AQA Higher Mathematics Revision Guide

Full worked solutions

Number

Integers, decimals and symbols

1 **a** $23 \times 8.7 = 200.1$

b $2.3 \times 0.87 = 2.001$

c $\frac{2.001}{0.87} = 2.3$

d $\frac{2001}{23} = 87$

2 **a** $4.86 \times 29 = 140.94$

b $0.486 \times 2.9 = 1.4094$

c $14094 \div 48.6 = 290$

d $140.94 \div 29 = 4.86$

3 Ascending order means going up in size.

$-0.5 \quad 0 \quad 0.012 \quad 0.12 \quad 12$

4 **a** $\frac{5}{0.5} = 10$

b $1\frac{5}{9} > \frac{4}{3}$

c $-3 < -1$

5 **a** $-2 + 7 = 5$

b $-3 - 5 = -8$

c $3 \times -5 = -15$

d $-12 \div -2 = 6$

e $-1 + 7 - 10 = -4$

Addition, subtraction, multiplication and division

1 **a** $1083 + 478 = 1561$

b $2445 + 89 + 513 = 3047$

c $66.55 + 3.38 = 69.93$

d $7.08 + 4.5 + 12.343 = 23.923$

2 **a** $4556 - 1737 = 2819$

b $674 - 387 = 287$

c $12.935 - 4.75 = 8.185$

d $5.77 - 0.369 = 5.401$

3 **a** $634 \times 47 = 29\,798$

b $7.7 \times 3.8 = 29.26$

c $8.32 \times 4.9 = 40.768$

4 **a** $1058 \div 23 = 46$

b $617.4 \div 1.8 = 343$

c $88.5 \div 2.5 = 35.4$

Using fractions

1 **a** $\frac{16}{5} = 3\frac{1}{5}$

b $1\frac{1}{5} = \frac{6}{5}$

c In ascending order, $\frac{5}{8} \quad \frac{3}{4} \quad \frac{9}{10} \quad 1\frac{1}{5} \quad \frac{16}{5}$

d $1\frac{1}{5} + \frac{16}{5} = 1\frac{1}{5} + 3\frac{1}{5} = 4\frac{2}{5}$

e $\frac{16}{5} - \frac{5}{8} = \frac{128}{40} - \frac{25}{40} = \frac{103}{40} = 2\frac{23}{40}$

2 $\frac{15}{45} = \frac{4}{12} = \frac{16}{48} = \frac{1}{3}$

3 **a** $3\frac{1}{3} \times 2\frac{1}{5} = \frac{10}{3} \times \frac{11}{5} = \frac{22}{3} = 7\frac{1}{3}$

b $1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1} = \frac{7}{2} = 3\frac{1}{2}$

4 Fraction of weekly wage spent $= \frac{1}{3} + \frac{1}{5} + \frac{1}{4}$

$= \frac{20 + 12 + 15}{60} = \frac{47}{60}$

Ravi has $1 - \frac{47}{60} = \frac{13}{60}$ of his wage left.

Different types of number

1 **a** 16 is one more than 15, which is a multiple of 5.

b 5 is one less than 6, which is a factor of 12.

c 16 is not a prime number as it has factors 1, 2, 4, 8 and 16.

2 $300 = 30 \times 10 = 15 \times 2 \times 5 \times 2 = 5 \times 3 \times 2 \times 5 \times 2$
$= 2 \times 2 \times 3 \times 5 \times 5$

It is best to write the product of prime factors in ascending order (i.e. smallest number first).

3 Find the multiples of each number.

12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156 ...

16, 32, 48, 64, 80, 96, 112, 128, 144, 160 ...

18, 36, 54, 72, 90, 108, 126, 144, 162 ...

Now look for the lowest number that appears in all three lists: 144.

This means that all three grandchildren will call on the same day every 144 days. (The lowest number to appear in only two lists is 36, so the earliest that only two grandchildren will call on the same day is after 36 days.)

4 **a** $756 = 252 \times 3 = 126 \times 2 \times 3 = 63 \times 2 \times 2 \times 3$
$= 7 \times 9 \times 2 \times 2 \times 3$
$= 7 \times 3 \times 3 \times 2 \times 2 \times 3 = 2^2 \times 3^3 \times 7$

b Comparing the two products and looking for the factors that are common we see that $2^2 \times 3^2 = 36$ Hence 36 is the highest common factor.

Listing strategies

1 There are 2 possible outcomes to tossing a coin.

There are 6 possible outcomes to rolling a six-sided dice.

So $2 \times 6 = 12$ different possible outcomes.

2 As the first number cannot be zero, it could be any number from 1 to 9 (i.e. any of 9 numbers). The second number could be any number from 0 to 9 (i.e. any of 10 numbers). For the whole 3-digit number to be divisible by 5 the last number must be 0 or 5 (i.e. 2 numbers).

So total number of 3-digit numbers that could be picked $= 9 \times 10 \times 2 = 180$.

The order of operations in calculations

1 **a** $10 + 4 \times 2 = 10 + 8 = 18$

b $5 \times 3 - 4 \div 2 = 15 - 2 = 13$

c $(7 - 4)^2 + (8 \div 2)^2 = 3^2 + 4^2 = 9 + 16 = 25$

2 **a** $2 + 3 \div \frac{1}{3} - 1 = 2 + 9 - 1 = 10$

Note that the division must be performed first and that $3 \div \frac{1}{3} = 3 \times \frac{3}{1} = 9$

b $15 - (4 - 6)^3 = 15 - (-2)^3 = 15 - (-8) = 15 + 8 = 23$

Brackets are worked out first.

c $\sqrt{4 - 3 \times (-7)} = \sqrt{4 + 21} = \sqrt{25} = 5$ or -5

Note that the whole calculation inside the square root sign should be completed first.

Indices

1 **a** $7^7 \times 7^3 = 7^{7+3} = 7^{10}$

b $3^{-2} \div 3^4 = 3^{(-2-4)} = 3^{-6}$

c $(5^4)^5 = 5^{4\times5} = 5^{20}$

2 **a** $\frac{5^4 \times 5^6}{5^3} = 5^{4+6-3} = 5^7$

b $6^{(-2)\times3} \div 6^{-3} = 6^{-6} \div 6^{-3} = 6^{-6-(-3)} = 6^{-3}$

c $2^8 \times 2^2 \times 5^4 \times 5^{-7} = 2^{10} \times 5^{-3}$

d $\frac{7^4 \times 7^6 \times 11^3}{11^4} = 7^{4+6} \times 11^{3-4} = 7^{10} \times 11^{-1}$

3 **a** $16^0 = 1$

b $100^{\frac{1}{2}} = \sqrt{100} = 10$ or -10

c $64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$

d $25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$ (or 0.2) or $-\frac{1}{5}$ (or -0.2)

e $27^{\frac{1}{3}} \times 36^{\frac{1}{2}} = \sqrt[3]{27} \times \sqrt{36} = 3 \times 6 = 18$ or 3×-6
$= -18$

4 $(3^{2x})^2 = 81$

$3^{4x} = 3^4$

The powers on each side must be the same, so

$4x = 4$

$x = 1$

Surds

1 **a** $\sqrt{3} \times \sqrt{2} = \sqrt{6}$

b $(\sqrt{5})^2 = 5$

c $2 \times 3 \times \sqrt{3} \times \sqrt{3} = 18$

d $(2\sqrt{5})^2 = 2 \times 2 \times \sqrt{5} \times \sqrt{5} = 20$

2 $\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$, hence $a = 2$

3 **a** $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

b $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$

4 **a** $\frac{16}{3\sqrt{2}} = \frac{16 \times \sqrt{2}}{3 \times \sqrt{2} \times \sqrt{2}} = \frac{16\sqrt{2}}{3 \times 2} = \frac{8\sqrt{2}}{3}$

b $\frac{18}{4 + \sqrt{7}} \times \frac{4 - \sqrt{7}}{4 - \sqrt{7}} = \frac{72 - 18\sqrt{7}}{16 - 7} = 8 - 2\sqrt{7}$

5 **a** $(1 + \sqrt{5})(1 - \sqrt{5}) = 1 - \sqrt{5} + \sqrt{5} - 5 = -4$

b $(2 + \sqrt{3})^2 = (2 + \sqrt{3})(2 + \sqrt{3}) = 4 + 2\sqrt{3} + 2\sqrt{3} + 3$
$= 7 + 4\sqrt{3}$

c $(1 + \sqrt{3})(2 + \sqrt{3}) = 2 + \sqrt{3} + 2\sqrt{3} + 3 = 5 + 3\sqrt{3}$

Standard form

1 **a** $5 \times 10^{-3} = 0.005$

b $5.65 \times 10^5 = 565\,000$

2 **a** $25\,000 = 2.5 \times 10^4$

b $0.00125 = 1.25 \times 10^{-3}$

c $0.05 \times 10^4 = 5 \times 10^2$

d $14 \times 10^{-3} = 1.4 \times 10^{-2}$

3 **a** $(3 \times 10^{-2})^2 = 9 \times 10^{-4}$

b $8 \times 10^4 \times 3 \times 10^{-2} = 24 \times 10^2 = 2.4 \times 10^3$

c $8 \times 10^4 \div 4 \times 10^2 = 2 \times 10^2$

d $8 \times 10^4 + 4 \times 10^2 = 10^2 (8 \times 10^2 + 4) = 10^2 \times 804$
$= 8.04 \times 10^4$

4 **a** 13.3 billion $= 1.33 \times 10^{10}$ pounds

b Number of people $= \frac{1.33 \times 10^{10}}{500} = 1.33 \times 2 \times 10^7$
$= 26\,600\,000$

5 **a** $15\,500 = 1.55 \times 10^4$

b $6.55 \times 10^5 = 655\,000$

c $\frac{1.25 \times 10^5}{2.5 \times 10^2} = \frac{12.5 \times 10^4}{2.5 \times 10^2} = 5 \times 10^2$

d $\sqrt{1.6 \times 10^7} = (1.6 \times 10^7)^{\frac{1}{2}} = 4 \times 10^3$

Converting between fractions and decimals

1 **a** $\frac{43}{100} = 0.43$

b $\frac{3}{8} = 0.375$

c $\frac{11}{20} = 0.55$

2 **a** $0.8 = \frac{8}{10} = \frac{4}{5}$

b $0.45 = \frac{45}{100} = \frac{9}{20}$

c $0.584 = \frac{584}{1000} = \frac{73}{125}$

3 **a** Let $x = 0.\dot{7} = 0.777777\ldots$ and $10x = 7.777777\ldots$

To eliminate the block of recurring digits after the decimal point we take x from $10x$.

$10x - x = 7.777777\ldots - 0.777777\ldots$

$9x = 7$

$x = \frac{7}{9}$

b Let $x = 0.0\dot{4} = 0.0444444...$

So $10x = 0.444444$ and $100x = 4.444444$

Subtract: $100x - 10x = 4.444444... - 0.444444...$

$$90x = 4$$

$$x = \frac{4}{90} = \frac{2}{45}$$

c Let $x = 0.9\dot{5}\dot{4} = 0.954545454...$

So $10x = 9.54545454...$ and $1000x = 954.545454...$

Subtract: $1000x - 10x = 954.545454... - 9.54545454...$

$$990x = 945$$

$$x = \frac{945}{990}$$

Dividing both parts of fraction by 5 and then by 9 gives $= \frac{21}{22}$

4 **a** Let $x = 0.\dot{5}1\dot{8} = 0.518518...$

$$1000x = 518.518518...$$

$$1000x - x = 518.518518... - 0.518518...$$

$$999x = 518$$

$$x = \frac{518}{999} = \frac{14}{27}$$

Dividing both parts of fraction by 37 gives $= \frac{14}{27}$

b $0.76 = \frac{76}{100} = \frac{19}{25}$

5 $0.\dot{7} = \frac{7}{9}$

So $0.\dot{7} + \frac{2}{9} = \frac{7}{9} + \frac{2}{9}$

$= \frac{9}{9}$

$= 1$

Converting between fractions and percentages

1 **a** $25\% = \frac{1}{4}$

b $85\% = \frac{17}{20}$

c $68\% = \frac{17}{25}$

2 Maths: $\frac{65}{80} = \frac{65}{80} \times 100\% = 81.25\%$

$81.25 > 80$

So Charlie did better at maths.

3 **a** $\frac{3}{10} = 30\%$

b $\frac{4}{25} = 16\%$

c $\frac{3}{7} = 42.9\%$ (to 3 s.f.)

Fractions and percentages as operators

1 **a** $\frac{3}{4}$ of £640 $= \frac{3}{4} \times 640 = 3 \times 160 =$ £480

b 15% of £30 $= \frac{15}{100} \times 30 =$ £4.50

c 95% of 80 kg $= \frac{95}{100} \times 80 = 76$ kg

2 Number of boys in School A = 56% of 600

$= \frac{56}{100} \times 600 = 336$

Percentage of boys in School B = 100 − 35 = 65%

Number of boys in School B = 65% of 700

$= \frac{65}{100} \times 700 = 455$

Standard measurement units

1 **a** 9.7 kg = 9.7 × 1000 = 9700 g

b 850 cm^3 = 850 ml = 850 ÷ 1000 = 0.85 litres

c 2.05 km = 2.05 × 1000 m = 2050 m

= 2050 × 100 cm

= 205 000 cm

2 1 day = 24 hours = 24 × 60 minutes

= 24 × 60 × 60 seconds = 86400 seconds

= 8.64×10^4 seconds

3 0.34 × 20 × 12 = £81.60

Rounding numbers

1 **a** 1259 = 1260 (correct to 3 s.f.)

b 14.919 = 14.9 (correct to 3 s.f.)

c 0.0003079 = 0.000308 (correct to 3 s.f.)

> Remember, you don't start counting the numbers for significant figures until the first non-zero number.

d 9 084 097 = 9 080 000 (correct to 3 s.f.)

e $1.8099 \times 10^{-4} = 1.81 \times 10^{-4}$ (correct to 3 s.f.)

2 **a** 10.565 = 10.6 (correct to 1 decimal place)

b 123.9765 = 123.977 (correct to 3 decimal places)

c 0.02557 = 0.03 (correct to 2 decimal places)

d 3.9707 = 3.971 (correct to 3 decimal places)

e 0.00195 = 0.002 (correct to 3 decimal places)

f 4.098 = 4.10 (correct to 2 decimal places)

3 **a** 1989 = 2000 (correct to 1 significant figure)

b 1989 = 2000 (correct to 2 significant figures)

c 1989 = 1990 (correct to 3 significant figures)

4 $3.755 \times 10^{-4} = 0.0003755 = 0.0004$ (correct to 4 decimal places)

Estimation

1 **a** $\frac{5.9 \times 189}{0.54} \approx \frac{6 \times 200}{0.5} \approx \frac{1200}{0.5} \approx 2400$

> Write each number to 1 significant figure.

> There are twice as many halves as units in 1200.

b $\sqrt{4.65 + 28.9 \div 6} \approx \sqrt{5 + 30 \div 6} \approx \sqrt{5 + 5} \approx \sqrt{10} \approx 3$

2

	Question	Estimation	Answer
a	3.45 × 2.78 × 0.09	≈ 3 × 3 × 0.09 ≈ 0.8	A
b	12.56 × 1.87 × 0.45	≈ 10 × 2 × 0.5 ≈ 10	C
c	120 ÷ 0.45	≈ 100 ÷ 0.5 ≈ 200	B
d	0.01 × 0.15 × 109	≈ 0.01 × 0.2 × 100 ≈ 0.2	B
e	0.12 × 300 × 0.53	≈ 0.1 × 300 × 0.5 ≈ 15	A
f	6.07 × 3.67 × 0.1	≈ 6 × 4 × 0.1 ≈ 2	B
g	20.75 ÷ 6.98	≈ 20 ÷ 7 ≈ 3	C
h	0.01 × 145 × 35	≈ 0.01 × 100 × 40 ≈ 40	A
i	6.5 × 0.3 × 0.01	≈ 7 × 0.3 × 0.01 ≈ 0.02	B
j	65 ÷ 1050	≈ 70 ÷ 1000 ≈ 0.07	A

3 **a** $\sqrt{36} < \sqrt{45} < \sqrt{49}$ so $\sqrt{45}$ is between 6 and 7.

$\sqrt{45} \approx 6.7$ (accept 6.5 to 6.9)

b $\sqrt{100} < \sqrt{104} < \sqrt{121}$ so $\sqrt{104}$ is between 10 and 11.

$\sqrt{104} \approx 10.2$ (accept 10.1 to 10.3)

c $(2)^3 < (2.3)^3 < (3)^3$ so $(2.3)^3$ is between 8 and 27.

$(2.3)^3 \approx 12$ (accept 10 to 14)

d $3^1 < 3^{1.4} < 3^2$ so $3^{1.4}$ is between 3 and 9.

$3^{1.4} \approx 5$ (accept 4 to 6)

Upper and lower bounds

1 **a** Half of the smallest unit is 0.5 cm

Lower bound = least length = 144.5 cm

b Upper bound = greatest length = 145.5 cm

c $144.5 \leq l < 145.5$ cm

Review it!

1 **a** 24 647.515

b 21.5443469 = 21.5 (1 d.p.)

2 **a** $8 - 1.5 = 6.5$

b $-1 + 4 + 1 = 4$

c $27 \times 3 = 81$

d $1.65 \times 3.6 = 4 \times 1.65 \times 9 \div 10 = 6.6 \times 9 \div 10 = (66 - 6.6) \div 10 = 59.4 \div 10 = 5.94$

3 **a** $\frac{64}{12} = \frac{16}{3} = 5\frac{1}{3}$

b $\frac{124}{13} = 9\frac{7}{13}$

4 $4\frac{3}{4} + \frac{1}{2}$

Use the largest integer as the whole number and the next largest as the numerator.

5 **a** Half the smallest unit is 0.005 cm.

Least width = 4.545 cm and least length = 2.225 cm

Least area = $4.545 \times 2.225 = 10.112625$ cm² = 10.113 cm² (3 d.p.)

Greatest width = 4.555 cm and greatest length = 2.235 cm

Greatest area = $4.555 \times 2.235 = 10.180425$ cm² = 10.180 cm² (3 d.p.)

b The upper and lower bounds for area are the same when written to the nearest cm² (5 and 2).

Area = $5 \times 2 = 10$ cm²

6 **a** $6.02 \times 10^{23} \div 18 \times 10 = 3.34 \times 10^{23}$ molecules (3 s.f.)

b 18 g = 0.018 kg

$0.018 \div 6.02 \times 10^{23} = 2.99 \times 10^{-26}$ kg (3 s.f.)

7 $(0.45 \times 0.78)^2 \approx (0.5 \times 0.8)^2 \approx 0.4^2 \approx 0.16$

8 **a** $7^0 = 1$

b $9^{\frac{1}{2}} = \sqrt{9} = 3$ or -3

c $8^{-2} = \frac{1}{8^2} = \frac{1}{64}$

d $64^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$ or $-\frac{1}{4}$

9 Half the smallest unit = $0.1 \div 2 = 0.05$

Upper bound = 5.6 + 0.05 = 5.65

Lower bound = 5.6 − 0.05 = 5.55

Error interval is $5.55 \leq y < 5.65$

10 $\frac{1-\sqrt{2}}{1+\sqrt{2}} = \frac{1-\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-2\sqrt{2}+2}{1-2} = \frac{3-2\sqrt{2}}{-1} = 2\sqrt{2} - 3$

11 Upper bound = 112.5 cm and lower bound = 111.5 cm

Error interval is $111.5 \leq a < 112.5$

12 Let $x = 0.\dot{7}\dot{2} = 0.727272...$

So $100x = 72.727272...$

$100x - x = 72.727272... - 0.727272...$

$99x = 72$

$x = \frac{72}{99} = \frac{8}{11}$

13 **a** a: upper bound = 0.67545, lower bound = 0.67535

b: upper bound = 2.345, lower bound = 2.335

Greatest value of $c = \frac{\text{Upper bound of } a}{\text{Lower bound of } b} = \frac{0.67545}{2.335}$

= 0.289272

Least value of $c = \frac{\text{Lower bound of } a}{\text{Upper bound of } b} = \frac{0.67535}{2.345}$

= 0.287996

Error interval is $0.287996 \leq c < 0.289272$

b The value of c is the same when rounded to 2 significant figures.

So $c = 0.29$ (to 2 s.f.)

14 **a** $\frac{3}{25} \div \frac{9}{50} = \frac{3}{25} \times \frac{50}{9} = \frac{1}{1} \times \frac{2}{3} = \frac{2}{3}$

b $25 \div \frac{5}{16} = \frac{25}{1} \times \frac{16}{5} = \frac{5}{1} \times \frac{16}{1} = 80$

15 **a** $0.00000045 = 4.5 \times 10^{-7}$

b 12 million = 12 000 000 = 1.2×10^7

c $5640 = 5.64 \times 10^3$

16 $(8 \times 10^{-5}) \times (4 \times 10^3) = 8 \times 4 \times 10^{-5} \times 10^3 = 32 \times 10^{-2} = 3.2 \times 10^{-1}$

17 **a** Factors of 64: 1, 2, 4, 8, 16, 32, 64

b Factors of 100: 1, 2, 4, 5, 10, 20, 25, 50, 100

Highest common factor = 4

Algebra

Simple algebraic techniques

1 **a** Formula

b Identity

c Expression

d Identity

e Equation

2 **a** $15x^2 - 4x + x^2 + 9x - x - 6x^2 = 10x^2 + 4x$

b $7a + 5b - b - 4a - 5b = 3a - b$

c $8yx + 5x^2 + 2xy - 8x^2 = -3x^2 + 10xy$ (or $10xy - 3x^2$)

d $x^3 + 3x - 5 + 2x^3 - 4x = 3x^3 - x - 5$

3 $P = I^2R = \left(\frac{2}{3}\right)^2 \times 36 = \frac{4}{9} \times 36 = 16$

4 $v = u + at = 20 + (-8)(2) = 20 - 16 = 4$

Removing brackets

1 **a** $2x + 8$

b $63x + 21$

c $-1 + x$ or $x - 1$

d $3x^2 - x$

e $3x^2 + 3x$

f $20x^2 - 8x$

2 **a** $2(x + 3) + 3(x + 2) = 2x + 6 + 3x + 6 = 5x + 12$

b $6(x + 4) - 3(x - 7) = 6x + 24 - 3x + 21 = 3x + 45$

c $3x^2 + x + x^2 + x = 4x^2 + 2x$

d $3x^2 - 4x - 6x + 8 = 3x^2 - 10x + 8$

3 **a** $(t + 3)(t + 5) = t^2 + 5t + 3t + 15 = t^2 + 8t + 15$

b $(x - 3)(x + 3) = x^2 + 3x - 3x - 9 = x^2 - 9$

c $(2y + 9)(3y + 7) = 6y^2 + 14y + 27y + 63$
$= 6y^2 + 41y + 63$

d $(2x - 1)^2 = (2x - 1)(2x - 1) = 4x^2 - 2x - 2x + 1$
$= 4x^2 - 4x + 1$

4 **a** $(x + 7)(x + 2)(2x + 3) = (x^2 + 9x + 14)(2x + 3)$
$= 2x^3 + 21x^2 + 55x + 42$

b $(2x - 1)(3x - 2)(4x - 3) = (6x^2 - 7x + 2)(4x - 3)$
$= 24x^3 - 28x^2 + 8x - 18x^2 + 21x - 6$
$= 24x^3 - 46x^2 + 29x - 6$

Factorising

1 **a** $24t + 18 = 6(4t + 3)$

b $9a - 2ab = a(9 - 2b)$

c $5xy + 15yz = 5y(x + 3z)$

d $24x^3y^2 + 6xy^2 = 6xy^2(4x^2 + 1)$

2 **a** $(x + 7)(x + 3)$

b $(x + 5)(x - 3)$

c To get 6, use factors 2 and 3, and to get 10 use factors 2 and 5. This gives $2x \times 2 = 4x$ and $3x \times 5 = 15x$, total $19x$; so solution is $(2x + 5)(3x + 2)$

d Difference of two squares. Factorises to $(2x + 7)(2x - 7)$

3 $\frac{1}{x - 7} - \frac{x + 10}{2x^2 - 11x - 21} = \frac{1}{x - 7} - \frac{x + 10}{(2x + 3)(x - 7)}$

Factorise the denominator of the 2nd fraction.

Make both denominators the same.

$= \frac{2x + 3}{(x - 7)(2x + 3)} - \frac{x + 10}{(2x + 3)(x - 7)}$

Combine into one fraction and simplify

$= \frac{2x + 3 - x - 10}{(x - 7)(2x + 3)}$

$= \frac{x - 7}{(x - 7)(2x + 3)}$

$= \frac{1}{2x + 3}$

Changing the subject of a formula

1 **a** $A = \pi r^2$ $\quad \frac{A}{\pi} = r^2$

$r = \sqrt{\frac{A}{\pi}}$

b $A = 4\pi r^2$

$\frac{A}{4\pi} = r^2$

$r = \sqrt{\frac{A}{4\pi}}$

c $V = \frac{4}{3}\pi r^3$

$3V = 4\pi r^3$

$\frac{3V}{4\pi} = r^3$

$r = \sqrt[3]{\frac{3V}{4\pi}}$

2 **a** $y = mx + c \quad (c)$ Subtract mx from both sides.

$c = y - mx$

b $v = u + at \quad (u)$ Subtract at from both sides.

$u = v - at$

c $v = u + at \quad (a)$ Subtract u from both sides.

$v - u = at$ Divide both sides by t.

$a = \frac{v - u}{t}$

d $v^2 = 2as \quad (s)$ Divide both sides by $2a$.

$s = \frac{v^2}{2a}$

e $v^2 = u^2 + 2as \quad (u)$ Subtract $2as$ from both sides.

$v^2 - 2as = u^2$ Square root both sides.

$u = \sqrt{v^2 - 2as}$

f $s = \frac{1}{2}(u + v)t \quad (t)$ Multiply both sides by 2.

$2s = (u + v)t$ Divide both sides by $(u + v)$.

$t = \frac{2s}{u + v}$

Solving linear equations

1 **a** $x - 7 = -4$

$x = -4 + 7 = 3$

b $9x = 27$

$x = 27 \div 9 = 3$

c $\frac{x}{5} = 4$

$x = 4 \times 5 = 20$

2 **a** $3x + 1 = 16$

$3x = 15$

$x = 5$

b $\frac{2x}{3} = 12$

$2x = 36$

$x = 18$

c $\frac{3x}{5} + 4 = 16$

$\frac{3x}{5} = 12$

$3x = 60$

$x = 20$

3 **a** $5(1 - x) = 15$

$5 - 5x = 15$

$-5x = 10$

$x = -2$

b $2m - 4 = m - 3$

$m - 4 = -3$

$m = 1$

c $9(4x - 3) = 3(2x + 3)$

$36x - 27 = 6x + 9$

$30x - 27 = 9$

$30x = 36$

$x = \frac{36}{30} = \frac{6}{5}, 1\frac{1}{5}$ or 1.2

Always cancel fractions so that they are in their lowest terms. Here both top and bottom can be divided by 6.

Solving quadratic equations using factorisation

1 **a** $(x + 3)(x + 2) = 0$ giving $x = -2$ or -3

b $(x + 3)(x - 4) = 0$ giving $x = -3$ or 4

c $(2x + 7)(x + 5) = 0$ giving $x = -\frac{7}{2}$ or $x = -5$

2 **a** Area of triangle $= \frac{1}{2} \times$ base $\times$ height

$\frac{1}{2}(2x + 3)(x + 4) = 9$

$2x^2 + 11x + 12 = 18$

$2x^2 + 11x - 6 = 0$

b $2x^2 + 11x - 6 = 0$

$(2x - 1)(x + 6) = 0$

So $x = \frac{1}{2}$ or $x = -6$

Since x represents a height, only the positive value is valid.

$x = \frac{1}{2}$

c $x = 0.5$, so base is $2 \times 0.5 + 3 = 4$ cm and height is $0.5 + 4 = 4.5$ cm

3 By Pythagoras' theorem

$(x + 1)^2 + (x + 8)^2 = 13^2$

$x^2 + 2x + 1 + x^2 + 16x + 64 = 169$

$2x^2 + 18x - 104 = 0$

Dividing by 2 gives

$x^2 + 9x - 52 = 0$

$(x - 4)(x + 13) = 0$

so $x = 4$ or $x = -13$

(Disregard $x = -13$ as x is a length.)

Hence, $x = 4$ cm

(This also means the sides of the triangle are 5, 12 and 13 cm.)

Solving quadratic equations using the formula

1 Comparing the equation given, with $ax^2 + bx + c$ gives $a = 2, b = -1$ and $c = -7$.

Substituting these values into the quadratic equation formula gives:

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-7)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{57}}{4}$$

$$= \frac{1 + 7.550}{4} \text{ or } \frac{1 - 7.550}{4} \text{ (to 4 s.f.)}$$

$x = 2.14$ or -1.64 (to 3 s.f.)

2 **a** $\frac{2x + 3}{x + 2} = 3x + 1$

$2x + 3 = (x + 2)(3x + 1)$

$2x + 3 = 3x^2 + x + 6x + 2$

$0 = 3x^2 + 5x - 1$

or $3x^2 + 5x - 1 = 0$

b Comparing the equation given, with $ax^2 + bx + c = 0$ gives $a = 3$, $b = 5$ and $c = -1$

Substituting these values into the quadratic equation formula gives:

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-5 \pm \sqrt{25 + 12}}{6} = \frac{-5 \pm \sqrt{37}}{6} = \frac{-5 + \sqrt{37}}{6} \text{ or } \frac{-5 - \sqrt{37}}{6}$$

Hence $x = 0.18$ or -1.85 (2 d.p.)

Solving simultaneous equations

1 **a** Firstly write the second equation so that in both equations the x value and the numerical value are aligned.

$y = 3x - 7$ (1)

$y = -2x + 3$ (2)

Notice that the coefficient of y (the number multiplying y, i.e. 1) is the same for both equations. We can eliminate y by subtracting equation (2) from equation (1).

Subtracting (1) − (2) we obtain

$0 = 5x - 10$

$5x = 10$

$x = 2$

Substituting $x = 2$ into equation (1) we obtain

$y = 3(2) - 7$

$= 6 - 7$

$= -1$

Checking by substituting $x = 2$ into equation (2) we obtain

$y = -2x + 3$

$= -2(2) + 3$

$= -1$

Hence solutions are $x = 2$ and $y = -1$.

b $y = 2x - 6$ (1)

$y = -3x + 14$ (2)

Subtracting (1) − (2) we obtain

$0 = 5x - 20$

$x = 4$

$y = 2 \times 4 - 6$ (1)

$y = 2.$

2 Equating expressions for y gives

$10x^2 - 5x - 2 = 2x - 3$

$10x^2 - 7x + 1 = 0$

Factorising this quadratic gives

$(5x - 1)(2x - 1) = 0$

Hence $x = \frac{1}{5}$ or $x = \frac{1}{2}$

Substituting $x = \frac{1}{5}$ into $y = 2x - 3$ gives

$y = -2\frac{3}{5}$

Substituting $x = \frac{1}{2}$ into $y = 2x - 3$ gives

$y = -2$

Hence $x = \frac{1}{5}$ and $y = -2\frac{3}{5}$ or $x = \frac{1}{2}$ and $y = -2$

3 Equating the y values gives

$x^2 + 5x - 4 = 6x + 2$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or -2

When $x = 3$, $y = 6 \times 3 + 2 = 20$

When $x = -2$, $y = 6 \times (-2) + 2 = -10$

Points are (3, 20) and (−2, −10)

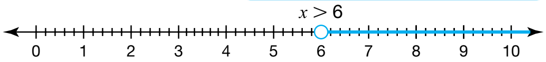
Solving inequalities

1 **a** $1 - 2x < -11$

$-2x < -12$

$x > 6$

Subtract 1 from both sides. Divide both sides by –2 and reverse sign.



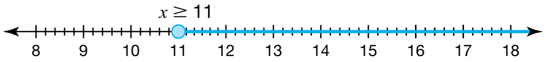
$\{x: x > 6\}$

b $2x - 7 \geq 15$

$2x \geq 22$

$x \geq 11$

Add 7 to both sides. Divide both sides by 2.



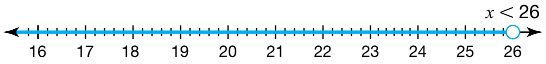
$\{x: x \geq 11\}$

c $\frac{x - 5}{3} < 7$

$x - 5 < 21$

$x < 26$

Multiply both sides by 3. Add 5 to both sides.



$\{x: x < 26\}$

2 **a** $2x - 4 > x + 6$

$x - 4 > 6$

$x > 10$

b $4 + x < 6 - 4x$

$4 < 6 - 5x$

$-2 < -5x$

$\frac{-2}{-5} > x$

$x < 0.4$ or $\frac{2}{5}$

c $2x + 9 \geq 5(x - 3)$

$2x + 9 \geq 5x - 15$

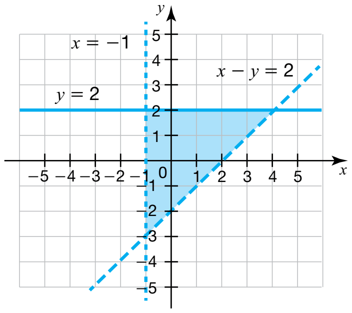
$9 \geq 3x - 15$

$24 \geq 3x$

$8 \geq x$

or $x \leq 8$

3 **a**



b (0, 2), (0,1), (0, 0), (0, −1), (1, 2), (1, 1), (1, 0), (2, 2), (2, 1), (3, 2)

4 $x^2 > 3x + 10$

$x^2 - 3x - 10 > 0$

$(x - 5)(x + 2) > 0$

$x < -2$ and $x > 5$

Problem solving using algebra

1 Let the larger number $= x$ and the smaller number $= y$.

$x + y = 77$

$x - y = 25$

Adding gives $2x = 102$ which gives $x = 51$ so y must be 26.

2 Let the number added $= x$

$\frac{15 + x}{31 + x} = \frac{5}{6}$

$6(15 + x) = 5(31 + x)$

$90 + 6x = 155 + 5x$

$x = 65$

Check the answer $\frac{15 + 65}{31 + 65} = \frac{80}{96} = \frac{5}{6}$

3 Perimeter: $2x + 2y = 24$ so $x + y = 12$ (1)

Area: $xy = 27$ (2)

From equation (1) $y = 12 - x$

Substitute into equation (2):

$x(12 - x) = 27$

So, $12x - x^2 = 27$

Hence, $x^2 - 12x + 27 = 0$

Factorising gives $(x - 3)(x - 9) = 0$

So $x = 3$ or $x = 9$

Substituting each of these values into equation (1) we have

$3 + y = 12$ or $9 + y = 12$, giving $y = 9$ or $y = 3$.

Hence, length = 9 cm and width = 3 cm.

Use of functions

1 **a** $f(0) = \frac{1}{0 - 1} = -1$

b $f(-\frac{1}{2}) = \frac{1}{-\frac{1}{2} - 1} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$

c Let $y = \frac{1}{x - 1}$

$y(x - 1) = 1$

$xy - y = 1$

$xy = y + 1$

$x = \frac{y + 1}{y}$

$f^{-1}(x) = \frac{x + 1}{x}$

2 a $\text{fg}(x) = \sqrt{((x+4)^2 - 9)}$

$= \sqrt{x^2 + 8x + 16 - 9}$

$= \sqrt{x^2 + 8x + 7}$

b $\text{gf}(x) = \sqrt{(x^2 - 9)} + 4$

c $\text{gf}(3) = \sqrt{(3^2 - 9)} + 4$

$= 4$

Iterative methods

1 $x_0 = 1.5$

$x_1 = 1.5182945$

$x_2 = 1.5209353$

$x_3 = 1.5213157$

$x_4 = 1.5213705 \approx 1.521$ (correct to three decimal places)

Check value of $x^3 - x - 2$ for $x = 1.5205, 1.5215$

x	$f(x)$
1.5205	−0.005225
1.5215	0.0007151

Since there is a change of sign, $a = 1.521$ is correct to three decimal places.

Equation of a straight line

1 a $2y = 4x - 5$

$y = 2x - \frac{5}{2}$

Comparing this to $y = mx + c$ we have gradient, $m = 2$

b Gradient $= -\frac{1}{m} = -\frac{1}{2}$

c $y = -\frac{1}{2}x + 5$ or $2y = -x + 10$

2 $y - y_1 = m(x - x_1)$ where $m = 3$ and $(x_1, y_1) = (2, 3)$.

$y - 3 = 3(x - 2)$

$y - 3 = 3x - 6$

$y = 3x - 3$

3 $y - y_1 = m(x - x_1)$ where $m = 2$ and $(x_1, y_1) = (-1, 0)$

$y - 0 = 2(x - (-1))$

$y = 2(x + 1)$

$y = 2x + 2$

$-y + 2x + 2 = 0$ (or $2x - y + 2 = 0$)

4 a Gradient $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{6 - (-2)} = \frac{4}{8} = \frac{1}{2}$

b $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 6}{2}, \frac{0 + 4}{2}\right) = (2, 2)$

c **i** Gradient $= -2$ (i.e invert $\frac{1}{2}$ and change the sign)

ii $y - y_1 = m(x - x_1)$

$y - 2 = -2(x - 2)$

$y - 2 = -2x + 4$

$y = -2x + 6$

Quadratic graphs

1 a $2x^2 - 12x + 1 = 2\left[x^2 - 6x + \frac{1}{2}\right]$

$= 2\left[(x - 3)^2 - 9 + \frac{1}{2}\right]$

$= 2\left[(x - 3)^2 - \frac{17}{2}\right]$

$= 2(x - 3)^2 - 17$

b **i** Turning point is at $(3, -17)$

ii At the roots,

$2(x - 3)^2 - 17 = 0$

$2(x - 3)^2 = 17$

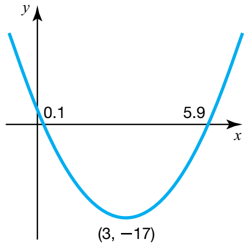
$(x - 3)^2 = \frac{17}{2}$

$x - 3 = \sqrt{\frac{17}{2}}$

$x = \sqrt{\frac{17}{2}} + 3$

Roots are $x = 0.1$ and $x = 5.9$ (1 d.p.)

c



2 a $y = (x + 1)(x - 5)$ or $y = x^2 - 4x - 5$

b $y = -(x - 2)(x - 7)$ or $y = -x^2 + 9x - 14$

3 a $x^2 + 12x - 16 = (x + 6)^2 - 36 - 16$

$= (x + 6)^2 - 52$

b Turning point is at $(-6, -52)$

Recognising and sketching graphs of functions

1 a B

b F

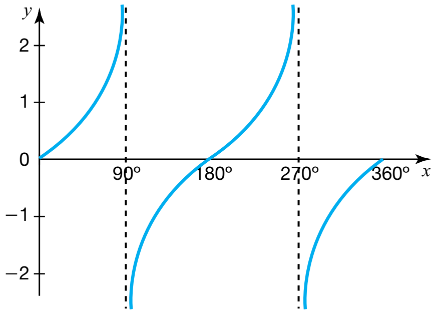
c E

d A

e D

f C

2 a



b Read up from 60° to the graph, then read across until you hit the graph again.

$x = 240°$

3 a A

b G

c F

d E

Translations and reflections of functions

1 a (3, 5) (i.e. a movement of one unit to the right)

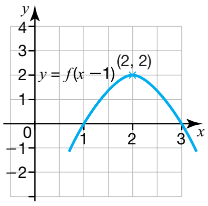
b (−1, 5) (i.e. a movement of three units to the left)

c (2, −5) (i.e. a reflection in the x-axis)

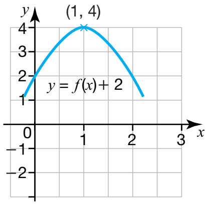
d (−2, 5) (i.e. a reflection in the y-axis)

2

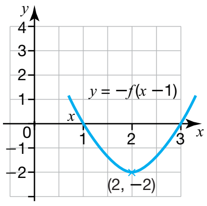
a



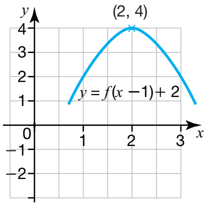
b



c



d



Equation of a circle and tangent to a circle

1 **a** Centre is (0, 0)

b radius = $\sqrt{49} = 7$

2 **a** $x^2 + y^2 = 100$

b Gradient of radius to (8, 6) = $\frac{6}{8} = \frac{3}{4}$

Gradient of tangent = $-\frac{4}{3}$

c $y - y_1 = m(x - x_1)$

$y - 6 = -\frac{4}{3}(x - 8)$

$y = -\frac{4}{3}x + 16\frac{2}{3}$ or $3y = -4x + 50$

Real-life graphs

1 **a** 08:00 to 09:00 is 1 hour (h), which is 1 unit on x axis.

Average speed = gradient = $\frac{2.5}{0.5}$ = 5 km/h

b 15 mins = 0.25 hours

c Average speed = gradient between 09:30 and 09:45

= $\frac{6}{0.25}$ = 24 km/h

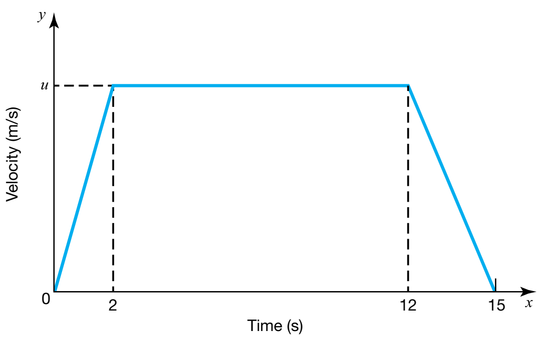
2

NAILIT!

When drawing a velocity-time graph, ensure that the axes are labelled with quantities and units.

Any values and letters for quantities that need to be found should be labelled on the graph.

a



b Total distance travelled = Area under the velocity-time graph

Use the formula for the area of a trapezium:

Distance = $\frac{1}{2}(15 + 10) \times u$

$= 12.5u$

Use the formula for the area of a trapezium.

The total distance travelled = 50 m

Hence $50 = 12.5u$

$u = 4$ m/s

c Velocity = 4 m/s and time for deceleration = 3 s

Deceleration = gradient = $\frac{4}{3}$ = 1.33 m/s^2

Since deceleration is negative acceleration, a positive answer is appropriate.

Generating sequences

1 **a** 17: sequence goes up by 3

b 3.0: sequence goes up by 0.2

c −12: sequence goes down by 3

d 432: last term is multiplied by 6

e $\frac{1}{48}$: last term is multiplied by $\frac{1}{2}$

f $-\frac{1}{16}$: last term is multiplied by $-\frac{1}{2}$

2 Second term is $(-4)^2 + 1 = 17$ and third term is $17^2 + 1 = 290$

Second term is 17, third term is 290.

3 Reverse the process: to find the preceding term, subtract 1 and halve.

Second term is $(12 - 1) \div 2 = \frac{11}{2} = 5.5$

First term is $(5.5 - 1) \div 2 = \frac{4.5}{2} = 2.25$

First term is 2.25, second term is 5.5

The nth term

1 **a** When $n = 1$, $50 - 3(1) = 47$

When $n = 2$, $50 - 3(2) = 44$

When $n = 3$, $50 - 3(3) = 41$

First three terms are 47, 44, 41

b Use the nth term formula to find the value of n when the nth term = 34

$50 - 3n = 34$

$3n = 16$

$n = 16 \div 3$

The value of n is not an integer so 34 is not a number in the sequence.

c Use the nth term formula to find the value of n when the nth term is less than zero (i.e. negative).

$50 - 3n < 0$ (subtracting 50 from both sides)

$-3n < -50$ (dividing both sides and reversing the inequality sign)

$n > \frac{50}{3}$

$n > 16\frac{2}{3}$

As n has to be an integer, its lowest possible value is $n = 17$.

Check that you get a negative term when $n = 17$ is put back into the nth term formula.

17th term = $50 - 3 \times 17 = 50 - 51 = -1$

2 **a** The first four terms are: $2 \times 3^1, 2 \times 3^2, 2 \times 3^3, 2 \times 3^4$

$= 6, 18, 54, 162$

b As the nth term formula is 2×3^n both 2 and 3 are factors, so 6 must also be a factor.

3 a Common difference between terms = 2 so formula will start with $2n$.

When $n = 1$, you need to subtract 3 from $2n$ to get an answer of -1.

Therefore nth term $= 2n - 3$

b $59 = 2x - 3$

$2x = 62$

$x = 31$

4 4, 17, 38, 67

First differences 13 21 29

Second differences 8 8

As a second difference is needed before a constant difference is found, there is an n^2 term in the nth term. The number in front of this n^2 will be $\frac{8}{2} = 4$.

So first part of the nth term will be $4n^2$.

n	1	2	3	4
Term	4	17	38	67
$4n^2$	4	16	36	64
Term $- 4n^2$	0	1	2	3

Use this set of information to work out the linear part of the sequence (the part with an n term and a number).

Difference 1 1 1

This means that the linear sequence will start with n.

When $n = 2$, 'Term $- 4n^2$' is 1, not 2, so if n is in the term you also need to subtract 1.

This makes the linear part of the sequence $n - 1$.

Check it with a different value of n. When $n = 3$, $n - 1$ equals 2. This is the correct value for 'Term $- 4n^2$'.

Combining the terms gives nth term $= 4n^2 + n - 1$

Arguments and proofs

1 a $2n$ is always even as it has 2 as a factor. Adding 1 to an even number always gives an odd number. The statement is true.

b $x^2 - 9 = 0$ so $x^2 = 9$ and $x = \sqrt{9} = \pm 3$

The statement is false.

c n could be a decimal such as 4.25 so squaring it would not give an integer.

The statement is false.

d If n was 1, or a fraction smaller than 1, this would not be true.

The statement is false.

2 Let the consecutive integers be n, $n + 1$, $n + 2$ and $n + 3$, where n is an integer that can be either odd or even.

Sum of the integers $= n + n + 1 + n + 2 + n + 3$

$= 4n + 6 = 2(2n + 3)$

As 2 is a factor of this expression, the sum of four consecutive integers must be a multiple of 2, and therefore even.

3 Let the consecutive integers be x, $x + 1$ and $x + 2$, where x is an integer that can be either odd or even.

Sum of the integers $= x + x + 1 + x + 2 = 3x + 3$

$= 3(x + 1)$

As 3 is a factor of this expression, the sum of three consecutive integers must be a multiple of 3.

4 a The numerator is larger than the denominator so the fraction will always be greater than 1. The statement is false.

b As a is larger than b, squaring a will result in a larger number than squaring b. Hence $a^2 > b^2$ so the statement is false.

c The square root of a number can have two values, one positive and the other negative so, this statement is false.

Review it!

1 a $-3(3x - 4) = -9x + 12$

b $4x + 3(x + 2) - (x + 2) = 4x + 3x + 6 - x - 2$

$= 6x + 4$

c $(x + 3)(2x - 1)(3x + 5) = (2x^2 + 5x - 3)(3x + 5)$

$= 6x^3 + 25x^2 + 16x - 15$

2 a $2x^2 + 7x - 4 = (2x - 1)(x + 4)$

b $2x^2 + 7x - 4 = 0$

$x = \frac{1}{2}$ or $x = -4$

3 a $(2x^2y)^3 = 8x^6y^3$

b $2x^{-3} \times 3x^4 = 6x$

c $\frac{15a^3b}{3a^3b^2} = \frac{5}{b}$

4 $3x + 2y = 8$ (1)

$5x + y = 11$ (2)

(2) × 2: $10x + 2y = 22$ (3)

(3) − (1): $7x = 14$

$x = 2$

Substitute into (2) to find y

$5 \times 2 + y = 11$

$y = 11 - 10$

$y = 1$

5 a $\frac{3}{x + 7} = \frac{2 - x}{x + 1}$

$3(x + 1) = (2 - x)(x + 7)$

$3x + 3 = 2x + 14 - x^2 - 7x$

$3x + 3 = -x^2 - 5x + 14$

$x^2 + 8x - 11 = 0$

b $x^2 + 8x - 11 = 0$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4 \times 1 \times (-11)}}{2 \times 1}$$

$$x = \frac{-8 \pm \sqrt{108}}{2}$$

$$x = \frac{-8 \pm 6\sqrt{3}}{2}$$

$x = -4 + 3\sqrt{3}$ or $x = -4 - 3\sqrt{3}$

So $x = 1.20$ or $x = -9.20$ (to 2 d.p.)

6 $\frac{3y - x}{z} = ax + 2 \quad (x)$

$3y - x = z(ax + 2)$

$3y - x = axz + 2z$

$3y - 2z = axz + x$

$3y - 2z = x(az + 1)$

$$x = \frac{3y - 2z}{az + 1}$$

7 a $y = \frac{x}{3} + 5$

$3y = x + 15$

$x = 3y - 15$

Now replace x with $f^{-1}(x)$ and y with x.

$f^{-1}(x) = 3x - 15$ or $f^{-1}(x) = 3(x - 5)$

b $fg(x) = \frac{(2(x)^2 + k)}{3} + 5$

So $fg(2) = \frac{(8 + k)}{3} + 5$

We know that $fg(2) = 10$

So $\frac{(8 + k)}{3} + 5 = 10$

$(8 + k) + 15 = 30$

$8 + k = 15$

$k = 7$

8 a Let $n = 1$: $30 - 4 \times 1 = 26$

Let $n = 2$: $30 - 4 \times 2 = 22$

Let $n = 3$: $30 - 4 \times 3 = 18$

First three terms are 26, 22, 18.

b $30 - 4n < 0$

$-4n < -30$

$n > \frac{-30}{-4}$

$n > 7.5$

n must be an integer, so the lowest possible value of n is $n = 8$

Therefore the first negative term of the sequence is: $30 - 4 \times 8 = -2$

9 $x = 4, y = 3$

$(4)^2 + (3)^2 = 16 + 9 = 25$

So $x^2 + y^2 > 21$

Hence the point (4, 3) lies outside the circle.

10 $(\sqrt{x} + \sqrt{9y})(\sqrt{x} - 3\sqrt{y})$

Simplify terms inside the brackets if possible

$(\sqrt{x} + 3\sqrt{y})(\sqrt{x} - 3\sqrt{y}) = x + 3\sqrt{xy} - 3\sqrt{xy} - 9y$

$= x - 9y$

11 a $2x^2 + 8x + 1 = 2(x^2 + 4x) + 1$

$= 2(x + 2)^2 - 8 + 1$

$= 2(x + 2)^2 - 7$

b **i** Turning point is $(-2, -7)$.

ii For $2x^2 + 8x + 1 = 0$

$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 2 \times 1}}{2 \times 2}$

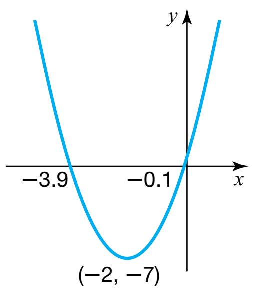
$x = \frac{-8 \pm \sqrt{56}}{4}$

$x = \frac{-8 \pm 2\sqrt{14}}{4}$

$x = \frac{-4 \pm \sqrt{14}}{2}$

So roots are at $x = -3.9$ and $x = -0.1$ (1 d.p.)

c



12 Perimeter of $ABCD = 2 \times (4x + (2x - 3)) = 12x - 6$

Perimeter of $EFG = 2x - 1 + x + 9 + 5x - 2 = 8x + 6$

Equate the perimeters to find x

$12x - 6 = 8x + 6$

$4x = 12$

$x = 3$

The height of the triangle, $EF = 2 \times 3 - 1 = 5$ cm

The base of the triangle, $EG = 3 + 9 = 12$ cm

Area of the triangle $= \frac{1}{2} \times 12 \times 5 = 30\,\text{cm}^2$

13 Side AB is parallel to side CD, so $k = 5$.

Gradient of $BD = \frac{5 - (-2)}{-1 - (-2)} = \frac{7}{1} = 7$

Using point B $(-2, -2)$

$y - (-2) = 7(x - (-2))$

$y + 2 = 7x + 14$

Equation of BD is $y = 7x + 12$

14 $x^2 + y^2 = 4$ (1)

$2y - x = 2$ (2)

Rearrange (2) for y: $y = \frac{1}{2}x + 1$ (3)

Substitute (3) into (1):

$x^2 + \left(\frac{1}{2}x + 1\right)^2 = 4$

$x^2 + \frac{x^2}{4} + x + 1 = 4$

$5x^2 + 4x + 4 = 16$

$5x^2 + 4x - 12 = 0$

$(5x - 6)(x + 2) = 0$

$x = \frac{6}{5} = 1.2$ or $x = -2$

Substitute into (2) to find y

So $x = \frac{6}{5}, y = \frac{8}{5}$ or $x = -2, y = 0$

Ratio, proportion and rates of change

Introduction to ratios

1 **a** 2 : 6 = 1: 3 (divide both sides by 2)

b 25 : 60 = 5 : 12 (divide both sides by 5)

c 1.6 : 3.6 = 4 : 9 (divide both sides by 0.4, or multiply by ten then divide by four)

2 **a** 250 g : 2 kg = 250 g : 2000 g = 250 : 2000 = 1 : 8

b 25 m : 250 mm = 25 000 mm : 250 mm
= 25 000 : 250 = 100 : 1

c 2 cl : 1 l = 2 cl : 100 cl = 2 : 100 = 1 : 50

3 Ratio = 3.5 : 2.1 = 35 : 21 = 5 : 3

Total shares = 5 + 3 = 8

1 share = £$\frac{400}{8}$ = £50

5 shares = 5 × £50 = £250

3 shares = 3 × £50 = £150

4 4 parts = 180

1 part = 45   (Dividing both sides by 4)

3 parts = 3 × 45
= 135

Hence there are 180 + 135 = 315 members of the gym.

5 The ratio is 21 : 25 : 29

Total shares = 21 + 25 + 29 = 75

One share = £$\frac{150\,000}{75}$ = £2000

Youngest daughter receives 21 × 2000 = £42 000

6 Total number of parts in the ratio = 5 + 2 = 7

Now pick a number that is divisible by 7. We will choose 70.

Dividing this into the ratio 5 : 2 gives 50 male guests and 20 female guests.

60% of male guests are under 40 and 70% of female guests are under 40.

60% of 50 = 30 (males under 40)

70% of 20 = 14 (females under 40)

So if there were 70 guests, 44 of them would be under 40 years old. Use this information to work out the correct percentage, whatever the number of guests:

$\frac{44}{70}$ × 100 = 62.9%

7 There are 2 more parts of the ratio for 20p coins, and 6 more 20p coins.

So 2 parts = 6 coins

1 part = 3 coins.

Hence there are 5 × 3 = 15 10p coins, and 7 × 3 = 21 20p coins

Total amount (£) in the money box = 15 × 0.1 + 21 × 0.2
= 1.5 + 4.2 = £5.70

8 Let x be the number of yellow marbles.

So $5x$ = number of red marbles, and $2 \times 5x = 10x$ = number of blue marbles.

Hence the ratio of blue to red to yellow marbles = $10x : 5x : x$ = 10 : 5 : 1

Scale diagrams and maps

1 First convert 150 km to cm.

150 km = 150 000 m = 15 000 000 cm

500 000 cm is equivalent to 1 cm on the map.

Distance in cm on the map = $\frac{15\,000\,000}{500\,000}$ = 30 cm

2 **a** Distance between the ship and the port = 2 cm.

This is measured using a ruler on the map.

We now need to get the units the same.

10 km = 10 000 m = 1 000 000 cm

The scale is 2 : 1 000 000

Dividing both sides of the ratio by 2 gives

1: 500 000

b Measuring the actual distance between the two ships gives 1.2 cm

Actual distance = 1.2 × 500 000 = 600 000 cm

Divide this by 100 and then 1000 to give the actual distance in km.

600 000 cm = 6 km

Percentage problems

1 $\frac{8}{300} \times 100 = 2.\dot{6}$ = 2.67% to 2 d.p.

2 Increase = Final earnings − Initial earnings
= 1 100 000 − 600 000 = £500 000

% increase = $\frac{\textit{Increase}}{\textit{original value}} \times 100$

= $\frac{500\,000}{600\,000} \times 100$

= 83.3% to 1 d.p.*

3 3.5% = $\frac{3.5}{100}$ = 0.035

Add 1 to create a multiplier for the original number: 1.035

New salary = 1.035 × 38 000 = £39 330

4 18% = $\frac{18}{100}$ = 0.18

1 − 0.18 = 0.82

82% of original price = £291.92

1% of original price = $\frac{291.92}{82}$ = 3.56

100% of original price = 3.56 × 100 = 356

Original price = £356

5 Amount of interest in one year = 3.5% of £12 000
= $\frac{3.5}{100}$ × 12000 = £420

Total interest paid over 6 years = 6 × £420 = £2520

Direct and inverse proportion

1 Inverse proportion means that if one quantity doubles the other quantity halves.

2 **a** $y = kx$

b $8 = k \times 3$ giving $k = \frac{8}{3}$

$y = \frac{8}{3}x$

When $x = 4$, $\frac{8}{3} \times 4 = \frac{32}{3}$ = 10.7 (1 d.p.)

---

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

3 **a** Find the equivalent price in £

$€\frac{120}{1.27} = £94.49$

The sunglasses are cheaper in the UK.

b $£94.49 - £89 = £5.49$

4 $V \propto r^3$ so $V = kr^3$

When $V = 33.5$, $r = 2$ so $33.5 = k \times 2^3$

$k = \frac{33.5}{8} = 4.1875$

Substituting this value of k back into the equation gives

$V = 4.1875r^3$

When $r = 4$, $V = 4.1875 \times 4^3 = 268$ cm^3

5 $P \propto \frac{1}{V}$ so $P = \frac{k}{V}$

Hence $100\,000 = \frac{k}{1}$, giving $k = 100\,000$

Formula is $P = \frac{100\,000}{V}$

When $V = 3$,

$P = \frac{100\,000}{3}$

$= 33\,333$ (to the nearest whole number)

6 $a = kb^2$

When $a = 96$, $b = 4$ so $96 = k \times 4^2$ giving $k = 6$

Hence $a = 6b^2$ and when $b = 5$, $a = 6 \times 5^2 = 150$

7 **a** A square of side x cm has an area of x^2.

$A = kx^2$

As there are 6 faces to a cube, surface area $= 6x^2$

Hence $A = 6x^2$

So constant of proportionality $k = 6$

b $A = 6x^2$

When $x = 4$ cm

$A = 6 \times 4^2 = 96$ cm^2

Graphs of direct and inverse proportion and rates of change

1 Graph C

2 Graph B

3 As P and V are inversely proportional, $P = \frac{k}{V}$

At point A, when $P = 12$, $V = 3$ so $12 = \frac{k}{3}$ hence $k = 36$

Substituting this value of k back into the equation we have $P = \frac{36}{V}$

When $V = 6$, $P = \frac{36}{6} = 6$

Hence $a = 6$

4 As x and y are inversely proportional, $y \propto \frac{1}{x}$, so $y = \frac{k}{x}$

When $x = 1$, $y = 4$ so $4 = \frac{k}{1}$ so $k = 4$

The equation of the curve is now $y = \frac{4}{x}$

When $x = 4$, $y = \frac{4}{4} = 1$ so $a = 1$

When $y = 0.8$, $0.8 = \frac{4}{x}$ giving $x = 5$ so $b = 5$.

Hence $a = 1$ and $b = 5$.

5 $y = kx^2$

When $x = 2$, $y = 16$ so $16 = k \times 2^2$ giving $k = 4$.

$y = 4x^2$

Hence when $y = 36$,

$36 = 4x^2$ so $x = 3$ or -3

Since a is positive,

$a = 3$

Growth and decay

1 For a, b and c, multiplier is $\frac{100\% + \text{percentage given}}{100}$

a 1.05

b 1.25

c 1.0375

d Multiplier is $\frac{100\% - \text{percentage given}}{100} = 0.79$

2 Multiplier $= 1 - \frac{18}{100} = 0.82$

Value at the end of n years $= A_0 \times (\text{multiplier})^n$ where A_0 is the initial value.

Value at the end of 3 years $= 9000 \times (0.82)^3 = £4962.312$

$= £4962$ (nearest whole number)

3 Number of restaurants after n years, $R_n = R_0 \times (\text{multiplier})^n$

Multiplier $= 1.25$, $n = 3$, $R_n = 4000$

So $R_0 = \frac{4000}{(1.25)^3} = 2048$

There were 2048 restaurants 3 years ago.

Ratios of lengths, areas and volumes

1 Scale factor $= \frac{6}{9} = \frac{2}{3}$

$x = \frac{2}{3} \times 10 = \frac{20}{3} = 6\frac{2}{3}$ cm

2 As the shapes are similar $\frac{V_A}{V_B} = \left(\frac{r_A}{r_B}\right)^3$

Cube rooting both sides gives $\frac{r_A}{r_B} = \sqrt[3]{\frac{V_A}{V_B}}$

$\frac{r_A}{r_B} = \sqrt[3]{\frac{27}{64}}$

$= \frac{3}{4}$

$\frac{\text{radius of cylinder A}}{\text{radius of cylinder B}} = \frac{3}{4}$

$\frac{\text{surface area of cylinder A}}{\text{surface area of cylinder B}} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

So surface area of cylinder A = surface area of cylinder B $\times \frac{9}{16}$

Surface area of cylinder A $= 96 \times \frac{9}{16} = 54$ cm^2

3 **a** As *BE* is parallel to *CD* all the corresponding angles in both triangles are the same so triangles *ABE* and *ACD* are similar.

$\frac{BE}{8} = \frac{5}{10}$ giving $BE = 4$ cm

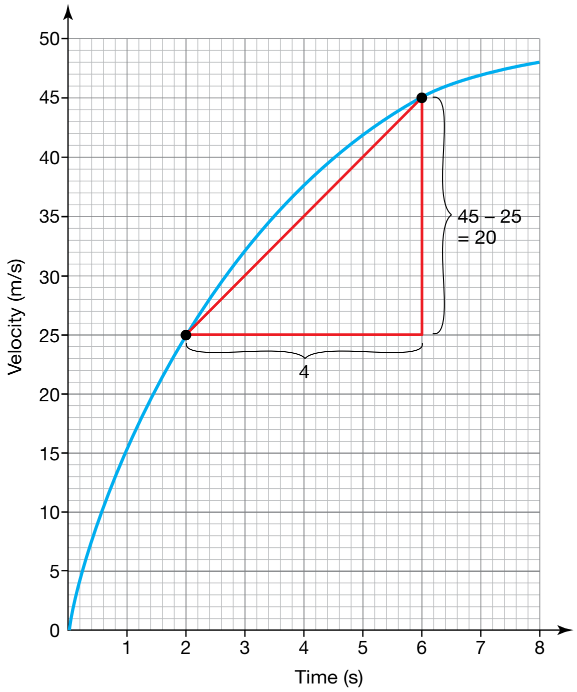
b $BE = 4$ cm and $AB = 5$ cm. Angle *ABE* is a right angle because angle *ACD* is a right angle and the triangles are similar.

Hence area $ABE = \frac{1}{2} \times 4 \times 5 = 10$ cm^2

Gradient of a curve and rate of change

1 **a** The gradient represents the acceleration.

b Find the gradient between when time = 2 s and time = 6 s.



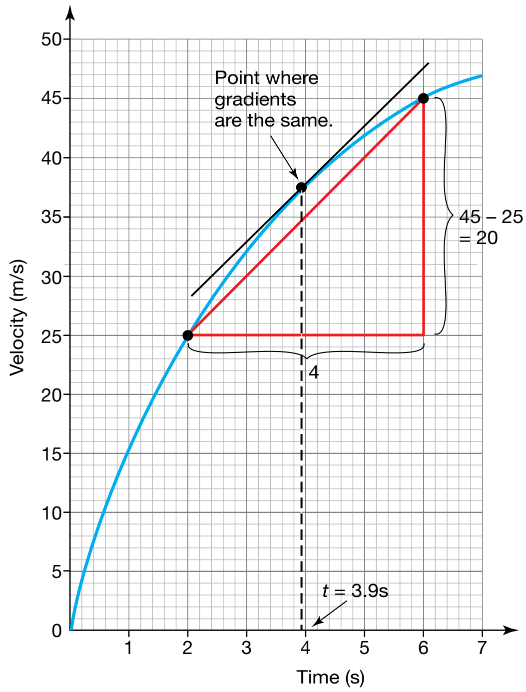
From the graph, acceleration between time = 2 s and time = 6 s

$$= \frac{45 - 25}{6 - 2}$$

$$= \frac{20}{4}$$

$= 5$ m/s^2.

c We need to find a point on the graph where the gradient equals that of the tangent already drawn (5 m/s^2).



Time when the instantaneous acceleration is the same as the average acceleration = 3.9 s

Converting units of areas and volumes, and compound units

1 **a** Surface area $= 2 \times 4 \times 6 + 2 \times 4 \times 5 + 2 \times 6 \times 5$

$= 148$ cm^2

i 1cm^2 = 10×10 mm^2 = 100 mm^2

148 cm^2 = 148×100 mm^2 = 14 800 mm^2

ii 1 m^2 = 100×100 cm^2 = 10 000 cm^2

148 cm^2 = $148 \div 10\,000$ m^2 = 0.0148 m^2

b Volume $= 6 \times 5 \times 4 = 120$ cm^3

1 m^3 = $100 \times 100 \times 100$ cm^3 = 1 000 000 cm^3

120 cm^3 = $120 \div 1\,000\,000 = 0.00012$ cm^3

2 Density $= \frac{\text{mass}}{\text{volume}} = \frac{1159}{600} = 1.932$ g/cm^3 (3 d.p.)

3 First get the units the same.

1m^3 = $100 \times 100 \times 100$ cm^3 = 1 000 000 cm^3

Number of ball bearings $= \frac{1\,000\,000}{0.5} = 2\,000\,000$

(i.e. 2 million)

4 50 km/h = 50 000 m/h

1 h = 60×60 s = 3600 s

Now as fewer m/s will be covered compared to m/h we divide by 3600 to convert the speed to m/s.

Hence 50 000 m/h $= \frac{50\,000}{3600} = 13.89$* m/s (2 d.p.)

5 **a** Distance = speed × time = $70 \times 4 = 280$ km

Mary's speed = 280/5 = 56 km/h

b If Mary's route was longer, her average speed would increase. If her route was shorter, her average speed would decrease.

Review it!

1 80% of the original price = £76.80

10% of the original price $= \frac{76.80}{8} =$ £9.60

100% of the original price = £9.60 × 10 = £96

2 **a** $A_1 = 1.02 \times A_0$

$= 1.02 \times 5$

$= 5.10$ m^2 (2 d.p.)

b $A_2 = 1.02 \times A_1 = 1.02 \times 5.1 = 5.202$ m^2

$A_3 = 1.02A_2 = 1.02 \times 5.202 = 5.31$ m^2 (2 d.p.)

3 $y = \frac{k}{x}$

$4 = \frac{k}{2.5}$

$k = 10$

Hence $y = \frac{10}{x}$

When $x = 5$, $y = \frac{10}{5} = 2$

4 2 parts of the ratio = 54 students, so 1 part = 27 students.

Students in the whole year = 2 + 7 parts = 9 parts.

In Year 11, there are 9 × 27 = 243 students.

5 The price of the house has more than doubled.

Multiplier = 1 + 1.2 = 2.2

So £220 000 × 2.2 = £484 000

The value of the house = £485 000 to the nearest £5000

---

*In the first edition of our Revision Guide, an answer of '13.8 m/s' has been given, which is not to 2 d.p. The answer provided here is correct to 2 d.p., as stipulated in the question.

6 **a** Simple interest: One year $= 2000 \times 0.025 = £50$

$50 \times 5 = 250$

After 5 years, there will be £2250 in the account.

b Compound interest: $2000 \times 1.025^5 = £2262.82$

7 **a** $400 \times 8.55 = 3420$ yuan

b Travel agent: $800 \div 8.6 = £93.02$

Commission $= 93.02 \times 0.025 = £2.33$

So Tom would get $£93.02 - £2.33 = £90.69$

From the post office, Tom would get
$800 \div 8.9 = £89.89$

Tom will get a better deal from the travel agent.

8 The difference between Luke's and Amy's payouts was £4000. The difference in parts is $7 - 5 = 2$.
So 2 parts $= £4000$ and 1 part $= £2000$

The profits are split into $3 + 5 + 7 = 15$ parts.

Total profits $= 15 \times 2000 = £30\,000$

9 $\dfrac{\text{Surface area A}}{\text{Surface area B}} = \left(\dfrac{\text{Length A}}{\text{Length B}}\right)^2$

So $\dfrac{25}{4} = \dfrac{5^2}{2^2}$

Similarly, $\dfrac{\text{Volume A}}{\text{Volume B}} = \left(\dfrac{\text{Length A}}{\text{Length B}}\right)^3$

$\dfrac{10}{\text{Volume B}} = \dfrac{5^3}{2^3}$

So Volume B $= \dfrac{10 \times 8}{125} = \dfrac{80}{125} = 0.64\,\text{cm}^3$

10 $7 + 4 = 11$ parts in the ratio.

Let this be 1100 members.

There would then be 700 male club members: 25% of this is $0.25 \times 700 = 175$ junior members.

There would be 400 female club members: 10% of this is $0.1 \times 400 = 40$ junior members.

In total, there would be $175 + 40 = 215$ junior members.

As a percentage, this is $\dfrac{215}{1100} \times 100 = 19.54$
$= 20\%$ to the nearest integer.

11 Each cm^3 of the alloy contains 9 parts ($= 0.9\ \text{cm}^3$) copper and 1 part ($= 0.1\ \text{cm}^3$) tin.

So the mass of copper in $1\ \text{cm}^3 = 8.9 \times 0.9 = 8.01$ g

The mass of tin in $1\ \text{cm}^3 = 7.3 \times 0.1 = 0.73$ g

Mass of $1\ \text{cm}^3$ of alloy $= 8.01 + 0.73 = 8.74$ g

So the density of the alloy $= 8.7\ \text{g/cm}^3$ (1 d.p.)

Geometry and measures

2D shapes

1 **a** True

b False

c True

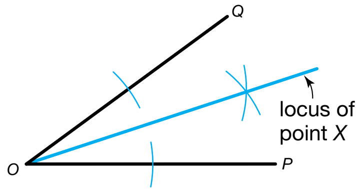
d False

e True

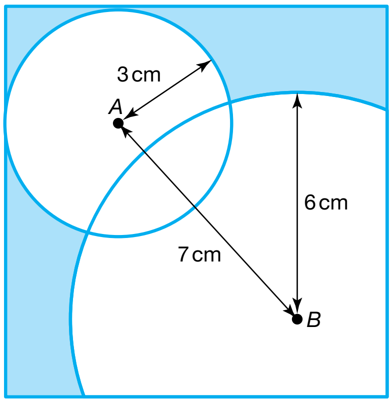
f True

Constructions and loci

1 The line which bisects the angle will be the locus of point *X*.



2 Draw two circles: one with a radius of 3 cm from *A*, another with a radius of 6 cm from *B*. Shade the area outside the two circles. The area outside the two circles shows all possible locations for the wind farm.



Properties of angles

1 Exterior angle $= \dfrac{360°}{\text{number of sides}} = \dfrac{360}{10} = 36°$

Interior angle + exterior angle $= 180°$, so interior angle $= 144°$

The interior and exterior angle always add up to 180°.

2 **a** Number of sides $= \dfrac{360°}{\text{exterior angle}} = \dfrac{360}{40} = 9$

b Interior angle $= 180° - \text{exterior angle}$
$= 180 - 40 = 140°$

As there are 9 sides, total of interior angles $= 140 \times 9 = 1260°$

3 Angle *BAC* $= 50°$ (corresponding angles)

Angle *ABC* $= 180 - (50 + 60) = 70°$ (angles in a triangle add up to 180°)

$x = 180 - (90 + 70) = 20°$ (Angles in triangle *ABD* add up to 180°)

4 There are several different ways of working the answers out and the method shown is only one of them.

a Angle x is an alternate angle to angle *EBC*. Hence $x = 55°$.

b Angle *EHI* = 180 − (85 + 55) = 40° as angles in a triangle add up to 180°.

Angle *DEH* is an alternate angle to angle *EHI* so it is 40°.

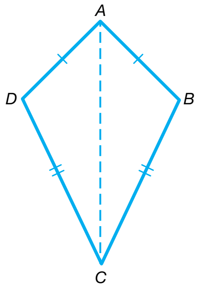
5 Opposite angles in a rhombus are equal.

$$\begin{aligned} \text{So } 2(3a + 5) + 2(5a - 17) &= 360 \\ 6a + 10 + 10a - 34 &= 360 \\ 16a - 24 &= 360 \\ 16a &= 384 \\ a &= \frac{384}{16} \\ a &= 24° \end{aligned}$$

There are two angles of 3 × 24 + 5 = 77°, and two angles of 180 − 77 = 103°

Congruent triangles

1



There are two triangles with a common side *AC*.

AB = *AD* (given in the question)

BC = *CD* (given in the question)

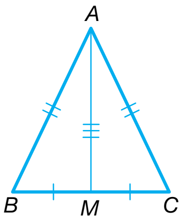
Triangles *ACB* and *ACD* are congruent (SSS).

Hence, angle *ABC* = angle *ADC*.

2 *AB* = *AC* (given in the question)

BM = *MC* (as M is the midpoint of *BC*)

AM = *AM* (common to both triangles)



There are three pairs of equal corresponding sides so the triangles *ABM* and *ACM* are congruent (SSS).

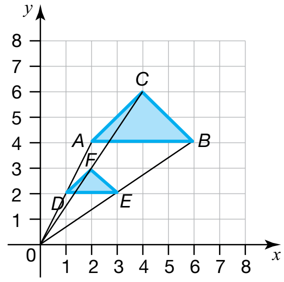
Hence angle *AMB* = angle *AMC*.

Angle *AMB* + angle *AMC* = 180°, (angles on a straight line)

So angle $AMB = \frac{180}{2} = 90°$

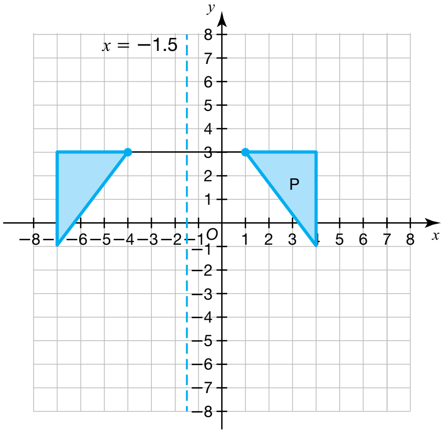
Transformations

1 **a** and **b**

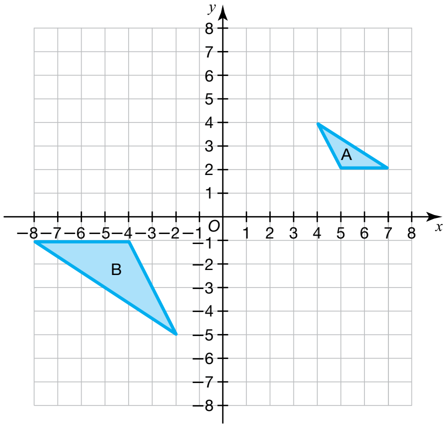


c Enlargement, scale factor 2, centre (0, 0)

2



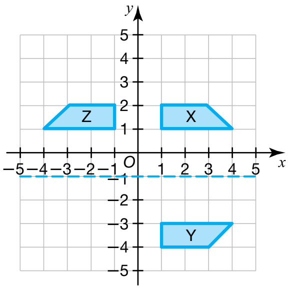
3



4 A translation by $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$

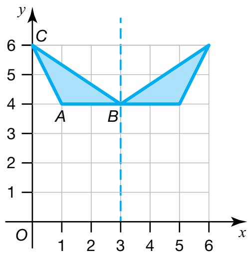
Invariance and combined transformations

1 **a, b, c**



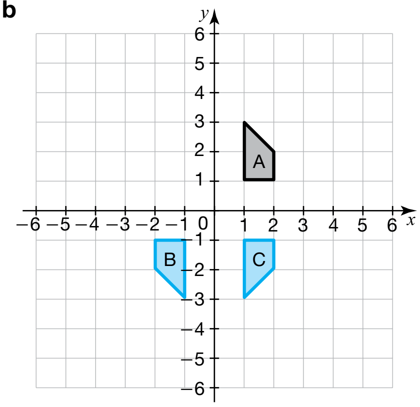
d Reflection in the y-axis, or reflection in the line $x = 0$

2 a, b



c Invariant point is (3, 4)

3 a, b



c A reflection in the x-axis.

3D shapes

1 a

Shape	Number of vertices, *V*	Number of faces, *F*	Number of edges, *E*
Triangular-based pyramid	4	4	6
Cone	1	2	1
Cuboid	8	6	12
Hexagonal prism	12	8	18

b $V = 16, F = 10, E = 24; V + F - E = 16 + 10 - 24 = 2$

Parts of a circle

1 a radius

b chord

c minor arc

d minor segment

e minor sector

Circle theorems

1 a Angle *ACB* = 30° as it is equal to angle *YAB* (alternate segment theorem)

b Angle *ABC* = 90° (angle in a semicircle is always a right angle)

c Angle *ADC* = 90° (angle in a semicircle is always a right angle)

2 a Angle *OAX* = 90° (angle formed by a tangent to a radius or diameter is always a right angle)

b Angle *AOX* = 180 − (90 + 30) = 60° (angles in a triangle add up to 180°)

c Angle *ACB* = 60 ÷ 2 = 30° (angle at the centre is twice the angle at the circumference standing on the same arc)

3 a Angle *ADB* = 40° as it is equal to angle *ACB* (angles at the circumference on the same arc are equal)

b Angle *ABD* = angle *ADE* = 50° (alternate segment theorem)

Angle *EDB* = 90° (i.e. 40° + 50°) which means *BD* must be a diameter, as a tangent and a diameter are at right angles to each other.

As *BD* is a diameter, angle *BAD* is 90° as it is the angle in a semicircle.

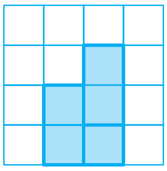
4 Angle *ACB* = 46° (angles bounded by the same chord in the same segment are equal)

Angle *ABC* = 90° (angle in a semicircle is a right-angle)

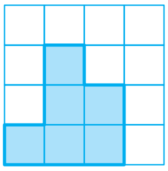
Angle *BAC* = 180 − (90 + 46) = 44° (angles in a triangle add up to 180°)

Projections

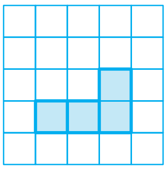
1 a



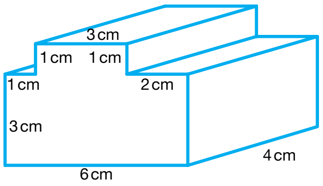
b



c

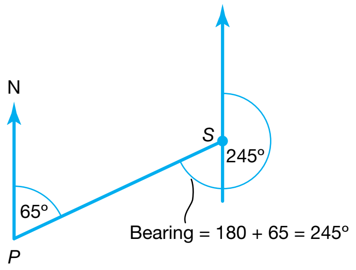


2



Bearings

1



Bearing = 245°

2 a Bearing of P from Q = 180 + 45 = 225°

b Bearing of Q from R = 180 + 140 = 320°

Pythagoras' theorem

1 a By Pythagoras' theorem

$x^2 = 5^2 + 9^2 = 106$

$x = \sqrt{106} = 10.3$ cm (1 d.p.)

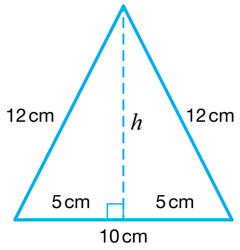
b By Pythagoras' theorem

$13.5^2 = 10.2^2 + x^2$

$x^2 = 182.25 - 104.04 = 78.21$

$x = \sqrt{78.21} = 8.8$ cm (1 d.p.)

2



By Pythagoras' theorem

$12^2 = 5^2 + h^2$

$144 = 25 + h^2$

$h^2 = 119$

$h = \sqrt{119}$

$= 10.9087$ cm (4 d.p.)

Area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 10 \times 10.9087$

$= 54.54$ cm^2 (2 d.p.)

3 $AC^2 = 3^2 + 11^2 = 130$

$x^2 = 14^2 - AC^2 = 196 - 130 = 66$

$x = \sqrt{66} = 8.12$ cm (2 d.p.)

4 Let the perpendicular height of the triangle $= h$

Area of triangle $= \frac{1}{2} \times 14 \times h$

Hence $\frac{1}{2} \times 14 \times h = 90$

$h = \frac{90 \times 2}{14} = \frac{180}{14} = \frac{90}{7}$

$h = 12.8571$ cm (4 d.p.)

By Pythagoras' theorem, $AC^2 = 7^2 + 12.8571^2$

$AC = \sqrt{214.305}$ (3 d.p.)

$AC = 14.6$ cm (3 s.f.)

Area of 2D shapes

1 Right angle and parallel lines show that the office is a trapezium.

Area of a trapezium $= \frac{1}{2} \times (a + b) \times h = \frac{1}{2} \times (9 + 12) \times 4$

Office area $= 42$ m^2

Cost of new floor = £38 × 42 = £1596

2 Length of side of square $= \frac{\text{perimeter}}{4} = \frac{20}{4} = 5$ cm

Area of square $= 5 \times 5 = 25$ cm^2

Area of small triangle $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 2.5 \times 2.5 = 3.125$ cm^2

Area of large triangle $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 2.5 \times 5 = 6.25$ cm^2

Shaded area $= 25 - (3.125 + 6.25) = 15.625$

Proportion shaded $= \frac{15.625}{25}$

Multiply by 8 to get rid of the decimal:

$= \frac{125}{200}$

Cancel fraction (divide by 25):

$= \frac{5}{8}$ of the square.

3 **a** Area of semicircle $= \frac{1}{2} \times \pi \times x^2 = \frac{\pi x^2}{2}$

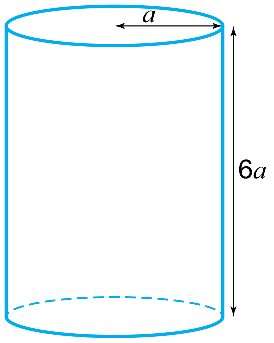
Area of rectangle $= 4x \times 2x = 8x^2$

Area of shape $= \frac{\pi x^2}{2} + 8x^2 = x^2(8 + \frac{\pi}{2})$

b Perimeter $= 2x + 4x + 4x + \frac{1}{2} \times 2\pi x = 10x + \pi x = x(10 + \pi)$. An exact answer is required, so you should leave π in your answer.

Volume and surface area of 3D shapes

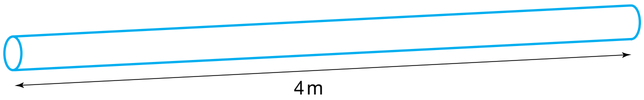
1



Volume of cylinder = cross-sectional area × length

$= \pi a^2 \times 6a$

$= 6\pi a^3$



Volume of rod = cross-sectional area × length

$= \pi r^2 \times 4$

$= 4\pi r^2$

The two volumes are the same so we can write

$4\pi r^2 = 6\pi a^3$ (Dividing both sides by π)

$4r^2 = 6a^3$ (Dividing both sides by 2)

$2r^2 = 3a^3$ (Dividing both sides by 2)

$r^2 = \frac{3}{2}a^3$ (Square rooting both sides)

$r = \sqrt{\frac{3}{2}a^3}$

2 **a** Area of trapezium $= \frac{1}{2}(a + b)h = \frac{1}{2}(0.75 + 1.75) \times 10 = 12.5$ m^2

Cross-sectional area $= 12.5$ m^2

b Volume = Cross-sectional area × length

$= 12.5 \times 5 = 62.5$ m^3

c Total volume per minute of water entering $= 2 \times 0.05 = 0.1$ m^3 per minute

Number of minutes it takes to fill $= \frac{62.5}{0.1}$

$= 625$ mins

Number of hours it takes to fill $= \frac{625}{60}$

$= 10.42$ hours (2 d.p.)

$= 10$ hours (nearest hour)

3 Need to first find the slant height, l, as this is needed for the formula.

By Pythagoras' theorem $l^2 = 4^2 + 15^2 = 241$

$l = 15.524$ m (3 d.p.)

Curved surface area of a cone $= \pi rl = \pi \times 15 \times 15.524$

$= 731.55$ m^2 (2 d.p.)

Curved surface area of a cylinder $= 2\pi rh$

$= 2 \times \pi \times 15 \times 5$

$= 471.24$ m^2 (2 d.p.)

Total surface area of the tent $= 731.55 + 471.24$

$= 1202.79$ m^2

The tent uses 1200 m^2 of fabric (to 3 s.f.)

4 Volume = volume of large cone – volume of small cone

$$= \frac{1}{3} \times \pi \times 5^2 \times 20 - \frac{1}{3} \times \pi \times 3.5^2 \times 14$$

$$= \frac{500}{3}\pi - \frac{171.5}{3}\pi$$

$$= \frac{328.5}{3}\pi$$

$$= 109.5\pi \text{ cm}^3$$

Trigonometric ratios

1 **a** $\cos 30° = \frac{15}{x}$

x is the hypotenuse and the 15cm side is the adjacent so we use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.

$$x = \frac{15}{\cos 30°} = 17.32 \text{ cm (2 d.p.)}$$

b $\cos 40° = \frac{x}{12}$

x is the adjacent and the 12 cm side is the hypotenuse so we use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.

Hence, $x = 12 \cos 40°$

$$= 9.19 \text{ cm (2 d.p.)}$$

2 **a** $\tan\theta = \frac{10}{3}$

$$\theta = \tan^{-1}\left(\frac{10}{3}\right)$$

$$= 73.3° \text{ (nearest 0.1°)}$$

b $\sin\theta = \frac{10}{13}$

The 13cm side is the hypotenuse and the 10cm side is the opposite, so we use $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$.

$$\theta = \sin^{-1}\left(\frac{10}{13}\right)$$

$$= 50.3° \text{ (nearest 0.1°)}$$

3 $\sin\theta = \frac{b}{c}$ (i.e. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$)

and $\cos\theta = \frac{a}{c}$ (i.e. $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$)

Remember that when you divide by fractions you turn the bottom fraction upside down and replace the division by a multiplication.

Hence $\frac{\sin\theta}{\cos\theta} = \frac{\frac{b}{c}}{\frac{a}{c}}$

$$= \frac{b}{c} \times \frac{c}{a}$$

$$= \frac{b}{a}$$

Now, $\frac{b}{a} = \frac{\text{opposite}}{\text{adjacent}} = \tan\theta$

Hence, $\frac{\sin\theta}{\cos\theta} = \tan\theta$

4 **a** By Pythagoras' theorem, $BD^2 = AB^2 + AD^2$

$$= 10^2 + 7^2$$

$$= 149$$

$$BD = \sqrt{149}$$

$$= 12.207 \text{ cm (3 d.p.)}$$

Applying Pythagoras' theorem to triangle BCD

$$CD^2 = BD^2 + BC^2$$

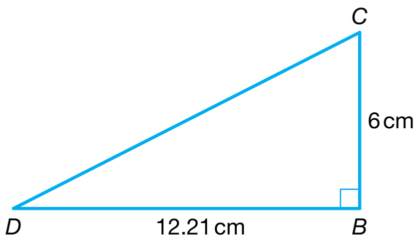
$$= 12.207^2 + 6^2$$

$$= 185.011 \text{ (3 d.p.)}$$

$$CD = \sqrt{185.011}$$

$$= 13.60 \text{ cm (2 d.p.)}$$

b



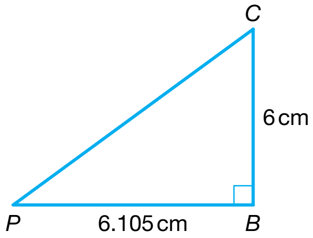
Let angle BCD = x

$$\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{12.21}{6}$$

$$x = \tan^{-1}\left(\frac{12.21}{6}\right)$$

So $x = 63.8°$ to 1 decimal place

c $BP = \frac{1}{2} \times BD = \frac{1}{2} \times 12.21 = 6.105$ cm

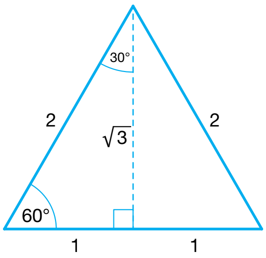


By Pythagoras' theorem, $PC^2 = 6.105^2 + 6^2$

$$PC = 8.56 \text{ cm (2 d.p.)}$$

Exact values of sin, cos and tan

1 First draw an equilateral triangle with sides 2 cm and mark on the following sides and angles.



From the diagram $\tan 30° = \frac{1}{\sqrt{3}}$ and $\cos 60° = \frac{1}{2}$

$$\sqrt{3}\tan 30° + \cos 60° = \sqrt{3} \times \frac{1}{\sqrt{3}} + \frac{1}{2} = 1 + \frac{1}{2} = \frac{3}{2}$$

where $a = 3$ and $b = 2$.

2 $\sin 30 = \frac{1}{2}$ and $\cos 30 = \frac{\sqrt{3}}{2}$

$\sin^2 30 = \frac{1}{4}$ and $\cos^2 30 = \frac{3}{4}$

$\sin^2 30 + \cos^2 30 = \frac{1}{4} + \frac{3}{4} = 1$

Sectors of circles

1 $l = \frac{\theta}{360} \times 2\pi r$

$$10 = \frac{\theta}{360} \times 2\pi \times 12$$

$$\theta = \frac{10 \times 360}{2\pi \times 12} = \frac{5 \times 30}{\pi}$$

$$= 47.7° \text{ (1 d.p.)}$$

2 **a** Perimeter of logo = 2 × arc length of sector + 2 × radius of sector + base of triangle

Arc length $= \frac{\theta}{360} \times 2\pi r = \frac{40}{360} \times 2 \times \pi \times 2 = 1.396$cm

So perimeter $= 2 \times 1.396 + 2 \times 2 + 1.2$

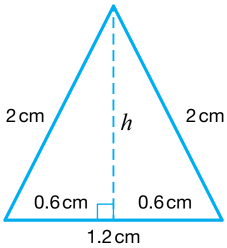
$$= 7.99 \text{ cm (3 s.f.)}$$

b Area of logo = 2 × area of sector + area of triangle

Area of sector $= \frac{\theta}{360} \times \pi r^2 = \frac{40}{360} \times \pi \times 2^2$

$= 1.396\,\text{cm}^2$ (3 d.p.)

Find the height of the isosceles triangle to find its area.



Use Pythagoras' theorem

$2^2 = 0.6^2 + h^2$

So $h = 1.9079$ cm (4 d.p.)

Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 1.2 \times 1.9079 = 1.1447\,\text{cm}^2$

Area of logo $= 2 \times 1.396 + 1.1447 = 3.9367$

$= 3.94\,\text{cm}^2$ (3 s.f.)

3 Area of sector $= \frac{85}{360} \times \pi \times r^2$

Area $= 25\pi$ so $\frac{85}{360} \times \pi \times r^2 = 25\pi$

$r^2 = \frac{25\pi \times 360}{85\pi} = 105.88$ (2 d.p.)

$r = OA = \sqrt{105.88} = 10.3$ cm (1 d.p.)

4 First find the radius of the circle using the length of arc.

$\frac{85}{360} \times \pi \times d = 10$

$d = \frac{3600}{85\pi}$

$r = \frac{3600}{85\pi} \div 2$

$= 6.7407$ (4 d.p.)

Area of sector $= \frac{275}{360} \times \pi \times 6.7407^2 = 109\,\text{cm}^2$ to the nearest whole number

Sine and cosine rules

1 **a** Area of triangle $ABC = \frac{1}{2}\,b\,c \sin A$

$= \frac{1}{2} \times 10 \times 6 \sin 150°$

$= 15\,\text{cm}^2$

b Using the cosine Rule

$BC^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \cos 150°$

$BC^2 = 36 + 100 - 120 \cos 150°$

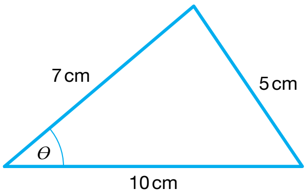
$BC^2 = 136 + 103.92$

$BC = \sqrt{239.92}$

$BC = 15.5$ cm (to 3 s.f.)

2 The smallest angle of any triangle is always opposite the smallest side.

Draw a sketch of the triangle and let the smallest angle be θ.



Use the cosine Rule

$5^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \cos \theta$

$25 = 49 + 100 - 140 \cos \theta$

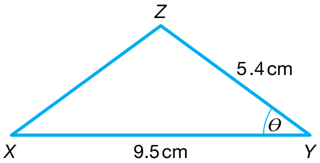
$140 \cos \theta = 124$

$\cos \theta = 0.8857$

$\theta = \cos^{-1}(0.8857)$

$= 27.7°$ (nearest 0.1°)

3 **a**



Let angle $XYZ = \theta$

Area $= \frac{1}{2} \times XY \times YZ \times \sin \theta$

From the question, area = 16 cm²

So $16 = \frac{1}{2} \times 9.5 \times 5.4 \times \sin \theta$

$\sin \theta = \frac{16 \times 2}{9.5 \times 5.4}$

$\theta = \sin^{-1}\left(\frac{32}{51.3}\right)$

So $\theta = 38.6°$ (1 d.p.)

b If angle XZY is not obtuse, then angle XYZ can be obtuse.

So an alternative answer for $\theta = 180 - 38.6$

$= 141.4°$ (1 d.p).

Vectors

1 **a** $\mathbf{b} - \mathbf{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$

b $3\mathbf{a} + 5\mathbf{b} = 3\begin{pmatrix} 4 \\ -5 \end{pmatrix} + 5\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 12 \\ -15 \end{pmatrix} + \begin{pmatrix} -10 \\ 15 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

2 **a** $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\mathbf{a} + \mathbf{b}$

$= \mathbf{b} - \mathbf{a}$

b $\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AB}$

$= \frac{3}{5}(\mathbf{b} - \mathbf{a})$

c $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$ ← Multiply out the bracket and then simplify the terms.

$= \mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{3}{5}\mathbf{a}$

$= \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

$= \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$

3 **a** $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$\overrightarrow{BA} = -\overrightarrow{AB} = -\mathbf{a}$

$\overrightarrow{AC} = \overrightarrow{AP} + \overrightarrow{PC} = -2\mathbf{b} - \mathbf{b} = -3\mathbf{b}$

So $\overrightarrow{BC} = -\mathbf{a} - 3\mathbf{b}$

b If M is the midpoint of BC, $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BC}$

$= \frac{1}{2}(-\mathbf{a} - 3\mathbf{b}) = -\frac{1}{2}(\mathbf{a} + 3\mathbf{b})$

If B is the midpoint of AD, $AB = BD = \mathbf{a}$

So $\overrightarrow{PM} = \overrightarrow{PA} + \overrightarrow{AB} + \overrightarrow{BM}$

$= 2\mathbf{b} + \mathbf{a} - \frac{1}{2}(\mathbf{a} + 3\mathbf{b})$

$= 2\mathbf{b} + \mathbf{a} - \frac{1}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}$

$= \frac{1}{2}(\mathbf{a} + \mathbf{b})$

Similarly, $\overrightarrow{MD} = \overrightarrow{MB} + \overrightarrow{BD}$

$= \frac{1}{2}(\mathbf{a} + 3\mathbf{b}) + \mathbf{a}$

$= \frac{1}{2}\mathbf{a} + \frac{3}{2}\mathbf{b} + \mathbf{a}$

$= \frac{3}{2}(\mathbf{a} + \mathbf{b})$

$\overrightarrow{PM}$ and $\overrightarrow{MD}$ have the same vector part, $(\mathbf{a} + \mathbf{b})$, therefore they are parallel. Both lines pass through point M and parallel lines cannot pass through the same point unless they are the same line. Hence PMD is a straight line.

Review it!

1 Angle $ADE = y°$ (alternate segment theorem)

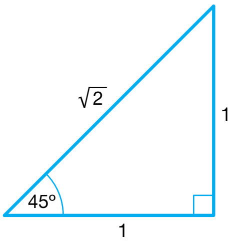
Angle $ADC = (180 - x)°$ (opposite angles in cyclic quadrilateral add up to 180°)

EDF is a tangent, so it is a straight line.

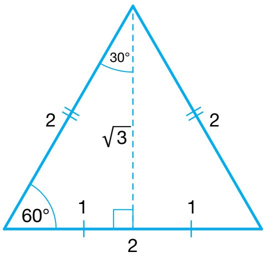
So angle $CDF = 180 -$ angle $ADE -$ angle ADC

$= 180 - y - (180 - x) = (x - y)°$

2 Draw the following triangles and mark on the angles.



$\cos 45° = \frac{1}{\sqrt{2}}$



$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 45° + \sin 60° = \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}$

$= \frac{1}{2}(\sqrt{2} + \sqrt{3})$

3 Using Pythagoras' theorem we obtain

$(4x - 3)^2 = (x + 1)^2 + (3x)^2$

$16x^2 - 24x + 9 = x^2 + 2x + 1 + 9x^2$

So $6x^2 - 26x + 8 = 0$

Dividing both sides by 2 we obtain

$3x^2 - 13x + 4 = 0$

$(3x - 1)(x - 4) = 0$

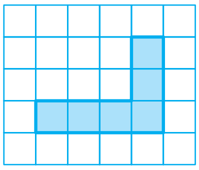
$3x - 1 = 0$ or $x - 4 = 0$

Hence $x = \frac{1}{3}$ or $x = 4$.

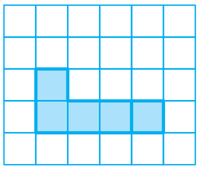
We need to put both values into each of the expressions for the sides to see if any of the sides end up negative. When $x = \frac{1}{3}$ is substituted into $4x - 3$ the result is negative. As you cannot have a negative length for a side, $x = \frac{1}{3}$ is disregarded. Hence $x = 4$.

When x is 4 the sides are 5, 12 and 13.

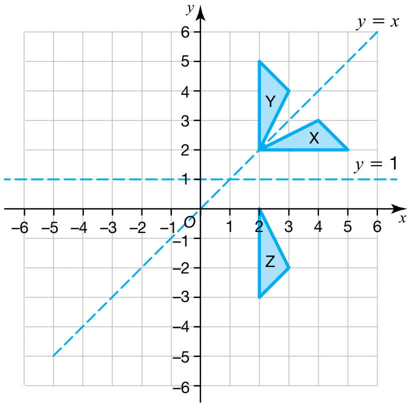
4 a



b



5 a, b i and **c**



b ii Invariant point (vertex) is (2, 2)

d Rotation 90° clockwise about (1, 1)

6 a Work out angle ABC in the triangle.

The angle between AB and due south at B is 70° (alternate with 70° marked at A).

The angle between due south at B and BC is 30° (angles on a straight line add up to 180°).

So angle $ABC = 100°$

Using the Cosine Rule:

$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos B$

$AC^2 = 15^2 + 10^2 - 2 \times 15 \times 10 \times \cos 100°$

$AC^2 = 325 - (-52.094)$

So $AC = \sqrt{377.094} = 19.4$ km (3 s.f.)*

b Let angle $= \theta$

Using the Sine Rule, $\frac{19.4}{\sin 100} = \frac{10}{\sin \theta}$

So $\sin \theta = \frac{10 \sin 100}{19.4} = \frac{9.84}{19.4}$

$\theta = \sin^{-1}\left(\frac{9.84}{19.4}\right)$

$\theta = 30.48°$

Bearing of C from $A = 70 + 30.48 = 100.48°$

So bearing of A from $C = 100.48 + 180 = 280°$ to the nearest degree.

*In the first edition of our Revision Guide, the answer is incorrectly stated as being correct to 1 s.f. The answer is correct to 3 s.f., as stipulated in the question.

Probability

The basics of probability

1 There are two possible even numbers (i.e. 2, 4)

Probability of landing on an even number

$$= \frac{\text{Number of ways something can happen}}{\text{Total of number of possible outcomes}} = \frac{2}{5}$$

Probability of getting an even number on each of three spins $= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$

2 a As there are no white counters, the probability of picking one = 0

b Probability of picking a black counter $= \frac{4}{20} = \frac{1}{5}$

c Probability of picking a green counter $= \frac{9}{20}$

Prob of picking a green counter + prob of not picking a green counter = 1

Prob of not picking a green counter

$= 1 - \frac{9}{20} = \frac{11}{20}$

Probability experiments

1 a Relative frequency for a score of 3 $= \frac{25}{120}$

$= 0.21$ (2 d.p.)

b Relative frequency for a score of 6 $= \frac{17}{120}$

$= 0.14$ (2 d.p.)

c Sean is wrong. 120 spins is a small number of spins and it is only over a very large number of spins that the relative frequencies may start to be nearly the same.

2 a Estimated probability = relative frequency

$$= \frac{\text{Frequency}}{\text{Total Frequency}} = \frac{20}{500} = 0.04$$

b Number of cans containing less than 330 ml = 0.04 × 15 000 = 600

Another way to do this is to see how many times 500 divides into 15 000. This is 30. So there will be 30 × 20 = 600 cans containing less than 30 ml.

3 Expected frequency = Probability of the event × number of events

$= \frac{3}{40} \times 600$

$= 45$ apples

The AND and OR rules

1 a Independent events are events where the probability of one event does not influence the probability of another event occurring. Here it means that the probability of the first set of traffic lights being red does not affect the probability of the second set being red.

b P(A AND B) = P(A) × P(B)

P(stopped at first AND stopped at second)

= P(stopped first) × P(stopped second)

$= 0.2 \times 0.3$

$= 0.06$

c $P = 0.8 \times 0.7 = 0.56$

You can work out the probability of lights not being on red by subtracting the probability of being red from 1. So the probability of not being red at the first set is 1 – 0.2 = 0.8 and for the second set it is 1 – 0.3 = 0.7

2 a Probability of all events taking place $= \frac{1}{4} \times \frac{2}{3} \times \frac{7}{8}$

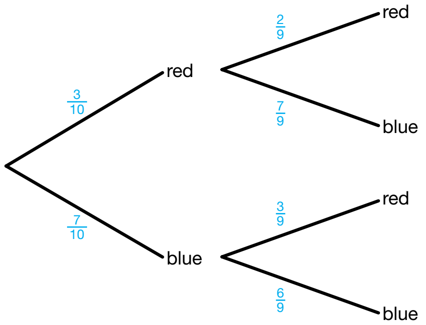
$= \frac{2 \times 7}{12 \times 8} = \frac{7}{48}$

b Probability of none of the events taking place

$= \frac{3}{4} \times \frac{1}{3} \times \frac{1}{8} = \frac{3}{3 \times 32} = \frac{1}{32}$

Tree diagrams

1 a The following tree diagram is drawn.



P(red AND red) $= \frac{3}{10} \times \frac{2}{9} = \frac{6}{90} = \frac{1}{15}$

b P(red AND blue) = P(RB) + P(BR)

Note that red and blue does not specify an order. There are two paths that need to be considered on the tree diagram.

$= \frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{3}{9}$

$= \frac{7}{30} + \frac{7}{30}$

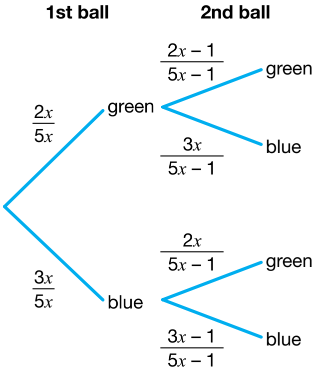
$= \frac{14}{30}$

$= \frac{7}{15}$

Remember to fully cancel fractions. Use your calculator to help you.

2 Let the number of green balls in the bag be $2x$. Let the number of blue balls be $3x$. So the total number of balls in the bag is $5x$.

Put these values into a tree diagram:



P(blue AND blue) $= \frac{3x}{5x} \times \frac{3x-1}{5x-1} = \frac{33}{95}$

Divide both sides by $\frac{3}{5}$ and cancel the x top and bottom on the left.

So $\frac{3x-1}{5x-1} = \frac{11}{19}$

$19(3x - 1) = 11(5x - 1)$

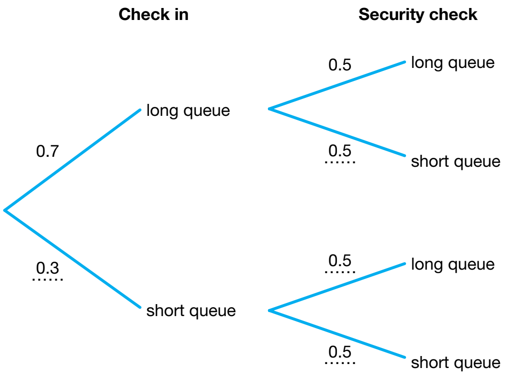
$57x - 19 = 55x - 11$

$2x = 8$

$x = 4$

Hannah put $5x = 5 \times 4 = 20$ balls into the bag.

3 **a**



b Probability $= 0.3 \times 0.5 = 0.15$

c Probability $= 1 -$ probability of short queue at both $= 1 - 0.15 = 0.85$

4 **a** Total number of students in school $= 450 + 500 = 950$

Number of male students in upper school

$= 0.6 \times 450 = 270$

Number of female students in upper school

$= 450 - 270 = 180$

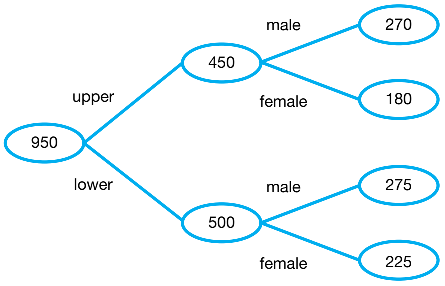
Number of male students in lower school

$= 0.55 \times 500 = 275$

Number of female students in lower school

$= 500 - 275 = 225$

This information is added to the frequency tree.



b Total males $= 270 + 275$

$= 545$

P(student is male) $= \frac{545}{950}$

$= 0.57$

Venn diagrams and probability

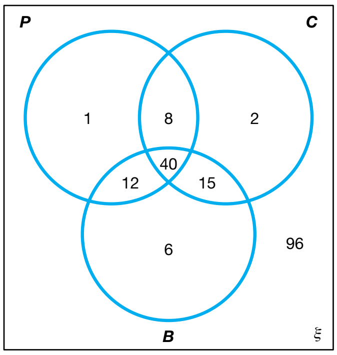
1 **a** **i** $A \cup B = \{1, 3, 4, 5, 8, 9, 10, 11\}$

ii $A \cap B = \{8, 9\}$

iii $A' = \{2, 5, 6, 10, 13\}$

b $P(B') = \frac{7}{11}$

2 Complete the diagram, using the information you are given to work out the unknown areas.

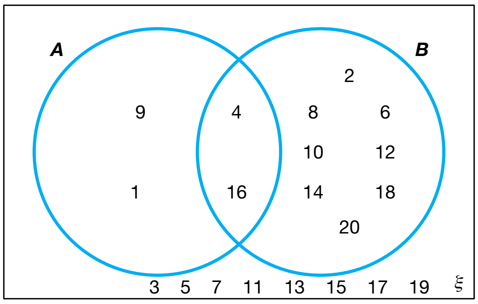


a P(all 3 sciences) $= \frac{40}{84} = \frac{10}{21}$

b P(only one science) $= \frac{(1 + 2 + 6)}{84} = \frac{9}{84} = \frac{3}{28}$

c P(chemistry if study physics) $= \frac{(8 + 40)}{(8 + 40 + 12 + 1)} = \frac{48}{61}$

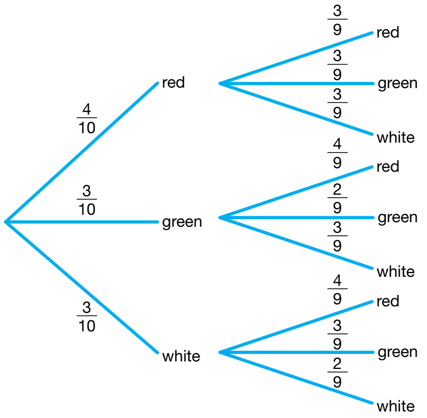
3 **a**



b $P(A \cup B)' = \frac{8}{20} = \frac{2}{5}$

Review it!

1 **a**



b Probability the same colour

$= P\ (2\ \text{red}) + P(2\ \text{green}) + P(2\ \text{white})$

$= \left(\frac{4}{10} \times \frac{3}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right)$

$= \frac{12}{90} + \frac{6}{90} + \frac{6}{90}$

$= \frac{24}{90}$

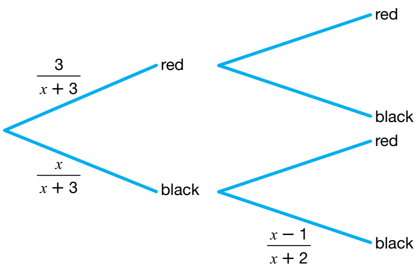
$= \frac{4}{15}$

c Probability different colours = 1 − probability of the same colour

$$= 1 - \frac{4}{15}$$

$$= \frac{11}{15}$$

2 a



Probability two black balls chosen $= \left(\frac{x}{x+3}\right) \times \left(\frac{x-1}{x+2}\right)$

Also, Probability two black balls chosen $= \frac{7}{15}$

$\left(\frac{x}{x+3}\right) \times \left(\frac{x-1}{x+2}\right) = \frac{7}{15}$

$15x(x - 1) = 7(x + 3)(x + 2)$

$15x^2 - 15x = 7x^2 + 35x + 42$

$8x^2 - 50x - 42 = 0$

$4x^2 - 25x - 21 = 0$

b Solving the quadratic equation

$4x^2 - 25x - 21 = 0$

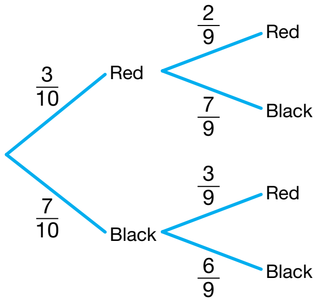
$(4x + 3)(x - 7) = 0$

$x = -\frac{3}{4}$ (which is impossible as x has to be a positive integer) or $x = 7$.

Hence $x = 7$

Total number of balls in the bag = 3 + 7 = 10

c Producing a new tree diagram now that x is known.



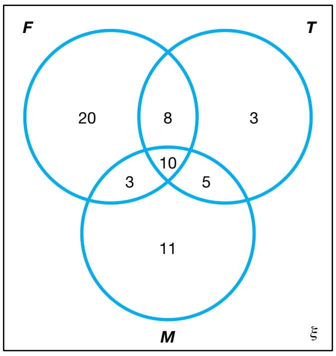
Probability of two different colours

$= \left(\frac{3}{10} \times \frac{7}{9}\right) + \left(\frac{7}{10} \times \frac{3}{9}\right)$

$= \frac{21}{90} + \frac{21}{90}$

$= \frac{7}{15}$

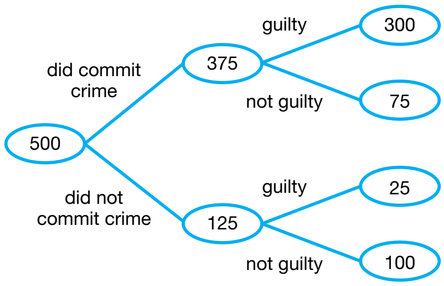
3



a P(only liked motor racing) $= \frac{11}{60}$

b P(student who liked motor racing also liked tennis) $= \frac{15}{29}$

4 a



b P(random defendant found guilty) $= \frac{300 + 25}{500}$

$= \frac{13}{20}$

c P(defendant who did not commit crime found guilty) $= \frac{25}{125} = \frac{1}{5}$

Statistics

Sampling

1 Find the % of males in the youth club

$$= \frac{\text{Number of males}}{\text{Total number of members}} \times 100$$

The total number of members is 185 + 165 = 350

$$= \frac{185}{350} \times 100$$

$$= 52.86\%$$

Now find this same % of the sample = 52.86% of 50 = 0.5286 × 50 = 26 (nearest whole number)

A quicker way to do this would be to notice that 50 = 350 ÷ 7, so:

Males chosen $= \frac{185}{7} = 26$ to nearest whole number.

Two-way tables and pie charts

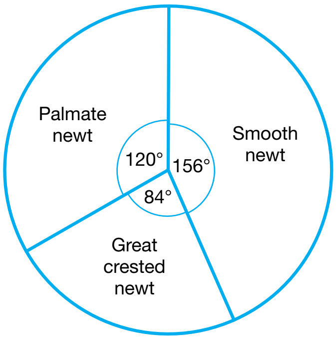
1 a Total frequency = 13 + 7 + 10 = 30

30 newts = 360° so 1 newt represents $\frac{360}{30} = 12°$

Smooth newt angle = 13 × 12 = 156°

Great crested newt angle = 7 × 12 = 84°

Palmate newt angle = 10 × 12 = 120°

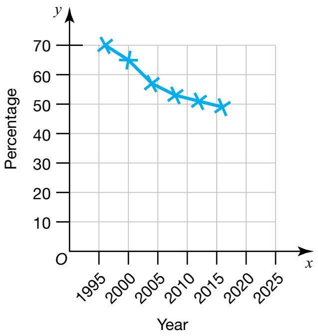


b No. The other pond might have had more newts in total. The proportion of smooth newts in the second pond is lower, but there may be more newts.

Line graphs for time series data

1 As these values are decreasing with time the trend is decreasing sales.

2 a



b The percentage of people visiting the local shop is decreasing.

c Extending the line gives 44%

d There are no points, so you would be trying to predict the future. There may be a change of ownership or a refurbishment, making it more popular. It could even close down before then.

Averages and spread

1 Mean $= \frac{\text{Total age of members}}{\text{Number of members}}$

Rearranging, we have:

Total age of members = Mean × Number of members

Total age for boys = 13 × 10 = 130

Total age for girls = 14 × 12 = 168

Total age for boys and girls = 130 + 168 = 298

Total number of boys and girls in club = 10 + 12 = 22

Mean $= \frac{\text{Total age of members}}{\text{Number of members}} = \frac{298}{22} = 13.5$ years

2 Mean $= \frac{\text{Total number of marks}}{\text{Number of students}}$

Total mark for boys = 50 × 10 = 500

Total mark for girls = 62 × 15 = 930

Total mark for the whole class of 25 = 500 + 930
= 1430

Mean for whole class $= \frac{\text{Total number of marks}}{\text{Number of students}} = \frac{1430}{25}$
$= 57.2\%$

Joshua is wrong, because he didn't take account of the fact that there were different numbers of boys and girls.

3 a

Cost (£C)	Frequency	Mid-interval value	Frequency × mid-interval value
$0 < C \leq 4$	12	2	24
$4 < C \leq 8$	8	6	48
$8 < C \leq 12$	10	10	100
$12 < C \leq 16$	5	14	70
$16 < C \leq 20$	2	18	36

b Estimated mean $= \frac{24 + 48 + 100 + 70 + 36}{37} =$ £7.51 (to nearest penny)

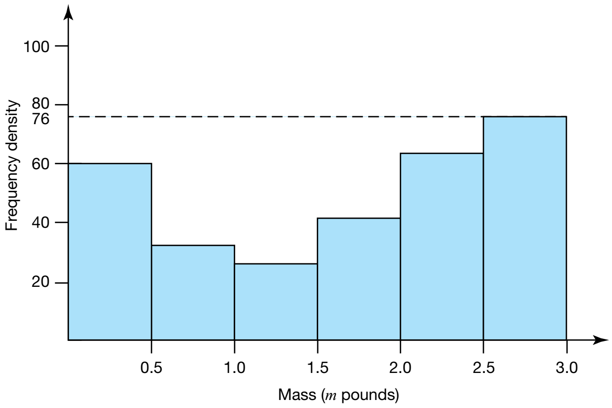
Histograms

1 Notice there is no scale for frequency density on the histogram. It is necessary to use a bar on the histogram where there is a known frequency so the frequency density can be found which will then enable the scale to be found.

Take the class $2.5 < m \leq 3.0$ which has a high frequency of 38.

Frequency density $= \frac{\text{Frequency}}{\text{Class width}} = \frac{38}{0.5} = 76$

This bar has a height of 76 and is also the highest bar so the scale can be worked out.



For the class $0.5 < m \leq 1.0$, the frequency density can be obtained from the graph (i.e. 32).

Frequency = frequency density × class width

= 32 × 0.5 = 16

For the class $2.0 < m \leq 2.5$, the frequency density from the graph is 64.

Frequency = frequency density × class width

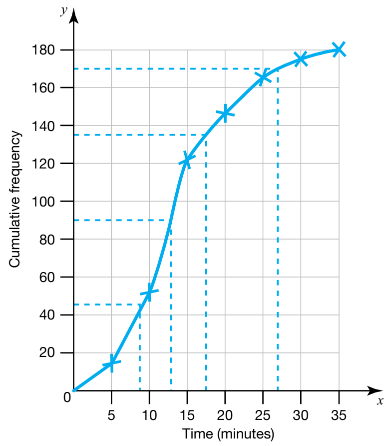
= 64 × 0.5 = 32

The table can now be completed with the new frequencies.

Mass (m pounds)	Frequency
$0.0 < m \leq 0.5$	30
$0.5 < m \leq 1.0$	**16**
$1.0 < m \leq 1.5$	13
$1.5 < m \leq 2.0$	21
$2.0 < m \leq 2.5$	**32**
$2.5 < m \leq 3.0$	38

Cumulative frequency graphs

1 a



b To read off the median find half-way through the cumulative frequency and then draw a horizontal line and where it meets the curve draw a vertical line down. The median is the value where this line cuts the horizontal axis.

Hence median = 13 minutes

c **i** To find the upper quartile read off $\frac{3}{4}$ of the way through the cumulative frequency to the curve and read off the value on the horizontal axis. Upper quartile = 17.5 minutes

ii To find the lower quartile read off $\frac{1}{4}$ of the way through the cumulative frequency to the curve and read off the value on the horizontal axis. Lower quartile = 9 minutes

iii Interquartile range = upper quartile − lower quartile = 17.5 − 9 = 8.5 minutes

d Read vertically up from 27 minutes to the curve and then across and read off the cumulative frequency. This gives 170 which means 180 − 170 = 10 waited more than 27 minutes.

$\frac{\text{Number of people waiting 27 minutes or more}}{\text{Total number of people}} \times 100 = \frac{10}{180} \times 100 = 5.6\%$ (1 d.p.)

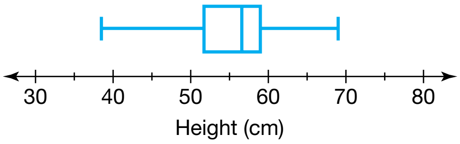
So 5.6% of passengers waited for 27 minutes or more at security.

Comparing sets of data

1

	Height (cm)
Lowest height	38
Lower quartile	**52**
Median	**57**
Upper quartile	**59**
Highest height	69

Measurements from the scale on the box plot are transferred to the table.



The lowest height and the highest height are added to complete the box plot.

2 a **i** Range = 120 − 0 = 120 marks

ii Median = 65 marks

iii Upper quartile = 75 marks

iv Lower quartile = 51 marks

v Interquartile range = 75 − 51 = 24 marks

b For the girls:

You need to find the median and upper and lower quartiles from the graph for the girls.

Median mark = 74 marks

Upper quartile = 89 marks

Lower quartile = 58 marks

Interquartile range = 89 − 58 = 31 marks

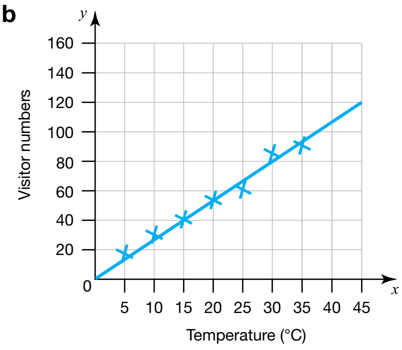
Comparison:

The median mark for the girls is higher.

The interquartile range is lower for the boys showing that their marks are less spread out for the middle half of the marks.

Scatter graphs

1 a, b



c 71 visitors

d **i** 120 visitors

ii There are no points near this temperature so you cannot assume the trend continues. As the temperature gets extremely high people might choose to stay away from the beach.

Review it!

1 **a** Number of students in class = 6 + 15 + 9 + 6 = 36

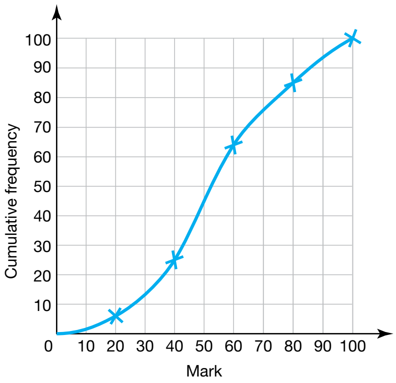
b

Amount of pocket money (£a per week)	Frequency	Mid-interval value	Frequency × mid-interval value
$2 < a \leq 4$	6	3	6 × 3 = 18
$4 < a \leq 6$	15	5	15 × 5 = 75
$6 < a \leq 8$	9	7	9 × 7 = 63
$8 < a \leq 10$	6	9	6 × 9 = 54

Estimate for mean = $\frac{18 + 75 + 63 + 54}{36}$ = £5.83 (to nearest penny)

c Modal class interval is $4 < a \leq 6$

2 **a**



b Median = 54 (i.e. the mark corresponding to half-way through the cumulative frequency)

c The top 75% would pass so the bottom 25% would fail. This 25% corresponds to a cumulative frequency of 25 which gives a pass mark of 40.

3 **a** D (the median mark is furthest to the right)

b C (the largest gap between the quartiles)

c D. The median mark is the highest and the interquartile range is small, which means 50% of pupils got near to the median mark.

4 **a** The sample needs to be representative of the people living on the street, so they should take a stratified sample that includes different groups such as males and females, adults and children, in the right proportions.

b $\frac{15}{40} \times 320 = 120$

They should buy lemonade for 120 people.

AQA Higher Mathematics Exam Practice Book

Full worked solutions

Number

Integers, decimals and symbols

1 $(-1)^3 = -1$

$(0.1)^2 = 0.01$

$\frac{1}{1000} = 0.001$

0.1

$\frac{1}{0.01} = 100$

Descending order: $\frac{1}{0.01}$ 0.1 $(0.1)^2$ $\frac{1}{1000}$ $(-1)^3$

2 **a** $0.035 \times 1000 = 35$

b $12.85 \div 1000 = 0.01285$

c $(-3) \times 0.09 \times 1000 = -0.27 \times 1000 = -270$

d $(-1) \times (-0.4) \times 100 = 0.4 \times 100 = 40$

3 **a** $86 \times 54 = (0.86 \times 100) \times 54 = 46.44 \times 100 = 4644$

b $8.6 \times 540 = (0.86 \times 10) \times (54 \times 10) = 46.44 \times 100 = 4644$

c $\frac{4644}{54} = \frac{46.44 \times 100}{54} = 0.86 \times 100 = 86$

d $\frac{46.44}{0.086} = \frac{46.44}{0.86 \div 10} = 54 \times 10 = 540$

4 **a** $12.56 \times 3.45 = 0.1256 \times 345$

b $(-8)^2 = 64$, so $(-8)^2 > -64$

c $6 - 12 = -6$ and $8 - 14 = -6$, so $6 - 12 = 8 - 14$

d $(-7) \times (0) = 0$ and $(-7) \times (-3) = 21$,

so $(-7) \times (0) < (-7) \times (-3)$

Addition, subtraction, multiplication and division

1 **a** $67.78 + 8.985 = 76.765$

b $124.706 + 76.9 + 0.04 = 201.646$

c $93.1 - 1.77 = 91.33$

d $23.7 + 8.94 - 22.076 = 32.64 - 22.076 = 10.564$

2 **a** $147 \times 8 = 1176$

b $57 \times 38 = 2166$

c $9.7 \times 4.6 = 44.62$

d $1.24 \times 0.53 = 0.6572$

e $486 \div 18 = 27$

f $94.5 \div 1.5 = 945 \div 15 = 63$

3 **a** $34^2 = 34 \times 34 = 1156$

b $\frac{1.5 \times 2.5}{0.5} = 3 \times 2.5 = 7.5$

c $2.4^2 = 2.4 \times 2.4 = 5.76$

Using fractions

1 $\frac{2}{5} = \frac{16}{40} = \frac{30}{75} = \frac{50}{125}$

2 **a** $\frac{64}{12} = 5\frac{4}{12} = 5\frac{1}{3}$

b $\frac{124}{13} = 9\frac{7}{13}$

3 **a** $4\frac{1}{4} \times 1\frac{2}{3} = \frac{17}{4} \times \frac{5}{3} = \frac{85}{12} = 7\frac{1}{12}$

b $1\frac{7}{8} \div \frac{1}{4} = \frac{15}{8} \times \frac{4}{1} = \frac{60}{8} = \frac{15}{2} = 7\frac{1}{2}$

c $3\frac{1}{5} - \frac{3}{4} = 3\frac{4}{20} - \frac{15}{20} = 2\frac{24}{20} - \frac{15}{20} = 2\frac{9}{20}$

4 $1 - \left(\frac{2}{7} + \frac{3}{8} + \frac{1}{4}\right) = 1 - \frac{2 \times 8 + 3 \times 7 + 1 \times 14}{56} = 1 - \frac{51}{56} = \frac{5}{56}$

$\frac{5}{56}$ of the students travel by car.

5 The lowest common multiple of all the denominators is 24.

$\frac{2}{3} = \frac{16}{24}$ $\frac{3}{4} = \frac{18}{24}$ $\frac{7}{8} = \frac{21}{24}$ $\frac{1}{2} = \frac{12}{24}$ $\frac{7}{12} = \frac{14}{24}$

Order with the smallest first:

$\frac{1}{2}$ $\frac{7}{12}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{7}{8}$

Different types of number

1 **a** 7

b 49

c 2

d 6

e 6

2 **a** $693 = 3^2 \times 7 \times 11$

b $945 = 3^3 \times 5 \times 7$

3^2 and 7 are common to both lists.

Highest common factor $= 3^2 \times 7 = 63$

c Lowest common multiple $= 3^3 \times 5 \times 7 \times 11 = 10\,395$

3 $49 = 7^2$

$63 = 9 \times 7 = 3 \times 3 \times 7 = 3^2 \times 7$

Lowest common multiple $= 3^2 \times 7^2 = 441$

4 $15 = 3 \times 5$

$20 = 4 \times 5 = 2^2 \times 5$

$25 = 5^2$

Lowest common multiple $= 2^2 \times 3 \times 5^2 = 300$ seconds $= 5$ minutes

They will make a note together after 5 minutes.

Listing strategies

1 Multiples of 14:

14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154, 168, 182, 196, 210, …

Multiples of 15:

15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, 195, 210, …

210 is common to both lists, so they will sound together after 210 seconds.

2 Multiples of 6:

6, 12, 18, 24, 30, 36, 42, …

Multiples of 6 add 1:

7, 13, 19, 25, 31, 37, 43, …

Multiples of 7:

7, 14, 21, 28, 35, 42, 49, 56, …

Multiples of 7 minus 4:

3, 10, 17, 24, 31, 38, 45, 52

31 is common to both lists. This means 6 chocolates could be given to 5 friends with one chocolate left over.

Hence number of friends = 5

3 List all the ratios that are equivalent to 5 : 6, find the difference between them and look for a difference of 100.

Since this is a college, the numbers are likely to be large.

Male	**Female**	**Difference**
50	60	10
500	600	100

There are a total of $500 + 600 = 1100$ students.

4 Call the students A, B, C, D, E and F.

Possible pairs beginning with A:

AB, AC, AD, AE, AF

Possible pairs beginning with B (excluding pairs in first list):

BC, BD, BE, BF

Possible pairs beginning with C (excluding pairs in previous lists):

CD, CE, CF

Possible pairs beginning with D (excluding pairs in previous lists):

DE, DF

Possible pairs beginning with E (excluding pairs in previous lists):

EF

Total number of pairs = 5 + 4 + 3 + 2 + 1 = 15

The order of operations in calculations

1 **a** Ravi has worked out the expression from left to right, instead of using BIDMAS. He should have performed the division and multiplication before the addition.

b $24 \div 3 + 8 \times 4 = 8 + 32$

$= 40$

2 **a** $9 \times 7 \times 2 - 24 \div 6 = 126 - 4 = 122$

b $2 - (-27) \div (-3) + 4 = 2 - 9 + 4 = -3$

c $(4 - 16)^2 \div 4 + 32 \div 8 = (-12)^2 \div 4 + 4 = 144 \div 4 + 4$
$= 36 + 4 = 40$

3 **a** $1 + 4 \div \frac{1}{2} - 3 = 1 + 8 - 3 = 6$

b $15 - (1 - 2)^2 = 15 - (-1)^2 = 15 - 1 = 14$

c $\sqrt{4 \times 12 - 2 \times (-8)} = \sqrt{48 + 16} = \sqrt{64} = 8$

Indices

1 **a** $10^5 \times 10 = 10^{5+1} = 10^6$

b $(10^4)^2 = 10^{4 \times 2} = 10^8$

c $\frac{10^5 \times 10^3}{10^2} = 10^{5+3-2} = 10^6$

d $(10^6)^{\frac{1}{2}} = 10^{6 \times \frac{1}{2}} = 10^3$

2 **a** $5^0 = 1$

b $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

c $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

d $49^{\frac{1}{2}} = \sqrt{49} = 7$

3 **a** $\left(\frac{8}{27}\right)^{-\frac{1}{3}} = \left(\frac{27}{8}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2}$

b $\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2 = 4^2 = 16$

c $36^{-\frac{1}{2}} = \frac{1}{\sqrt{36}} = \frac{1}{6}$

d $16^{\frac{3}{2}} = \left(16^{\frac{1}{2}}\right)^3 = (\sqrt{16})^3 = 4^3 = 64$

4 $125 = 5^3$, so $5^{2x} = 5^3$

$2x = 3$ giving $x = 1.5$

Surds

1 **a** $(\sqrt{5})^2 = \sqrt{5} \times \sqrt{5} = 5$

b $3\sqrt{2} \times 5\sqrt{2} = 3 \times 5 \times \sqrt{2} \times \sqrt{2} = 15 \times 2 = 30$

c $(3\sqrt{2})^2 = 3^2 \times (\sqrt{2})^2 = 9 \times 2 = 18$

2 $\frac{15}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{15 \times \sqrt{3}}{4 \times 3} = \frac{5\sqrt{3}}{4}$

3 $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 2\sqrt{3} + 2\sqrt{3} - 3 = 1$

4 $\sqrt{3}\,(3\sqrt{24} + 4\sqrt{6}) = 3\sqrt{72} + 4\sqrt{18}$

$= 3\sqrt{36 \times 2} + 4\sqrt{9 \times 2}$

$= 3 \times 6\sqrt{2} + 4 \times 3\sqrt{2}$

$= 18\sqrt{2} + 12\sqrt{2}$

$= 30\sqrt{2}$

$a = 30$

5 $(1 - \sqrt{5})(3 + 2\sqrt{5}) = 3 + 2\sqrt{5} - 3\sqrt{5} - 10 = -\sqrt{5} - 7$

6 $\frac{1}{\sqrt{2}} + \frac{1}{4} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{1}{4} = \frac{\sqrt{2}}{2} + \frac{1}{4} = \frac{2\sqrt{2}}{4} + \frac{1}{4} = \frac{1 + 2\sqrt{2}}{4}$

7 $\frac{2}{1 - 1/\sqrt{2}} = \frac{2}{\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}}}$

$= \frac{2}{\frac{\sqrt{2} - 1}{\sqrt{2}}}$

$= \frac{2\sqrt{2}}{\sqrt{2} - 1}$

$= \frac{2\sqrt{2}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$

$= \frac{4 + 2\sqrt{2}}{2 - 1}$

$= 4 + 2\sqrt{2}$

8 $\frac{3}{\sqrt{3}} + \sqrt{75} + (\sqrt{2} \times \sqrt{6}) = \frac{3\sqrt{3}}{3} + \sqrt{3 \times 25} + \sqrt{12}$

$= \sqrt{3} + 5\sqrt{3} + \sqrt{3 + 4}$

$= \sqrt{3} + 5\sqrt{3} + 2\sqrt{3}$

$= 8\sqrt{3}$

Standard form

1 **a** 2.55×10^{-3}

b 1.006×10^{10}

c 8.9×10^{-8}

2 **a** $3 \times 10^6 \times 2 \times 10^8 = 3 \times 2 \times 10^{6+8} = 6 \times 10^{14}$

b $5.5 \times 10^{-3} \times 2 \times 10^8 = 5.5 \times 2 \times 10^{-3+8} = 11 \times 10^5$
$= 1.1 \times 10^6$

c $\frac{8 \times 10^5}{4 \times 10^3} = \frac{8}{4} \times 10^{5-3} = 2 \times 10^2$

d $\frac{5 \times 10^{-3}}{0.5} = \frac{5}{0.5} \times 10^{-3} = 10 \times 10^{-3} = 1 \times 10^{-2}$

e $\frac{4.5 \times 10^{-6}}{0.5 \times 10^{-3}} = \frac{4.5}{0.5} \times \frac{10^{-6}}{10^{-3}} = 9 \times 10^{-6-(-3)} = 9 \times 10^{-3}$

3 $2.4 \times 10^3 + 2.8 \times 10^2 = 2400 + 280 = 2680$

4 $3.3 \times 10^4 = 33\,000$ and $3.3 \times 10^2 = 330$

$3.3 \times 10^4 + 3.3 \times 10^2 = 33\,330$

$a = 3.3$

Converting between fractions and decimals

1 **a** $\frac{11}{20} = \frac{55}{100} = 0.55$

b $\frac{3}{8} = 3 \div 8 = 0.375$

2 **a** $16 = 2 \times 2 \times 2 \times 2$

All the prime factors in the denominator are 2, so it will produce a terminating fraction.

b The prime factors of the denominators are 7 and 1 so this will produce a recurring decimal.

c Prime factors of 35 are 7 and 5 so this will produce a recurring decimal.

3 Let $x = 0.\dot{4}0\dot{2} = 0.402402402\ldots$

$1000x = 402.402402\ldots$

$1000x - x = 402.402402\ldots - 0.402402402\ldots$

$999x = 402$

$x = \frac{402}{999} = \frac{134}{333}$

Hence $0.\dot{4}0\dot{2} = \frac{134}{333}$

4 Let $x = 0.6\dot{5}\dot{2} = 0.652525252\ldots$

$10x = 6.52525252\ldots$

$$1000x = 652.52525252\ldots$$
$$1000x - 10x = 652.52525252\ldots - 6.52525252\ldots$$
$$990x = 646$$
$$x = \frac{646}{990} = \frac{323}{495}$$

Hence $0.6\dot{5}\dot{2} = \frac{323}{495}$

Converting between fractions and percentages

1 **a** $35\% = \frac{35}{100} = \frac{7}{20}$

b $7\% = \frac{7}{100}$

c $76\% = \frac{76}{100} = \frac{19}{25}$

d $12.5\% = \frac{12.5}{100} = \frac{125}{1000} = \frac{1}{8}$

2 **a** $\frac{1}{5} = \frac{1 \times 20}{5 \times 20} = \frac{20}{100} = 20\%$

b $\frac{17}{25} = \frac{17 \times 4}{25 \times 4} = \frac{68}{100} = 68\%$

c $\frac{150}{60} = \frac{50}{20} = \frac{250}{100} = 250\%$

d $\frac{7}{40} = 7 \div 40 = 0.7 \div 4 = 0.175 = 17.5\%$

3 $\frac{8}{15} = \frac{8}{15} \times 100\% = 53.33\%$ (to 2 d.p.)

4 $\frac{66}{90} = \frac{66}{90} \times 100\% = 73.3\%$ (to 1 d.p.)

Jake did better in chemistry.

Fractions and percentages as operators

1 70% of £49.70 = 0.7×49.70 = £34.79

2 8% of 600 = $\frac{8}{100} \times 600 = 48$

48 apples are bad.

3 12% of 8000 = $0.12 \times 8000 = 960$ components were rejected.

$8000 - 960 = 7040$ components were accepted.

4 **a** VAT = 20% of £12 000 = $0.2 \times 12\,000$ = £2400

Total cost = 12 000 + 2400 = £14 400

b Deposit = 20% of £14 400 = $0.2 \times 14\,400$ = £2880

Remainder = 14 400 − 2880 = £11 520

Monthly payment = $\frac{11\,520}{36}$ = £320

5 $\frac{2}{3}$ of $\frac{7}{11} = \frac{2}{3} \times \frac{7}{11} = \frac{14}{33}$

Standard measurement units

1 1.75 km = 1.75×1000 m = 1750 m

1 m = 100 cm, so 1750 m = 1750×100 = 175 000 cm

Hence 1.75 km = 175 000 cm

2 3 litres = 3000 ml

Number of glasses = $\frac{3000}{175} = 17.14$

Number of complete glasses = 17

3 900 litres = 900×1000 = 900 000 cm^3

Number of containers needed to fill pool = $\frac{900\,000}{700} = 1286$ (to nearest whole number)

4 **a** Mass of one atom = $\frac{12}{6.02 \times 10^{23}} = 1.99 \times 10^{-23}$ g (to 3 s.f.)

b Mass of one atom = $\frac{12 \times 10^{-3}}{6.02 \times 10^{23}} = 1.99 \times 10^{-26}$ kg (to 3 s.f.)

5 $79 \times 9.11 \times 10^{-31}$ kg = $79 \times 9.11 \times 10^{-31} \times 10^{3}$ g
= 7.20×10^{-26} g (to 3 s.f.)

Rounding numbers

1 **a** 35 **c** 0 **e** 2
b 101 **d** 0

2 **a** 34.88 **b** 34.877

3 **a** 12 800

b 0.011

c 7×10^{-5}

4 **a** $(0.18 \times 0.046)^2 - 0.01 = -0.00993$ (to 3 s.f.)

b $\frac{1200 \times 1.865}{2.6 \times 25} = 34.4$ (to 3 s.f.)

c $\frac{36}{0.07} \times 12 \div \frac{1}{2} = 12\,300$ (to 3 s.f.)

Estimation

1 $4.6 \times 9.8 \times 3.1 \approx 5 \times 10 \times 3 = 150 \approx 200$ (to 1 s.f.)

2 **a** 236.229 8627

b $19.87^2 - \sqrt{404} \times 7.89 \approx 20^2 - \sqrt{400} \times 8$

$= 400 - 20 \times 8 = 240$

3 $(0.52 \times 0.83)^2 \approx (0.5 \times 0.8)^2 = (0.4)^2 = 0.16$

4 $\sqrt{5.08 + 4.10 \times 5.45} \approx \sqrt{5 + 4 \times 5} = \sqrt{25} = 5$

5 $\sqrt{100} = 10$ and $\sqrt{121} = 11$

$\sqrt{112}$ is slightly over halfway between the two roots.

Hence $\sqrt{112} = 10.6$ (to 1 d.p.) is a reasonable estimate.

6 $\frac{0.89 \times 7.51 \times 19.76}{2.08 \times 5.44 \times 3.78} \approx \frac{1 \times 8 \times 20}{2 \times 5 \times 4} = \frac{160}{40} = 4$

7 **a** mass $\approx 9 \times 10^{-31} \times 6 \times 100 = 5.4 \times 10^{-28} \approx 5 \times 10^{-28}$ kg

b This will be an underestimate, as the mass of one electron has been rounded down.

Upper and lower bounds

1 upper bound = 2.345 kg

lower bound = 2.335 kg

$2.335 \leq l < 2.345$ kg

2 **a** **i** $V = \frac{P}{I}$

The upper bound for V will be when P has its upper bound and I has its lower bound.

upper bound for $P = 3.0525$

lower bound for $I = 1.235$

upper bound for $V = \frac{3.0525}{1.235} = 2.472$

ii The lower bound for V will be when P has its lower bound and I has its upper bound.

lower bound of $P = 3.0515$

upper bound of $I = 1.245$

lower bound for $V = \frac{3.0515}{1.245} = 2.451$

b The upper bound and the lower bound are the same when the numbers are given to 2 significant figures.

$V = 2.5$ (to 2 s.f.)

3 lower bound for shelf length = 1.05 m = 105 cm

upper bound for book thickness = 3.05 cm

number of books on shelf = $\frac{\text{length of shelf}}{\text{thickness of book}}$

number of books that will definitely fit onto shelf =

minimum value for $\frac{\text{length of shelf}}{\text{thickness of book}}$

minimum value for $\frac{\text{length of shelf}}{\text{thickness of book}}$ is when shelf length is at its lower bound and book thickness is at its upper bound.

$\frac{\text{minimum length of shelf}}{\text{maximum thickness of book}} = \frac{105}{3.05} = 34.4$

The answer must be a whole number, so 34 books will definitely fit on the shelf.

Algebra

Simple algebraic techniques

1 **a** formula **c** expression **e** formula
b identity **d** identity

2 $4x + 3x \times 2x - 3x = 4x + 6x^2 - 3x = x + 6x^2$

3 $y^3 - y = (1)^3 - 1 = 0$ so $y = 1$ is correct.
$y^3 - y = (-1)^3 - (-1) = -1 + 1 = 0$ so $y = -1$ is correct.

4 **a** $6x - (-4x) = 6x + 4x = 10x$
b $x^2 - 2x - 4x + 3x^2 = 4x^2 - 6x$
c $(-2x)^2 + 6x \times 3x - 4x^2 = 4x^2 + 18x^2 - 4x^2 = 18x^2$

5 **a** $s = \frac{3^2 - 1^2}{2 \times 2} = \frac{8}{4} = 2$
b $s = \frac{(-4)^2 - 3^2}{2 \times 4} = \frac{7}{8}$
c $s = \frac{5^2 - (-2)^2}{2 \times (-7)} = \frac{21}{-14} = -\frac{3}{2}$

Removing brackets

1 **a** $$\begin{aligned}8(3x - 7) &= 8 \times 3x - 8 \times 7\\ &= 24x - 56\end{aligned}$$
b $$\begin{aligned}-3(2x - 4) &= -3 \times 2x - 3 \times (-4)\\ &= -6x + 12\end{aligned}$$

2 **a** $$\begin{aligned}3(2x - 1) - 3(x - 4) &= 6x - 3 - 3x + 12\\ &= 3x + 9\end{aligned}$$
b $$\begin{aligned}4y(2x + 1) + 6(x - y) &= 8xy + 4y + 6x - 6y\\ &= 8xy + 6x - 2y\end{aligned}$$
c $5ab(2a - b) = 10a^2b - 5ab^2$
d $x^2y^3(2x + 3y) = 2x^3y^3 + 3x^2y^4$

3 **a** $$\begin{aligned}(m - 3)(m + 8) &= m^2 + 8m - 3m - 24\\ &= m^2 + 5m - 24\end{aligned}$$
b $$\begin{aligned}(4x - 1)(2x + 7) &= 8x^2 + 28x - 2x - 7\\ &= 8x^2 + 26x - 7\end{aligned}$$
c $$\begin{aligned}(3x - 1)^2 &= (3x - 1)(3x - 1)\\ &= 9x^2 - 3x - 3x + 1\\ &= 9x^2 - 6x + 1\end{aligned}$$
d $$\begin{aligned}(2x + y)(3x - y) &= 6x^2 - 2xy + 3xy - y^2\\ &= 6x^2 + xy - y^2\end{aligned}$$

4 **a** $$\begin{aligned}(x + 5)(x + 2) &= x^2 + 2x + 5x + 10\\ &= x^2 + 7x + 10\end{aligned}$$
b $$\begin{aligned}(x + 4)(x - 4) &= x^2 - 4x + 4x - 16\\ &= x^2 - 16\end{aligned}$$
c $$\begin{aligned}(x - 7)(x + 1) &= x^2 + x - 7x - 7\\ &= x^2 - 6x - 7\end{aligned}$$
d $$\begin{aligned}(3x + 1)(5x + 3) &= 15x^2 + 9x + 5x + 3\\ &= 15x^2 + 14x + 3\end{aligned}$$

5 **a** $$\begin{aligned}(x + 3)(x - 1)(x + 4) &= (x^2 - x + 3x - 3)(x + 4)\\ &= (x^2 + 2x - 3)(x + 4)\\ &= x^3 + 4x^2 + 2x^2 + 8x - 3x - 12\\ &= x^3 + 6x^2 + 5x - 12\end{aligned}$$
b $$\begin{aligned}(3x - 4)(2x - 5)(3x + 1) &= (6x^2 - 15x - 8x + 20)(3x + 1)\\ &= (6x^2 - 23x + 20)(3x + 1)\\ &= 18x^3 + 6x^2 - 69x^2 - 23x + 60x\\ &\quad + 20\\ &= 18x^3 - 63x^2 + 37x + 20\end{aligned}$$

Factorising

1 **a** $25x^2 - 5xy = 5x(5x - y)$
b $4\pi r^2 + 6\pi x = 2\pi(2r^2 + 3x)$
c $6a^3b^2 + 12ab^2 = 6ab^2(a^2 + 2)$

2 **a** $9x^2 - 1 = (3x + 1)(3x - 1)$
b $$\begin{aligned}16x^2 - 4 &= (4x + 2)(4x - 2)\\ &= 4(2x + 1)(2x - 1)\end{aligned}$$

3 **a** $a^2 + 12a + 32 = (a + 4)(a + 8)$
b $p^2 - 10p + 24 = (p - 6)(p - 4)$

4 **a** $a^2 + 12a = a(a + 12)$
b $b^2 - 9 = (b + 3)(b - 3)$
c $x^2 - 11x + 30 = (x - 5)(x - 6)$

5 **a** $3x^2 + 20x + 32 = (3x + 8)(x + 4)$
b $3x^2 + 10x - 13 = (3x + 13)(x - 1)$
c $2x^2 - x - 10 = (2x - 5)(x + 2)$

6 $$\begin{aligned}\frac{x + 15}{2x^2 - 3x - 9} + \frac{3}{2x + 3} &= \frac{x + 15}{(2x + 3)(x - 3)} + \frac{3}{(2x + 3)}\\ &= \frac{x + 15 + 3(x - 3)}{(2x + 3)(x - 3)}\\ &= \frac{4x + 6}{(2x + 3)(x - 3)}\\ &= \frac{2(2x + 3)}{(2x + 3)(x - 3)}\\ &= \frac{2}{(x - 3)}\end{aligned}$$

7 $$\begin{aligned}\frac{1}{8x^2 - 2x - 1} \div \frac{1}{4x^2 - 4x + 1} &= \frac{1}{8x^2 - 2x - 1} \times (4x^2 - 4x + 1)\\ &= \frac{1}{(4x + 1)(2x - 1)} \times (2x - 1)(2x - 1)\\ &= \frac{2x - 1}{4x + 1}\end{aligned}$$

Changing the subject of a formula

1 $PV = nRT$
$T = \frac{PV}{nR}$

2 $$\begin{aligned}2y + 4x - 1 &= 0\\ 2y &= 1 - 4x\\ y &= \frac{1 - 4x}{2}\end{aligned}$$

3 $$\begin{aligned}v &= u + at\\ at &= v - u\\ a &= \frac{(v - u)}{t}\end{aligned}$$

4 $$\begin{aligned}y &= \frac{x}{5} - m\\ \frac{x}{5} &= y + m\\ x &= 5(y + m)\end{aligned}$$

5 $$\begin{aligned}E &= \frac{1}{2}mv^2\\ v^2 &= \frac{2E}{m}\\ v &= \sqrt{\frac{2E}{m}}\end{aligned}$$

6 **a** $$\begin{aligned}V &= \frac{1}{3}\pi r^2h\\ r^2 &= \frac{3V}{\pi h}\\ r &= \sqrt{\frac{3V}{\pi h}}\end{aligned}$$
b $$\begin{aligned}r &= \sqrt{\frac{3 \times 100}{\pi \times 8}}\\ &= 3.45\,\text{cm (to 2 d.p.)}\end{aligned}$$

7 **a** $y = 3x - 9$

$3x = y + 9$

$x = \frac{y + 9}{3}$

b $x = \frac{3 + 9}{4}$

$= 4$

8 $3y - x = ax + 2$

$3y - x - 2 = ax$

$3y - 2 = ax + x$

$3y - 2 = x(a + 1)$

$x = \frac{3y - 2}{a + 1}$

9 **a** $\mathrm{c}^2 = \frac{(16a^2b^4c^2)^{\frac{1}{2}}}{4a^2b}$

$c^2 = \frac{4ab^2c}{4a^2b}$

$c^2 = \frac{bc}{a}$

$c = \frac{b}{a}$

b upper bound of $a = 2.85$ lower bound of $a = 2.75$

upper bound of $b = 3.25$ lower bound of $b = 3.15$

upper bound for $c = \frac{\text{upper bound for } b}{\text{lower bound for } a} = \frac{3.25}{2.75} = 1.18$ (to 3 s.f.)

lower bound for $c = \frac{\text{lower bound for } b}{\text{upper bound for } a} = \frac{3.15}{2.85} = 1.11 =$ (to 3 s.f.)

Solving linear equations

1 **a** $2x + 11 = 25$

$2x = 14$

$x = 7$

b $3x - 5 = 10$

$3x = 15$

$x = 5$

c $15x = 60$

$x = 4$

d $\frac{x}{4} = 8$

$x = 32$

e $\frac{4x}{5} = 20$

$4x = 100$

$x = 25$

f $\frac{2x}{3} = -6$

$2x = -18$

$x = -9$

g $5 - x = 7$

$5 = 7 + x$

$-2 = x$

$x = -2$

h $\frac{x}{7} - 9 = 3$

$\frac{x}{7} = 12$

$x = 84$

2 $5x - 1 = 2x + 1$

$5x = 2x + 2$

$3x = 2$

$x = \frac{2}{3}$

3 **a** $\frac{1}{4}(2x-1) = 3(2x-1)$

$2x - 1 = 12(2x - 1)$

$2x - 1 = 24x - 12$

$11 = 22x$

$x = \frac{11}{22}$

$x = \frac{1}{2}$

b $5(3x + 1) = 2(5x - 3) + 3$

$15x + 5 = 10x - 6 + 3$

$15x + 5 = 10x - 3$

$5x = -8$

$x = -\frac{8}{5}$

Solving quadratic equations using factorisation

1 **a** $x^2 - 7x + 12 = (x - 3)(x - 4)$

b $x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

So $x - 3 = 0$ or $x - 4 = 0$, giving $x = 3$ or $x = 4$

2 **a** $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

b $2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

So $2x - 1 = 0$ or $x + 3 = 0$, giving $x = \frac{1}{2}$ or $x = -3$

3 $x^2 - 3x - 20 = x - 8$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

So $x + 2 = 0$ or $x - 6 = 0$, giving $x = -2$ or $x = 6$

4 **a** $x(x - 8) - 7 = x(5 - x)$

$x^2 - 8x - 7 = 5x - x^2$

$2x^2 - 13x - 7 = 0$

b $2x^2 - 13x - 7 = 0$

$(2x + 1)(x - 7) = 0$

So $2x + 1 = 0$ or $x - 7 = 0$, giving $x = -\frac{1}{2}$ or $x = 7$

5 area of trapezium $= \frac{1}{2}(a + b)h$

$= \frac{1}{2}(x + 4 + x + 8)x$

$= \frac{1}{2}(2x + 12)x$

$= (x + 6)x$

$= x^2 + 6x$

area = 16 cm² so $x^2 + 6x = 16$

$x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$

Solving gives $x = -8$ or $x = 2$

$x = -8$ is impossible as x is the height and so cannot be negative.

Hence $x = 2$ cm

Solving quadratic equations using the formula

1 **a** $\frac{3}{x + 7} = \frac{2 - x}{x + 1}$

$3(x + 1) = (2 - x)(x + 7)$

$3x + 3 = 2x + 14 - x^2 - 7x$

$3x + 3 = -x^2 - 5x + 14$

$x^2 + 8x - 11 = 0$

b $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times (-11)}}{2 \times 1}$

$= \frac{-8 \pm \sqrt{64 + 44}}{2}$

$= \frac{-8 \pm \sqrt{108}}{2}$

$= \frac{-8 + \sqrt{108}}{2}$ or $\frac{-8 - \sqrt{108}}{2}$

$= 1.1962$ or -9.1962

$x = 1.20$ or -9.20 (to 2 d.p.)

2 area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2}(3x + 1)(2x + 3)$

$= \frac{1}{2}(6x^2 + 9x + 2x + 3)$

$= \frac{1}{2}(6x^2 + 11x + 3)$

$= 3x^2 + 5.5x + 1.5$

area = 40, so $3x^2 + 5.5x + 1.5 = 40$

Hence, $3x^2 + 5.5x - 38.5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-5.5 \pm \sqrt{5.5^2 - 4 \times 3 \times (-38.5)}}{2 \times 3}$$
$$= \frac{-5.5 \pm \sqrt{492.25}}{6}$$
$$= \frac{-5.5 + \sqrt{492.25}}{6} = \text{or } \frac{-5.5 - \sqrt{492.25}}{6}$$

= 2.78 or −4.61 (to 2 d.p.)

$x = -4.61$ would give negative lengths, which are impossible.

$x = 2.78$ cm (to 2 d.p.)

3 $x^2 - 2x - 9 = x - 8$

$x^2 - 3x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$
$$= \frac{3 \pm \sqrt{9 + 4}}{2}$$
$$= \frac{3 \pm \sqrt{13}}{2}$$
$$= \frac{3 + \sqrt{13}}{2} = \text{or } \frac{3 - \sqrt{13}}{2}$$

$x = 3.30$ or -0.30 (to 2 d.p.)

Solving simultaneous equations

1 $2x - 3y = -5$ (1)

$5x + 2y = 16$ (2)

Equation (1) × 2 and equation (2) × 3 gives:

$4x - 6y = -10$ (3)

$15x + 6y = 48$ (4)

Equation (3) + equation (4) gives:

$19x = 38$

$x = 2$

Substituting $x = 2$ into equation (1):

$2 \times 2 - 3y = -5$

$4 - 3y = -5$

$3y = 9$

$y = 3$

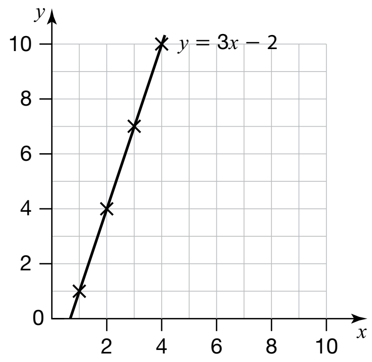
Checking by substituting $x = 2$ and $y = 3$ into equation (2) gives:

$5 \times 2 + 2 \times 3 = 10 + 6 = 16$

$x = 2$ and $y = 3$

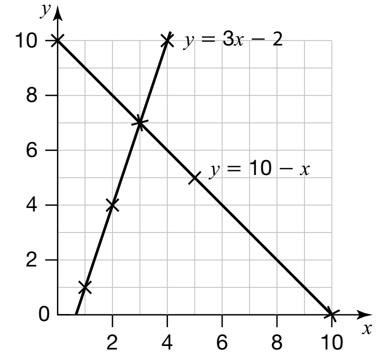
2 **a** Table of values for plotting graph of $y = 3x - 2$:

x	1	2	3	4
y	1	4	7	10



b Table of values for plotting graph of $y = 10 - x$:

x	0	5	10
y	10	5	0



The graphs intersect at (3, 7).

$x = 3, y = 7$

3 $x - y = 3$ (1)

$x^2 + y^2 = 9$ (2)

Rearrange equation (1) as $y = x - 3$.

Substitute in equation (2):

$x^2 + (x - 3)^2 = 9$

$x^2 + x^2 - 6x + 9 = 9$

$2x^2 - 6x = 0$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0$ or $x = 3$

Substituting into equation (1) gives:

$x = 0, y = -3$ or $x = 3, y = 0$

Solving inequalities

1 **a** $\frac{x + 5}{4} \geq -1$

$x + 5 \geq -4$

$x \geq -9$

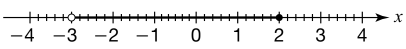
b $3x - 4 > 4x + 8$

$-x - 4 > 8$

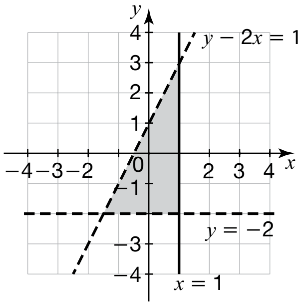
$-x > 12$

$x < -12$

2



3 **a**



b Coordinates of points that lie in the shaded region or on the solid line:

(1, 2), (1, 1), (1, 0), (1, −1), (0, 0), (0, −1),

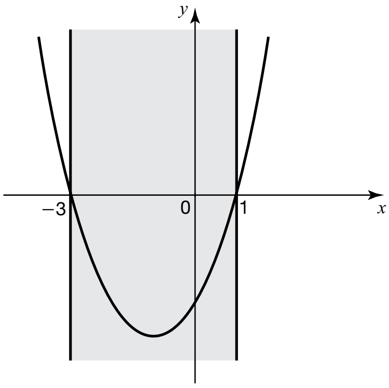
4 $x^2 + 2x \leq 3$

$x^2 + 2x - 3 \leq 0$

Solving $x^2 + 2x - 3 = 0$:

$(x - 1)(x + 3) = 0$, giving $x = 1$ and $x = -3$

Sketch of the curve $y = x^2 + 2x - 3$:



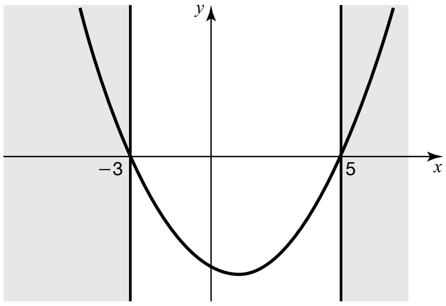
From graph, $x^2 + 2x - 3 \leq 0$ when:

$-3 \leq x \leq 1$

5 Solving $x^2 - 2x - 15 = 0$:

$(x - 5)(x + 3) = 0$, giving $x = 5$ and $x = -3$

Sketch of the curve $y = x^2 - 2x - 15$:



From graph, $x^2 - 2x - 15 > 0$ when:

$x < -3$ and $5 < x$

Problem solving using algebra

1 Let width $= x$

length $= x + 1$

perimeter $= x + x + 1 + x + x + 1 = 4x + 2$

perimeter $= 26$ so

$4x + 2 = 26$

$4x = 24$

$x = 6$

width $= 6$ m and length $= 7$ m

area $= 6 \times 7 = 42\,\text{m}^2$

2 Let cost of adult ticket $= x$ and cost of child ticket $= y$

$2x + 5y = 35$ (1)

$3x + 4y = 38.5$ (2)

Equation (1) × 3 and equation (2) × 2 gives:

$6x + 15y = 105$ (3)

$6x + 8y = 77$ (4)

Equation (3) − equation (4) gives:

$7y = 28$

$y = 4$

Substituting $y = 4$ into equation (1) gives:

$2x + 20 = 35$

$2x = 15$

$x = 7.5$

cost of adult ticket = £7.50

cost of child ticket = £4

3 **a** Let Rachel be x years and Hannah be y years.

$xy = 63$

$(x + 2)(y + 2) = 99$

$xy + 2x + 2y + 4 = 99$

Substitute for xy:

$63 + 2x + 2y + 4 = 99$

$2x + 2y = 32$

$x + y = 16$

The sum of their ages is 16 years.

b $y = x - 2$

$x + x - 2 = x + y$

$2x - 2 = 16$

$2x = 18$

$x = 9$

Rachel is 9 years old.

Use of functions

1 **a** $f(3) = 5 \times 3 + 4 = 19$

b Set $f(x) = -1$

$5x + 4 = -1$

$5x = -5$

$x = -1$

2 **a** $fg(x) = f(g(x)) = f(x - 6) = (x - 6)^2$

b $gf(x) = g(f(x)) = g(x^2) = x^2 - 6$

3 **a** $f(5) = \sqrt{5 + 4} = \sqrt{9} = 3$ or -3

b $gf(x) = 2(\sqrt{x + 4})^2 - 3$

$= 2(x + 4) - 3$

$= 2x + 5$

4 Let $y = 5x^2 + 3$

$x = \sqrt{\frac{y - 3}{5}}$

$f^{-1}(x) = \sqrt{\frac{x - 3}{5}}$

Iterative methods

1 Let $f(x) = 2x^3 - 2x + 1$

$f(-1) = 2(-1)^3 - 2(-1) + 1 = 1$

$f(-1.5) = 2(-1.5)^3 - 2(-1.5) + 1 = -2.75$

There is a sign change of $f(x)$, so there is a solution between $x = -1$ and $x = -1.5$.

2 $x_1 = (x_0)^3 + \frac{1}{9} = (0.1)^3 + \frac{1}{9} = 0.1121111111$

$x_2 = (x_1)^3 + \frac{1}{9} = (0.1121111111)^3 + \frac{1}{9} = 0.1125202246$

$x_3 = (x_2)^3 + \frac{1}{9} = (0.1125202246)^3 + \frac{1}{9} = 0.1125357073$

3 **a** $x_1 = 1.5182945$

$x_2 = 1.5209353$

$x_3 = 1.5213157$

$x_4 = 1.5213705 \approx 1.521$ (to 3 d.p.)

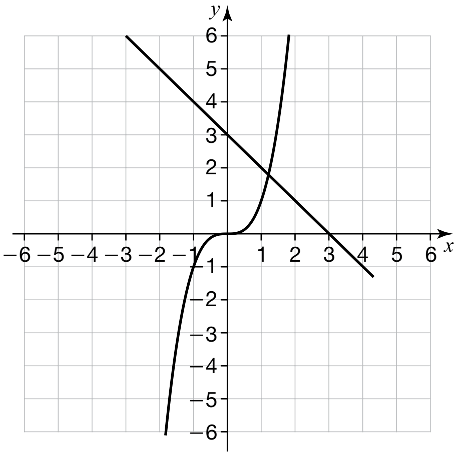
b Checking value of $x^3 - x - 2$ for $x = 1.5205, 1.5215$:

When $x = 1.5205$ f(1.5205)= −0.0052

$x = 1.5215$ f(1.5215) = 0.0007

Since there is a change of sign, the root is 1.521 correct to 3 decimal places.

4 **a** **i**



ii There is a real root of $x^3 + x - 3 = 0$ where the graphs of $y = x^3$ and $y = 3 - x$ intersect. The graphs intersect once so there is one real root of the equation $x^3 + x - 3 = 0$.

b $x_1 = 1.216440399$

$x_2 = 1.212725591$

$x_3 = 1.213566964$

$x_4 = 1.213376503$

$x_5 = 1.213419623$

$x_6 = 1.213409861 = 1.2134$ (to 4 d.p.)

Equation of a straight line

1 Comparing the equation with the equation of a straight line, $y = mx + c$:

$y = -2x + 3$

gradient of line, $m = -2$ (line has a negative gradient).

intercept on the y-axis, $c = 3$

Correct line is A.

2 **a** gradient, $m = \frac{\text{change in } y\text{−values}}{\text{change in } x\text{−values}} = -\frac{4}{3}$

b gradient of CD, $m = \frac{1 - 5}{5 - (-3)} = \frac{-4}{8} = -\frac{1}{2}$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 1 = -\frac{1}{2}(x - 5)$

$y = -\frac{1}{2}x + \frac{7}{2}$ or $x + 2y = 7$

c M is at $\left(\frac{-3+5}{2}, \frac{5+1}{2}\right) = (1, 3)$

gradient of perpendicular to $CD = \frac{-1}{-\frac{1}{2}} = 2$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 3 = 2(x - 1)$

$y = 2x + 1$

3 P is the point (x , y).

gradient of line $OP = \frac{y}{x} = 3$, so $y = 3x$

Using Pythagoras' theorem:

$OP^2 = x^2 + y^2$

$12^2 = x^2 + y^2$

$12^2 = x^2 + (3x)^2$

$144 = x^2 + 9x^2$

$144 = 10x^2$

$14.4 = x^2$

$x = 3.8$ (to 1 d.p.)

$y = 3x$

$= 3 \times 3.8$

$= 11.4$

P is the point (3.8, 11.4) (to 1 d.p.).

Quadratic graphs

1 **a** $x^2 + 4x + 1 = 0$

$(x + 2)^2 - 4 + 1 = 0$

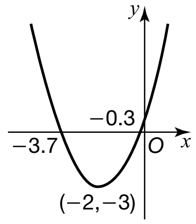
$(x + 2)^2 = 3$

$x + 2 = \pm\sqrt{3}$

$x = \sqrt{3} - 2$ or $-\sqrt{3} - 2$

$x = -0.3$ or -3.7 (to 1 d.p.)

b $x^2 + 4x + 1 = (x + 2)^2 - 3$, so turning point is at (−2, −3).



2 $5x^2 - 20x + 10 = 5[x^2 - 4x + 2]$

$= 5[(x - 2)^2 - 4 + 2]$

$= 5(x - 2)^2 - 10$

$a = 5$, $b = -2$ and $c = -10$

3 $2x^2 + 12x + 3 = 2[x^2 + 6x + \frac{3}{2}]$

$= 2[(x + 3)^2 - 9 + \frac{3}{2}]$

$= 2[(x + 3)^2 - \frac{15}{2}]$

$= 2(x + 3)^2 - 15$

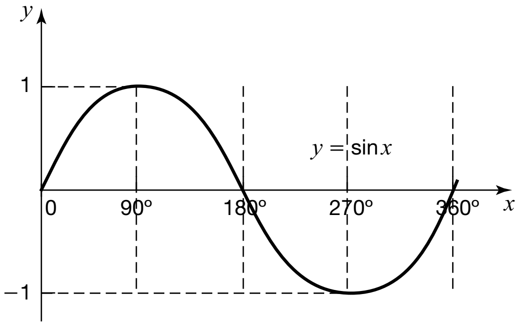
$a = 2$, $b = 3$ and $c = -15$

Recognising and sketching graphs of functions

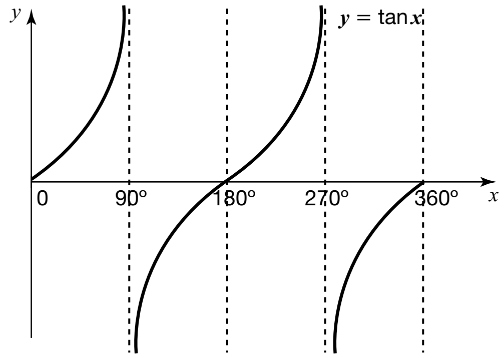
1

Equation	Graph
$y = x^2$	B
$y = 2^x$	D
$y = \sin x°$	E
$y = x^3$	C
$y = x^2 - 6x + 8$	A
$y = \cos x°$	F

2 a

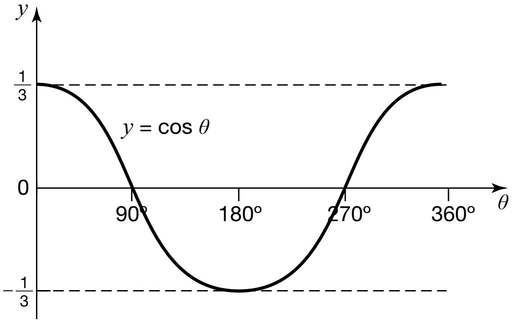


b



3 $3\cos\theta = 1$

$\cos\theta = \frac{1}{3}$



$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.5°$ (to 1 d.p.)

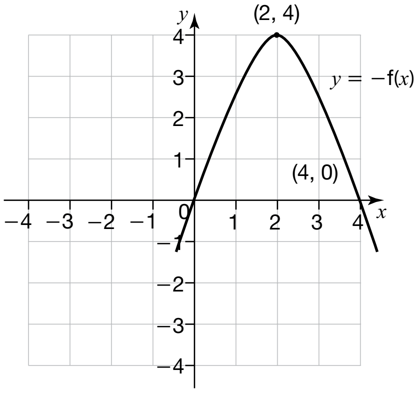
From the graph, $\cos\theta$ is also $\frac{1}{3}$ when $\theta = 360 - 70.5 = 289.5°$

$\theta = 70.5°$ or $289.5°$ (to 1 d.p.)

Translations and reflections of functions

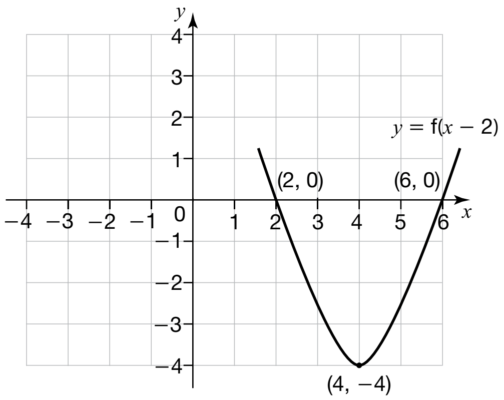
1 **a** $y = -f(x)$ is a reflection in the $x-$axis of the graph $y = f(x)$

The points on the $x-$axis stay in the same place and the turning point at (2, −4) is reflected to become a turning point at (2, 4).



b $y = f(x - 2)$ is a translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ of the graph $y = f(x)$.

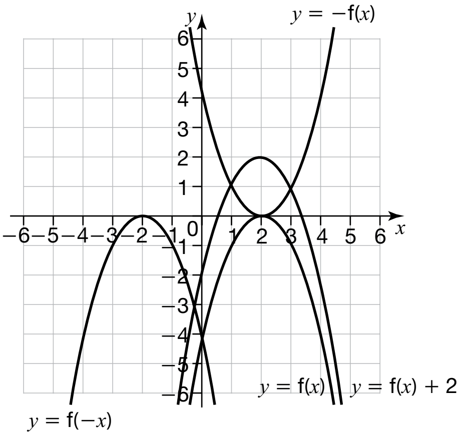
The y-coordinates stay the same but the x-coordinates are shifted to the right by 2 units.



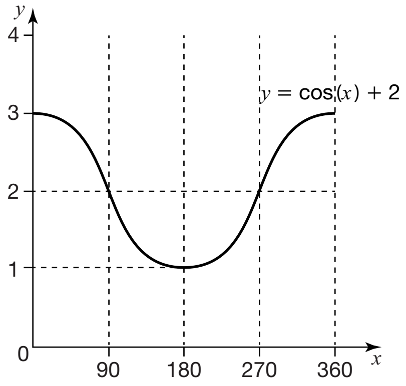
2 **a** $y = -f(x)$: reflection in the $x-$axis.

b $y = f(x) + 2$: translation of 2 units vertically upwards.

c $y = f(-x)$: reflection in the y-axis.



3 The cosine graph is shifted two units in the vertical direction, i.e. a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.



Equation of a circle and tangent to a circle

1 **a** $x^2 + y^2 = 25$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{25} = 5$

b $x^2 + y^2 - 49 = 0$

$x^2 + y^2 = 49$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{49} = 7$

c $4x^2 + 4y^2 = 16$

$x^2 + y^2 = 4$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{4} = 2$

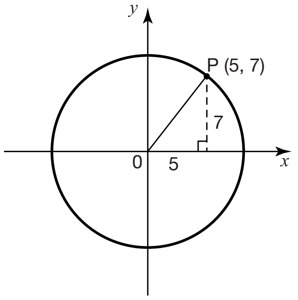
2 Radius of the circle $= \sqrt{21} = 4.58$

Distance of the point (4, 3) from the centre of the circle (0, 0)

$= \sqrt{16 + 9} = \sqrt{25} = 5$

This distance is greater than the radius of the circle, so the point lies outside the circle.

3 **a**



Using Pythagoras' theorem:

$OP^2 = 5^2 + 7^2$

$= 25 + 49$

$= 74$

OP = radius of the circle = $\sqrt{74}$

b equation of a circle, radius r, centre the origin: $x^2 + y^2 = r^2$

$x^2 + y^2 = 74$

c gradient of line $OP = \frac{7}{5}$

gradient of the tangent at $P = -\frac{5}{7}$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 7 = -\frac{5}{7}(x - 5)$

$7y - 49 = -5x + 25$

$7y = -5x + 74$

$y = -\frac{5}{7}x + \frac{74}{7}$

Real-life graphs

1 **a** acceleration = gradient of line = $\frac{10}{10} = 1\,\text{m/s}^2$

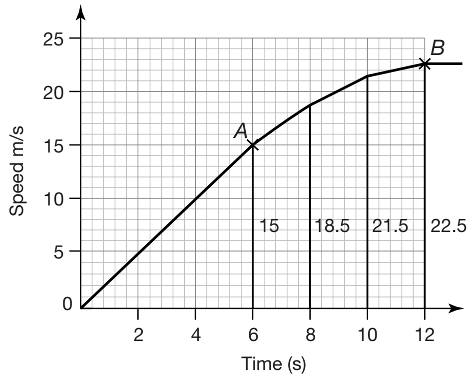
b total distance travelled = area under the velocity−time graph

$= \frac{1}{2}(30 + 15) \times 10 = 225\,\text{m}$

2 **a** The graph is a straight line starting at the origin, so this represents constant acceleration from rest of $\frac{15}{6} = 2.5\,\text{m/s}^2$.

b The gradient decreases to zero, so the acceleration decreases to zero.

c area = area of 3 trapeziums



area $= \frac{1}{2}(15 + 18.5) \times 2 + \frac{1}{2}(18.5+21.5) \times 2 + \frac{1}{2}(21.5 + 22.5) \times 2$

$= 117.5$

distance travelled between A and B = 118 m (to nearest integer)

d It will be a slight underestimate, as the curve is always above the straight lines forming the tops of the trapeziums.

Generating sequences

1 **a** **i** $\frac{1}{2}$ (term-to-term rule is divide by 2)

ii 243 (term-to-term rule is multiply by 3)

iii 21 (term-to-term rule is add 4)

b 4th term − 1st term = −12 − 27 = −39

common difference = −39 ÷ 3 = −13

missing terms are 14, 1

2 3rd term = 2 × 1 − 5 = −3

4th term = 2 × −3 − 5 = −11

3 **a** 25, 36 (square numbers)

b 15, 21 (triangular numbers)

c 8, 13 (Fibonacci numbers)

The *n*th term

1 **a**

n	1		2		3		4
Term	2		6		10		14
Difference		4		4		4	

nth term starts with $4n$

$4n$	4	8	12	16
Term − $4n$	−2	−2	−2	−2

nth term = $4n - 2$

b nth term = $4n - 2 = 2(2n - 1)$

2 is a factor, so the nth term is divisible by 2 and therefore is even.

c Let nth term = 236

$4n - 2 = 236$

$4n = 238$

$n = 59.5$

n is not an integer, so 236 is not a term in the sequence.

2 **a** 2nd term = $9 - 2^2 = 5$

b 20th term = $9 - 20^2 = 9 - 400 = -391$

c n^2 is always positive, so the largest value value $9 - n^2$ can take is 8 when $n = 1$. All values of n above 1 will make $9 - n^2$ smaller than 8. So 10 cannot be a term.

3

Term	1		1		3		7		13
First difference		0		2		4		6	
Second difference			2		2		2		

The formula starts n^2.

n	1		2		3		4		
Term	1		1		3		7		13
n^2	1		4		9		16		25
Term − n^2	0		−3		−6		−9		−12
Difference		−3		−3		−3		−3	

The linear part of the sequence starts with $-3n$.

$-3n$	−3	−6	−9	−12	−15
Linear term − $(-3n)$	3	3	3	3	3

nth term = $n^2 - 3n + 3$

Checking:

When $n = 1$, term is $1^2 - 3 \times 1 + 3 = 1$

$n = 2$, term is $2^2 - 3 \times 2 + 3 = 1$

$n = 3$, term is $3^2 - 3 \times 3 + 3 = 3$

Arguments and proofs

1 The only even prime number is 2.

Hence, statement is false because 2 is a prime number that is not odd.

2 **a** true: $n = 1$ is the smallest positive integer and this would give the smallest value of $2n + 1$, which is 3.

b true: 3 is a factor of $3(n + 1)$ so $3(n + 1)$ must be a multiple of 3.

c false: $2n$ is always even and subtracting 3 will give an odd number.

3 Let first number = x so next number = $x + 1$

Sum of consecutive integers = $x + x + 1 = 2x + 1$

Regardless of whether x is odd or even, $2x$ will always be even as it is divisible by 2.

Hence $2x + 1$ will always be odd.

4 $(2x - 1)^2 - (x - 2)^2 = 4x^2 - 4x + 1 - (x^2 - 4x + 4)$

$= 4x^2 - 4x + 1 - x^2 + 4x - 4$

$= 3x^2 - 3$

$= 3(x^2 - 1)$

The 3 outside the brackets shows that the result is a multiple of 3 for all integer values of x.

5 Let two consecutive odd numbers be $2n - 1$ and $2n + 1$.

$(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1)$

$= 8n$

Since 8 is a factor of $8n$, the difference between the squares of two consecutive odd numbers is always a multiple of 8.

(If you used $2n + 1$ and 2n + 3 for the two consecutive odd numbers, difference of squares = $8n + 8 = 8(n + 1)$.)

Ratio, proportion and rates of change

Introduction to ratios

1 3 parts = 18 black balls so 1 part = 6 black balls

There are 3 + 2 = 5 parts in total, so the total number of balls

= 5 × 6 = 30.

2 total shares = 15 + 17 + 18 = 50

50 shares = £25 000

1 share = $\frac{\text{£}25\,000}{50}$ = £500

15 year old will get 15 × £500 = £7500.

17 year old will get 17 × £500 = £8500.

18 year old will get 18 × £500 = £9000.

3 40% of 800 = = 320 acres

remainder of land area = 800 − 320 = 480 acres

total shares = 9 + 7 =16

1 share = $\frac{480}{16}$ = 30

area devoted to sheep = 7 shares = 7 × 30 = 210 acres

4 $\frac{3x + 1}{x + 4} = \frac{2}{3}$

$3(3x + 1) = 2(x + 4)$

$9x + 3 = 2x + 8$

$7x = 5$

$x = \frac{5}{7}$

5 For 2 oak trees there are 3 ash trees, so if there are 8 oak trees there would be 12 ash trees.

pine : oak: ash = 5 : 8 : 12

There are a total of 5 + 8 + 12 = 25 parts.

1 part = $\frac{300}{25}$ = 12

number of ash trees = 12 × 12 = 144

Scale diagrams and maps

1 10 cm on map = 10 × 50 000 cm = 500 000 cm actual distance

500 000 cm = 5000 m = 5 km

Towns are 5 km apart.

2 **a** length of road = 2.3 × 40 000 cm

= 92 000 cm

= 920 m

= 0.92 km

b length of road = 3 × 40000 mm

= 120000 mm

= 12 000 cm

= 120 m

= 0.12 km

3 5 cm : 40 m = 5 cm : 4000 cm

= 5 : 4000

= 1 : 800

scale of drawing = 1 : 800

4 length measured on map = 6 cm

Scale is:

6 cm : 12 km = 6 : 12 × 1000 × 100

= 6 : 1200000

= 1 : 200000

Percentage problems

1 increase = £35

percentage increase = $\frac{\text{increase}}{\text{original price}} \times 100$

$= \frac{35}{350} \times 100$

= 10%

2 increase = final earnings − initial earnings

= 1100000 − 600000

= £500000

% increase = $\frac{\text{increase}}{\text{original value}} \times 100$

$= \frac{500\,000}{600\,000} \times 100$

= 83.3%

3 multiplier = 1 − 0.28 = 0.72

value of car = 0.72 × 25000

= £18000

4 88% of the original price = £14300

1% of the original price = $\frac{14\,300}{88}$

100% of the original price = $\frac{14\,300}{88} \times 100 = 16250$

original price = £16250

5 interest earned = $\frac{2.8}{100} \times 8000 \times 4$

= £896

Direct and inverse proportion

1 **a** $P = kT$

b $P = kT$ so $k = \frac{P}{T} = \frac{200\,000}{540} = 370.370$

$P = 370.370T = 370.370 \times 200 = 74074$ pascals (to nearest whole number)

2 $C \propto r^2$

$C = kr^2$

$480 = k \times 3^2$

$k = 53.33$

$C = 53.33r^2$

cost = 53.33×4^2 = £853.28 = £853 (to nearest whole number)

3 **a** $c \propto \frac{1}{h}$ so $c = \frac{k}{h}$

$3 = \frac{k}{12}$, so $k = 36$

$c = \frac{36}{h}$

b $c = \frac{36}{h}$

$= \frac{36}{15}$

$= 2.4$

4 **a** 350 × 1.15 = €402.50

b 80 ÷ 1.11 = £72.07 (to nearest penny)

c The €80 she had left cost her 80 ÷ 1.15 = £69.57 before her holiday.

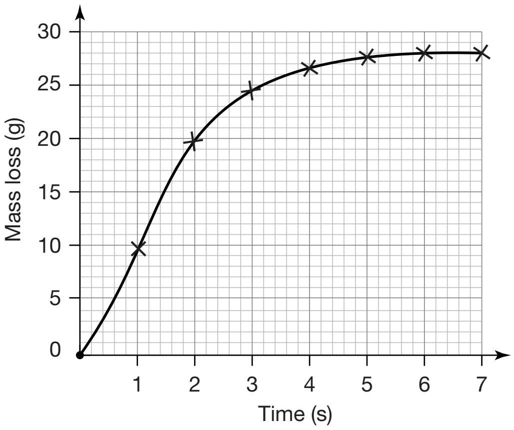
If she hadn't changed this amount, she would have saved 72.07 − 69.57 = £2.50

Graphs of direct and inverse proportion and rates of change

1 straight line through the origin: B

2 curve that gets close to but does not cross either axis: B

3 **a**



b **i** initial rate of change = gradient over first 2 minutes

$= \frac{19.6}{2}$

= 9.8 g/minute

ii initial rate of change = $\frac{9.8}{60}$ = 0.16 g/second (to 2 d.p.)

Growth and decay

1 **a** multiplier = $1 + \frac{\text{\% increase}}{100} = 1 + \frac{6}{100} = 1.06$

population after n years = A_0 × (multiplier)^n,

where A_0 = initial population and n = number of years

population after 3 years = 150 000 × 1.06^3 = 178 652

b Try $n = 5$ years: population = 150 000 × 1.06^5 = 200 733

Try $n = 4$ years: population = 150 000 × 1.06^4 = 189 372

After 5 years, the population will have risen to over 200 000.

2 multiplier = $1 - \frac{\text{\% decrease}}{100} = 1 - \frac{12}{100} = 0.88$

value of car after 4 years = 21 000 × 0.88^4 = £12 594

3 multiplier = $1 - \frac{\text{\% decrease for each time unit}}{100} = 1 - \frac{50}{100} = 0.5$

2 minutes = 120 seconds

number of time intervals, $n = \frac{120}{12} = 10$

amount at the end of n time intervals = A_0 × (multiplier)^n

= 100 × 0.5^{10}

= 0.1 (to 1 s.f.)

Ratios of lengths, areas and volumes

1 **a** scale factor for enlargement $\left(\frac{big}{small}\right)^3 = \left(\frac{12}{8}\right)^3 = 3.375$ or $\frac{27}{8}$

b $\frac{\text{area of triangle of large prism}}{\text{area of triangle of small prism}} = \left(\frac{12}{8}\right)^2$

area of triangle of large prism = $\left(\frac{12}{8}\right)^2 \times 10 = 22.5\,\text{cm}^2$

c $\frac{\text{volume of small prism}}{\text{volume of large prism}} = \left(\frac{8}{12}\right)^3$

volume of small prism = $\left(\frac{8}{12}\right)^3 \times 450$

= 133.33

= 133 cm^3 (to nearest whole number)

2 $\frac{\text{volume of larger cuboid}}{\text{volume of smaller cuboid}} = \left(\frac{h}{12}\right)^3$

$\frac{\text{volume of larger cuboid}}{\text{volume of smaller cuboid}} = 1.953$

$\left(\frac{h}{12}\right)^3 = 1.953$

$h^3 = 3374.784$

h = 15 cm (to nearest cm)

3 **a** **i** angle *XYZ* = angle *TUZ* (corresponding angles)

angle *ZXY* = angle *ZTU* (corresponding angles)

angle *XZY* = angle *TZU* (same angle)

Hence triangles *XYZ* and *TUZ* are similar.

$\frac{XY}{TU} = \frac{YZ}{UZ}$

$\frac{XY}{3} = \frac{15}{5}$

XY = 9 cm

ii angle *YUT* = angle *YZW* (corresponding angles)

angle *YTU* = angle *YWZ* (corresponding angles)

angle *TYU* = angle *WYZ* (same angle)

Hence triangles *YUT* and *YZW* are similar.

$\frac{WZ}{TU} = \frac{YZ}{YU}$

$\frac{WZ}{3} = \frac{15}{10}$

WZ = 4.5 cm

b angle *YTX* = angle *ZTW* (vertically opposite angles)

angle *YXT* = *TZW* (alternate angles)

Two angles of both triangles are equal so the third angles must also be equal.

Hence triangles *XYT* and *ZWT* are similar.

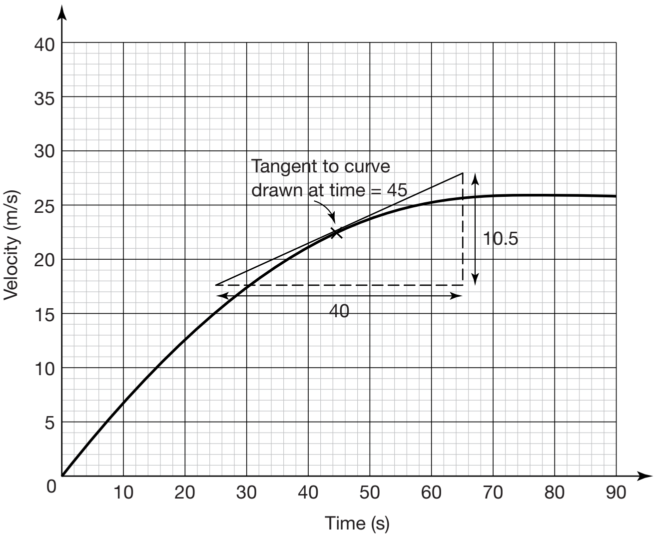
ratio of the areas of *TYX* and *TWZ* = $\left(\frac{XY}{WZ}\right)^2 = \left(\frac{9}{4.5}\right)^2 = 4$

ratio of areas = 4 : 1

Gradient of a curve and rate of change

1 **a** acceleration = gradient of straight line = $\frac{10-0}{15-0} = \frac{2}{3}$ m/s^2

b Instantaneous acceleration = gradient of the tangent to the curve at 45 s.



Gradient = $\frac{10.5}{40} = 0.2625$

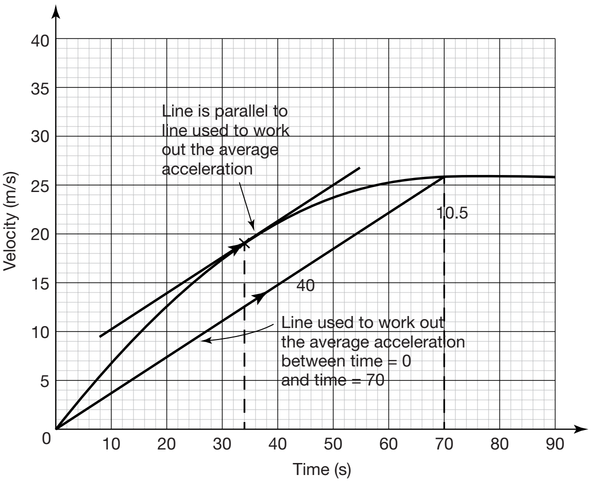
Instantaneous acceleration at 45 s = 0.26 m/s^2 (to 2 d.p.)

c average acceleration over the first 70 s = gradient of the line from (0, 0) to (70, 26)

gradient = $\frac{26-0}{70-0} = 0.3714\ldots$

acceleration = 0.37 m/s^2 (to 2 d.p.)

d



gradients of the two lines are parallel when time = 34 s

Converting units of areas and volumes, and compound units

1 pressure = $\frac{\text{force}}{\text{area}} = \frac{200}{0.4} = 500$ N/m^2

2 1 m^2 = 100 × 100 cm^2 = 10 000 cm^2

1 cm^2 = $\frac{1}{10000}$ m^2

200 cm^2 = $\frac{200}{(10\,000)}$ m^2 = 0.02 m^2

pressure = $\frac{\text{force}}{\text{area}} = \frac{500}{0.02} = 25\,000$ N/m^2

3 density = $\frac{\text{mass}}{\text{volume}}$

mass = volume × density

= 12 × 8.92

= 107.04 g

mass of wire = 107 g (to nearest g)

4 He has worked out the area in m^2 by dividing the area in cm^2 by 100, which is incorrect.

There are 100 × 100 = 10 000 cm^2 in 1 m^2, so the area should have been divided by 10 000.

Correct answer:

area in m^2 = $\frac{9018}{10\,000}$

= 0.9018

= 0.90 m^2 (to 2 d.p.)

5 distance in first two hours = time × speed

= 2 × 60

= 120 km

distance in next three hours = time × speed

= 3 × 80

= 240 km

total distance = 120 + 240

= 360 km

average speed = $\frac{\text{distance}}{\text{time}}$

= $\frac{360}{5}$

= 72 km/h

Geometry and measures

2D shapes

1 **a** true **c** true **e** true

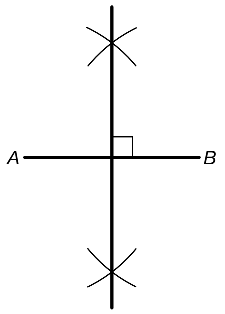
b false **d** true **f** false (this would be true only for a regular pentagon)

2 **a** rhombus **c** equilateral triangle

b parallelogram **d** kite

Constructions and loci

1 The locus of the point is the line that bisects and is perpendicular to *AB*.

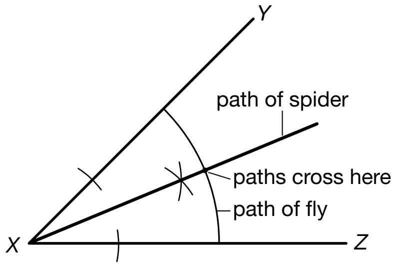


2 The line AB is 6 cm long, with semicircles of radius 1.5 cm at either end, joined by two lines parallel to and 1.5 cm from the line.

Not drawn to scale

3 **a** bisector of angle *YXZ*

b path of fly: arc with centre *X* of radius 6 cm between the two walls



Properties of angles

1 **a** angle *ACB* = angle *BAC* = 30° (base angles of isosceles triangle *ABC*, since *AB* and *BC* are equal sides of a rhombus)

b angle *AOB* = 90° (diagonals of a rhombus intersect at right angles)

c angle *ABO* = 180 − (90 + 30) = 60° (angle sum of a triangle)

angle *BDC* = angle *ABO* = 60° (alternate angles between parallel lines *AB* and *DC*)

2 angle *BAC* = $\frac{(180-36)}{2}$ = 72° (angle sum of a triangle and base angles of an isosceles triangle)

angle *BDC* = 180 − 90 = 90° (angle sum on a straight line)

angle *ABD* = 180 − (90 + 72) = 18° (angle sum of a triangle)

3 **a** $2x + 4x = 180$ (angle sum on a straight line)

$6x = 180$

$x = 30°$

b If lines *AB* and *CD* are parallel, the angles $4x$ and $3x + 30$ would be corresponding angles, and so equal.

$4x = 4 \times 30 = 120°$

$3x + 30 = 3 \times 30 + 30 = 120°$

These two angles are equal so lines *AB* and *CD* are parallel.

4 For the regular pentagon:

exterior angle = $\frac{360°}{\text{number of sides}} = \frac{360}{5} = 72°$

exterior angle of a square = interior angle of a square = 90°

angle $x = 90 + 72 = 162°$

Congruent triangles

1 *BD* common to triangles *ABD* and *CDB*

angle *ADB* = angle *CBD* (alternate angles)

angle *ABD* = angle *CDB* (alternate angles)

Therefore triangles *ABD* and *CDB* are congruent (ASA).

Hence angle *BAD* = angle *BCD*

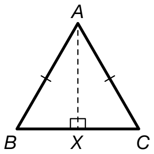
2 Draw the triangle and the perpendicular from *A* to *BC*.

AX = *AX* (common)

AB = *AC* (triangle *ABC* is isosceles)

angle *AXB* = *AXC* = 90° (given)

Therefore triangles *ABX* and *ACX* are congruent (RHS).



Hence *BX* = *XC*, so *X* bisects *BC*.

3 *OQC* = 90° (corresponding angles), so *PB* = *OQ* (perpendicular distance between 2 parallel lines).

AP = *PB* (given), so *AP* = *OQ*

PO = *QC* (*Q* is the midpoint of *BC*)

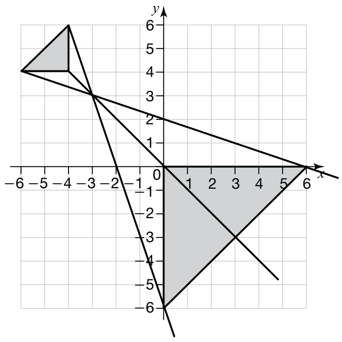
angle *ABC* = angle *APO* = angle *OQC* = 90° (*OQ* is parallel to *AB* and *OP* parallel to *BC*)

Therefore triangles *AOP* and *OCQ* are congruent (SAS).

Transformations

1 translation of $\begin{pmatrix}-7\\-5\end{pmatrix}$

2



3 **a** translation of $\begin{pmatrix}-7\\0\end{pmatrix}$

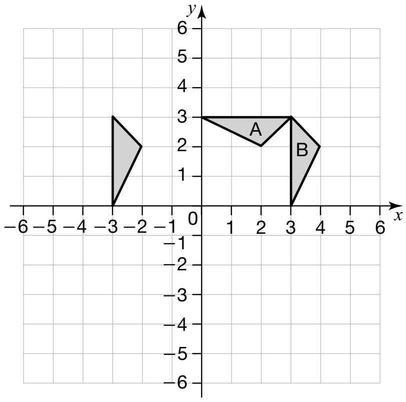
b reflection in the line $y = 3$

c rotation of 90° clockwise about (0, 1)

Invariance and combined transformations

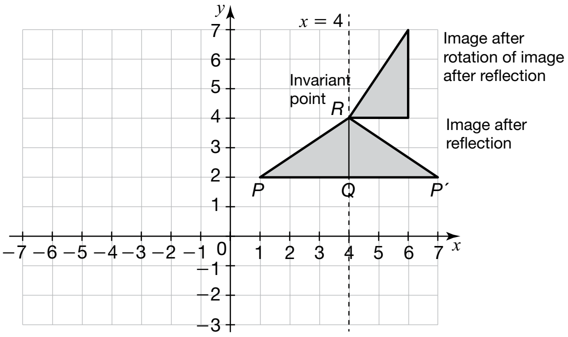
1 **a** 1

b **i** invariant point (3, 3)



ii rotation 90°anticlockwise about (3, 3)

2 **a** The shaded triangle is the image after the two transformations.



b invariant point is R (4, 4)

3D shapes

a	G	**c**	A, H	**e**	C
b	B	**d**	B	**f**	A, H

Parts of a circle

1	**a**	radius	**c**	chord
	b	diameter	**d**	arc
2	**a**	minor sector	**c**	major sector
	b	major segment	**d**	minor segment

Circle theorems

1 angle $OTB = 90°$ (angle between tangent and radius)

angle $BOT = 180 - (90 + 28) = 62°$ (angle sum in a triangle)

angle $AOT = 180 - 62 = 118°$ (angle sum on a straight line)

$AO = OT$ (radii), so triangle AOT is isosceles

angle $OAT = \frac{(180 - 118)}{2} = 31°$ (angle sum in a triangle)

2 **a** angle $ACB = 30°$ (angle at centre twice angle at circumference)

b angle BAC = angle $CBX = 70°$ (alternate segment theorem)

c $OA = OB$ (radii), so triangle AOB is isosceles

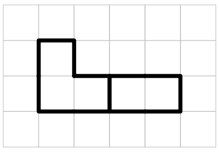
angle $AOB = 60°$, so triangle AOB is equilateral

angle $OAB = 60°$ (angle of equilateral triangle)

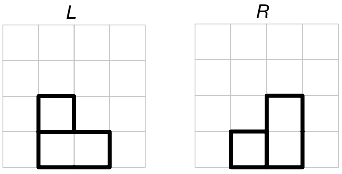
angle $CAO = 70 - 60 = 10°$

Projections

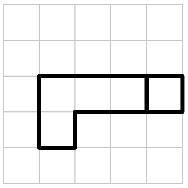
1



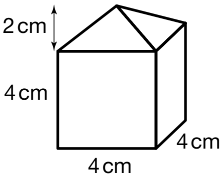
2 **a**



b

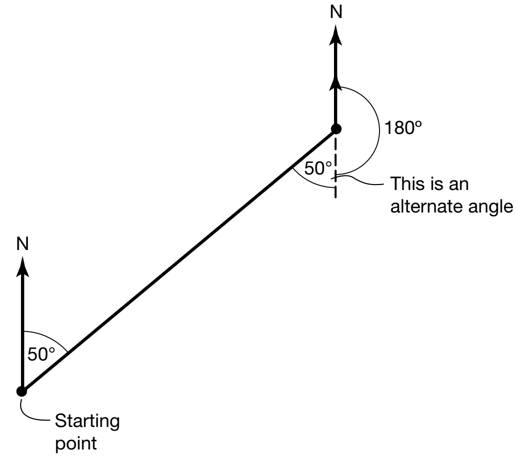


3



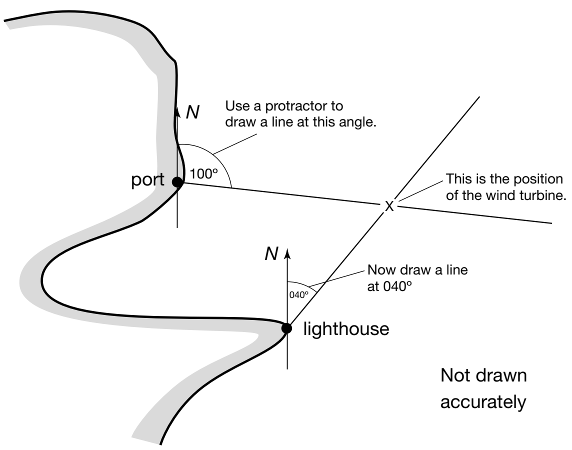
Bearings

1



bearing to return to starting point = 180 + 50 = 230°

2



Pythagoras' theorem

1 $AC^2 = AB^2 + BC^2 = 24^2 + 16^2 = 832$

$AD^2 = AC^2 - CD^2 = 832 - 9^2 = 751$

$AD = 27$ m (to nearest m)

perimeter = 24 + 16 + 9 + 27 = 76 m (to nearest m)

2 In triangle ABC, $AC^2 = AB^2 + BC^2 = 16^2 + 7^2 = 305$

In triangle ACG, $AG^2 = AC^2 + CG^2 = 305 + 5^2 = 330$

$AG = 18.2$ cm (to 1 d.p.)

extra distance = 16 + 7 + 5 − 18.2 = 9.8 cm (to 1 d.p.)

3 **a** angle $ADC = 90°$ (angle in a semicircle)

Using Pythagoras' theorem:

$10.8^2 = 5.8^2 + AD^2$

$AD = 9.11$ cm (to 2 d.p.)

b area of triangle $ACD = \frac{1}{2} \times 5.8 \times 9.11 = 26.419\text{ cm}^2$

angle ABC = 90° (angle in a semicircle)

$\frac{AB}{10.8} = \cos 65°$

$AB = 4.564$ cm

$\frac{BC}{10.8} = \sin 65°$

$BC = 9.788$ cm

area of triangle ABC $= \frac{1}{2} \times 4.564 \times 9.788 = 22.336\text{ cm}^2$

area of quadrilateral $ABCD = 26.419 + 22.336$
$= 48.76\text{ cm}^2$ (to 2 d.p.)

Area of 2D shapes

1 **a** **i** Using Pythagoras' theorem:

$7^2 + AE^2 = 9^2$

$49 + AE^2 = 81$

$AE^2 = 32$

$AE = \sqrt{32} = 5.6569 = 5.66$ cm* (to 2 d.p.)

ii area of triangle $ADE = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 5.6569 \times 7 = 19.80\text{ cm}^2$ (to 2 d.p.)

b Using Pythagoras' theorem:

$7^2 + BF^2 = 8^2$

$49 + BF^2 = 64$

$BF^2 = 15$

$BF = \sqrt{15} = 3.87$ cm (to 2 d.p.)

area of triangle $BCF = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 3.87 \times 7$
$= 13.56\text{ cm}^2$

---

*This answer differs from that in the Exam Practice book due to an error in our first edition. It has now been re-checked and corrected.

area of trapezium *CDEF* = area of rectangle − area of *ADE* − area of *BCF*

= 84 − 19.81 − 13.56

= 50.65 cm^2

2 **a** perimeter of triangle = $x + 1 + 2x + 1 + x + 2 = 4x + 4$

perimeter of rectangle = $x + 1 + 3 + x + 1 + 3 = 2x + 8$

perimeters are the same:

$4x + 4 = 2x + 8$

$2x = 4$

$x = 2\,\text{cm}$

area of *DEFG* = $3 \times (x + 1) = 9\,\text{cm}^2$

b area of triangle = $\frac{1}{2} \times 4 \times 3 = 6\,\text{cm}^2$

3 3 circles in stencil

a area = $3 \times \pi r^2 = 3 \times \pi \times 3^2 = 27\pi\,\text{cm}^2$

b perimeter = 3 × circumference + 2 × radius

$= 3 \times 2\pi r + 2r$

$= 3 \times 2 \times \pi \times 3 + 2 \times 3$

$= 18\pi + 6\,\text{cm}$

Volume and surface area of 3D shapes

1 **a** area of trapezium = $\frac{1}{2}(a + b)h$

$= \frac{1}{2}(3.5 + 1.8) \times 1.5$

$= 3.975\,\text{m}^2$

b volume of prism = area of cross−section × length

$= 3.975 \times 1.6$

$= 6.36\,\text{m}^3$

2 volume of sphere = $\frac{4}{3}\pi r^3$

volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi \times 4^3$

$= 134.04\,\text{cm}^3$

volume of drink in bottle = 750 ml = 750 cm^3

number of glasses = $\frac{750}{134.04} = 5.6$

A bottle will fill 5 glasses.

3 **a** Using Pythagoras’ theorem:

$l^2 = h^2 + r^2$

$= 5.6^2 + 4.8^2$

$= 54.4$

$l = 7.3756\ldots = 7.4\,\text{cm}$ (to 1 d.p.)

b surface area of cone = $\pi rl + \pi r^2$

$= \pi \times 4.8 \times 7.3756 + \pi \times 4.8^2$

$= 183.60\,\text{cm}^2$

surface area of sphere = $4\pi r^2 = 183.60$

$r = \sqrt{\frac{183.60}{4\pi}} = 3.8\,\text{cm}$ (to 1 d.p.)

4 volume of water in cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 2^2 \times 12$

$= 50.27\,\text{cm}^3$

volume of water in cylinder = $\pi r^2 h = 50.27\,\text{cm}^3$

$\pi \times 5^2 \times h = 50.27$

$h = 0.64\,\text{cm}$

depth of water in the cylinder = 0.64 cm

Trigonometric ratios

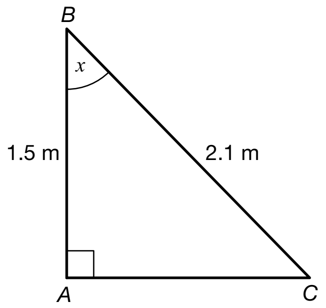
1 **a** $\cos 48° = \frac{x}{9}$

$x = 9\cos 48° = 6.0\,\text{cm}$ (to 1 d.p.)

b $\tan y = \frac{9}{12}$

$y = \tan^{-1}\left(\frac{9}{12}\right) = 36.9°$ (to 1 d.p.)

2



Let angle *ABC* = x

$\cos x = \frac{1.5}{2.1}$

$x = \cos^{-1}\left(\frac{1.5}{2.1}\right) = 44.4°$ (to 1 d.p.)

angle *ABC* = 44.4° (to 1 d.p.)

3 $\sin 70° = \frac{h}{10}$

$h = 10\sin 70° = 9.40\,\text{cm}$ (to 2 d.p.)

4 In triangle *ABC*:

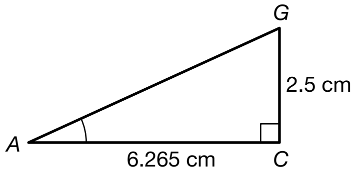
$AC^2 = AB^2 + BC^2$

$= 5.5^2 + 3^2$

$= 39.25$

$AC = \sqrt{39.25} = 6.265\,\text{cm}$

In triangle *ACG*, you know the lengths *AC* and *CG*, and you want to find the angle *CAG*, so use tan (*CG* is 'opposite' and *AC* is 'adjacent').

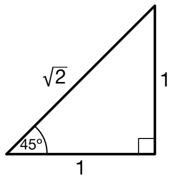


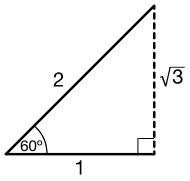
tan (angle *CAG*) = $\frac{CG}{AC} = \frac{2.5}{6.265} = 0.3990$

angle *CAG* = $\tan^{-1}(0.3990) = 21.8°$ (to 1 d.p.)

Exact values of sin, cos and tan

1





$\tan 45° = \frac{1}{1} = 1$

$\cos 60° = \frac{1}{2}$

Hence, $\tan 45° + \cos 60° = 1 + \frac{1}{2} = \frac{3}{2}$

2 **a** **i** $\sin 45° = \frac{1}{\sqrt{2}}$

ii $\cos 45° = \frac{1}{\sqrt{2}}$

b $\frac{\sin 45°}{\cos 45°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$\tan 45° = 1$

Hence $\frac{\sin 45°}{\cos 45°} = \tan 45°$

3 **a** Using Pythagoras’ theorem:

$AC^2 = 1^2 + 2^2$

$AC = \sqrt{5}$

b **i** $\sin x = \frac{1}{\sqrt{5}}$

ii $\cos x = \frac{2}{\sqrt{5}}$

c $(\sin x)^2 = \frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{1}{5}$

$(\cos x)^2 = \frac{2}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{4}{5}$

$(\sin x)^2 + (\cos x)^2 = \frac{1}{5} + \frac{4}{5}$

$= 1$

4 $\tan 30° + \tan 60° + \cos 30° = \frac{1}{\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{\sqrt{3}\,\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{3} + \sqrt{3} + \frac{\sqrt{3}}{2}$

$= \frac{2\sqrt{3} + 6\sqrt{3} + 3\sqrt{3}}{6}$

$= \frac{11\sqrt{3}}{6}$

Sectors of circles

1 $l = \frac{\theta}{360} \times 2\pi r$

$3 = \frac{\theta}{360} \times 2\pi \times 5$

$\theta = 34°$ (to nearest degree)

2 area of sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{290}{360} \times \pi \times 12^2$

$= 364.4\,\text{cm}^2$

3 a area of sector $= \frac{\theta}{360} \times \pi r^2$

area of sector A $= \frac{74}{360} \times \pi \times 3^2$

area of sector B $= \frac{360 - 122}{360} \pi r^2 = \frac{238}{360} \pi r^2$

$\frac{238}{360} \pi r^2 = \frac{74}{360} \times \pi \times 3^2$

$r^2 = \frac{74}{238} \times 3^2 = 2.798$

$r = 1.67\,\text{cm}$ (to 3 s.f.)

b perimeter of sector $= \frac{\theta}{360} \times 2\pi r + 2r$

perimeter of sector A $= \frac{74}{360} \times 2 \times \pi \times 3 + 2 \times 3$

$= 9.875\,\text{cm}^2$

perimeter of sector B $= \frac{238}{360} \times 2 \times \pi \times 1.67 + 2 \times 1.67$

$= 10.277\,\text{cm}^2$

perimeter A : perimeter B = 9.875 : 10.277 = 1 : 1.04

4 a Arc length $= \frac{\theta}{360} \times 2\pi r$

$5.4 = \frac{\theta}{360} \times 2 \times \pi \times 6$

$\theta = 51.5662°$

area of sector $= \frac{\theta}{360} \times \pi r^2$

Area $= \frac{51.5662}{360} \times \pi \times 6^2$

Area of minor sector $= 16.2\,\text{cm}^2$ (to 1 d.p.)

b Area of triangle $= \frac{1}{2} \times 6 \times 6 \times \sin 51.5662° = 14.09988\,\text{cm}^2$

Area of segment = area of sector − area of segment

$= 16.2 - 14.1$

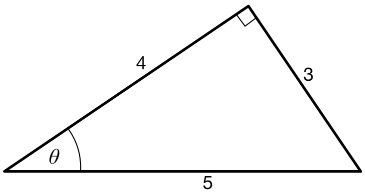
Area of segment $= 2.1\,\text{cm}^2$ (to 1 d.p.)

Sine and cosine rules

1 a area $= \frac{1}{2} \times 25 \times 30 \times \frac{3}{5}$

$= 225\,\text{cm}^2$

b Because of the value of $\sin \theta$, this must be a right-angled triangle with sides in the ratio 3 : 4 : 5. Draw the simplest form of this triangle.



adjacent = 4

$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$

c Using the cosine rule:

$AC^2 = 25^2 + 30^2 - 2 \times 25 \times 30 \times \frac{4}{5}$

$= 325$

$AC = 18.0\,\text{cm}$ (to 3 s.f.)

2 a Using the sine rule:

$\frac{x}{\sin 84°} = \frac{12}{\sin 40°}$

$x = \frac{12 \sin 84°}{\sin 40°}$

$= 18.5664$

$= 18.6\,\text{cm}$ (to 3 s.f.)

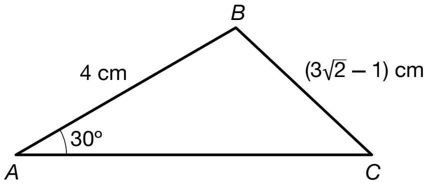
b angle $BAC = 180 - (84 + 40) = 56°$

area $= \frac{1}{2} \times 18.5664 \times 12 \times \sin 56°$

$= 92.4\,\text{cm}^2$ (to 3 s.f.)

If you use your answer from part a (to 3 s.f) you get 92.5, but taking x to 4 d.p. gives the answer of 92.4.

3



Using the sine rule:

$\frac{3\sqrt{2} - 1}{\sin 30°} = \frac{4}{\sin ACB}$

$\sin ACB = \frac{4 \sin 30°}{3\sqrt{2} - 1}$

$= \frac{2}{3\sqrt{2} - 1}$

$= \frac{2(3\sqrt{2} + 1)}{(3\sqrt{2} - 1)(3\sqrt{2} + 1)}$

$= \frac{6\sqrt{2} + 2}{18 - 1}$

$= \frac{2 + 6\sqrt{2}}{17}$

Vectors

1 a $\frac{1}{2}(\mathbf{p} + \mathbf{q}) = \frac{1}{2}\left(\begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}6\\-1\end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$

b $2\mathbf{p} - 3\mathbf{q} = 2\begin{pmatrix}-2\\3\end{pmatrix} - 3\begin{pmatrix}6\\-1\end{pmatrix} = \begin{pmatrix}2 \times -2 - 3 \times 6\\2 \times 3 - 3 \times -1\end{pmatrix}$

$= \begin{pmatrix}-22\\9\end{pmatrix}$

2 a $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$

b $\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AB} = \frac{3}{5}(\mathbf{b} - \mathbf{a})$

c $\overrightarrow{OQ} = \frac{2}{5}\overrightarrow{OA} = \frac{2}{5}\mathbf{a}$

$\overrightarrow{QP} = \overrightarrow{QA} + \overrightarrow{AP}$

$= \frac{3}{5}\mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$

$= \frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{3}{5}\mathbf{a}$

$= \frac{3}{5}\mathbf{b}$

As $\overrightarrow{QP} = \frac{3}{5}\mathbf{b}$ and $\overrightarrow{OB} = \mathbf{b}$ they both have the same vector part and so are parallel.

3 a $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= -3\mathbf{b} + \mathbf{a}$

$= \mathbf{a} - 3\mathbf{b}$

b $\overrightarrow{PB} = \frac{1}{3}\overrightarrow{AB} = \mathbf{b}$

$\overrightarrow{PM} = \overrightarrow{PB} + \overrightarrow{BM}$

$= \overrightarrow{PB} + \frac{1}{2}\overrightarrow{BC}$

$= \mathbf{b} + \frac{1}{2}(\mathbf{a} - 3\mathbf{b})$

$= \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

$= \frac{1}{2}(\mathbf{a} - \mathbf{b})$

$\overrightarrow{MD} = \overrightarrow{MC} + \overrightarrow{CD}$

$= \frac{1}{2}\overrightarrow{BC} + \overrightarrow{CD}$

$= \frac{1}{2}(\mathbf{a} - 3\mathbf{b}) + \mathbf{a}$

$= \frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}$

$= \frac{3}{2}(\mathbf{a} - \mathbf{b})$

Both $\overrightarrow{PM}$ and $\overrightarrow{MD}$ have the same vector part ($\mathbf{a} - \mathbf{b}$) so they are parallel. Since they both pass through *M*, they are parts of the same line, so *PMD* is a straight line.

Probability

The basics of probability

1 **a**

Dice 1

Dice 2

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b P(prime number) $= \frac{15}{36} = \frac{5}{12}$

c 7, as there are six 7s which is more than all the other scores.

2 **a** total number of chocolates $= 2x + 1 + x + 2x = 5x + 1$

P(mint) $= \frac{x}{5x+1} = \frac{4}{21}$

$21x = 20x + 4$

$x = 4$

total number of chocolates $= 5x + 1 = 5 \times 4 + 1 = 21$

b number of truffles $= 2 \times 4 + 1 = 9$

P(truffle) $= \frac{9}{21} = \frac{3}{7}$

3

Beth

Amy

	1	**2**	**3**	**4**	**5**	**6**
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

a number of possible outcomes = 36

number of pairs the same = 6

P(scores are equal) $= \frac{6}{36} = \frac{1}{6}$

b The possible scores where Amy's score is higher are:

(2, 1)

(3, 1), (3, 2)

(4, 1), (4, 2), (4, 3)

(5, 1), (5, 2), (5, 3), (5, 4)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5)

P(Amy's score is higher) $= \frac{15}{36} = \frac{5}{12}$

Probability experiments

1 **a** He is wrong because 100 spins is a very small number of trials. To approach the theoretical probability you would have to spin many more times. Only when the number of spins is extremely large will the frequencies start to become similar.

b relative frequency $= \frac{\text{frequency of event}}{\text{total frequency}} = \frac{22}{100} = \frac{11}{50}$

c Spinning 100 times gives a frequency of 19.

Spinning 500 times gives an estimate for the frequency $= 5 \times 19 = 95$.

2 **a** $3x + 0.05 + 2x + 0.25 + 0.20 + 0.1 = 5x + 0.6$

The relative frequencies have to add up to 1.

So $5x + 0.6 = 1$

$5x = 0.4$

$x = 0.08$

b relative frequency for a score of 1 $= 3 \times 0.08 = 0.24$

c number of times $= 0.20 \times 80 = 16$

The AND and OR rules

1 **a** P(picture card) $= \frac{12}{52} = \frac{3}{13}$

P(2 picture cards) $= \frac{3}{13} \times \frac{3}{13} = \frac{9}{169}$

b P(ace) $= \frac{4}{52} = \frac{1}{13}$

P(ace and picture card) $= \frac{1}{13} \times \frac{3}{13} = \frac{3}{169}$

c P(queen of hearts and queen of diamonds) $= \frac{1}{52} \times \frac{1}{52}$

$= \frac{1}{2704}$

2 **a** When an event has no effect on another event, they are said to be independent events. Here the colour of the first marble has no effect on the colour of the second marble.

b P(red) $= \frac{3}{10}$

P(2 reds) $= \frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$

c P(red then blue) $= \frac{3}{10} \times \frac{5}{10} = \frac{15}{100}$

P(blue then red) $= \frac{5}{10} \times \frac{3}{10} = \frac{15}{100}$

P(red and blue) $= \frac{15}{100} + \frac{15}{100} = \frac{30}{100} = \frac{3}{10}$

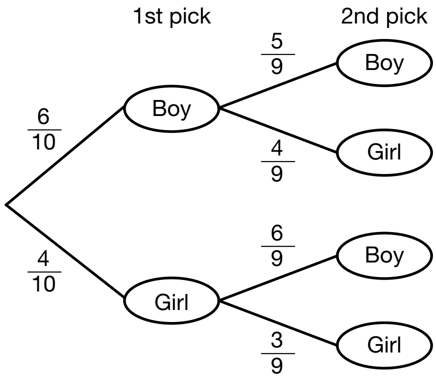
3 **a** P(homework in all 3 subjects) $= \frac{3}{5} \times \frac{3}{7} \times \frac{1}{4} = \frac{9}{140}$

b P(homework not given in any of the subjects)

$= \frac{2}{5} \times \frac{4}{7} \times \frac{3}{4} = \frac{24}{140} = \frac{6}{35}$

Tree diagrams

1



a P(2 girls) $= \frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$

b P(boy and girl) = P(BG) + P(GB)

$$= \left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9}\right)$$

$$= \frac{8}{15}$$

2 **a** number of boys = 0.56 × 450 = 252

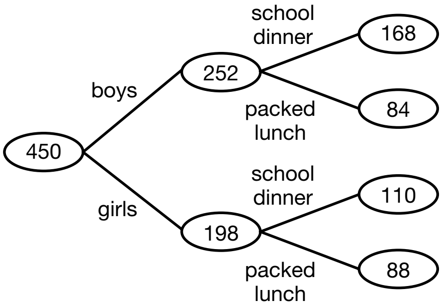
number of girls = 450 − 252 = 198

number of boys who have a packed lunch = $\frac{1}{3} \times 252 = 84$

number of boys who have a school dinner = 252 − 84 = 168

number of girls who have a school dinner = $\frac{5}{9} \times 198 = 110$

number of girls who have a packed lunch = 198 − 110 = 88



b P(girl who has a school dinner) = $\frac{110}{450} = \frac{11}{45}$

c P(boy who has a school dinner) = $\frac{168}{450}$

P(school dinner) = $\frac{110}{450} + \frac{168}{450} = \frac{278}{450} = \frac{139}{225}$

3 **a** Possible ways of one marble of each colour:

RGY RYG GYR GRY YGR YRG

P(RGY) = $\frac{5}{9} \times \frac{3}{8} \times \frac{1}{7} = \frac{15}{504}$

P(RYG) = $\frac{5}{9} \times \frac{1}{8} \times \frac{3}{7} = \frac{15}{504}$

P(one of each colour) = $6 \times \frac{5}{9} \times \frac{3}{8} \times \frac{1}{7} = 6 \times \frac{15}{504} = \frac{90}{504}$

$= \frac{5}{28} = 0.179$ (to 3 d.p.)

b P(no green) = $\frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{120}{504} = \frac{5}{21} = 0.238$ (to 3 d.p.)

c P(all red) = $\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{5}{42}$

P(all green) = $\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} = \frac{1}{84}$

P(same colour) = $\frac{5}{42} + \frac{1}{84} = \frac{11}{84} = 0.131$ (to 3 d.p.)

Venn diagrams and probability

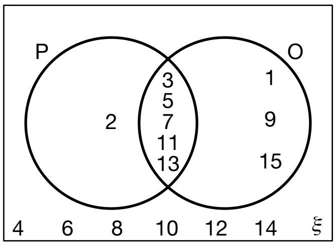
1 **a** 9, 8

b 1, 2, 3, 5, 7, 8, 9, 12, 15

c 1, 3, 4, 10, 12, 15

d 4, 10

2 **a**



b P(number in P ∩ O) = $\frac{5}{15} = \frac{1}{3}$

3 **a** P(team sports) = P(not only individual) = $1 - \frac{15}{100} = \frac{85}{100}$

$= \frac{17}{20}$

Alternatively, you could find the total of all those who play team sports.

b P(student playing large team also plays small team sports)

$= \frac{18}{73}$

Statistics

Sampling

1 **a** Ling's, as he has a larger sample so it is more likely to represent the whole population (i.e. students at the school).

b % of boys in school = $\frac{400}{900} \times 100 = 44.4\%$

number of boys in sample = $\frac{44.4}{100} \times 50 = 22.2 = 22$ (as number has to be an integer)

2 The sample should be taken randomly, with each member of the population having an equal chance of being chosen.

The sample size should be large enough to represent the population, since the larger the sample size, the more accurate the results.

3 **a** 52% males and 48% females.

12% males who smoke and 88% males who do not smoke

10% females who smoke and 90% females who do not smoke

number of females in sample = 48% of 400 = 192

number of females who do not smoke in sample = 0.9 × 192 = 172.8 =173

173 questionnaires needed

b number of males in sample = 52% of 400 = 208

number of males who smoke in sample = 0.12 × 208 = 24.96 = 25

25 questionnaires needed

Two-way tables and pie charts

1 **a**

	Coronation Street	EastEnders	Emmerdale	Total
Boys	12	31	20	63
Girls	18	12	7	37
Total	30	43	27	100

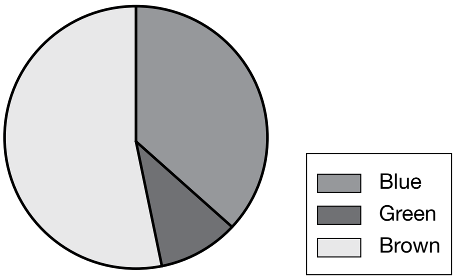
b P(student's favourite is Emmerdale) = $\frac{27}{100}$

c P(girl's favourite is Eastenders) = $\frac{12}{37}$

2 total frequency = 11 + 3 + 16 = 30

angle for one person = $\frac{360}{30} = 12°$

Colour	Frequency	Angle
Blue	11	11 × 12 =132°
Green	3	3 × 12 = 36°
Brown	16	16 × 12 = 192°



3 **a** The median is the 11th student, working from the smallest value.

median = 11 students

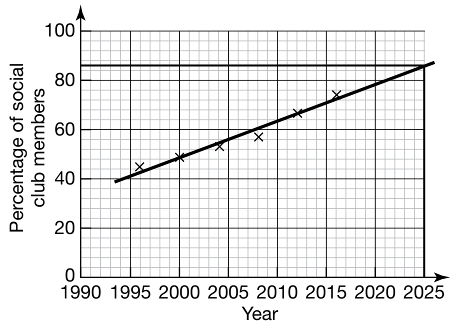
b range = highest value − lowest value = 26 − 3 = 23 students

Line graphs for time series data

1 **a** An upward trend, rising slowly at first, then rising quickly, then continuing to rise more slowly.

b Mode, because it shows that June is the month when the environment offers the largest sample of insects to study. The median would give May, when there are also a lot of insects.

2 **a**



b An upward trend – people are more likely to assume it should be an option.

c Reading from graph: 86% (or a close value).

d Future data may change so that the line of best fit is no longer accurate. Also, the relationship may not be best represented by a straight line, but by a curve.

Averages and spread

1 **a** $5 + 3 + 1 + 4 + 3 + 5 + 0 + 1 + 4 + 1 + 2 + 3 = 32$

b mean $= \frac{32}{12} = 2.7$ (to 1 d.p.)

c Ordering the data gives:

0 1 1 1 2 3 3 3 4 4 5 5

median = 3

2 mean mark $= \frac{\text{total marks}}{\text{total number of students}}$

total marks = mean mark × total number of students

For the group of 25 students:

total marks $= 60 \times 25 = 1500$

For the 30 students:

total marks $= 72 \times 30 = 2160$

total marks for both groups $= 1500 + 2160 = 3660$

mean mark for both groups $= \frac{3660}{55} = 66.5$ (to 1 d.p.)

3 **a**

Age (t years)	Frequency	Mid-interval value	Frequency × mid-interval value
$0 < t \le 4$	8	2	16
$4 < t \le 8$	10	6	60
$8 < t \le 12$	16	10	160
$12 < t \le 14$	1	13	13

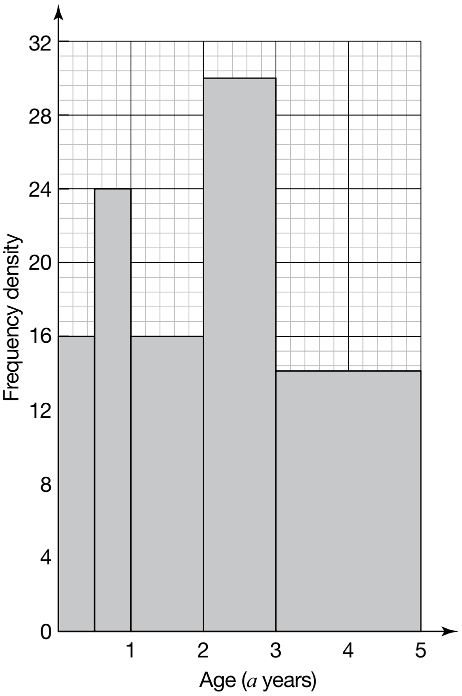
b 35

c estimated mean $= \frac{16 + 60 + 160 + 13}{35} = 7.1$ years (to 2 s.f.)

Histograms

1

Age (a years)	Frequency	Frequency density
$0 < a \le 0.5$	8	$\frac{8}{0.5} = 1$
$0.5 < a \le 1$	12	$\frac{12}{0.5} = 24$
$1 < a \le 2$	16	$\frac{16}{1} = 16$
$2 < a \le 3$	30	$\frac{30}{1} = 30$
$3 < a \le 5$	28	$\frac{28}{2} = 14$



2 The vertical scale of the histogram in your book is incorrect*. It should be marked 0 – 5.

Adding the scale to the vertical axis:

For the class interval $15 < w \le 25$, frequency density $= \frac{\text{frequency}}{\text{class width}} = \frac{30}{10} = 3$

class interval $0 < w \le 5$: frequency $= 5 \times 0.4 = 2$

class interval $25 < w \le 40$: frequency $= 15 \times 0.6 = 9$

Using the axis markings in the book would have given the values 10 and 45 (but these answers would not be correct in relation to the other values in the table).

Wingspan (w cm)	Frequency
$0 < w \le 5$	2
$5 < w \le 10$	6
$10 < w \le 15$	24
$15 < w \le 25$	30
$25 < w \le 40$	9

Cumulative frequency graphs

1 **a**

Number of items of junk mail per day (n)	Frequency	Cumulative frequency
$0 < n \le 2$	21	21
$2 < n \le 4$	18	39
$4 < n \le 6$	10	49
$6 < n \le 8$	8	57
$8 < n \le 10$	3	60
$10 < n \le 12$	1	61

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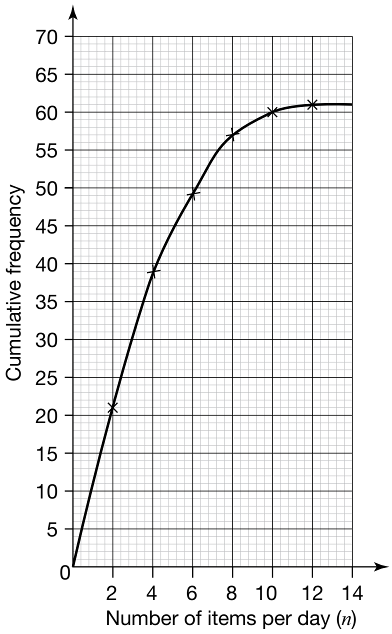
*This answer differs from that in the Exam Practice book due to an error in our first edition. It has now been re-checked and corrected.

b



c median = value at frequency of 30.5 = 3 (acceptable values: 2.8 to 3.0)

2 **a** interquartile range = 17.5 − 11 = 6.5 kg

b number of penguins with mass below 10 kg = 8 (from graph)

number of penguins with mass above 10 kg = 40 − 8 = 32

Comparing sets of data

1 **a** Chloe's scores in ascending order:

10 12 12 14 14 15 15 16 17 17 17 17 18 18 19

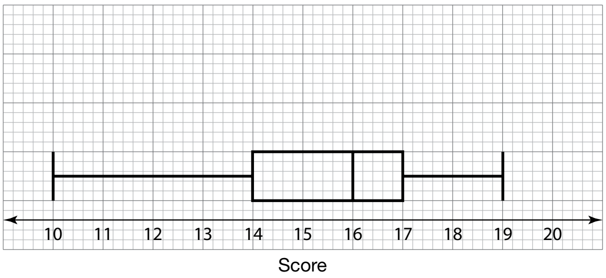
median = 16

range = 19 − 10 = 9

lower quartile = 14 (halfway between the median and the lowest data value)

upper quartile = 17 (halfway between the median and the highest data value)

interquartile range (IQR) = 17 − 14 = 3



b Sasha's median score is higher (17 compared to 16). The IQR for Sasha is 2 compared to Chloe's 3. The range for Sasha is 5 compared to Chloe's 9. Both these are measures of spread, which means that Sasha's scores are less spread out (i.e. more consistent).

2 **a** **i** Draw a horizontal line, from halfway up the vertical axis across to the curve, then draw a line down to the horizontal axis and read off your result:
median = 138 minutes

ii lower quartile = value at frequency of 25 = 128 minutes

upper quartile = value at frequency of 75 = 150 minutes

interquartile range = 150 − 128 = 22 minutes. (21 or 20 minutes are also acceptable answers, depending on your values for the upper and lower quartiles from the graph.)

b From the box plot for the female runners:

median = 145 minutes

range = 180 − 106 = 74 minutes

lower quartile = 134 minutes

upper quartile = 156 minutes

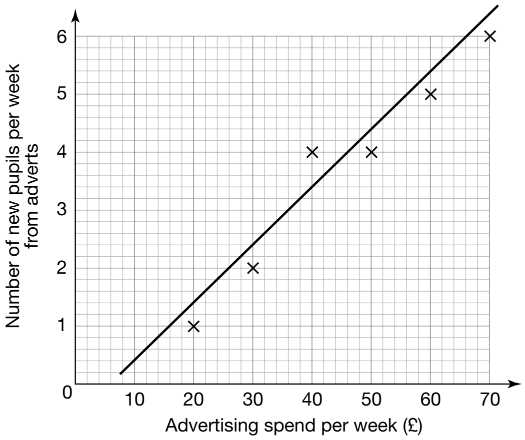
interquartile range = 156 − 134 = 22 minutes

On average the men were faster as the median is lower.

The variation in times were greater for the men as their range was greater, although the spread of the middle half of the data (the interquartile range) was slightly greater for the women.

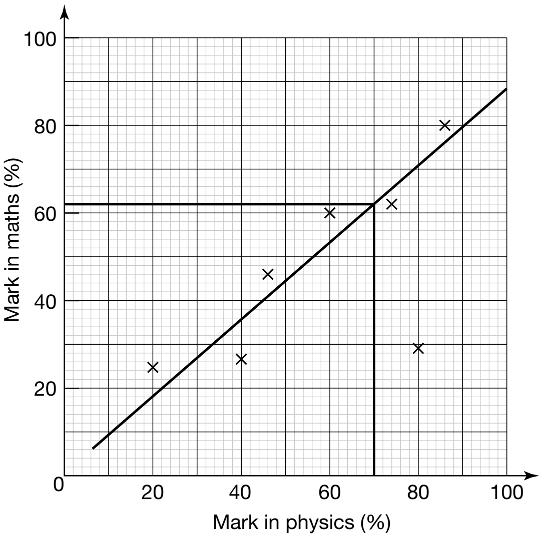
Scatter graphs

1 **a**, **b**



c positive correlation

2 **a**



b Reading from graph: 62%. A suitable range would be 60–63%.

c The data only goes up to a score of 86% in physics, and the score at 80% in physics is an outlier, so the line may not be accurate for higher scores in physics.

Practice paper 1 (non-calculator)

1 $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ The term-to-term rule is: multiply by $\frac{1}{2}$, therefore there is a constant ratio between each number and the one before, so this sequence is geometric.

2 The triangles are identical.

3 $2^2 \times 5 \times 13$

4 $y^2 = 25 - x^2$, because it can be written in the form $x^2 + y^2 = r^2$

5 $3\frac{1}{3} \times 1\frac{3}{8} = \frac{10}{3} \times \frac{11}{8}$

$= \frac{55}{12}$

$= 4\frac{7}{12}$

6 $2^3 : 1^3 = 8 : 1$

7

n	1	2	3	4
$2n + 1$	3	5	7	9
$n^2 + 1$	2	2	10	17

5 is in both sequences.

Alternative method:

Let the particular value for n that is true for both sequences $= a$.

$2a + 1 = a^2 + 1$

$2a = a^2$

$a = 2$

$2a + 1 = 5$

$a^2 + 1 = 5$

So 5 is the value in both sequences.

8 $5^4 \times 5^{-1} \times 5 = 5^4 \times \frac{1}{5} \times 5 = 5^4 \times \frac{5}{5}$

$= 5^4$

9 Area = length × width

$= 30 \times 20$

$= 600\,\text{cm}^2 = 0.06\ \text{m}^2$

10 **a** area of triangle $= \frac{1}{2} \times 3x \times 2x = 3x^2$

area of rectangle $= 4xy$

As both areas are equal $3x^2 = 4xy$

$3x^2 - 4xy = 0$

$x(3x - 4y) = 0$

b Any pairs of values such that $3x - 4y = 0$, for example $x = 4$, $y = 3$ and $x = 8$, $y = 6$

11 P(green) $= \frac{x - 3}{x + x - 3 + 2x} = \frac{x - 3}{4x - 3}$

$\frac{x - 3}{4x - 3} = \frac{1}{5}$

$5(x - 3) = 4x - 3$

$5x - 15 = 4x - 3$

$x = 12$

total number of counters $= 4x - 3 = 4 \times 12 - 3 = 45$

number of blue counters $= 2x = 24$

P(blue) $= \frac{24}{45} = \frac{8}{15}$

12 $3.6 \times 10^4 = 36\,000$ and $3.6 \times 10^2 = 360$ hence $y = 3.6$

So $y \times 10^4 - y \times 10^2 = 3.6 \times 10^4 - 3.6 \times 10^2$

$= 36\,000 - 360$

$= 35\,640$

$= 3.564 \times 10^4$

13 **a** LHS $= x^2 + y^2 = 4^2 + 3^2 = 25$

LHS = RHS so point satisfies the equation, so the point (4, 3) lies on the circle.

b gradient of $OP = \frac{3}{4}$

gradient of tangent $= -\frac{4}{3}$

Equation of the tangent is $y - 3 = -\frac{4}{3}(x - 4)$

$3(y - 3) = -4(x - 4)$

$3y - 9 = -4x + 16$

$3y + 4x = 25$

or $y = -\frac{4}{3}x + \frac{25}{3}$

14 $\frac{8}{3\sqrt{2}} = \frac{8}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{8\sqrt{2}}{3 \times 2} = \frac{8\sqrt{2}}{6}$

$= \frac{4\sqrt{2}}{3}$

15 **a** $(2x^2y)^3 = 2^3 \times x^{2 \times 3} \times y^{1 \times 3} = 8x^6y^3$

b $2x^{-3} \times 3x^4 = 2 \times 3 \times x^{-3 + 4} = 6x$

c $\frac{15a^3b}{3a^3b^2} = 15 \div 3 \times a^{3 - 3} \times b^{1 - 2} = 5b^{-1} = \frac{5}{b}$

16 **a** True; x is always smaller than y, so $\frac{x}{y} < 1$.

b True; a value of $x = -1$ would give $x^3 = -1$, but the question says that x is a positive integer so x can never be -1.

c False; x is always smaller than y, so $x - y < 0$.

d False; since $y > x$, y^2 can never be equal to x^2.

17 Let $x = 0.1\dot{3}\dot{6} = 0.136363636\ldots$

$10x = 1.36363636\ldots$

$1000x = 136.363636\ldots$

$1000x - 10x = 136.363636\ldots - 1.36363636\ldots$

$990x = 135$

$x = \frac{135}{990} = \frac{3}{22}$

18 $\tan 30° = \frac{1}{\sqrt{3}}$ and $\sin 60° = \frac{\sqrt{3}}{2}$

$\tan 30° + \sin 60° = \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2}$

$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{2}$

$= \frac{2\sqrt{3}}{6} + \frac{3\sqrt{3}}{6}$

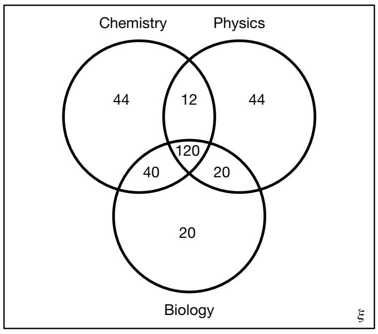
$= \frac{5\sqrt{3}}{6}$

19 $(x + 1)(x + 1)(x + 1) = (x + 1)(x^2 + 2x + 1)$

$= x^3 + 2x^2 + x + x^2 + 2x + 1$

$= x^3 + 3x^2 + 3x + 1$

20 **a**



b Let x = number who take only chemistry
(therefore the number who take only physics is also x)

$$120 + 40 + 20 + 20 + 12 + x + x = 300$$
$$212 + 2x = 300$$
$$2x = 88$$
$$x = 44$$

44 students take only chemistry (and 44 take only physics).

c total number who take biology = 200

number who take biology and chemistry = 160

P(biology student taking chemistry) $= \frac{160}{200} = \frac{4}{5}$

21 **a** angle $ACB = 55°$ (angle between tangent and chord equal to angle in the alternate segment)

b angle CAB = angle $ABC = \frac{180 - 55}{2} = 62.5°$ (base angles in isosceles triangle and angle sum in triangle)

angle $CBY = 180 - (62.5 + 55) = 62.5°$ (angles on a straight line add up to 180°)

22 **a** number of students in the class $= 8 + 5 + 3 + 4 + 2 + 3 + 1 + 1 = 27$

total number of pets $= (1 \times 8) + (2 \times 5) + (3 \times 3) + (4 \times 4) + (5 \times 2) + (6 \times 3) + (7 \times 1) + (10 \times 1)$
$= 88$

mean number of pets $= \frac{88}{27} = 3.\dot{2}5\dot{9}$
$= 3$ (to the nearest integer)

Amy has 3 pets.

b There are 27 students so the median is the 14th value when the number of pets is put in order.

From the graph, the 14th value is 3.

Jacob has 3 pets.

23 **a** **i** $\overrightarrow{AM} = \overrightarrow{AO} + \frac{1}{2}\overrightarrow{OB}$

$= -\mathbf{a} + \frac{\mathbf{b}}{2}$

$= \frac{\mathbf{b}}{2} - \mathbf{a}$

ii $\overrightarrow{AR} = \frac{2}{3}\overrightarrow{AM}$

$= \frac{2}{3}\left(\frac{\mathbf{b}}{2} - \mathbf{a}\right)$

$= \frac{\mathbf{b}}{3} - \frac{2\mathbf{a}}{3}$

b $\overrightarrow{BP} = \overrightarrow{BO} + \overrightarrow{OP} = -\mathbf{b} + \frac{\mathbf{a}}{2} = \frac{\mathbf{a}}{2} - \mathbf{b} = \frac{1}{2}(\mathbf{a} - 2\mathbf{b})$

$\overrightarrow{BR} = \overrightarrow{BA} + \overrightarrow{AR} = \mathbf{a} - \mathbf{b} + \frac{\mathbf{b}}{3} - \frac{2\mathbf{a}}{3} = \frac{\mathbf{a}}{3} - \frac{2\mathbf{b}}{3}$

$= \frac{1}{3}(\mathbf{a} - 2\mathbf{b})$

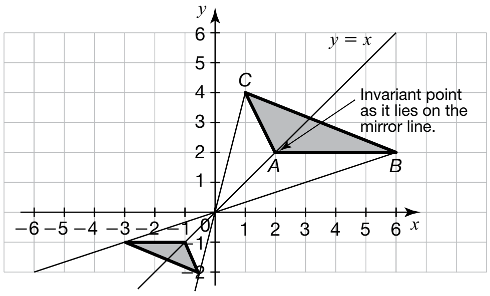
$\overrightarrow{BP}$ and $\overrightarrow{BR}$ have the same vector part ($\mathbf{a} - 2\mathbf{b}$) and are therefore parallel. As they both pass through point B, points P and R lie on the same straight line.

24 maximum attendance = 104 999

minimum attendance = 99 500

difference = 5499

25 **a**



b One invariant point (i.e. point A)

26 $6x + 5y = 35$ (1)

$x - 2y = 3$ (2)

Multiplying equation (1) by 2 and equation (2) by 5 gives:

$12x + 10y = 70$ (3)

$5x - 10y = 15$ (4)

(3) + (4) gives:

$17x = 85$, so $x = 5$

Substituting in (1): $30 + 5y = 35$, so $y = 1$

$x = 5$ and $y = 1$

27 $x^2 \leq 2x + 15$

$x^2 - 2x - 15 \leq 0$

Factorising $x^2 - 2x - 15 = 0$ gives

$(x - 5)(x + 3) = 0$: $x = 5$ or -3

As the coefficient of x^2 is positive, the graph of $y = x^2 - 2x - 15$ is U-shaped.

$x^2 - 2x - 15 \leq 0$ for the region below the x-axis (i.e. where $y < = 0$).

$-3 \leq x \leq 5$

28 **a** translation 3 units in the x-direction: turning point at (3, 6)

b reflection in x-axis: turning point at (0, −6)

29 gradient of $OA = \frac{4}{3}$

gradient of $BC = -\frac{3}{4}$

equation of BC is

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{3}{4}(x - 3)$

$4y - 16 = -3x + 9$

$3x + 4y - 25 = 0$

Practice paper 2 (calculator)

1 $2x^2 + \frac{3x^2}{x} + x = 2x^2 + 3x + x = 2x^2 + 4x$

2 $x - 3 = 0$ or $x + 4 = 0$, therefore $x = 3$ or -4

3 100

4 $3\mathrm{a} - 2\mathrm{b} = 3\begin{pmatrix}2\\1\end{pmatrix} - 2\begin{pmatrix}-1\\0\end{pmatrix} = \begin{pmatrix}3 \times 2 - 2 \times (-1)\\3 \times 1 - 2 \times 0\end{pmatrix} = \begin{pmatrix}8\\3\end{pmatrix}$

5 percentage increase $= \frac{\text{increase}}{\text{original value}} \times 100$
$= \frac{20\,000}{150\,000} \times 100$
$= 13.3\%$ (to 1 d.p.)

6 **a** 391.8954599

b $19.85^2 - \sqrt{98.67} \div 4.67 \approx 20^2 - \sqrt{100} \div 5$
$= 400 - 2$
$= 398$

7 Number of sides $= \frac{360}{60} = 6$, therefore the regular polygon is a hexagon.

8 The first digit could be from 1 to 9 (i.e. 9 options).

The second digit is fixed at 5 (i.e. 1 options).

The third digit could be any digit (i.e. 10 options).

The fourth digit would have to be even to make the whole number even so it would have to be 0, 2, 4, 6 or 8 (i.e. 5 options).

total number of possible numbers $= 9 \times 1 \times 10 \times 5 = 450$

9 C

$y = \frac{k}{x}$. Since x is the denominator of the fraction, the larger x gets, the smaller y will be. Similarly, y will be big when x is as small as possible. Graph C shows this relationship.

10 Using $y = mx + c$, the equation of the straight line is $y = -\frac{2}{3}x + 2$

11 a $245 = 5 \times 49 = 5 \times 7 \times 7 = 5 \times 7^2$

b The only shared factors are 5 and 7, so: highest common factor $= 5 \times 7 = 35$

12 $s = \frac{1}{2}at^2$

$= \frac{1}{2} \times 2.7 \times 10^8 \times (7.5 \times 10^{-3})^2$

$= 7593.75$

$= 7600$ to 2 s.f. (as the values used to calculate it were given to 2 s.f.)

13 $2^2 : 7^2 = 4 : 49$

14

Number of pets per household	Frequency	Midpoint	Frequency × midpoint
0–2	22	1	22
3–5	12	4	48
6–8	x	7	$7x$
9–11	3	10	30

total number of pets $= 22 + 48 + 7x + 30 = 100 + 7x$

total number of households $= 22 + 12 + x + 3 = 37 + x$

estimate of mean $= \frac{\text{total number of pets}}{\text{total number of households}} = \frac{100 + 7x}{37 + x}$

$\frac{100 + 7x}{37 + x} = 3.25$

$100 + 7x = 120.25 + 3.25x$

$3.75x = 20.25$

$x = 20.25 \div 3.75$

$x = 5.4$

5.4 is the correct answer for the figures given, but a frequency should be an integer so the question is not realistic. If the question gave midpoints of 1, 5, 6, 10, for example, the answer would be 3. Note that in the question, column 1 is also labelled as ‘Number of households’ when it should say ‘Number of pets’.*

15 a Multiply the coefficient of x^2 by the number term: $10 \times 3 = 30$

Now look for a factor pair of 30 that adds to give the coefficient of :

$2 + 15 = 17$

$10x^2 + 17x + 3 = 10x^2 + 15x + 2x + 3$
$= 5x(2x + 3) + (2x + 3)$

Therefore, $10x^2 + 17x + 3 = (5x + 1)(2x + 3)$

b $(5x + 1)(2x + 3) = 0$

so $5x + 1 = 0$ giving $x = -\frac{1}{5}$

or $2x + 3$ giving $x = -\frac{3}{2}$

16 a volume $= 8 \times 6 \times 4 = 192\,\text{cm}^3$

b $1\,\text{m}^3 = 1\,000\,000\,\text{cm}^3$

volume in $\text{m}^3 = \frac{192}{1\,000\,000} = 1.92 \times 10^{-4}\,\text{m}^3$

c density in $\text{kg/m}^3 = \frac{\text{mass in kg}}{\text{volume in m}^3}$

mass in kg = density in $\frac{\text{kg}}{\text{m}^3}$ × volume in m^3

$= 8940 \times 1.92 \times 10^{-4}$

$= 1.72$ kg (to 2 d.p.)

d Using the answer from c, force in N = mass in kg × 9.81

$= 1.72 \times 9.81$

$= 16.87$ N (to 2 d.p.)

Or, more accurately, using the answer from c without rounding first:

$8940 \times 1.92 \times 10^{-4} \times 9.81$

$= 16.84$ N (to 2 d.p.)

e The greatest value for pressure will occur when the block is on its smallest area.

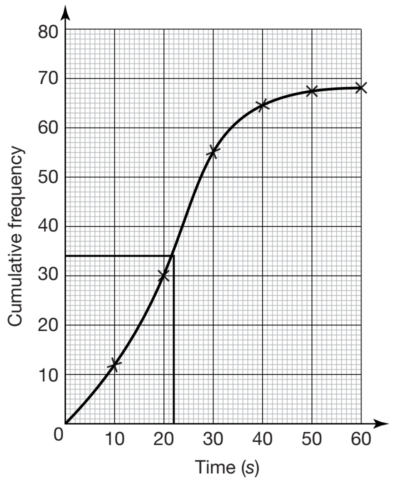
smallest area $= 6 \times 4 = 24\,\text{cm}^2 = \frac{24}{10\,000}\,\text{m}^2 = 0.0024\,\text{m}^2$

maximum pressure $= \frac{16.87}{0.0024} = 7030\,\text{N/m}^2$ (to 3 s.f.)

17 a

Time for call to be answered (t seconds)	Frequency	Cumulative frequency
$0 < t \leq 10$	12	12
$10 < t \leq 20$	18	30
$20 < t \leq 30$	25	55
$30 < t \leq 40$	10	65
$40 < t \leq 50$	2	67
$50 < t \leq 60$	1	68

b



c Reading from the graph above, the median time is 22 s, but any answer within the range of 21–22.5 is acceptable (depending on the curve drawn for part b).

18 a angle OAP = angle OBP = 90° (tangent and radius at right angle)

$OA = OB$ (radii of the circle)

Side OP is common to both triangles.

Both OAP and OBP are right-angled triangles and the two sides are the same, so by Pythagoras’ theorem the third sides BP and AP must be equal.

b All the corresponding sides are equal in length, so the two triangles are congruent (SSS).

19 Let the length of the edge of the original cube $= x$ cm

volume of original cube $= x^3$

The three sides of the cuboid are $x + 3$, $x - 2$ and x.

volume of cuboid $= (x + 3)(x - 2)x$

difference in volumes $= (x + 3)(x - 2)x - x^3$

$= (x^2 + x - 6)x - x^3$

$= x^3 + x^2 - 6x - x^3$

$= x^2 - 6x$

---

*This information differs from that given in the Exam Practice book due to an error in our first edition. This has now been re-checked and corrected.

$x^2 - 6x = 55$

$x^2 - 6x - 55 = 0$

$(x - 11)(x + 5) = 0$

$x = 11$ or -5

The cube cannot have a negative length of side, so length of side of original cube = 11 cm.

20 $2ay + 2c = 3 - y$

$2ay + y = 3 - 2c$

$y(2a + 1) = 3 - 2c$

$y = \frac{3 - 2c}{2a + 1}$

21 3 + 5 = 8 parts

8 parts = 500 g so 1 part = 62.5 g

metal A: mass = 3 × 62.5 = 187.5 g

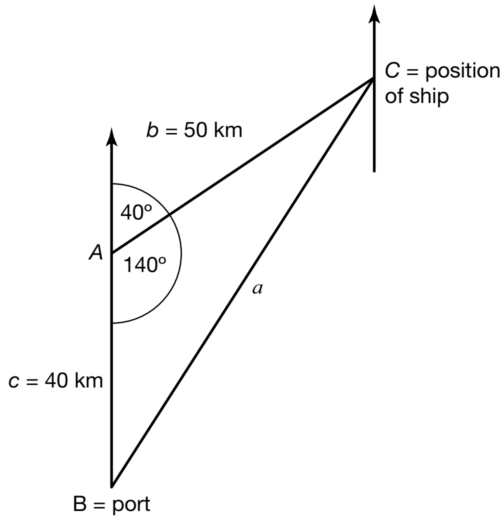
cost = $\frac{187.5}{1000} \times 50$ = £9.375

metal B: mass = 5 × 62.5 = 312.5 g

cost = $\frac{312.5}{1000} \times 140$ = £43.75

total cost = 9.375 + 43.75 = 53.125 = £53.13 (to nearest penny)

22 **a**



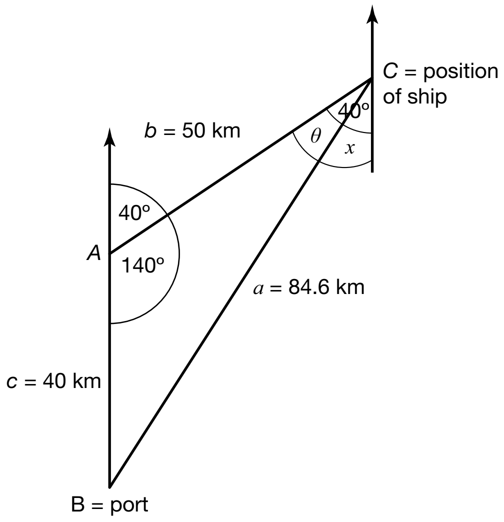
Using the cosine rule:

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 50^2 + 40^2 - 2 \times 50 \times 40 \times \cos 140°$

distance = 84.6 km (to 1 d.p.)

b



Using the sine rule:

$\frac{40}{\sin\theta} = \frac{84.6}{\sin 140°}$

$\sin\theta = \frac{40 \times \sin 140°}{84.6}$

$\theta = 17.7°$ (to 1 d.p.)

$x = 40 - \theta = 40 - 17.7 = 22.3°$

required angle = 180 + 22.3 = 202.3°

bearing = 202° to nearest degree.

23 **a** multiplier = $1 - \frac{\%\text{ evaporation}}{100}$

$= 1 - \frac{2}{100} = 0.98$

volume at the end of 4 weeks = $A_0 \times (\text{multiplier})^n$

$= 30 \times (0.98)^4$

$= 27.7\text{ m}^3$ (to 3 s.f.)

b Need to aim for a volume of 15 m³.

Try 14 weeks: volume = $30 \times (0.98)^{14} = 22.6$

Try 20 weeks: volume = $30 \times (0.98)^{20} = 20.0$

Try 24 weeks: volume = $30 \times (0.98)^{24} = 18.5$

Try 34 weeks: volume = $30 \times (0.98)^{34} = 15.1$

Try 35 weeks: volume = $30 \times (0.98)^{35} = 14.8$

Answer is 35 weeks.

24 a number of males = $\frac{4}{9} \times 36 = 16$

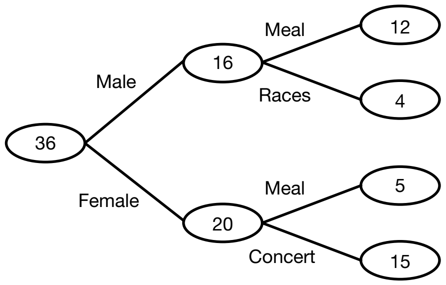
number of males + races = 25% of 16 = 4

number of males + meal = 16 − 4 = 12

number of females = 36 − 16 = 20

number of females + meal = $\frac{1}{4} \times 20 = 5$

number of females + concert = 20 − 5 = 15



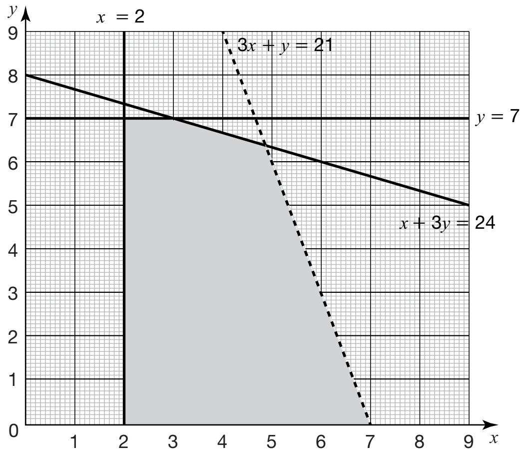
b P(meal) = $\frac{12 + 5}{36} = \frac{17}{36}$

25 $x + 3y = 24$: when $x = 0$, $y = 8$ and when $x = 6$, $y = 6$; required region below this line

$y + 3x = 21$: when $x = 5$, $y = 6$ and when $x = 7$, $y = 0$; required region below this line

$x \geq 2$: required region to the right of this line

$y \leq 7$: required region below this line



26 Using Pythagoras' theorem in triangle *ABC*:

$AC^2 = 2^2 + 3^2 = 13$

$AC = \sqrt{13}$ cm

Using Pythagoras' theorem in triangle *ACD*:

$AD^2 = (\sqrt{13})^2 + 3^2 = 13 + 9 = 22$

$AD = \sqrt{22}$ cm

27 **a** angle *PRQ* = 60° (angles on the same arc are equal)

b angle *PSR* = 90° (angle in a semicircle is 90°)

angle *PRS* = 180 − (90 + 21) = 69° (angles in a triangle add up to 180°)