2$
Third term: $\frac{1}{2} \times 3^2 = \frac{9}{2} = 4\frac{1}{2}$

b If 32 is in the sequence, then:
$\frac{1}{2}n^2 = 32$
$n^2 = 64$
$n = 8$
This gives n as a whole number, 8, so 32 is the 8th term in the sequence.

Show that...

1 LHS = $2x + 1$; RHS = $2x + 1$; LHS = RHS. Therefore, $2\left(x + \frac{1}{2}\right) \equiv x + x + 1$

2 LHS = $x^2 - 25 + 9 = x^2 - 16$; RHS = $x^2 - 16$

3 Let the three consecutive numbers be n, $n + 1$ and $n + 2$.
$n + n + 1 + n + 2 = 3n + 3 = 3(n+1)$. Therefore, the sum of three consecutive numbers is a multiple of 3.

4 **a** Width of pond = $x - y + x + x + x - y = 4x - 2y$
Length of pond = $4x$
Perimeter = $4x - 2y + 4x - 2y + 4x + 4x = 16x - 4y$

b Yes Sanjit is correct, because $16x - 4y = 4(4x - y)$, showing that when x and y are whole numbers, the perimeter is always a multiple of 4.

Functions

1 **a** when $x = 3$, $y = 3 \times 4 - 1 = 11$

b when $y = 23$, $4x - 1 = 23$, therefore $x = (23 + 1) \div 4 = 6$

c To get y you multiply x by 4 and subtract 1, so $y = 4x - 1$

2

x	Operations	y
−2	(−2) × 2 + 3	−1
0	0 × 2 + 3	3
3	(9 − 3) ÷ 2	9

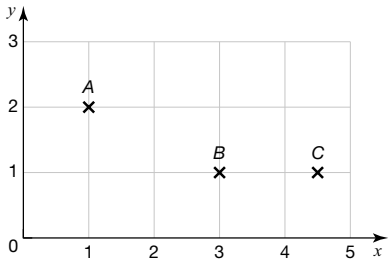
3

x	Operations	y
−2	(−2) ÷ 2 + 1	0
1	(1) ÷ 2 + 1	$1\frac{1}{2}$
8	(5 − 1) × 2	5

Coordinates and midpoints

1 **a** 1 along the x-axis and 2 up the y-axis: (1, 2)

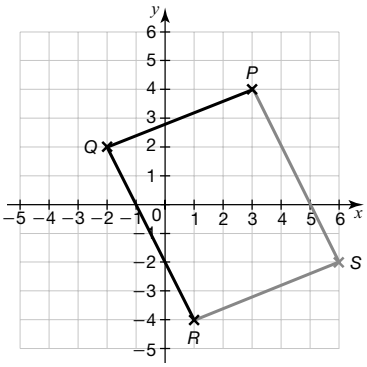
b



c $4\frac{1}{2}$ along the x-axis and 1 up the y-axis: $(4\frac{1}{2}, 1)$

2 **a** 1 along the x-axis and -4 'up' the y-axis: (1, −4)

b



S = (6, −2) to make a parallelogram

3 **a** x coordinate $= \frac{4 + (-2)}{2} = 1$

y coordinate $= \frac{5 + 1}{2} = 3$

Midpoint of XY is (1, 3).

b Midpoint of $XZ = \left(\frac{4 + 4}{2}, \frac{5 + (-4)}{2}\right) = (4, \frac{1}{2})$

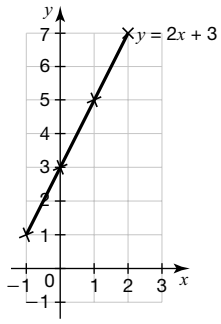
c Midpoint of $YZ = \left(\frac{(-2) + 4}{2}, \frac{1 + (-4)}{2}\right) = (1, -1\frac{1}{2})$

Straight line graphs

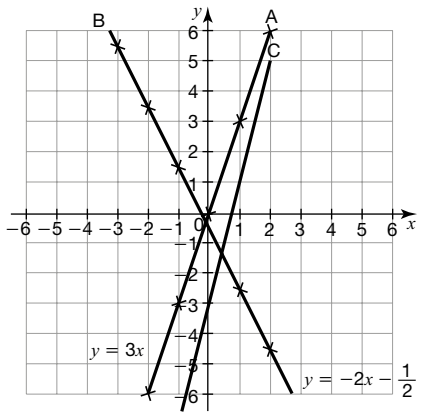
1 **a** For $y = 2x + 3$

x	−1	0	1	2
Operations	$2 \times (-1) + 3$	$2 \times (0) + 3$	$2 \times (1) + 3$	$2 \times (2) + 3$
y	**1**	**3**	**5**	**7**

b



2 **a** and **b**



a For $y = 3x$

x	−2	−1	0	1	2
Operations	$3 \times (-2)$	$3 \times (-1)$	$3 \times (0)$	$3 \times (1)$	$3 \times (2)$
y	**–6**	**–3**	**0**	**3**	**6**

b Rearrange the equation to give $y = -2x - \frac{1}{2}$

x	Operations	y
−3	$-2 \times (-3) - \frac{1}{2}$	$5\frac{1}{2}$
−2	$-2 \times (-2) - \frac{1}{2}$	$3\frac{1}{2}$
−1	$-2 \times (-1) - \frac{1}{2}$	$1\frac{1}{2}$
0	$-2 \times (0) - \frac{1}{2}$	$-\frac{1}{2}$
1	$-2 \times (1) - \frac{1}{2}$	$-2\frac{1}{2}$
2	$-2 \times (2) - \frac{1}{2}$	$-4\frac{1}{2}$

c Line C goes through points (0, −3), (1, 1) and (2, 5)

The y intercept is −3.

The gradient is $\frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}} = \frac{5 - 1}{2 - 1} = 4$

The equation of line C is $y = 4x - 3$.

3 **a** B and C, because they have the same gradient of 2.

b A and B, because they both have a y-intercept at (0, 1).

4 The gradient is $\frac{2 - (-6)}{3 - (-1)} = \frac{8}{4} = 2$

Using point (3, 2) and gradient m = 2:

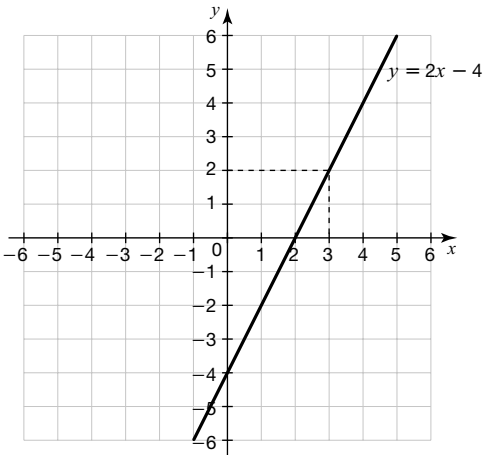
$y = 2x + c$

$2 = 2 \times 3 + c$

$c = -4$

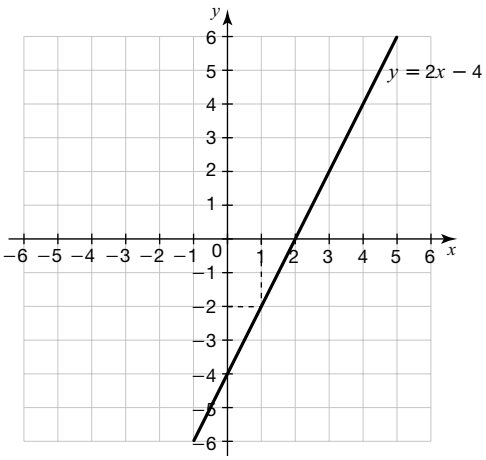
Equation of the line is $y = 2x - 4$

5 **a**



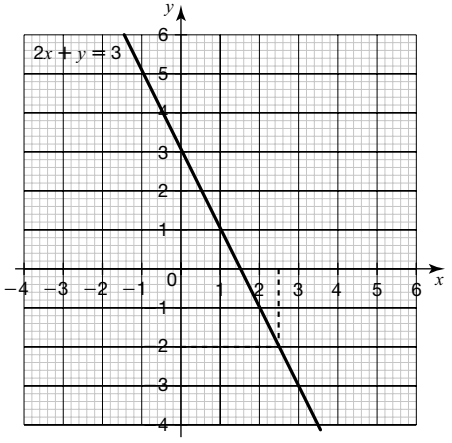
Draw a vertical line up from $x = 3$ to the graph, and then a horizontal line to the y-axis to read off the result: $y = 2$.

b



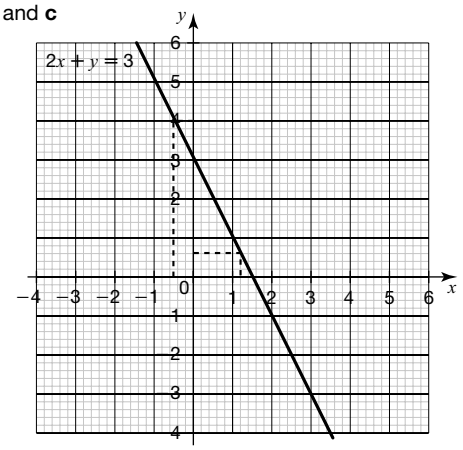
Draw a horizontal line across from $y = -2$ to the graph, and then a vertical line up to the x-axis to read off the result: $x = 1$.

6 **a**



Draw a horizontal line across from $y = -2$ to the graph, and then a vertical line up to the x-axis to read off the result: $x = 2.5$.

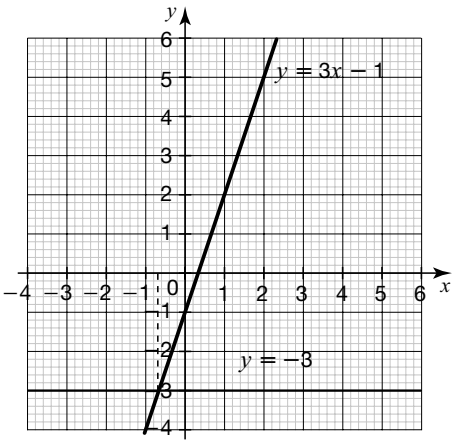
b and **c**



b Draw a vertical line up from $x = -0.5$ to the graph, and then a vertical line across to the y-axis to read off the result: $y = 4$.

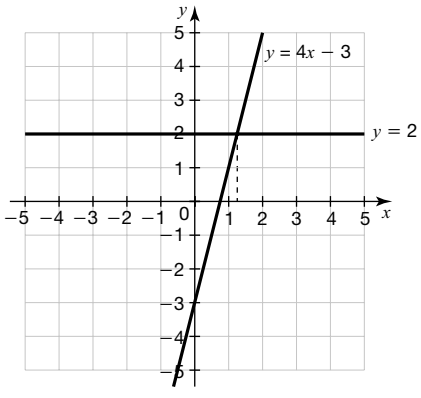
c Reading up from $x = 1.2$ and then across to y axis gives $y = 0.6$. Any value from 0.6 to 0.75 is acceptable.

7 **a** and **b**



b Where the graphs cross, draw a vertical line up to the x-axis. It meets the axis two thirds of the way between $x = -1$ and $x = 0$, so the solution is approximately $x = -0.67$. Any value from -0.6 to -0.7 is acceptable.

8



Compare the equations $y = 4x - 3$ and $4x - 3 = 2$.
y has been replaced with 2, so add line $y = 2$ to the graph.
The intersection point of the two graphs gives the solution to the equation $4x - 3 = 2$
$x = 1.25$. Any answer between 1.2 and 1.3 is acceptable.

Solving simultaneous equations

1 **a** Substituting $y = 2x$ into the first equation gives

$3x + 2x = 15$
$5x = 15$
$x = 3$

When $x = 3$, $y = 2 \times 3 = 6$

b $3x = 12$
$x = 4$
Substituting $x = 4$ into the first equation gives
$8 + y = 9$
$y = 1$

c $5x = 10$
$x = 2$
Substituting $x = 2$ into the first equation gives
$6 + y = 4$
$y = -2$

2 **a** $2x + 2y = 14$ (1)
$3x + y = 11$ (2)
(2) × 2 $6x + 2y = 22$
(3) − (1) $4x = 8$
$x = 2$
Substitute into (1) $4 + 2y = 14$
$2y = 10$
$y = 5$
Solution is $x = 2, y = 5$

b $4x - 2y = 2$ (1)
$2x - 3y = 7$ (2)
(2) × 2 $4x - 6y = 14$ (3)
(3) − (1) $-4y = 12$
$y = -3$
Substitute into (1) $4x + 6 = 2$
$4x = -4$
$x = -1$
Solution is $x = -1, y = -3$

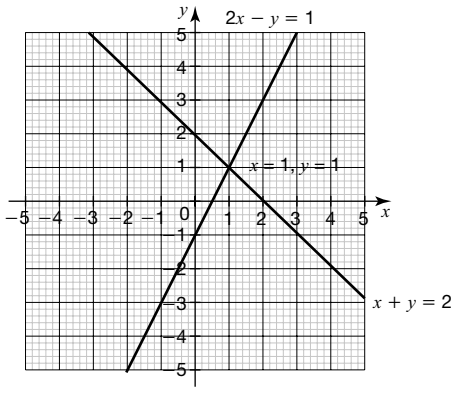
c $2x + 3y = 20$ (1)
$3x + 2y = 15$ (2)
(1) × 2 $4x + 6y = 40$ (3)
(2) × 3 $9x + 6y = 45$ (4)
(4) − (3) $5x = 5$
$x = 1$
Substituting $x = 1$ into equation (1) $2 + 3y = 20$
$3y = 18$
$y = 6$
Solution is $x = 1, y = 6$

3 **a** $x + y = 2$ $2x - y = 1$

x	0	2
y	2	0

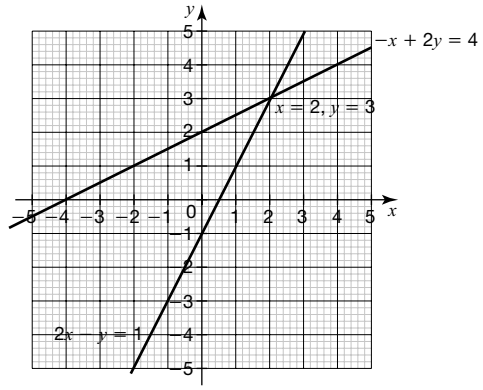
x	0	$\frac{1}{2}$
y	−1	0

b



From the intersection of the two lines, $x = 1, y = 1$.

c



From the intersection of the two lines, $x = 2$, $y = 3$.

Quadratic graphs

1 **a** C and D – they are straight lines so they are linear.

b E – it is quadratic, with a positive multiplier for x^2, and is symmetrical about the origin.

c A – it is quadratic, with a negative multiplier for x^2, and is symmetrical about the origin.

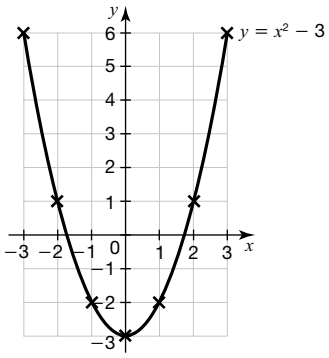
d D – the x coordinates are all different, but all the y coordinates on this line are 1.

e B – it is the same as E except that it has been moved 1 unit up the y-axis.

2 **a**

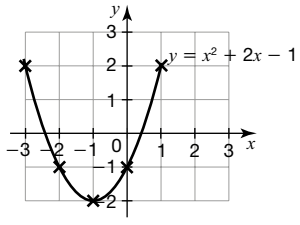
x	−3	−2	−1	0	1	2	3
y	6	1	−2	−3	−2	1	6

b



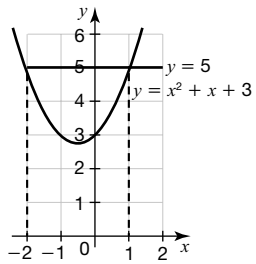
c $x = 0$ **d** (0, −3)

3 **a**



b $x = -1$ **c** (−1, −2)

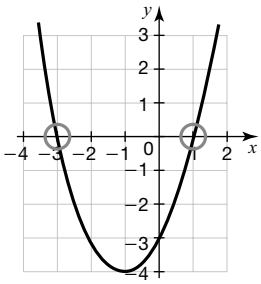
4 **a**



b $x = -2$ or $x = 1$

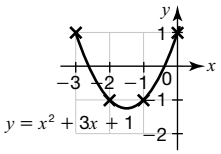
5 The roots are where the curve cuts the x-axis.

$x = -3$ or $x = 1$

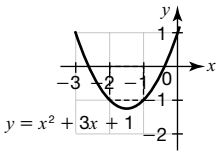


6 **a**

x	−3	−2	−1	0
$x^2 + 3x + 1$	$(-3)^2 + 3\times(-3) + 1$	$(-2)^2 + 3\times(-2) + 1$	$(-1)^2 + 3\times(-1) + 1$	$(0)^2 + 3\times(0) + 1$
y	**1**	**−1**	**−1**	**1**



b Compare the equations $y = x^2 + 3x + 1$ and $x^2 + 3x + 1 = -1$. y has been replaced with −1, so the solutions to the equation $x^2 + 3x + 1 = -1$ are where $y = -1$.



$x = -2$ and $x = -1$

c Compare the equations $y = x^2 + 3x + 1$ and $x^2 + 3x + 1 = 0$. y has been replaced with 0, so the solutions to the equation $x^2 + 3x + 1 = 0$ are where the graph crosses the x-axis.

$x = -2.6$ and $x = -0.38$ (any answer close to −0.4 is acceptable)

Solving quadratic equations

1 **a** $x(x + 6) = 0$
$x = 0$ or -6

b $y(y - 11) = 0$
$y = 0$ or 11

c $3d(d - 3) = 0$
$d = 0$ or 3

2 **a** $(x + 4)(x - 4) = 0$
$x = 4$ or -4

b $(a + 9)(a - 9) = 0$
$a = 9$ or -9

c $(z - 10)(z + 10) = 0$
$z = 10$ or -10

3 **a** $(x + 3)(x + 2) = 0$
$x = -2$ or -3

b $(x + 5)(x - 2) = 0$
$x = -5$ or 2

c $(x - 7)(x - 2) = 0$
$x = 2$ or 7

4 **a** $0 = x(x - 3)$
$x = 0$ or 3

b $0 = (x - 5)(x + 5)$
$x = 5$ or -5

c $0 = (x + 6)(x - 3)$
$x = -6$ or 3

Cubic and reciprocal graphs

1 **a** A, C and D – they are not continuous curves with two turning points (s-shaped curves).

b B – it has two turning points and has rotational symmetry about the origin.

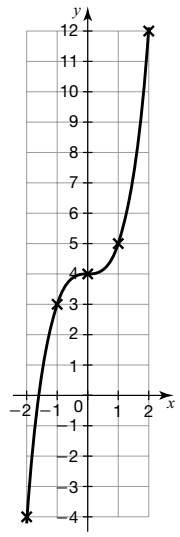
c E – it has two turning points and is a reflection of B, raised up one unit on the y-axis.

d D – this is the form for a reciprocal graph.

2 **a**

x	−2	−1	0	1	2
y	−4	3	4	5	12

b

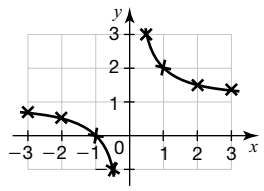


3 **a** cubic **b** (0, −8) **c** (2, 0)

4 **a**

x	−3	−2	−1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	3
y	$\frac{2}{3}$	$\frac{1}{2}$	0	−1	3	2	$1\frac{1}{2}$	$1\frac{1}{3}$

b



Drawing and interpreting real-life graphs

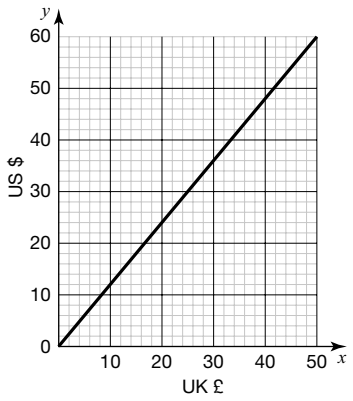
1 **a** The initial charge is the value at 0 miles, where the graph cuts the y-axis (the y-intercept): \$3.

b The charge per mile is given by the gradient of the graph.

Gradient $= \frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}} = \frac{23-3}{8-0} = \frac{20}{8} = 2.5$

Charge per mile is \$2.50

c



d From the conversion graph, £15 = \$18.

From the graph of the cost of a taxi in New York, \$18 allows you to travel 6 miles.

2 **a** You can read this from the highest point of the graph: 16 m.

b Read from the highest point down to the value on the horizontal axis: 4 seconds.

c This is when the ball reaches 0 height for the second time: 8 seconds.

d Read from 12 on the vertical axis across to the curve: 2 seconds and 6 seconds.

3 **a** This is the highest value on the vertical axis: 10 m/s.

b The cyclist is travelling at a constant speed of 10 m/s.

c The cyclist is decelerating, so it will be a negative value.

Acceleration $= \frac{-10}{20} = -\frac{1}{2}$ m/s^2

d The speed returns to 0: the cyclist stops.

4 **a** Assuming that water pours into the container at a constant rate,

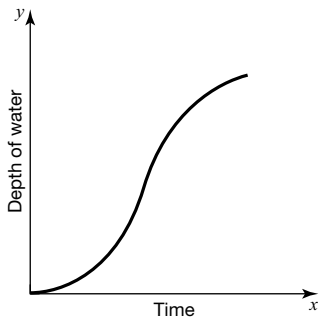
A: depth goes up increasingly slowly as the container widens, so 2.

B: depth rises steadily and fairly slowly in a broad container of consistent diameter, so 4.

C: depth rises quickly at first, then more slowly as the container widens, then more quickly as it gets narrow towards the top, so 3.

D: depth rises steadily and quickly in a narrow container of consistent diameter, so 1.

b



Ratio, proportion and rates of change

Units of measure

1 **a** 4 (m) × 100 = 400 cm

b 500 (g) ÷ 1000 = 5 kg

c 1.5 (l) × 1000 = 1500 ml

d 8250 (m) ÷ 1000 = 8.25 km

2 6 (litres) × 1000 = 6000 ml

6000 − 3500 = 2500

Sally had **2500 ml** of lemonade left.

3 **a** Luke: 240 seconds; Adam: 3 × 60 + 47 = 227 seconds. Adam arrived first.

or Luke: 240 ÷ 60 = 4 minutes; Adam: 3 minutes 47 seconds. Adam arrived first.

b 4 minutes − 3 minutes 47 seconds = 13 seconds. Adam waited **13 seconds** for Luke to arrive at school.

4 Ben = 1.25 m = 3.2 + 0.8 feet = 4 feet. Tom is taller.

or Tom = 4.8 feet = 3.2 + 1.6 feet = 1 + 0.5 metres = 1.5 metres. Tom is taller.

Ratio

1 12 stories and 8 colouring

Ratio of story: colouring = 12 : 8 = 3 : 2

2 **a** density of tin : density of copper = 2 : 1

b 10 g is 1 part

So 90 g is $\frac{90}{10}$ = 9 parts

9

c weight of tin : weight of copper = 1 : 9.

density of tin: density of copper = 2 : 1

density $= \frac{\text{weight}}{\text{volume}}$ and so volume $= \frac{\text{weight}}{\text{density}}$

So ratio of volume of tin to copper in the bronze $= \frac{1}{2} : \frac{9}{1}$

Multiply both parts by 2:

= 1 : 18

3 Total parts = 1 + 2 + 3 = 6
1 part = $\frac{60}{6}$ = 10
Amount given to charity = 3 × 10 = £30

4 Ratio of blue to yellow required is 3 : 7.
There are 3 + 7 = 10 parts. He needs to make 5 litres.
10 parts = 5000 ml
1 part = 500 ml
Phil needs 3 × 500 ml = 1500 ml = 1.5 litres of blue paint. He has 2 litres of blue paint.
Phil needs 7 × 500 ml = 3500 ml = 3.5 litres of yellow paint. He has 3 litres of yellow paint.
Phil has enough blue paint, but does not have enough yellow paint.

Scale diagrams and maps

1 1 cm on the map is **10 000 cm** in real life.
This means 1 cm on the map is **100 m** in real life.

2 1 cm on the map represents 50 m in real life.
3 × 50 m = 150 m, so the bus stop is **3 cm** from the village shop on the map.

3 Measure the distance between the trees on the diagram = 5 cm
1 cm on the diagram represents 4 m in real life.
5 × 4 = 20
The trees are **20 m** apart.

4 A scale of 1 : 400 means 1 cm on the model represents 4 m (= 400 cm) in real life.
96 ÷ 4 = 24
The scale mode is **24 cm** tall.

Fractions, percentages and proportion

1 1 + 3 = 4 parts so Bess receives $\frac{3}{4}$

2 **a** 1 : 3 : 6

b 1 + 3 + 6 = 10 items in the basket
Fruit = $\frac{3}{10}$

c Tins = $\frac{6}{10}$ = 60%

3 $\frac{50}{4000} = \frac{1}{80}$

4 Total parts = 3 + 8 + 14 = 25
$\frac{8}{25} = \frac{32}{100}$ = 32%

Direct proportion

1 **a** One ticket costs £80 ÷ 5 = £16

b Nine tickets cost 9 × £16 = £144

2 **a** Read up from 6 packs on the horizontal axis, to the line, then across to the vertical axis to find the cost: £1.20

b There are 10 pencils in a pack, so 1 pencil is 0.1 of a pack. Reading off the graph using this value, the price is 2p.

c It is a straight-line graph; the graph passes through the origin (0, 0).

3 **a** Sally needs to make 28 ÷ 4 = 7 lots of the recipe.
She will need 1 × 7 = 7 teaspoons of turmeric,
2 × 7 = 14 teaspoons of chilli powder and
$2\frac{1}{2} \times 7 = 17\frac{1}{2}$ teaspoons of cumin.

b Sally has 75 g of chilli powder. That is 75 ÷ 3 = 25 teaspoons.
Sally needs 14 teaspoons to make the curry for her class. She does have enough.

Inverse proportion

1 **a** Start from 5 on the x-axis, read up to the graph, then left to the scale on the y-axis.
5 winners will each get **£400**.

b Start from 200 on the y-axis, read right to the graph, then down to the scale on the x-axis. 10 winners each get £200, so there are **9** other winners.

c Multiply the number of winners by the amount each one gets to find the total prize money. e.g. 5 × £400 = **£2000**.

2 **a** The total time needed to decorate the room is 3 × 2 = 6 hours.

b 6 hours ÷ 12 people = 0.5 hours = **30 minutes** (or 0.5 hours).

3 The printer can print 240 ÷ 4 = 60 pages per minute.
600 ÷ 60 = 10, so it would take **10 minutes** to print the larger document.

4 $y = 3x$ means that as x increases, y increases (3 times as quickly).
$y = \frac{3}{x}$ means that as x increases, 3 is divided by a bigger number, so y gets smaller.
$y = x - 3$ means that as x increases, y increases but y is always 3 smaller than x.
$y = \frac{x}{3} = \frac{1}{3} \times x$ means that as x increases, y increases ($\frac{1}{3}$ as quickly).
$y = x + 3$ means that as x increases, y increases but y is always 3 bigger than x.
So the answer is $y = \frac{3}{x}$

Working with percentages

1 125 − 75 = 50
$\frac{50}{125} \times 100 = 40\%$

2 **a** Amount of increase = 24 − 15 = £9 million
$\frac{9}{15} \times 100 =$ **60%** increase in sales for Company X

b 125% = £35 million
35 ÷ 125 × 100 = **£28 million** sales in 2006.

3 **a** $\frac{2}{100} \times 265 = 5.30$ and 3 × 5.30 = 15.90
15.90 + 265 = £280.90

b 265 × $(1.02)^3$ = £281.22

Compound units

1 $\frac{3 \text{ km}}{\text{minute}} = \frac{3000 \text{ m}}{\text{minute}} = \frac{3000 \text{ m}}{60 \text{ seconds}} = 50$ m/s

2 20 ÷ 5 = 4 minutes to fill the tank.

3 Pressure = $\frac{300}{0.05}$ = 6000 Newtons/m^2 (**or** 6000 N/m^2)

4 On Saturday Sami drove 4 × 50 = 200 miles; on Sunday Sami drove 356 ÷ 8 × 5 = 222.5 miles. Sami drove further on Sunday.

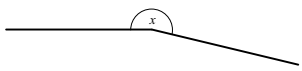
Geometry and measures

Measuring and drawing angles

1 **a** 123°

b Use a protractor to measure angle (angle *ABC*): 42°

c 331°

2 

3 **a** 100°, 120°, 140°, 160°

b First angle + second angle = 87°. This means both angles are less than 87°, and so they both must be acute.

Using the properties of angles

1 **a** x = 360 − 111 − 102 − 94
= 53° (angles around a point sum to 360°)

b x = 180 − 49
= 131° (alternate angles are equal and angles on a straight line add up to 180°)

c Angle *ACB* = 52° (Angles in a triangle add up to 180°)
x = 128° (Angles on a straight line add up to 180°)

d Angle $ADC = 86°$ (Angles in a quadrilateral add up to 360°)
$x = 94°$ (Angles on a straight line add up to 180°)

2 Angles in a triangle add up to 180°, so apex of Meg's teepee $= 180° - 2\theta =$ apex of Jonah's teepee.

Since Jonah's teepee is symmetrical, both its other angles are equal. So each angle is $\frac{2\theta}{2} = \theta$.

So the outlines of both teepees have all three angles the same and are congruent.

3 **a** Angle $BED = 39°$ (Alternate angles are equal)
Angle $BDE = 39°$ (Base angles in an isosceles triangle are equal)
$x = 102°$ (Angles in a triangle add up to 180°)

b Angle $DCF = 98°$ (Vertically opposite angles are equal)
$x = 98°$ (Corresponding angles are equal)

4 Angle $CFG = 62°$ (Co-interior angles add up to 180°)
$x = 66°$ (Angles on a straight line add up to 180°)

5 $x + 40 + 3x + 5x - 40 = 180°$
$9x = 180°$
$x = 20°$
Angle $BAC = x + 40 = 20 + 40 = 60°$
Angle $ACB = 3x = 3 \times 20 = 60°$
Angle $ABC = 5x - 40 = 5 \times 20 - 40 = 60°$
Triangle ABC has equal angles of 60°. Therefore, it is an equilateral triangle.

6 angle ABC = angle ***XAB*** (alternate angles)
angle ACB = angle ***YAC*** (alternate angles)
angle BAC + angle ***XAB*** + angle ***YAC*** = 180° (angles on a straight line add up to 180°)
So angle BAC + angle ABC + angle $ACB = 180°$

Using the properties of polygons

1 **a** $180° \times (6 - 2) = 720°$
b $720° \div 6 = 120°$
c $180° - 120° = 60°$ or $360° \div 6 = 60°$

2 **a** It is an octagon because it has eight sides.
b All angles are equal; all sides are equal.
c $180° \times (8 - 2) = 1080°$
$1080° \div 8 = 135°$
or $180° - (360° \div 8) = 135°$

3 exterior angle $= 180° - 144° = 36°$
number of sides $= 360 \div 36 = 10$
Therefore it is a decagon.

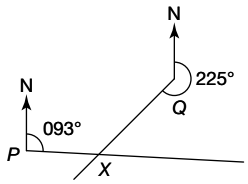
Using bearings

1 **a** This is the angle measured clockwise from North at A: 065°.
b $180 - 138 = 42°$ This is the acute angle at C.
Bearing of B from $C = 360 - 42° = 318°$
c $180 - 65 = 115°$ This is the angle between the north line at B and AB, measured anticlockwise.
Bearing of A from $B = 360 - 115 = 245°$

2 To find a reciprocal bearing, subtract 180 from the original bearing (or add 180 to it).
The bearing of O from $X = 276 - 180 = 096°$

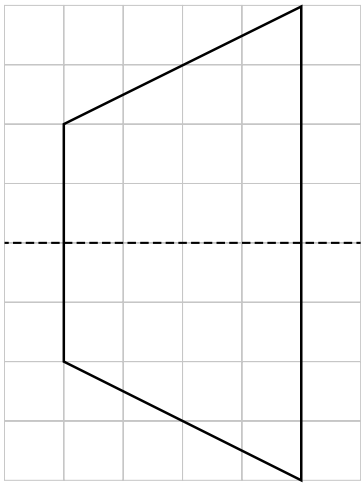
3 **a** Draw a North line at P, then join P to Q and measure the angle between the North line and this line: 060° (any value 058° to 062° accepted).

b



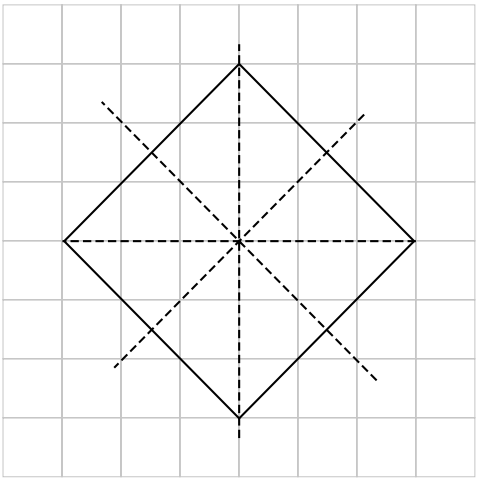
Properties of 2D shapes

1 **a**



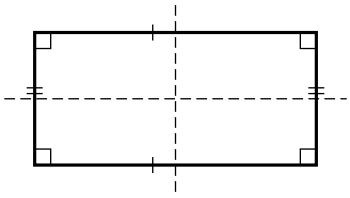
b trapezium
c one pair of parallel sides

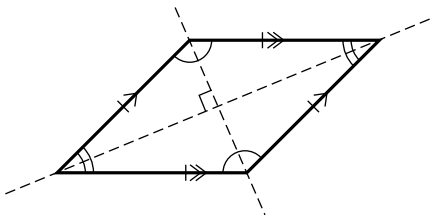
2 **a**



b 4
c square
d Two from: all sides equal in length; all angles are 90°; diagonals are equal; diagonals bisect each other at 90°.

3 **a** rectangle, rhombus
b





Congruent shapes

1 D and F are exactly the same as A – they are the same size and shape (it doesn't matter that they are rotated). E is similar to A, not congruent – it is smaller.

2 If the triangles are congruent, all three angles must be the same in both. You know that two of the angles are 35° and 82°, so $x = 180 - 35 - 82 = 63°$.

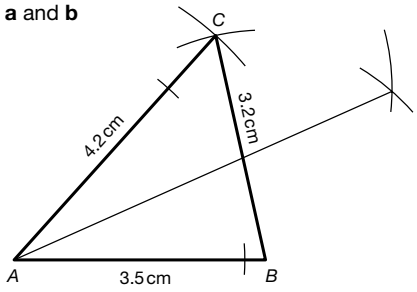
3 **a** Identify what values match: SAS (side, angle, side – two sides and the angle between them).

b Identify what values match: ASA (angle, side angle – two angles and a corresponding side).

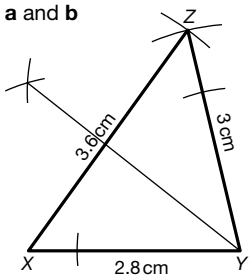
4 No, they are not congruent. They have the same angles, but the sides may not be the same size (one triangle could be an enlargement of the other).

Constructions

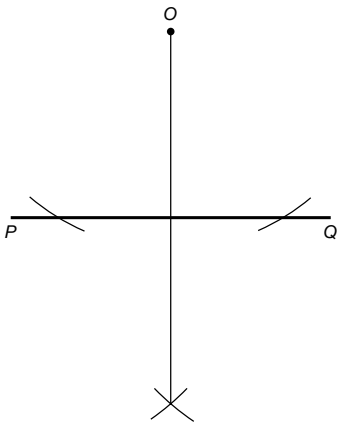
1 **a** and **b**



2 **a** and **b**



3 **a**



b Distance on the diagram = 2.5 cm

$2.5 \times 100 = 250$ cm in real life = 2.5 m

Drawing circles and parts of circles

1 **a–d**

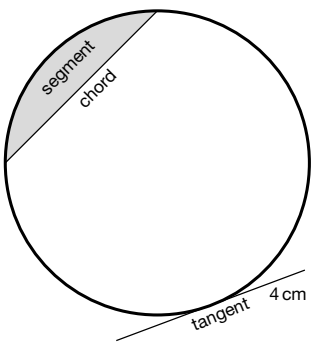


Diagram not to scale

2 **a** and **b**

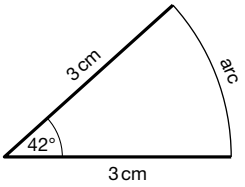
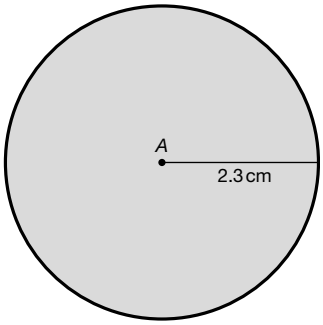


Diagram not to scale

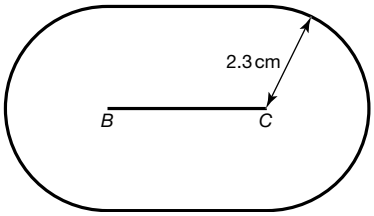
3 No, Donald is not correct. A segment of a circle is the area enclosed by a chord and an arc; a sector of a circle is the area enclosed by two radii and the arc between them.

Loci

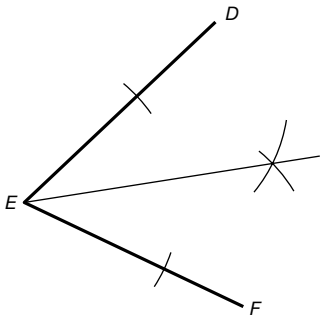
1 **a**



b



c



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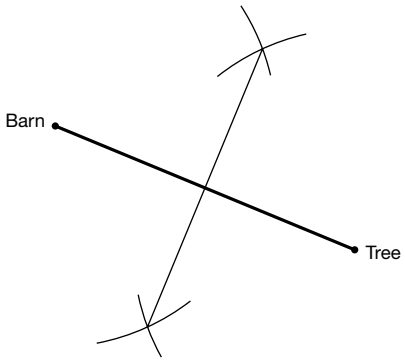

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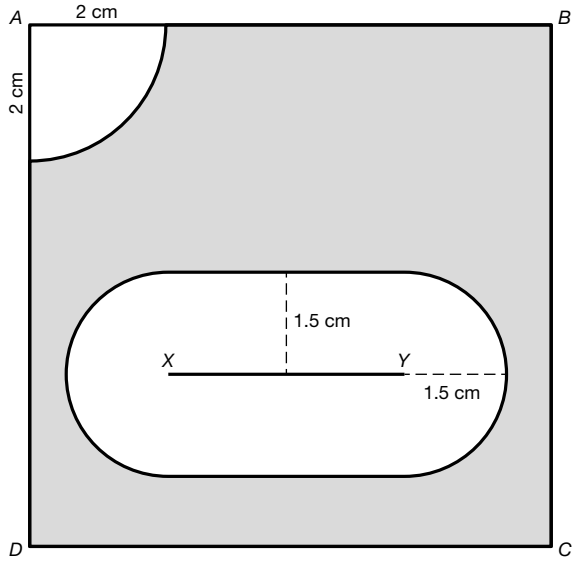

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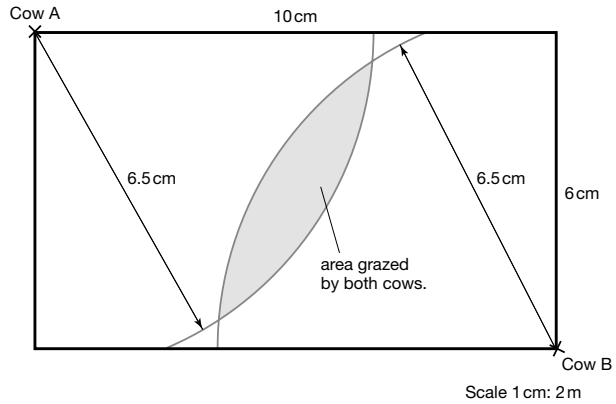


Scale 1 cm: 2 m

Perimeter

1 A hexagon has 6 sides so perimeter = $6 \times 9 = 54$ cm

2 Missing vertical length = $20 - 5 - 5 - 4 = 6$ mm

Missing horizontal lengths are all equal = $25 - 13 = 12$ mm each

Perimeter = $20 + 25 + 6 + 12 + 4 + 12 + 5 + 12 + 5 + 13$
$= 114$ mm

$114 \div 10 = 11.4$ cm

3 Perimeter of cushion = $\frac{1}{2} \times 2 \times \pi \times 24 + 30 + 48 + 30$
$= 183$ cm (to nearest cm)
$= 1.83$ m. So no, Greta does not have enough lace.

Area

1 **a** Area = $12 \times 6 = 72$ cm^2

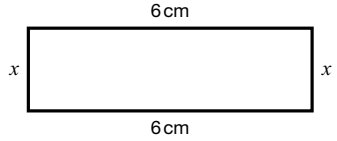
b Area = $\frac{1}{2} \times (3 + 8) \times 4 = 22$ cm^2

c Area of rectangle = $2 \times 10 = 20$ cm^2

Area of trapezium = $\frac{1}{2}(a + b)h = \frac{1}{2}(2.5 + 10)10$
$= 62.5$ cm^2

Area of shape = $20 + 62.5 = 82.5$ cm^2

2 First draw a diagram. Two sides are equal, and are 6 cm. The two other sides are equal, and are x cm.



$x + x + 6 + 6 = 16$

$2x + 12 = 16$

$x = \frac{16 - 12}{2} = 2.$

Area = $6 \times 2 = 12$ cm^2

3 Radius of circle = 3.5 cm

Area of circle = $\pi r^2 = \pi \times 3.5^2 = 38.48$ cm^2

Area of square = 49 cm^2

Area of shaded part = $\frac{49 - 38.48}{4} = 2.63$ cm^2

Sectors

1 **a** $\frac{120}{360} = \frac{1}{3}$

b Area = $\frac{1}{3} \times \pi \times 3^2 = 3\pi$ cm^2

2 **a** Area = $\frac{1}{4} \times \pi \times 2.8^2 = 6.2$ cm^2 (1 d.p.)

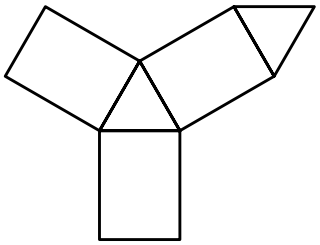
b Perimeter = $2.8 + 2.8 + \frac{1}{4} \times 2 \times \pi \times 2.8$
$= 10.0$ cm (1 d.p.)

3 **a** Area = $\frac{40}{360} \times \pi \times 5^2 = 8.73$ cm^2 (2 d.p.)

b Arc $AB = \frac{40}{360} \times 2 \times \pi \times 5 = 3.49$ cm (2 d.p.)

3D shapes

1 **a**

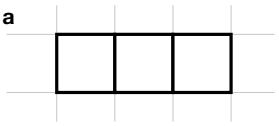


b triangular prism

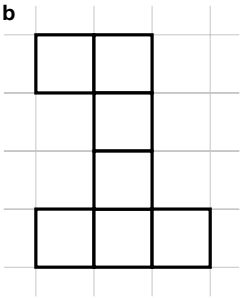
c

Number of faces	Number of edges	Number of vertices
5	9	6

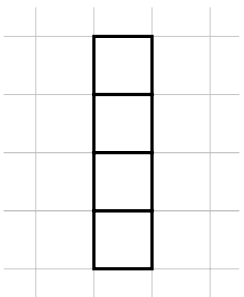
2 **a**



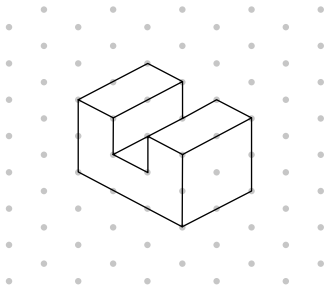
b



c



3 **a**



3 **b** The front elevation shows 5 cubes and the side shows that the shape is 2 cubes deep. $5 \times 2 = 10$, so 10 cubes make up the shape.

Volume

1 The front elevation shows 5 cubes and the side shows that the shape is 4 cubes deep.Volume = $5 \times 4 = 20$ cm^3

2 **a** Volume = area of cross-section × length = $\pi \times 5^2 \times 12$
$= 942$ cm^3 (to 3 s.f.)

b Volume = $\frac{1}{3}$ × area of cross-section × length
$= \frac{1}{3} \times \pi \times 7^2 \times 15 = 770$ cm^3 (to nearest cm)

3 Volume = $\frac{1}{2} \times \frac{4}{3} \pi r^3 = \frac{2}{3} \times \pi \times 8^3 = 1072.33$ cm^3 (to 2 d.p.)

4 Volume of tank = $40 \times 40 \times 60 = 96\,000$ cm^3

Volume of water in tank, 80% full = $0.8 \times 96\,000$
$= 76\,800$ cm^3

Height of water in pond (1st fill) = $76\,800 \div (80 \times 60) = 16$ cm

Height of water in pond (2nd fill) = $16 \times 2 = 32$ cm

Height of water in pond (3rd fill) = $16 \times 3 = 48$ cm

Three tanks of water are needed to fill the pond.

Alternative method: divide volume of pond by volume of water in tank.

$\frac{80 \times 60 \times 48}{76\,800} = 3$

Surface area

1 **a** 6 faces

b Surface area = $60 + 60 + 5 + 5 + 3 + 3 = 136$ cm^2

2 surface area = area of triangular side × 4 + area of square base

$= \left(\frac{1}{2} \times 6 \times 5\right) \times 4 + 6^2$

$= (15 \times 4) + 36$

Surface area = 96 cm^2

3 **a** Surface area of a sphere = $4\pi r^2 = 4 \times \pi \times 14^2$
$= 2463.01$ cm^2 (to 2 d.p.)

b Surface area of a cone = $\pi rl + \pi r^2 = \pi \times 6 \times 10 + \pi \times 6^2$
$= 301.59$ cm^2 (to 2 d.p.)

4 Area of cylinder = $2\pi rh = 2\pi \times 6 \times 1.5 = 56.55$ cm^2

Area of circular base = $\pi r^2 = \pi \times 6^2 = 113.10$ cm^2

Area of curved surface area of cone = $\pi rl = \pi \times 6 \times 11.5$
$= 216.77$ cm^2

Total surface area = $56.55 + 113.10 + 216.77 = 386.42$ cm^2 (to 2 d.p.)

Using Pythagoras' theorem

1 $x^2 = 3^2 + 4^2$
$= 9 + 16$
$= 25$
$x = \sqrt{25}$
$= 5$ cm

$15^2 = y^2 + 12^2$
$225 = y^2 + 144$
$81 = y^2$
$y = \sqrt{81}$
$= 9$ cm

2 $6^2 = 4.5^2 + w^2$
$36 = 20.25 + w^2$
$w = \sqrt{15.75}$
$= 3.97$ cm

Area = $l \times w = 4.5 \times 3.97 = 17.9$ cm^2

3 $AB^2 = 2^2 + 4^2$
$= 4 + 16$
$= 20$
$AB = \sqrt{20}$
$= \sqrt{4 \times 5}$
$= 2\sqrt{5}$ units

4 Square of diagonal of doorway = $70^2 + 190^2 = 41\,000$

Diagonal of doorway = $\sqrt{41\,000}$

$= 202.48$ cm $= 2.0248$ m $= 2.02$ m (2 d.p.)

Yes, the artwork will fit through the diagonal of the doorway.

Trigonometry

1 $\tan x = \frac{8}{13}$
$x = 31.6°$

2 $\cos 42 = \frac{17}{AC}$
$AC = \frac{17}{\cos 42} = 22.9$ cm

3 $\sin 49 = \frac{h}{6}$
$h = 6 \sin 49$
$= 4.53$ m

Exact trigonometric values

1 **a** $\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{1} = 1$

b $x = \tan^{-1}(1) = 45°$

2 $\cos 30 = \frac{\sqrt{3}}{PR}$
$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{PR}$
$PR = 2$ cm

3 $\frac{YZ}{20} = \sin 30$
$YZ = 20 \sin 30$
$= 20 \times \frac{1}{2}$
$= 10$ cm

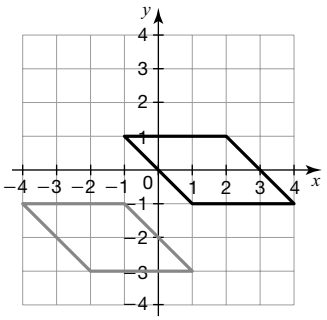
4 $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$

So the second angle in the triangle must be 45°.

The third angle will be $180 - 90 - 45 = 45°$.

Transformations

1



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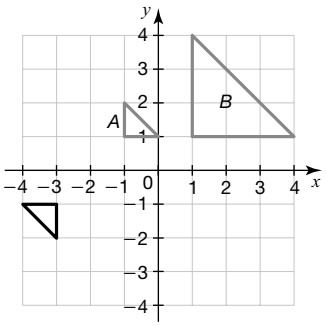
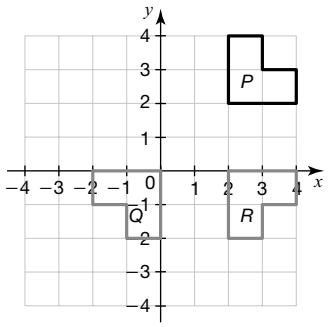


3 Rotation 90° clockwise about (1, −1); or rotation 270° anticlockwise about (1, −1).

4 **a** and **b**



c Reflection in $y = 1$

Similar shapes

1 **a** *YZW*

b Scale of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{6.3}{2.1} = 3$

c $WZ = 4 \times 3 = 12\,\text{cm}$

2 **a** 37.5°

b Scale of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{5}{2.5} = 2$

Length of $AB = 8 \div 2 = 4$ cm

c They are isosceles, because they have two equal sides and two equal angles. Note that the diagrams are not drawn to scale, as is common practice in maths questions – you have to go by the numbers.

d Length of BC = length of $AC = 2.5\,\text{cm}$

3 **a** Scale of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{4}{6} = \frac{2}{3}$

b Length of $RT = 4.5 \times \frac{2}{3} = 3\,\text{cm}$

4 **a** Scale of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{2.2}{4.4} = \frac{1}{2}$

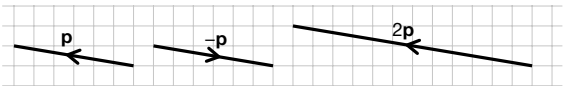
b Length of CE = length of $CD \div \frac{1}{2} = 2.8 \times 2 = 5.6\,\text{cm}$

c Angle $ACE = 180 - 70 - 64 = 46°$

Vectors

1 $\mathbf{a} = \begin{pmatrix}3\\2\end{pmatrix}$ $\mathbf{b} = \begin{pmatrix}-3\\3\end{pmatrix}$ $\mathbf{c} = \begin{pmatrix}2\\-4\end{pmatrix}$ $\mathbf{d} = \begin{pmatrix}-4\\-2\end{pmatrix}$

2 **a**



b $-\mathbf{p} = \begin{pmatrix}6\\-1\end{pmatrix}$

c $\mathbf{2p} = \begin{pmatrix}-12\\2\end{pmatrix}$

d $\mathbf{2p} + \mathbf{p} = \begin{pmatrix}-12\\2\end{pmatrix} + \begin{pmatrix}-6\\1\end{pmatrix} = \begin{pmatrix}-18\\3\end{pmatrix}$

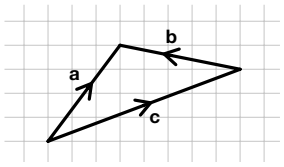
$\mathbf{3p} = 3 \times \begin{pmatrix}-6\\1\end{pmatrix} = \begin{pmatrix}-18\\3\end{pmatrix}$

Therefore, $\mathbf{2p} + \mathbf{p} = \mathbf{3p}$

3 **a** $\mathbf{a} + \mathbf{b} = \begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}-5\\1\end{pmatrix} = \begin{pmatrix}3-5\\4+1\end{pmatrix} = \begin{pmatrix}-2\\5\end{pmatrix}$

b $\mathbf{c} = \mathbf{a} - \mathbf{b} = \begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}-5\\1\end{pmatrix} = \begin{pmatrix}3-(-5)\\4-1\end{pmatrix} = \begin{pmatrix}8\\3\end{pmatrix}$

c



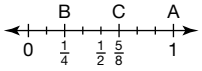
Probability

Basic probability

1 Probability $= \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}} = \frac{1}{10}$ (**or** 0.1 **or** 10%)

2 P(not rain) = 1 − P(rain) = 1 − 0.6 = 0.4

3 **a b c**



a P(a number from 1 to 8) $= \frac{8}{8} = 1$

b P(a multiple of 3) $= \frac{2}{8} = \frac{1}{4}$

c P(a number greater than 3) $= \frac{5}{8}$

4 $2p - 0.1 + 2p + 0.1 + p = 1$

$5p = 1$

$p = 0.2$

Outcome	Red	Blue	Green
Probability	$2p - 0.1$ $= 2 \times 0.2 - 0.1$ $= 0.3$	$2p + 0.1$ $= 2 \times 0.2 + 0.1$ $= 0.5$	$p = 0.2$

Blue is most likely.

Two-way tables and sample space diagrams

1 Work out the missing values one by one, for example in the order shown from first to seventh. (There is more than one order you can do it in.)

	Single	Double	King	Totals
Oak	2	Fourth: $42 - 12 - 14 = 16$	Fifth: $30 - 16 - 2 = 12$	30
Pine	First: $54 - 14 - 17 = 23$	14	17	54
Walnut	1	12	Sixth: $32 - 12 - 17 = 3$	Seventh: $1 + 12 + 3 = 16$
Totals	Second: $2 + 23 + 1 = 26$	Third: $100 - 26 - 32 = 42$	32	100

2 **a**

		Spinner			
		1	2	3	4
Coin	**Heads**	1, H	2, H	3, H	4, H
	Tails	1, T	2, T	3, T	4, T

b $P(1, T) = \frac{1}{8}$

c $P(2, H) + P(3, H) + P(4, H) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

3 **a**

	Study sciences	Do not study sciences	Totals
Boys	$\frac{1}{5} \times 120 = 24$	$45 - 24 = 21$	$\frac{3}{8} \times 120 = 45$
Girls	$75 - 40 = 35$	$\frac{1}{3} \times 120 = 40$	$120 - 45 = 75$
Totals	$24 + 35 = 59$	$21 + 40 = 61$	120

b 21 boys do not study science, so probability $= \frac{21}{120}$ **or** $\frac{7}{40}$

c There are 75 girls and 35 of them study science, so probability $= \frac{35}{75} = \frac{7}{15}$

Sets and Venn diagrams

1 **a** $\xi = \{21, 22, 23, 24, 25, 26, 27, 28, 29\}$

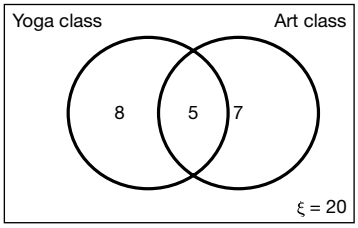
b $A = \{21, 24, 27\}$

c $B = \{24, 28\}$

d $A \cup B = \{21, 24, 27, 28\}$ – A 'union' B means **all** the values in A and **all** the values in B

e $A \cap B = \{24\}$ – A 'intersect' B means only those value that are in **both** A and B.

2 **a**



b There are 7 adults who only go to art class, so probability $= \frac{7}{20}$

c 8 adults only go to yoga class and 7 adults only go to art class.

Probability $= \frac{8+7}{20} = \frac{3}{4}$

3 **a** −3 is not included, but 2 is:

−2, −1, 0, 1, 2

b Upper and lower bounds are not integers. Values are: 8, 9

c Upper and lower bounds are not integers. Values are: 2, 3, 4, 5, 6

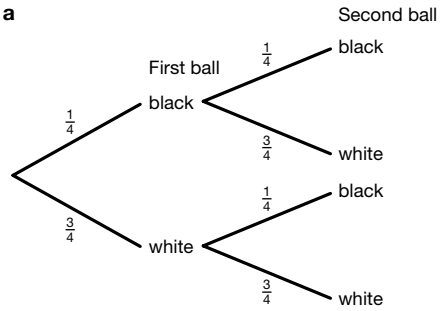
4 a Total number of respondents $= \xi$
$= 25 + 11 + 2 + 12 + 0 + 7 + 3 = 60$
P(supermarket only) $= \frac{25}{60} = \frac{5}{12}$

b P(F) $= \frac{12 + 2 + 7 + 3}{60} = \frac{24}{60} = \frac{2}{5}$

c $S = \{25 + 11 + 2 + 12\} = \{50\}$
$S' = \{60 - 50\} = \{10\}$
$\xi = \{50 + 0 + 3 + 7\} = \{60\}$
P(S′) $= \frac{10}{60}$
$= \frac{1}{6}$
(This is the probability that the person never shops at a supermarket.)

Frequency trees and tree diagrams

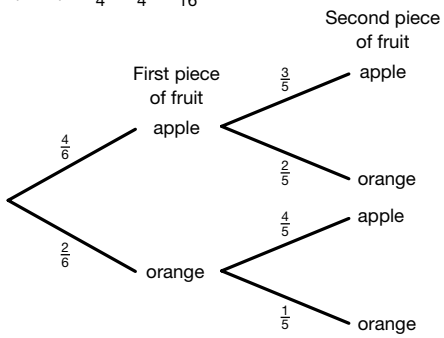
1 a



b P(W, B) $= \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$

c P(B, B) $= \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

2 a

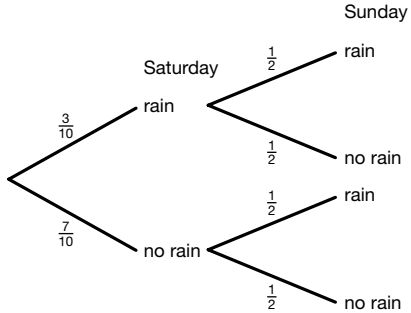


b From tree diagram:
P(A, O) $= \frac{4}{6} \times \frac{2}{5}$
$= \frac{8}{30}$
$= \frac{4}{15}$

c From tree diagram:
P(O, O) $= \frac{2}{6} \times \frac{1}{5}$
$= \frac{2}{30}$
$= \frac{1}{15}$

d P(O, O) + P(A, A) $= \left(\frac{2}{6} \times \frac{1}{5}\right) + \left(\frac{4}{6} \times \frac{3}{5}\right) = \frac{2}{30} + \frac{12}{30} = \frac{14}{30}$
$= \frac{7}{15}$

3 a



b P(R, R) $= \frac{3}{10} \times \frac{1}{2} = \frac{3}{20}$

c P(R, R′) + P(R′, R) $= \left(\frac{3}{10} \times \frac{1}{2}\right) + \left(\frac{7}{10} \times \frac{1}{2}\right)$
$= \frac{3}{20} + \frac{7}{20} = \frac{10}{20} = \frac{1}{2}$
The probability of one day having rain and one day having no rain is 50%.

Expected outcomes and experimental probability

1 a The spinner was spun $12 + 13 + 10 + 15 = 50$ times.

b Estimated probability of blue $= \frac{13}{50}$

c Estimated probability of yellow $= \frac{15}{50} = \frac{3}{10}$

d You would expect $\frac{10}{15} \times 100 =$ **20** green outcomes from 100 spins.

2 $0.75 \times 20 =$ **15** students would be expected to pass the exam.

3 a Total number of customers $= 26 + 20 + 6 + 5 + 3 = 60$
Estimated probability that someone will buy stamps
$= \frac{20}{60} = \frac{1}{3}$

b $\frac{1}{3} \times 450 =$ **150** customers buy stamps each day

c $\frac{6}{60} \times 450 =$ **45** customers buy foreign currency each day

d $450 \times 6 =$ **2700** customers each week
$\frac{5}{60} \times 2700 =$ **225** customers use the post office for banking each week

Statistics

Data and sampling

1 65, because that is 10% of 650 (the entire population).

2 $\frac{50}{25000} \times 100 = 0.2\%$. The sample is not big enough.
People in the town centre may not be the only ones using buses. For example, some people may take buses to the local train station, school or hospital.

3 a $\frac{9}{45} \times 400 = 80$ people like carrot cake
Sam needs to make 80 cakes.

b Assumptions
Assumed that these are individual carrot cakes. If instead each cake is a large one divided into 8 slices, then only $80 \div 8 = 10$ carrot cakes would be needed.
Assumed the sample is representative of the population; this could affect the answer because not all the 400 people who have accepted the invitation may turn up.

Frequency tables

1 a $1 + 11 + 9 + 6 + 1 + 2 = 30$ tables

b $(1 \times 1) + (2 \times 11) + (3 \times 9) + (4 \times 6) + (5 \times 1) + (6 \times 2) = 91$ people

2

Number of electronic devices	Tally	Frequency
0–1	|||	3
2–3	𝍸 𝍸	10
4–5	𝍸	5
6–7	𝍸	5
8–9	|	1

3 a Continuous

b

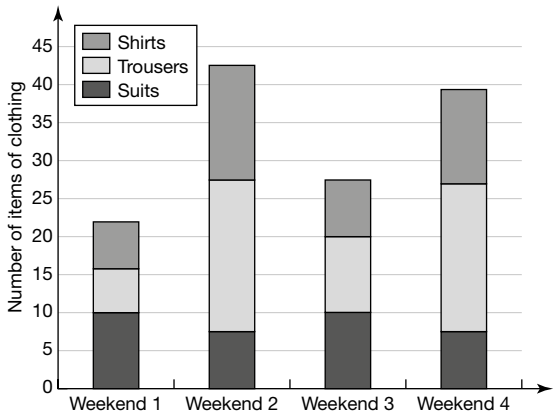
Mass, m (kg)	Tally	Frequency
$50 \leq m < 60$	|||	3
$60 \leq m < 70$	𝍸	5
$70 \leq m < 80$	||||	4
$80 \leq m < 90$	𝍸	5
$90 \leq m < 100$	|||	3

Bar charts and pictograms

1 **a** $9 - 4 = $ **5** more boys than girls prefer squash

b $15 + 6 + 9 + 7 + 4 = $ **41** girls were surveyed

2



3 **a** $3 + 3 + 3 + 3 + 2 = 14$ hours of sunshine

b Yorkshire gets 9 hours, Inverness-shire gets 7 hours. Yorkshire gets 2 more hours of sunshine each day than Inverness-shire.

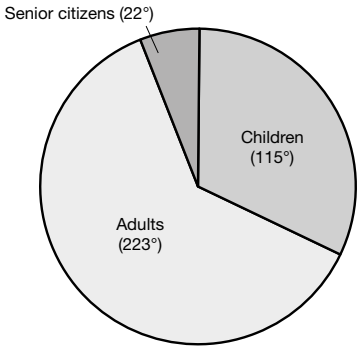
Pie charts

1 **a** $\frac{150}{360} \times 300 = $ **125** people last saw an action movie

b Romance = 90°, so $\frac{3}{4} \times 300 = $ **225** people did not see a romance movie.

2

	Children	Adults	Senior citizens
Calculation	$\frac{80}{250} \times 360$ $= 115.2$	$\frac{155}{250} \times 360$ $= 223.2$	$\frac{15}{250} \times 360$ $= 21.6$
Angle	115°	223°	22°



3 Angle for 8:30–9pm = 72°

$\frac{360}{72} \times 12 = $ **60** people were surveyed

Stem and leaf diagrams

1 **a** 71 cm – this is the highest stem value combined with the highest leaf value

b 20 – count the number of leaf values

2 **a** longest – shortest = 4.26 – 2.03 = 2.23 m

b 3 girls jumped more than 3.5 m (3.65 m, 3.74 m and 4.06 m)

3 Write all the amounts in order, and convert them to pounds.

Year 9s	0.50	0.80	0.95	1.30	1.75	2.00	2.65	3.05
Year 10s	0.65	0.75	1.50	1.85	2.70	3.10		

Year 9s		Year 10s
95 80 50	0	65 75
75 30	1	50 85
65 00	2	70
05	3	10

Key	
Year 9s	Year 10s
50|0 means £0.50	0|65 means £0.65

Measures of central tendency: mode

1 There are two modes: 3 minutes and 4 minutes, since each appears twice. An alternative correct answer is to say that there is no mode.

2 This is the one with the highest frequency: $12 < a \leq 13$.

3 This is the class with the biggest slice of the pie chart: £10–£20.

4 Look for where there are most repeated digits to the right of the vertical line: 5, 5, 5.

Use key to work out these numbers: 25, 25, 25.

So **25 kg** is the modal weight.

5 The mode is 108, so this number must have the highest frequency.

2 of the numbers are 54, so 3 of the numbers must be 108.

The numbers are 54, 54, 108, 108, 108, 120.

The 'other 3 numbers' are all 108.

Measures of central tendency: median

1 Ages in order: 11 11 12 13 13 13 13 14 15 15 16 16 17 17 18 18

Median = $14\frac{1}{2}$ years old

2 Total frequency = 5 + 12 + 17 + 10 + 6 = 50

Median = $\frac{50 + 1}{2} = 25.5$th person

Median class = $12 < a \leq 13$

3 The median is the middle value.

Counting the digits in each row:

2 + 5 + 14 + 6 + 4 + 2 + 5 + 2 = 40

The median is between the 20th and 21st values.

2 + 5 + 14 = 21, so the 20th and 21st values are the last two values in the 3rd row:

28 and 29 (using the key)

median = $\frac{28 + 29}{2}$

$= 28.5$ or $28\frac{1}{2}$

Measures of central tendency: mean

1 Mean age = $\frac{6 + 7 + 11 + 13 + 18}{5} = 11$ years old

2 Number of bedrooms = $(1 \times 4) + (2 \times 7) + (3 \times 13) + (4 \times 17) = 125$

Number of houses = 4 + 7 + 13 + 17 = 41

Mean number of bedrooms = 125 ÷ 41 = 3.05 ≈ 3 bedrooms

3 Number of holidays = $(0 \times 4) + (1 \times 21) + (2 \times 9) + (3 \times 2) = 45$

Number of employees = 4 + 21 + 9 + 2 = 36

Mean number of holidays = 45 ÷ 36 = 1.25 ≈ 1 holiday

4

Age of patients, a	Midpoint	Frequency	Midpoint × frequency
$0 < a \leq 10$	5	3	15
$10 < a \leq 20$	15	18	270
$20 < a \leq 30$	25	6	150
$30 < a \leq 40$	35	11	385
$40 < a \leq 50$	45	10	450
$50 < a \leq 60$	55	19	1045
$60 < a \leq 70$	65	16	1040
$70 < a \leq 80$	75	17	1275
		Total = 100	Total = 4630

Mean age of patients $= \frac{4630}{100} = 46.3 \approx 46$ years old

5 Let mean of first three measurements $= m$.

$3m = 3 \times 75.6 = 226.8$

There is now an extra measurement (4 in total).

New mean is:

$\frac{226.8 + 75.2}{4} = 75.5$ cm

Range

1 **a** Range of boys' ages $= 4 - 2 = 2$ years

b Range of boys' ages $= 4 - 1 = 3$ years

2 **a** Range in temperatures for Resort A $= 25 - 16 = 9°C$

b Range in temperatures for Resort B $= 28 - 13 = 15°C$

3 **a** Business A: Range $= 45\,816 - 23\,561 = £22\,255$

Mean profit $= \frac{23\,561 + 30\,485 + 39210 + 45\,816}{4} = £34\,768$

b Business B: Range $= 63\,248 - 17\,894 = £45\,354$

Mean profit $= \frac{32\,820 + 40\,328 + 17\,894 + 63\,248}{4} = £38\,572.50$

c **Either** Business A because its range in profit is lower and the profit is increasing each year, and so it shows a more consistent performance.

or Business B because its mean profit is higher, and its most recent profit (in Year 4) is £17 432 more than Business A.

Comparing data using measures of central tendency and range

1 **a** Mean time for bus journey $= \frac{32 + 30 + 39 + 32 + 43 + 31}{6} =$ 34.5 minutes

b Range for bus journey $= 43 - 30 = 13$ minutes

c Mean time for train journey $= \frac{16 + 24 + 18 + 26 + 70 + 17}{6} =$ 28.5 minutes

d Range for train journey $= 70 - 16 = 54$ minutes

e **Either** The bus is better because although it takes longer (on average), the range is lower, and so you can predict the time it takes for the journey.

or The train is better because it is quicker than the bus (on average), although the range suggests it may be less reliable.

2 **a** Mean = 13. This does not represent the age of the people using the playground. In fact, those using the playground are small children (under 10) and their parents (over 25).

b There are five modes (3, 4, 5, 7, 8), and so the mode does not represent the age of the people using the playground.

c Ages in order: 3 3 4 4 5 5 7 7 8 8 26 30 33 39

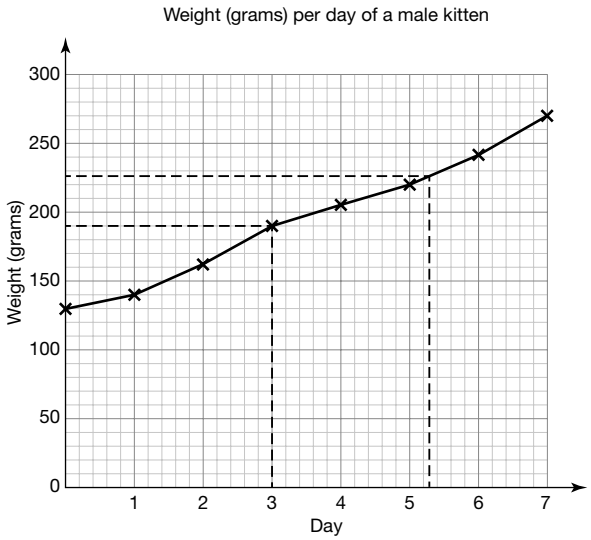
Median position $= \frac{14 + 1}{2} = 7.5$th value

Median age = 7 years old

3 Mode = 0; Median = 0; Mean = 2 days. Mode or median are the best averages to use, because the mean is skewed by the student who is absent due to sickness for 24 days.

Time series graphs

1 **a** and **b**



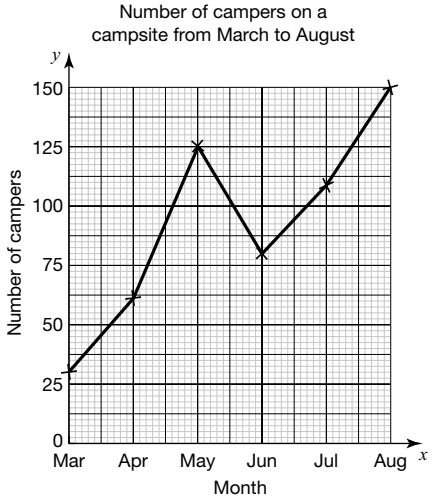
a Start at Day 3, read up to the graph line then left to the weight scale: 190 g

b According to the graph the kitten was just over 5 days old: 5 days (to the nearest day)

c Mean weight gained $= \frac{\text{total weight gained}}{\text{number of days}} = \frac{270 - 130}{7}$

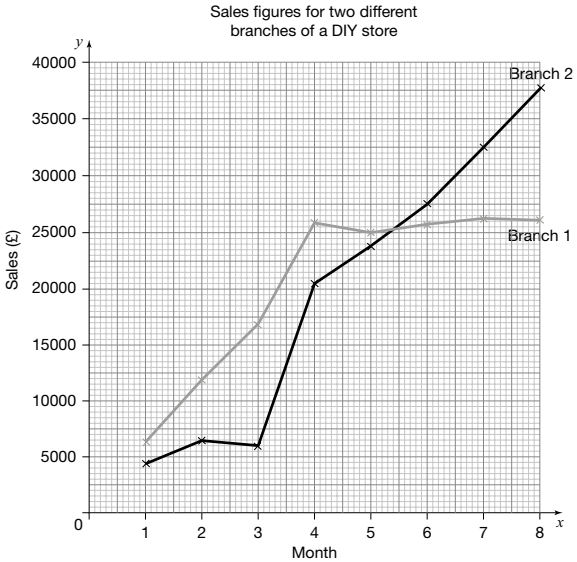
$= 20$ g per day

2 **a***



b There is an increase in campers in May. This may be due to May bank holidays, or May half-term, or perhaps there was some very sunny weather.

3 **a**



*This answer differs from the one in the Exam Practice Book due to an error in our first edition. This answer has now been re-checked and corrected.

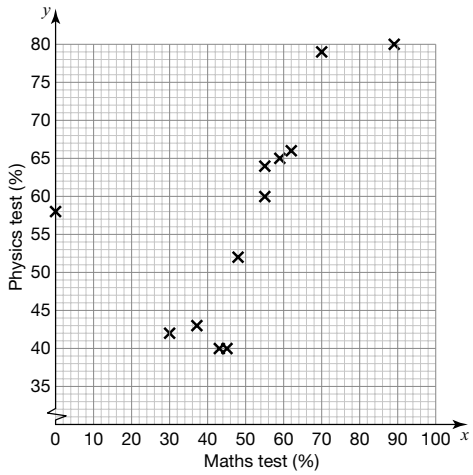
b Branch 1 had a steady increase in sales for the first four months. Then sales levelled off to stay at around £25 000. Branch 2 had a slow start to its sales in the first three months. Then perhaps it had a promotion, because sales increased a lot in month 4. Sales have been increasing ever since.

Scatter graphs

1 **a** Positive correlation. This means as the temperature rises, more pairs of flip flops are sold.

b Negative correlation. This means as the temperature rises, fewer wellington boots are sold.

2 **a**

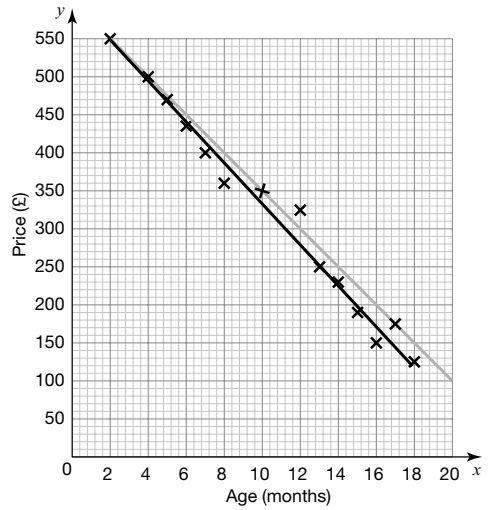


b The scatter diagram shows a positive correlation between students' maths and physics test percentages. Therefore, the students who got a low percentage in the maths test got the lower percentages in the physics test; the students who got a high percentage in the maths test got the higher percentages in the physics test.

c The outlier is the point marked at (0, 58).

d The student was absent for the maths test.

3 **a**



The black line shows the line of best fit.

The grey line shows the line where a laptop loses £150 every 6 months.

The shop owner is not correct. The line of best fit shows on average a laptop loses approximately £159/£160 every 6 months.

b The line of best fit cannot make a prediction outside the available data. The data only goes as far as 18 months.

Graphical misrepresentation

1 The pictogram suggests the weather is mostly sunny.

In the key, each symbol represents a different number of days. Also, it is not good practice to use a mixture of different symbols.

If drawn accurately, the pictogram would use symbols that all have the same value, showing that the weather is mostly cloudy (15 days), it rains on 9 days, and it is sunny for 7 days.

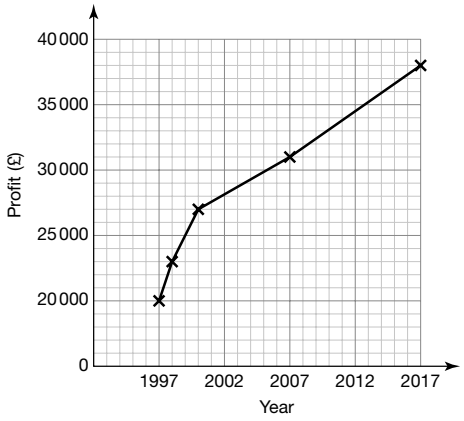
2 The bar chart suggests most people say yes to the supermarket, but actually, 40% of people said no, and just over 50% said yes.

On the vertical axis, the scale is only labelled from 40 to 50 percent. If the axes were less misleadingly labelled, the chart would show that the responses were reasonably close.

3 **a** No, his claim is not accurate.

The graph suggests that the growth in profits has been accelerating, but the years are not equally spread.

b



c Profits are increasing, but not as quickly as they did in the 1990s.

Practice papers

Non-calculator

1 It is in the 100 000s column, so 700 000.

2 $10\% = 70 \div 10 = 7$

$30\% = 3 \times 7 = 21$

3 No, Sandeep is not correct:

$\frac{2}{5} = \frac{4}{10} = 0.4$

But $4\% = 0.04$

4 $35 = 5 \times 7$

5 and 7

5 Distance on diagram between lamp post A and lamp post B = 6.1 cm

$6.1 \times 20 = 122$ m*

6 E appears twice (2 times) out of 7 times.

$P(E) = \frac{2}{7}$

7 Yes. Fun run + music festival = £9689 + £9689 + £6370

= £25 748

8 Total number of parts = 5 + 7 = 12

Fraction that are fiction = $\frac{5}{12}$

9 Prize A = $8 \times 4 = 32$ tickets

Prize B = 2 + 4 + 8 + 16 = 30 tickets

Prize A gives more tickets.

10 **a** There are 2 triangles in Pattern 1, and 4 more are added for each pattern.

Number of triangles = $4p - 2$

In Pattern 8, there are $4 \times 8 - 2 = 30$ triangles

b No, Harry is incorrect. The number of triangles is not the pattern number multiplied by 4. Rather, it is add 4 triangles each time.

11 **a** £3.50

b £5.00 – appears the most times

12 call out = 55

fee for hours worked = $2 \times 40 = 80$

total before VAT = 55 + 80 = 135

VAT at 20% = $135 \div 10 \times 2 = 13.5 \times 2 = 27$

total bill = 135 + 27 = 162

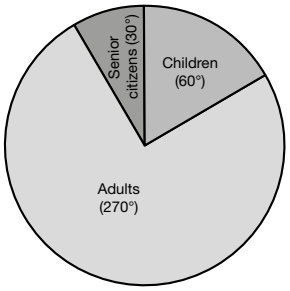
£162

*This answer differs from the one in the Exam Practice Book due to an error in our first edition. This answer has now been re-checked and corrected.

13 Bus 2A will stop at: 10:00, 10:15, 10:30, 10:45, 11:00
Bus 2B will stop at: 10:00, 10:12, 10:24, 10:36, 10:48, 11:00
They next arrive at the bus stop together at 11 am.

14 Total number of patients = 10 + 45 + 5 = 60
Angle per patient = 360 ÷ 60 = 6°

	Children	Adults	Senior citizens
Number	10	45	5
Angle	10 × 6 = 60°	45 × 6 = 270°	5 × 6 = 30°



15 Cost of T shirts = 8 × 12 = £96 (for 48 shirts)
Saturday: $\frac{3}{8} \times 48 = 18$ 18 × 5 = £90
Sunday: 30 shirts for £3 each 30 × 3 = £90
Profit = 90 + 90 − 96 = £84

16 **a** $\frac{5}{8} - \frac{7}{12} = \frac{15}{24} - \frac{14}{24} = \frac{1}{24}$

b $2\frac{2}{3} \div \frac{4}{9} = \frac{8}{3} \times \frac{9}{4} = 2 \times 3 = 6$

17 **a** $13 - 4x$

b $(x + 3)(x - 4) = x^2 - 4x + 3x - 12 = x^2 - x - 12$

18 £21 = 25%
100% is original price.
4 × 25% = 100%
4 × 21 = 84
£84

19 **a** 3 along and 4 down (or 3 in the x direction and −4 in the y direction)
(3, −4)

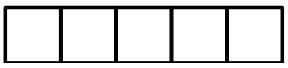
b Coordinates of A are −3 in the x direction and 0 in the y direction: (−3, 0).
Coordinates of C are (3, −4).
Coordinates of midpoint are:
the average (mean) of both x coordinates and the average (mean) of both y coordinates:
$\left(\frac{3 + -3}{2}, \frac{-4 + 0}{2}\right)$
$= \left(\frac{0}{2}, \frac{-4}{2}\right)$
$= (0, -2)$
Check on the diagram to see if this looks sensible – it does.

20 Notice there are 2 + 1 = 3 decimal places in total.
Ignore the decimal points while you work out the numbers:

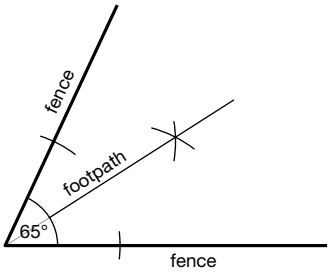
$$\begin{array}{r} 635 \\ \times\ 12 \\ \hline 1270 \\ 6350 \\ \hline 7620 \end{array}$$

Now put the 3 decimal places back in, to give your answer the correct place values:
7.620 = 7.62

21 Plan:



22



23 $\frac{8.02 \times 3.76}{15.98} \approx \frac{8 \times 4}{16} \approx 2$

24 **a** $K = \frac{1}{2} \times 11 \times 3^2$
$K = 49.5$

b Rearrange the formula:
$K = \frac{1}{2} mv^2$
$v^2 = \frac{2K}{m}$
$v = \sqrt{\frac{2K}{m}}$
Substitute values for K and m to find v
$v = \sqrt{\frac{2 \times 180}{10}}$
$v = 6$ or -6

25 $x = 180 - (90 + 36) = 54°$

26 **a** The first graph shows only a small part of the vertical scale to exaggerate the increase, she is using graph A.

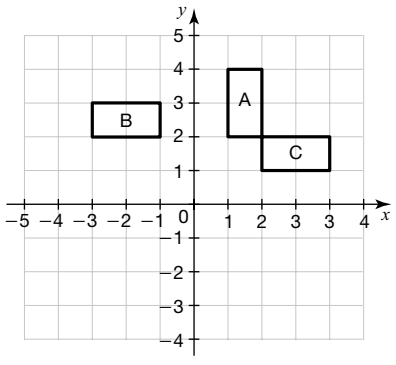
Note that although the scale on graph B gives a truer overall impression of the sales figures, the points are not plotted quite accurately to match the data in graph A – all but the first one are a little too high.* The answers to **b** and **c** below are based on graph A.

b actual increase = 11 000 − 10 000 = 1000
£1000

c percentage increase = $\frac{1000 \times 100}{10000} = \frac{1000}{100} = 10\%$

27 $5x + 3 = 6x - 7$ (as base angles of an isosceles triangle are equal)
$10 = x$
Hence base angles are 53°
Other angle = 180 − (53 + 53) = 74°
So $74° = 7y - 10$
Solving gives $y = 12°$
Hence $x = 10°$ and $y = 12°$

28 **a** and **b**



c Reflection in $x = y$, or rotation of 90° anticlockwise about the point (1.5, 1.5)

*This answer differs from the one in the Exam Practice Book due to an error in our first edition. This answer has now been re-checked and corrected.

29 $3\mathbf{a} = 3 \times \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

(Notice it's not the same as multiplying a fraction – you multiply both parts of a vector because they are like coordinates.)

$\mathbf{b} - 3\mathbf{a} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} - \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 3+3 \\ 8-6 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

Calculator

1 Enter 3 ÷ 8 into your calculator:

0.375

2 Enter 450 × 1.3 (or 450 × 130%) into your calculator:

£585

3 46.25 ÷ 5 = 9.25

Rate is £9.25 per hour.

9.25 × 37 = 342.25

£342.25

4 255 × 1.16 = 295.8 so the price in France is €295.80. The games console is cheaper in the UK.

5 On your calculator: $3\sqrt{13.824} + (4.5 - 0.38)^2 = 19.4$ (to 3 s.f.)

6 Using Pythagoras' theorem:

$YZ^2 = XY^2 + XZ^2$

$5.7^2 = XY^2 + 4.2^2$

$XY^2 = 5.7^2 - 4.2^2$

$= 32.49 - 17.64$

$= 14.85$

$XY = \sqrt{14.85}$

$= 3.9\,\text{cm}$ (1 d.p.)

7 **a** Friday

b On Thursday Jeff received 18 work emails and 2 personal emails.

$\frac{2}{20} \times 100 = 10\%$ of his emails on Thursday were personal.

c Total number of emails received = 8 + 16 + 4 + 15 + 5 + 16 + 2 + 18 + 11 + 10 = 105

Total number of work emails received = 16 + 15 + 16 + 18 + 10 = 75

$\frac{75}{105} = \frac{5}{7}$ of his emails that week were about work.

8 Factors of 15 are: 1, 3, 5 and 15.

Multiples of 8 are: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80…

Let answer = a.

$75 < a < 85$

So a could be 76, 77, 78, 79, 80, 81, 82, 83, 84.

Of these, 80 is a multiple of 8.

80 × 1 = 80, and 1 is a factor of 15.

So one pair is 1 and 80.

Which possible solutions for a have 3 or 5 as factors?

78, 81 and 84 have 3 as factors: 26 × 3 = 78, 27 × 3 = 81 and 28 × 3 = 84.

But 26, 27 and 28 are not multiples of 8.

80 has 5 as a factor: 5 × 16 = 80, and 16 is a multiple of 8.

So another pair is 5 and 16.

Pairs are 5, 16 and 1, 80.

9 −1, 0, 1, 2, 3, 4 – the range is less than 5 so this is not included

10 Area to be painted = $16 \times 1.8 \times 1.8 = 51.84\,\text{m}^2$

2 tins are enough for $2 \times 20 = 40\,\text{m}^2$ and 3 tins are enough for $3 \times 20 = 60\,\text{m}^2$.

$40 < 51.84 < 60$

So Geraldine needs 3 tins, at a price of 3 × 22.50 = £67.50.

11 $6 \times 9 \times 7 + \frac{1}{2} \times 6 \times 8 \times 7 = 546\,\text{cm}^3$

12 $84\,\text{km/h} = \frac{84 \times 1000}{60 \times 60}\,\text{m/s} = 23.\dot{3}\,\text{m/s}$

13 Felicity gets 5 parts, Ian gets 3.

£22 = 2 parts (difference between Felicity's share and Ian's)

1 part = £11

11 × 5 = 55

Felicity gets £55.

14 $PQ^2 = 2^2 + 4^2$

$PQ^2 = 20$

$PQ = \sqrt{20}$

$= \sqrt{4 \times 5}$

$= 2\sqrt{5}$

15 $\frac{2}{3}$ are the 42 women so $\frac{1}{3}$ is 21 women and

$\frac{3}{3} = 21 \times 3 = 63$

So there are 63 men and women and this represents 50% of the total.

Hence there are 126 people in the audience.

16 58 million $= 58 \times 10^6 = 5.8 \times 10^7$ km

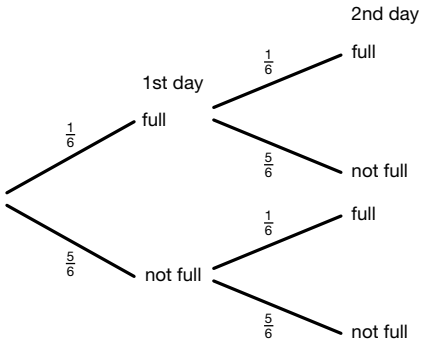
17 Let number by Seren = x

Number by Tom = $x + 3$

Number by Rohan = $x + 3 + 2 = x + 5$

Total number = $x + x + 3 + x + 5 = 3x + 8$

18 **a**



b P(full and not full) + P(not full and full) = $\left(\frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{5}{6} \times \frac{1}{6}\right) = \frac{10}{36} = \frac{5}{18}$

19 $\tan x = \frac{8}{5}$

$x = \tan^{-1}\frac{8}{5}$

$x = 58°$

$\cos 59° = \frac{y}{11}$

$y = 11 \cos 59°$

$= 5.7\,\text{cm}$ (to 1 d.p.)

20 Mean for Team A $= \frac{(0 \times 3) + (1 \times 4) + (2 \times 4) + (3 \times 2) + (4 \times 1) + (5 \times 2)}{16}$

$= \frac{32}{16} = 2$

Mean for Team B $= \frac{(0 \times 3) + (1 \times 4) + (2 \times 8) + (3 \times 4) + (4 \times 1)}{20}$

$= \frac{36}{20} = 1.8$

Team A scored more goals on average.

21 **a** Option A is best value if you plan to visit the gym once per week, as in one month 4 visits cost £20 (or 5 visits cost £25). This is less than the monthly fee of £30.

b The graphs intersect at 6, so the cost is the same for 6 visits.

It is cheaper to pay the monthly fee if you visit the gym more than 6 times.

22 Area of semicircle = $\frac{1}{2}\pi r^2$

Area of large semicircle = $\frac{1}{2} \times \pi \times 7.5^2 = \frac{225\pi}{8}$

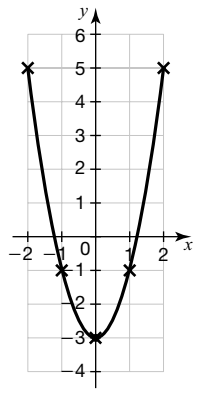
Area of small semicircles = $3 \times \left(\frac{1}{2} \times \pi \times 2.5^2\right) = 3 \times \frac{25\pi}{8} = \frac{75\pi}{8}$

Area of lawn = $\frac{225\pi}{8} - \frac{75\pi}{8} = \frac{150\pi}{8} = 58.9\,\text{m}^2$ (to 3 s.f.)

23 a

x	Operation	y
−2	$2 \times (-2)^2 - 3 =$	5
−1	$2 \times (-1)^2 - 3 =$	−1
0	$2 \times (0)^2 - 3 =$	−3
1	$2 \times (1)^2 - 3 =$	−1
2	$2 \times (2)^2 - 3 =$	5

b



24 a Draw a line of best fit to show that, yes, the car dealer is correct.

b Start at 55 000 miles and read up to your line of best fit, then read across to the price scale. The car dealer should charge £3500 to £4000. (Value varies with students' own graphs.)

25 $(n + 5)(n - 3) = n^2 + 2n - 15$