All Boards Foundation Mathematics Revision Guide

Full worked solutions

Number

Basic number techniques

1 **a** false **c** true **e** true
b true **d** true

2 −0.3, −1.5, −2.5, −4.2, −7.2

3 0.049, 0.124, 0.412, 0.442, 1.002

4 **a** < **b** < **c** >

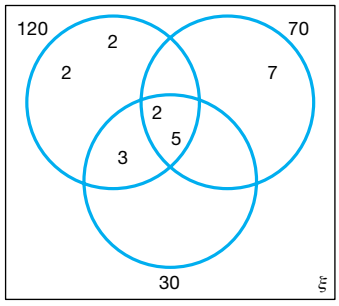
Factors, multiples and primes

1 **a** 5 **b** 1, 12 **c** 1, 5, 45 **d** 1, 5

2 $70 = 2 \times 5 \times 7$
$150 = 2 \times 3 \times 5 \times 5$
HCF = 10, LCM = 1050

3 $2 \times 3^2 \times 5$

4



a $2 \times 5 = 10$ **b** $2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840$

5 12 and 18

Calculating with negative numbers

Stretch it! Multiplying three negative numbers together always gives a negative answer.

1 **a** $-8 - 3 = -11$ **d** $14 + 4 = 18$
b 99 **e** 0
c −6 **f** $12 + 15 - 2 = 25$

2 −8 and 9

3 32°C

Division and multiplication

Stretch it!

$$\begin{array}{r} 621 \\ \times\ 239 \\ \hline 5589 \times 9 \\ 18630 \times 30 \\ 124200 \times 200 \\ \hline 148419 \end{array}$$

148419

1 **a**

$$\begin{array}{r} 235 \\ \times\ 9 \\ \hline 2115 \end{array}$$

2115

b

$$\begin{array}{r} 924 \\ \times\ 61 \\ \hline 924 \times 1 \\ 55440 \times 60 \\ \hline 56364 \end{array}$$

56364

2 **a** $5\overline{)235}$ = 47 **b** $8\overline{)4128}$ = 516

c

$$\begin{array}{r} 126 \\ 17\overline{)2146} \\ 17 \\ \hline 44 \\ 34 \\ \hline 106 \\ 102 \\ \hline 4 \end{array}$$

126 remainder 4, or $126\frac{4}{17}$

3 $8\overline{)2665}$ = 331 remainder 1 **a** 33 boxes **b** 1 pencil

4 $4\overline{)3651.00}$ = £91.25

5

$$\begin{array}{r} 32 \\ \times\ 9 \\ \hline 288 \end{array}$$

£288

6 $3\overline{)9207.66}$ = 307.6 = $307\frac{2}{3}$

7

$$\begin{array}{r} 823 \\ \times\ 35 \\ \hline 4115 \times 5 \\ 24690 \times 30 \\ \hline 28805 \end{array}$$

28 805

8 a $239 + ? = 921$

Missing number $= 921 - 239$
$= 682$

b $? \times 87 = 1131$

Missing number $= 1131 \div 87$
$= 13$

c $23 \times ? + 8 = 123$

$23 \times ? = 123 - 8 = 115$

Missing number $= 115 \div 23$
$= 5$

9

$$\begin{array}{r} 037 \\ 12\overline{)450} \\ 36 \\ \hline 90 \\ 84 \\ \hline 6 \end{array}$$

37 remainder 6, so
37 boxes

10 He has not placed a zero in the ones column before multiplying through by 5. The × 50 line should have 5 digits: 36300, so his final three rows of working should look like this:

$$\begin{array}{r} 1452 \times 2 \\ 36300 \times 50 \\ \hline 37752 \end{array}$$

Calculating with decimals

Stretch it! $3.2 + 7.5 \times 2 = 3.2 + 15 = 18.2$

1 a

$$\begin{array}{r} {}^{3}{}^{1} \\ 3.40 \\ -1.07 \\ \hline 2.33 \end{array}$$

2.33

b

$$\begin{array}{r} 19.300 \\ +\ 5.091 \\ \hline 24.391 \\ {\scriptstyle 1} \end{array}$$

24.391

c $5 \times 7 = 35$

$0.05 \times 0.7 = 0.035$

d

$$\begin{array}{r} 321 \\ \times\ 19 \\ \hline 2889 \times 9 \\ 3210 \times 10 \\ \hline 6099 \\ {\scriptstyle 1} \end{array}$$

$3.21 \times 1.9 = 6.099$

e

$$5\overline{)7.^{2}8^{3}1^{1}5} \quad 1.563$$

1.563

f $\frac{0.18}{0.3} = \frac{18}{30} = \frac{3}{5} = 0.6$

g $\frac{-0.3}{0.05} = \frac{-30}{5} = -6$

h $72 \times -7 = -504$

Therefore: 7.2×-0.7
$= -5.04$

2

$$\begin{array}{r} 64 \\ \times\ 24 \\ \hline 256 \\ 1280 \\ \hline 1536 \\ {\scriptstyle 1} \end{array}$$

$24 \times 64 = 1536$
$24 \times £0.64 = £15.36$
$£20 - £15.36 = £4.64$

3

$$3\overline{)8^{2}2.^{1}3^{1}8} \quad 27.46$$

Erica $2 \times 27.46 = £54.92$
Freya $82.38 - 54.92 = £27.46$

Rounding and estimation

Stretch it! a 1.0 **b** 1.00 **c** 1.000
All the answers are 1

Stretch it! $6.5 \times 8.5 = 55.25\,m^2$ – an overestimate.

1 a 0.35 **c** 32.6
b 10 **d** 33100

2 a $150 \leq x < 250$ **c** $3.15 \leq x < 3.25$
b $5.5 \leq x < 6.5$ **d** $5.055 \leq x < 5.065$

3 $\frac{30}{0.5 \times 6} = 10$

4 a 23580 **c** 23600 **e** 20000
b 23580 **d** 24000

5 b is false since $18 \times 1 = 18$ so 18×0.9 cannot be 1.62.

c is false because if you divide by a number smaller than 1, the answer will be larger.

6 Night-time low tariff: 2.32 units × 1.622p = 3.76p
7.151 units × 2.315p = 16.55p
20.31p

One tariff: $2.320 + 7.151 = 9.471$ units
9.471 units $\times$ 1.923p = 18.21p

Tarik should choose One tariff, since it is cheaper.

Converting between fractions, decimals and percentages

Stretch it! $0.\dot{1}, 0.\dot{2}, 0.\dot{3}, \ldots 0.\dot{4}, 0.\dot{5}$. The number of ninths is the same as the digit that recurs. The exception is $\frac{9}{9}$ which is the same as 1.

1 a $\frac{32}{100} = \frac{8}{25}$ **c** $\frac{33}{100}$
b $1\frac{24}{100} = 1\frac{6}{25}$ **d** $\frac{95}{100} = \frac{19}{20}$

2 a

$$12\overline{)5.^{5}0^{2}0^{8}0^{8}0^{8}0} \quad 0.41666\ldots$$

$0.41\dot{6}$

b

$$8\overline{)3.^{3}0^{6}0^{4}0\,0} \quad 0.375$$

0.375

c 0.49

d 0.185

e

$$7\overline{)3.^{3}0^{2}0^{6}0^{4}0^{5}0^{1}0^{3}0^{2}0} \quad 0.42857142\ldots$$

$0.\dot{4}28571\dot{4}2\ldots$

3 a $\frac{91}{100} = 91\%$

b $\frac{3}{10} = \frac{30}{100} = 30\%$

c $\frac{4}{5} = \frac{80}{100} = 80\%$

d $\frac{9}{15} = \frac{3}{5} = \frac{6}{10} = \frac{60}{100} = 60\%$

4 Divide 3 by 8:

$$8\overline{)3.^{3}0^{6}0^{4}0\,0} \quad 0.375$$

$= 0.375 = 37.5\%$

5 0.35 $\frac{2}{5} = \frac{4}{10} = 0.4$ $30\% = 0.3$

30%, 0.35, $\frac{2}{5}$

6 $\frac{15}{20} = \frac{75}{100} = 75\%$ – Amy

Rudi's mark was higher.

Ordering fractions, decimals and percentages

1 $\frac{1}{3} = \frac{8}{24}$ $\frac{3}{8} = \frac{9}{24}$ $\frac{7}{12} = \frac{14}{24}$
$\frac{7}{12}, \frac{3}{8}, \frac{1}{3}$

2 -2.2, 7, $\frac{1}{5} = 0.2$, $-\frac{1}{10} = -0.1$, 15% = 0.15, 1% = 0.01, 0.1
In order, this is: -2.2, $-\frac{1}{10}$, 1%, 0.1, 15%, $\frac{1}{5}$, 7.
The middle value is 0.1

3 Yes. If the numerator of a fraction is half the denominator then the fraction is equivalent to $\frac{1}{2}$.
If the numerator is smaller than this the fraction must be less than $\frac{1}{2}$.

Calculating with fractions

Stretch it! No – you could add the whole number parts, and then add the fraction parts, giving:

$1 + 2 = 3$

$\frac{3}{5} + \frac{1}{4} = \frac{17}{20}$

$= 3\frac{17}{20}$

1 a $2\frac{3}{8} - \frac{3}{4} = \frac{19}{8} - \frac{3}{4} = \frac{19}{8} - \frac{6}{8} = \frac{13}{8} = 1\frac{5}{8}$

b $\frac{\cancel{15}^{3}}{17} \times \frac{2}{\cancel{5}_{1}} = \frac{6}{17}$

c $\frac{1}{7} \times 3\frac{1}{3} = \frac{1}{7} \times \frac{10}{3} = \frac{10}{21}$

d $2\frac{2}{5} + 5\frac{3}{4} = 7 + \frac{2}{5} + \frac{3}{4} = 7 + \frac{8}{20} + \frac{15}{20}$

$= 7\frac{23}{20}$

$= 8\frac{3}{20}$

e $\frac{1}{5} \div 2\frac{1}{2} = \frac{1}{5} \div \frac{5}{2} = \frac{1}{5} \times \frac{2}{5} = \frac{2}{25}$

2 a $30 \div 5 = 6$
$6 \times 2 = 12$

b $40 \div 8 = 5$
$5 \times 7 = £35$

c $1818 \div 9 = 202$
$202 \times 4 = 808\,\text{mm}$

3 $35 \div 7 = 5$
$3 \times 7 = 15$
$35 - 15 = 20$

4 The number must be a multiple of 5, and $\frac{2}{5}$ of it must be a multiple of 2.
$\frac{2}{5}$ of 45 = 18
$\frac{2}{5}$ of 40 = 16
$\frac{2}{5}$ of 35 = 14
$\frac{2}{5}$ of 30 = 12
$\frac{2}{5}$ of the number must be greater than 12, so the number is 35

Percentages

1 a $18 \div 100 = 0.18\,\text{cm}$
$0.18 \times 10 = 1.8\,\text{cm}$

b $1.20 \div 100 = £0.012$
$0.012 \times 25 = £0.30$

c $200\,\text{ml} \div 100 = 2\,\text{ml}$
$2\,\text{ml} \times 2 = 4\,\text{ml}$

2 a $1.1 \times 30 = 33$

b $1.08 \times 500 = 540$

c $1.12 \times 91 = 101.92$, so £101.92

3 a $0.8 \times 600 = 480$

b $0.95 \times 140 = 133$

c $0.81 \times 18 = 14.58$, so £14.58

4 $1.09 \times 2800 = 3052$

5 $0.65 \times 22\,000 = £14\,300$

Order of operations

1 a 7

b $0.9 + 3.2 - \sqrt{36}$
$= 0.9 + 3.2 - 6$
$= -1.9$

c $(-1)^2 - 14$
$= 1 - 14$
$= -13$

2 30

3 $(8 - 3 + 5) \times 4$

Exact solutions

1 a π **b** 36π **c** $2\frac{1}{2}\pi$ or $\frac{5}{2}\pi$

2 a 7π **b** $\frac{5}{8}\pi$

3 Area $= \frac{2}{7} \times \frac{3}{4} = \frac{6}{28} = \frac{3}{14}\,\text{cm}^2$

Perimeter $= \left(2 \times \frac{3}{4}\right) + \left(2 \times \frac{2}{7}\right) = \frac{3}{2} + \frac{4}{7} = \frac{21+8}{14} = \frac{29}{14}$

$= 2\frac{1}{14}\,\text{cm}$

4 a $2 \times 9 \times \pi = 18\pi\,\text{cm}$

b $12^2 \times \pi = 144\pi\,\text{cm}^2$

5 Circumference $= 2 \times \pi \times 1 = 2\pi\,\text{cm}$
Length of one side of square $= 2\pi \div 4 = \frac{1}{2}\pi\,\text{cm}$

Indices and roots

1 a $\frac{1}{3}$ **b** $\frac{1}{0.4} = \frac{10}{4} = 2\frac{1}{2}$

c $\frac{1}{0.9} = \frac{10}{9} = 1\frac{1}{9}$

2 $3^2 = 9$ $1^3 = 1$ $\sqrt[3]{27} = 3$ $\sqrt[3]{8} = 2\frac{1}{12} = 0.083\dot{3}$
In order, this gives $\frac{1}{12}$, 1^3, $\sqrt[3]{8}$, $\sqrt[3]{27}$, 3.7, 3^2

3 a -8 **b** 1 **c** 81 **d** 1

4 a $\frac{1}{4}$ **b** $\frac{1}{7^2} = \frac{1}{49}$

c $\frac{1}{1^4} = 1$ **d** $\frac{1}{3}$

5 $\frac{5^9}{5^5} = 5^4$

6 $(0.01 \times 798)^2 = 7.98^2$
$\approx 8^2$
$= 64$

Standard form

1 a 45 000 000 **b** 0.091

2 a 6.45×10^8 **b** 7.9×10^{-8}

3 $350\,000 - 4200 = 345\,800$

4 $3.2 \times 10^2 = 320$ $3.1 \times 10^{-2} = 0.031$
$3.09 \times 10 = 30.9$ $3 + (2.1 \times 10^2) = 213$
In order, this gives:
3.1×10^{-2} 3.09×10
$3 + (2.1 \times 10^2)$ 3.2×10^2

5 3×10^8 m/s

6 $200 \times 1.1 \times 10^{-4} = 2.2 \times 10^{-2} = 0.022$ m $= 2.2$ cm

Listing strategies

Stretch it!

red + small, red + medium, red + large,

green + small, green + medium, green + large,

blue + small, blue + medium, blue + large.

1 111,
112, 121, 211, 113, 131, 311,
222
221, 212, 122, 223, 232, 322
333
331 313 133 332 323 233
123 132 213 231 312 321

2 444 446 449
464 466 469
494 496 499

3 Small A, Small B, Small C, Small D
Medium A, Medium B, Medium C, Medium D
Large A, Large B, Large C, Large D.

Review it!

1 Total tickets sold = 34 592
Sold online = 21 298

$$\begin{array}{r} 3\,4\,\overset{4}{\cancel{5}}\,\overset{18}{\cancel{9}}\,\overset{1}{2} \\ -\,2\,1\,2\,9\,8 \\ \hline 1\,3\,2\,9\,4 \end{array}$$

Sold at station = 13 294

2 7 and 6 (or 11 and 2, where both are prime and 2 is also a factor of 12)

3 $620 = 2 \times 2 \times 5 \times 31 = 2^2 \times 5 \times 31$

4 $18 = 2 \times 3 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
$40 = 2 \times 2 \times 2 \times 5$
HCF = 2

5 −11.5, −8.3, −3.5, −3.2, 1.4

6 a

$$\begin{array}{r} 3\,2\cdot 9\,9 \\ +\,1\,8\cdot 7\,4 \\ \hline 5\,1\cdot 7\,3 \\ \scriptstyle 1\;1\;\;1 \end{array}$$

£51.73

b

$$3\overline{)54.99}\quad 18.33$$

£18.33

7 a

$$\begin{array}{r} 2\,3 \\ \times\,1\,4 \\ \hline 9\,2 \times 4 \\ 2\,3\,0 \times 10 \\ \hline 3\,2\,2 \\ \scriptstyle 1 \end{array}$$

$23 \times 0.14 = 3.22$

b

$$\begin{array}{r} 1\,4\,9 \\ \times\,2\,7 \\ \hline 1\,0\,4\,3 \times 7 \\ 2\,9\,8\,0 \times 20 \\ \hline 4\,0\,2\,3 \\ \scriptstyle 1\;1 \end{array}$$

4023

c $-3 \times 4 = -12$

There are two decimal places to put in, so $-0.3 \times 0.4 = -0.12$

d $-13.5 + 8.7 = -13.5 + 9 - 0.3$
$= -4.5 - 0.3$
$= -4.8$

e $\frac{-1.2}{0.3} = \frac{-12}{3} = -4$

8 $30 \times \sqrt{16} + 17 = 30 \times \pm 4 + 17$
$= \pm 120 + 17$
$= 137$ or -103

9 $81 \div 3 = 27$

10

$$11\overline{)345}\quad 031 \text{ remainder } 4$$

$31\frac{4}{11}$

11 a

$$8\overline{)3.000}\quad 0.375$$

0.375

b $0.7 \times 100 = 70\%$

12 a $70\% = \frac{70}{100} = \frac{7}{10}$

b $0.8 = \frac{8}{10} = \frac{4}{5}$

13 $\frac{1}{2} = \frac{2}{4}$ $\frac{1}{2}$ is larger

$\frac{2}{7} = \frac{8}{28}$ $\frac{1}{4} = \frac{7}{28}$ $\frac{2}{7}$ is larger

$\frac{3}{11} = \frac{12}{44}$ $\frac{1}{4} = \frac{11}{44}$ $\frac{3}{11}$ is larger

$\frac{2}{5} = \frac{8}{20}$ $\frac{1}{4} = \frac{5}{20}$ $\frac{2}{5}$ is larger

All of them.

14 a $\frac{3}{5} + \frac{1}{7} = \frac{21}{35} + \frac{5}{35} = \frac{26}{35}$

b $2\frac{1}{5} - \frac{7}{10} = \frac{11}{5} - \frac{7}{10} = \frac{22}{10} - \frac{7}{10} = \frac{15}{10} = 1\frac{1}{2}$

c $\frac{2}{3} \div \frac{4}{9} = \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}} = \frac{3}{2} = 1\frac{1}{2}$

15 $0.25 - 0.07 = 0.18 = \frac{18}{100} = \frac{108}{600}$

$\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6} = \frac{100}{600}$

0.25 − 0.07 is larger

16 $\frac{3}{\cancel{5}} \times \frac{\cancel{5}}{4} = \frac{3}{4}$

17 $\frac{45}{1000} = \frac{9}{200}$

18 $8.6 \div 100 = 0.086$
$0.086 \times 25 = £2.15$

19 a 9 **b** 5

20 a 3.4×10^9 **b** 3.04×10^{-7}

21 $37.55 \leq x < 37.65$

22 a 51 **b** 12, 15, 21, 51, 25, 52

23 a $200 \times 9 \times 10 = 18000 = £180.00$

b Underestimate since all numbers were rounded down.

24 40% of 600 = 240

$\frac{1}{5}$ of 600 = 120

600 − (240 + 120) = 240

25 More than 33%, less than 50%, multiple of 5. 35%

26 No, since 2 is even and a prime number, and odd + odd + even = even.

27 $0.8 \times 349 = £279.20$

28 a 3.1 **b** 3.05

29 a 325 000 **b** 320 000

30 $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$

31 $26.25 + 18.23 + (4 \times 5.5) = £66.48$

$£66.48 \div 4 = £16.62$

32 $0.19 \times 18\,000 = 3420$

33 $102.3 \times 1.1 = 112.53$

The price in 2017 was £112.53

Algebra

Understanding expressions, equations, formulae and identities

1 a $3a + 6 = 10$ (It can be solved to find the value of a.)

b $C = \pi D$ (The value of C can be worked out if the value of D is known.)

c $3(a + 2)$ (It does not have an equals sign.)

d $3ab + 2ab = 5ab$ (Collecting the like terms on the left-hand side gives $5ab$ which is equal to the right-hand side.)

2 James is correct.

$4x - 2 = 2x$ can be solved to find the value of x so it is an equation.

Or, the two sides of $4x - 2 = 2x$ are not equal for all values of x so it cannot be an identity. For example, when $x = 2$:

(Left-hand side) $4x - 2 = 4 \times 2 - 2 = 6$

(Right-hand side) $2x = 2 \times 2 = 4$

$6 \neq 4$

Simplifying expressions

Stretch it!

The expressions must all contain algebra, so each part must include t.

There are four possible combinations that make $12t^3$: $12t \times t \times t$, $2t \times 6t \times t$, $2t \times 3t \times 2t$, $3t \times 4t \times t$.

1 a p^3

b $4 \times b \times c \times 7 = 4 \times 7 \times b \times c = 28bc$

c $4a \times 3b = 4 \times 3 \times a \times b = 12ab$

d $5x \times 4x = 5 \times 4 \times x \times x = 20x^2$

e $2g \times (-4g) = 2 \times (-4) \times g \times g = -8g^2$

f $2p \times 3q \times r = 2 \times 3 \times p \times q \times r = 6pqr$

2 a $10x \div 2 = \frac{10x}{2} = 5x$

b $\frac{14w}{-2} = -7w$

c $6p \div p = \frac{6p}{p} = 6$

d $8mn \div 2m = \frac{8mn}{2m} = 4n$

e $\frac{12xy}{3y} = 4x$

f $9abc \div bc = \frac{9abc}{bc} = 9a$

Collecting like terms

1 a $5f$

b $7b$

c $5mn$

d $4a + 6 - a - 5 = 4a - a + 6 - 5 = 3a + 1$

e $3d + 4e + d - 6e = 3d + d + 4e - 6e = 4d - 2e$

f $2x + 5y + 3x - 2y - 2 = 2x + 3x + 5y - 2y - 2$
$= 5x + 3y - 2$

g $3a - 2b + 4a + 7b = 3a + 4a - 2b + 7b =$
$7a + 5b$

h $2a - b - 5a - 3 = 2a - 5a - b - 3 = -3a - b - 3$

i $x^2 + x^2 = 2 \times x^2 = 2x^2$

j $2t^3 + 4 - t^3 - 4 = 2t^3 - t^3 + 4 - 4 = t^3$

k $2a + b^2$

l $(4 + 3)\sqrt{x} = 7\sqrt{x}$

m $(7 - 4)\sqrt{x} = 3\sqrt{x}$

n $(12 - 1 - 4)\sqrt{x} = 7\sqrt{x}$

Using indices

1 a $x^5 \times x^4 = x^{5+4} = x^9$

b $p \times p^4 = p^{1+4} = p^5$

c $2m^4 \times 3m^4 = 2 \times 3 \times m^4 \times m^4 = 6 \times m^{4+4} = 6m^8$

d $3m^4n \times 5m^2n^3$
$= 3 \times 5 \times m^4 \times m^2 \times n \times n^3$
$= 15 \times m^{4+2} \times n^{1+3} = 15m^6n^4$

e $u^{-2} \times u^5 = u^{-2+5} = u^3$

f $t^7 \times t^{-6} = t^{7+(-6)} = t$

2 a $x^4 \div x^2 = x^{4-2} = x^2$

b $\frac{y^7}{y^3} = y^{7-3} = y^4$

c $\frac{p^9}{p^8} = p^{9-8} = p$

d $8x^6 \div 4x^3 = \frac{8x^6}{4x^3}$

$(8 \div 4) \times (x^6 \div x^3) = 2 \times x^{6-3} = 2x^3$

e $m^3 \div m^5 = m^{3-5} = m^{-2} = \frac{1}{m^2}$

f $\frac{5x^8}{15x^4} = \frac{5}{15} \times \frac{x^8}{x^4} = \frac{1}{3} \times x^{8-4} = \frac{x^4}{3}$

g $3x^2 \div 9x = \frac{3x^2}{9x} = \frac{3}{9} \times \frac{x^2}{x} = \frac{1}{3} \times x^{2-1} = \frac{x}{3}$

3 a $(x^2)^3 = x^{2\times3} = x^6$

b $(y^4)^4 = y^{4\times4} = y^{16}$

c $(p^5)^2 = p^{5\times2} = p^{10}$

d $(4m^5)^2 = 4^2 \times (m^5)^2 \times 16 \times m^{5\times2} = 16m^{10}$

e $(x^2)^{-3} = x^{2\times(-3)} = x^{-6} = \frac{1}{x^6}$

f $(n^{-4})^{-2} = n^{-4\times(-2)} = n^8$

4 **a** $4x \times 3x^2 = 4 \times 3 \times x \times x^2 = 12 \times x^{1+2} = 12x^3$

b $\frac{5x^4}{x} = 5x^{4-1} = 5x^3$

c $\frac{1}{y^2}$

d $a^3b^2 \times a^2b = a^3 \times a^2 \times b^2 \times b = a^{3+2}b^{2+1} = a^5b^3$

5 $\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4} = x^{8-4} = x^4$

Expanding brackets

Stretch it!

a $a\sqrt{3} + a^2$ **or** $\sqrt{3}a + a^2$

b $b\sqrt{5} - b^2$ **or** $\sqrt{5}b - b^2$

c $c + d$

Stretch it!

1 $2 + \mathbf{4} = 6$ and $2 \times \mathbf{4} = 8$, so the numbers are 4 and 8.
$(x+2)(x+4) = x^2 + 6x + 8$

2 **a** $(x + 3)(2x + 2)$
$= 2x^2 + 6x + 2x + 6$
$= 2x^2 + 8x + 6$

b $(3x - 2)(x + 4)$
$= 3x^2 - 2x + 12x - 8$
$= 3x^2 + 10x - 8$

c $(2x + 3)(3x - 1)$
$= 6x^2 + 9x - 2x - 3$
$= 6x^2 + 7x - 3$

d $(x + 2y)(x - y)$
$= x^2 - xy + 2xy - 2y^2$
$= x^2 + xy - 2y^2$

e $(2x - y)(3x + y)$
$= 6x^2 + 2xy - 3xy - y^2$
$= 6x^2 - xy - y^2$

1 **a** $3a + 6$ **e** $4x + 4y + 8$ **i** $3x + 6y$

b $4b - 16$ **f** $-2y - 4$ **j** $-2a + 2b$ or $2b - 2a$

c $10c + 25$ **g** $x^2 - 2x$

d $6 - 2e$ **h** $2a^2 + 10a$

2 **a** $6a - (3a + 5)$
$= 6a - 3a - 5$
$= 3a - 5$

b $4x - 6 + 2(x + 5)$
$= 4x - 6 + 2x + 10$
$= 4x + 2x - 6 + 10$
$= 6x + 4$

3 **a** $2(2x + 3) + 4(x + 5) = 4x + 6 + 4x + 20 = 8x + 26$

b $3(3y + 1) + 2(4y - 3) = 9y + 3 + 8y - 6 = 17y - 3$

c $4(2m + 4) - 3(2m - 5) = 8m + 16 - 6m + 15 = 2m + 31$

4 **a** $(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$

b $(y - 3)(y + 4) = y^2 + 4y - 3y - 12 = y^2 + y - 12$

c $(a + 3)(a - 7) = a^2 - 7a + 3a - 21 = a^2 - 4a - 21$

d $(m - 1)(m - 6) = m^2 - 6m - m + 6 = m^2 - 7m + 6$

5 **a** $(x + 1)^2 = (x + 1)(x + 1)$
$= x^2 + x + x + 1 = x^2 + 2x + 1$

b $(x - 1)^2 = (x - 1)(x - 1)$
$= x^2 - x - x + 1 = x^2 - 2x + 1$

c $(m - 2)^2 = (m - 2)(m - 2)$
$= m^2 - 2m - 2m + 4 = m^2 - 4m + 4$

d $(y + 3)^2 = (y + 3)(y + 3)$
$= y^2 + 3y + 3y + 9 = y^2 + 6y + 9$

Factorising

Stretch it!

The width of the rectangle $= x + 1$, since $x^2 + 3x + 2$
$= (x + 2)(x + 1)$

Stretch it!

a $a^2 - 3 = a^2 - (\sqrt{3})^2$
$= (a + \sqrt{3})(a - \sqrt{3})$

b $b^2 - 5 = b^2 - (\sqrt{5})^2$
$= (b + \sqrt{5})(b - \sqrt{5})$

1 **a** $3(a + 3)$ **b** $5(b - 2)$

c $7(1 + 2c)$ **d** $d(d - 2)$

2 **a** $4(2a + 5)$ **b** $4(b - 3)$

c $9(2 + c)$ **d** $d(2d - 3)$

3 **a** $2(2x - 3y)$ **b** $m(a + b)$

c $4a(3a + 2)$ **d** $x(4x + 3y)$

e $n(2 - 9n)$ **f** $5x(1 + 2y)$

g $4p(q - 3)$ **h** $4y(x^2 - 2)$

4 $4(x - 3) + 3(2x + 6)$
$= 4x - 12 + 6x + 18$
$= 4x + 6x - 12 + 18$
$= 10x + 6$
$= 2(5x + 3)$
Compare $2(5x + 3)$ with $a(5x + b)$
$a = 2, b = 3$

5 **a** $(x + 1)(x + 7)$ **b** $(x - 1)(x + 5)$

c $(x + 2)(x - 4)$ **d** $(x - 2)(x - 3)$

e $(x - 3)(x - 3)$ **f** $(x + 3)(x + 4)$

g $(x - 2)(x + 5)$ **h** $(x + 4)(x - 5)$

6 **a** $x^2 - 16 = x^2 - 4^2 = (x + 4)(x - 4)$

b $x^2 - 36 = x^2 - 6^2 = (x + 6)(x - 6)$

c $x^2 - 81 = x^2 - 9^2 = (x + 9)(x - 9)$

d $y^2 - 100 = y^2 - 10^2 = (y + 10)(y - 10)$

Substituting into expressions

1 When $a = 3$ and $b = -2$,
$5a + 2b = 5 \times 3 + 2 \times (-2) = 15 + (-4) = 11$

2 **a** $2 - 2 \times (-4) = 2 - (-8) = 10$

b $3 \times 2 \times (-4) = -24$

c $4 \times (-4) - 3 \times 2 = -16 - 6 = -22$

d $2^2 + (-4)^2 = 4 + 16 = 20$

e $2 \times 2 + 4(2 - (-4)) = 2 \times 2 + 4 \times 6 = 4 + 24 = 28$

f $\frac{1}{2}(2 + (-4)) = \frac{1}{2} \times -2 = -1$

3 false
When $a = 3$: $3a^2 = 3 \times 3^2 = 3 \times 9 = 27$

4 When $p = \frac{1}{2}$ and $q = -4$,

a $10pq = 10 \times \frac{1}{2} \times (-4) = -20$

b $8p^2 = 8 \times \left(\frac{1}{2}\right)^2 = 8 \times \frac{1}{4} = 2$

c $\frac{q}{p} = \frac{-4}{\frac{1}{2}} = -4 \times 2 = -8$

d $2q^2 - 12p = 2 \times (-4)^2 - 12 \times \frac{1}{2}$

$= 2 \times 16 - 12 \times \frac{1}{2}$

$= 32 - 6$

$= 26$

5 When $d = 7$, $e = -3$ and $f = 10$,

$\frac{d(e-2)}{f} = \frac{7 \times (-3-2)}{10}$

$= \frac{7 \times (-5)}{10}$

$= \frac{-35}{10}$

$= -3.5$

Writing expressions

Stretch it!

a perimeter $= x + 2 + 2x + 4 + 5x - 2 = 8x + 4$

b perimeter of square = perimeter of triangle

$= 8x + 4$

$= 4(2x + 1)$

side of square $= \frac{4(2x+1)}{4} = 2x + 1$

1 **a** $4 - q$ **b** $n + m$ (or $m + n$)

c xy (or yx) **d** p^2

2 $x + y$

3 $\frac{y}{8}$

4 $100n + 75b$

5 Perimeter $= 3a + 2a + 4 + 4a - 2 = 9a + 2$

6 Area $= \frac{1}{2} \times 4 \times (2a + 5)$

$= 2 \times (2a + 5)$

$= 4a + 10$

Solving linear equations

1 **a** $5a = 35$

$a = \frac{35}{5}$

$a = 7$

b $b - 9 = 8$

$b = 8 + 9$

$b = 17$

c $\frac{c}{4} = 4$

$c = 4 \times 4$

$c = 16$

d $d + 4 = 2$

$d = 2 - 4$

$d = -2$

2 **a** $2x + 3 = 13$

$2x = 10$

$x = 5$

b $3y - 4 = 11$

$3y = 15$

$y = 5$

c $2p + 9 = 1$

$2p = -8$

$p = -4$

d $\frac{f}{3} - 7 = 4$

$\frac{f}{3} = 11$

$f = 33$

e $\frac{x+5}{2} = 8$

$x + 5 = 16$

$x = 11$

f $\frac{f-7}{3} = 4$

$f - 7 = 12$

$f = 19$

3 **a** $9 - m = 7$

$9 = 7 + m$

$m = 2$

b $10 - 3x = 1$

$10 = 1 + 3x$

$9 = 3x$

$3 = x$

c $7 - 2x = 2$

$7 = 2 + 2x$

$5 = 2x$

$x = \frac{5}{2}$

(**or** $x = 2.5$, or $x = 2\frac{1}{2}$)

d $5 = 1 - 2f$

$5 + 2f = 1$

$2f = -4$

$f = -2$

4 Hannah has not subtracted 4 from *both* sides.

Correct working:

$2x + 4 = 8$

$2x = 4$

$x = 2$

5 **a** $3(a + 2) = 15$

$3a + 6 = 15$

$3a = 9$

$a = 3$

b $4(b - 2) = 4$

$4b - 8 = 4$

$4b = 12$

$b = 3$

c $3(4c - 9) = 9$

$12c - 27 = 9$

$12c = 36$

$c = 3$

d $2(d + 3) + 4 = 2$

$2d + 6 + 4 = 2$

2d + 10 = 2

$2d = -8$

$d = -4$

e $4(2x + 3) - 2 = 6$

$8x + 12 - 2 = 6$

$8x + 10 = 6$

$8x = -4$

$x = -\frac{4}{8} = -\frac{1}{2}$

(**or** $x = -0.5$)

6 a $3m = m + 6$
$2m = 6$
$m = 3$

b $5t - 6 = 2t + 3$
$3t - 6 = 3$
$3t = 9$
$t = 3$

c $4x + 3 = 2x + 8$
$2x + 3 = 8$
$2x = 5$
$x = \frac{5}{2}$
(Or $x = 2.5$ or $x = 2\frac{1}{2}$)

d $3 - 2p = 6 - 3p$
$3 + p = 6$
$p = 3$

e $3y - 8 = 5y + 4$
$-8 = 2y + 4$
$-12 = 2y$
$-6 = y$

7 a $2(x + 5) = x + 6$
$2x + 10 = x + 6$
$x + 10 = 6$
$x = -4$

b $5(a - 1) = 4 - a$
$5a - 5 = 4 - a$
$6a - 5 = 4$
$6a = 9$
$a = \frac{9}{6} = \frac{3}{2}$ (or 1.5)

c $7b - 2 = 2(b + 4)$
$7b - 2 = 2b + 8$
$5b - 2 = 8$
$5b = 10$
$b = 2$

d $4(2y + 1) = 3(5y - 1)$
$8y + 4 = 15y - 3$
$4 = 7y - 3$
$7 = 7y$
$y = 1$

e $2x - 1 = 8 - 4x$
$6x - 1 = 8$
$6x = 9$
$x = \frac{9}{6} = \frac{3}{2}$
(Or $x = 1.5$ or $x = 1\frac{1}{2}$)

Writing linear equations

Stretch it!

$3(x + 3) = 5x - 12$
$3x + 9 = 5x - 12$
$12 + 9 = 5x - 3x$
$21 = 2x$
$x = \frac{21}{2} = 10.5$ cm

1 a Perimeter $= 4 \times (2s + 3) = 8s + 12$
(Or, Perimeter $= 2s + 3 + 2s + 3 + 2s + 3 + 2s + 3 = 8s + 12$)

b $8s + 12 = 84$
$8s = 72$
$s = 9$ cm

2 a Angles in a quadrilateral add up to 360° so:
$x + 20 + 2x - 15 + x + 65 + 2x - 10 = 360$
$6x + 60 = 360$
$6x = 300$
$x = 50$

b Largest angle: $x + 65 = 50 + 65 = 115°$
(Other angles: $x + 20 = 50 + 20 = 70°$;
$2x - 15 = 2 \times 50 - 15 = 85°$;
$2x - 10 = 2 \times 50 - 10 = 90°$)

3 Let a = Karen's age
Monica is 4 years younger: $a - 4$
$a + a - 4 = 64$
$2a - 4 = 64$
$2a = 68$
$a = 34$
Karen is 34 years old.
$a - 4 = 34 - 4 = 30$
Monica is 30 years old.

4 Let n = number.
$2n + 4 = 16 - n$
$3n + 4 = 16$
$3n = 12$
$n = 4$
The number is 4.

5 Let l = length of rectangle.
Width is 2 cm smaller: $l - 2$
Perimeter $= 2l + 2(l - 2)$
$= 2l + 2l - 4 = 4l - 4$
$4l - 4 = 36$
$4l = 40$
$l = 10$
Length is 10 cm.
$l - 2 = 10 - 2 = 8$
Width is 8 cm.

6 Base angles of an isosceles triangle are equal so:
$4a - 20 = 2a + 16$
$2a - 20 = 16$
$2a = 36$
$a = 18$
When $a = 18$: $4a - 20 = 4 \times 18 - 20 = 52$
So $2a + 16 = 52$
Angles in a triangle add up to 180° so:
$4b - 2a + 52 + 52 = 180$
$4b - 2a + 104 = 180$
$4b - 2a = 76$ (Substitute a = 18)
$4b - 2 \times 18 = 76$
$4b - 36 = 76$
$4b = 112$
$b = 28$

Linear inequalities

1 **a** $x = 3, 4, 5$

b $x = 2, 3, 4, 5$

c $x = 0, 1, 2, 3$

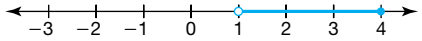
d $x = -3, -2, -1, 0, 1$

2 **a** $x < 3$ **b** $x \geq -2$

c $-1 \leq x \leq 5$

3 **a**

b



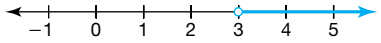
c

d

4 **a** $2x - 2 > 4$

$2x > 6$

$x > 3$



b $4x + 3 \leq 13$

$4x \leq 10$

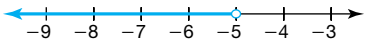
$x \leq \frac{10}{4} = \frac{5}{2}$

(**or** $x \leq 2.5$ or $x \leq 2\frac{1}{2}$)

c $4x < 2x - 10$

$2x < -10$

$x < -5$



d $7x + 2 \geq 3x - 2$

$4x + 2 \geq -2$

$4x \geq -4$

$x \geq -1$

5 Olivia has not multiplied all the terms in the bracket by the term outside.

Correct working:

$3(x + 4) > 22$

$3x + 12 > 22$

$3x > 10$

$x > \frac{10}{3}$ (**or** $x > 3\frac{1}{3}$)

6 **a** $-12 < 4x \leq 8$

$-3 < x \leq 2$

$x = -2, -1, 0, 1, 2$

b $-8 \leq 2x < 14$

$-4 \leq x < 7$

$x = -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6$

c $-6 < 6x \leq 18$

$-1 < x \leq 3$

$x = 0, 1, 2, 3$

d $9 \leq 3n \leq 15$

$3 \leq n \leq 5$

$n = 3, 4, 5$

7 **a** $2x + 3 < 4x + 8$

$3 < 2x + 8$

$-5 < 2x$

$-\frac{5}{2} < x$ or $x > -\frac{5}{2}$ (**or** $x > -2.5$)

b Smallest integer value of x that satisfies the inequality is -2.

8 **a** $4 - x \leq 1$

$4 \leq x + 1$

$3 \leq x$ (**or** $x \geq 3$)

Alternative method:

$4 - x \leq 1$

$-x \leq -3$

$x \geq 3$

b $6 - 3x > 9$

$6 > 3x + 9$

$-3 > 3x$

$-1 > x$ (**or** $x < -1$)

Alternative method:

$6 - 3x > 9$

$-3x > 3$

$x < -1$

c $8 - 2x \geq 7$

$8 \geq 2x + 7$

$1 \geq 2x$

$\frac{1}{2} \geq x$ (**or** $x \leq \frac{1}{2}$)

Alternative method:

$8 - 2x \geq 7$

$-2x \geq -1$

$x \leq \frac{1}{2}$

d $-2 < -x \leq 3$

$2 > x \geq -3$

(**or** $-3 \leq x < 2$)

Formulae

Stretch it!

$4a = 5(2b^2 - a)$

$4a = 10b^2 - 5a$

$9a = 10b^2$

$\frac{9a}{10} = b^2$

$b = \sqrt{\frac{9a}{10}}$

1 Pay $= 8 \times 35 + 25 = 280 + 25 = 305$

£305

2 $P = 2(8 + 5.5) = 2 \times 13.5 = 27$

3 $F = \frac{9}{5} \times 45 + 32$

$= 81 + 32 = 113$

113 °F

4 $v = 10 + (-20) \times 5 = 10 + (-100) = -90$

5 $v^2 = 2.5^2 + 2 \times -9.8 \times 0.2$

$v^2 = 6.25 - 3.92$

$v^2 = 2.33$

$v = \sqrt{2.33} = 1.5$ (to 1 d.p.)

6 $C = 25d + 50$

7 a Distance in kilometres $= \frac{8}{5} \times$ distance in miles

$k = \frac{8}{5}m$

b $k = \frac{8}{5} \times 200$

$= 320$ km

8 a $A = l \times l$ so

$A = l^2$

9 a $P = 2a + 2(a + 3) = 2a + 2a + 6 = 4a + 6$

(**or** $P = a + a + a + 3 + a + 3 = 4a + 6$)

b $P = 4 \times 6 + 6 = 24 + 6 = 30$

$P = 30$ cm

10 $-10 = \frac{D}{6.5}$

$D = -65$

11 a $v = u + at$

$v - u = at$

$\frac{v-u}{t} = a$

b $V = \frac{1}{3}Ah$

$3V = Ah$

$\frac{3V}{A} = h$

c $y = 3(x - 3)$

$y = 3x - 9$

$y + 9 = 3x$

$\frac{y+9}{3} = x$

(**or** $x = \frac{y}{3} + 3$)

d $v^2 = u^2 + 2as$

$v^2 - u^2 = 2as$

$\frac{v^2-u^2}{2a} = s$

e $s = \frac{1}{2}at^2$

$2s = at^2$

$\frac{2s}{a} = t^2$

$t = \sqrt{\frac{2s}{a}}$

f $T = \sqrt{\frac{2s}{g}}$

$T^2 = \frac{2s}{g}$

$gT^2 = 2s$

$g = \frac{2s}{T^2}$

g $4ax = 3 + a$

$4ax - a = 3$

$a(4x - 1) = 3$

$a = \frac{3}{4x-1}$

Linear sequences

1 a i 2, 5, 8, 11, **14**, **17** (rule is 'add 3')

ii 23, 19, 15, 11, **7**, **3** (rule is 'subtract 4')

iii 3, 9, 15, 21, **27**, **33** (rule is 'add 6')

iv 4, 9, 14, 19, **24**, **29** (rule is 'add 5')

b i 2, 5, 8, 11, 14, 17, 20, 23, 26, **29**

(**or**, 10th term $= 2 + (3 \times 9) = 29$)

ii 23, 19, 15, 11, 7, 3, −1, −5, −9, **−13**

(**or**, 10th term $= 23 - (4 \times 9) = -13$)

iii 3, 9, 15, 21, 27, 33, 39, 45, 51, **57**

(**or**, 10th term $= 3 + (6 \times 9) = 57$)

iv 4, 9, 14, 19, 24, 29, 34, 39, 44, **49**

(**or**, 10th term $= 4 + (5 \times 9) = 49$)

2 a 1st term $= 1 \times 4 - 2 = 2$

2nd term $= 2 \times 4 - 2 = 6$

3rd term $= 3 \times 4 - 2 = 10$

4th term $= 4 \times 4 - 2 = 14$

b 20th term $= 20 \times 4 - 2 = 78$

3 a Common difference is 7. Hence the term-to-term rule is add 7, so:

$-11 + 7 = -4$

$-4 + 7 = 3$

$3 + 7 = 10$

3, 10

b $15 - 2n < 0$

$-2n < -15$

$-n < -7.5$

$n > 7.5$

n is an integer.

8th term:

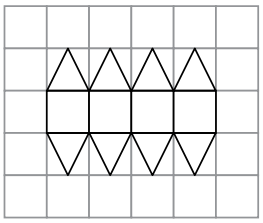
$15 - 2 \times 8 = -1$

Compare 7th term:

$15 - 2 \times 7 = 1$

So **8th term** is the first term with a negative value.

4 a



b Number of triangles: 2, 4, 6, 8, 10, 12, 14, 16

So there are 16 triangles in pattern number 8.

Or, $2 + 7 \times 2 = 16$ triangles

c No. The number of triangles forms an even number sequence and 35 is odd.

5 a 3, 7, 11, 15, 19

Common difference $= +4$

$4 \times$ term number $= 4, 8, 12, 16, 20$

-1 to get each term in the original sequence

So, nth term is $4n-1$

b If 99 is in the sequence then n will be an integer and:

$4n - 1 = 99$

$4n = 100$

$n = 25$

Yes, 99 is a term in the sequence because 25 is an integer.

Non-linear sequences

1 1, 3, 5, 7, 9, … Arithmetic sequence (term-to-term rule is 'add 2')

1, 2, 4, 8, 16, … Geometric sequence (term-to-term rule is 'multiply by 2', or 'double')

1, 4, 5, 9, 14, … Fibonacci-type sequence (next term of sequence is found by adding the previous two terms together)

1, 4, 9, 16, 25, … Square-number sequence (sequence of square numbers: 1^2, 2^2, 3^2, 4^2,…)

2 3, 6, 11, 18, 27, 38

+3, +5, +7, +9, +11

27, 38

3 **a** 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$ rule is 'divide by 2'

b 5, 0.5, 0.05, **0.005**, **0.0005** rule is 'divide by 10'

c $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ rule is 'divide by 2'

d $\frac{1}{9}$, $\frac{1}{3}$, 1, **3**, **9** rule is 'multiply by 3'

e −0.1, −0.2, −0.4, **−0.8**, **−1.6** rule is 'multiply by 2'

f 3, −6, 12, **−24**, **48** rule is 'multiply by −2'

4 4th term = 6

5th term = 10

6th term = 6 + 10 = 16

7th term = 10 + 16 = 26

8th term = 16 + 26 = **42**

5 1st term = $1^2 + 5 = 6$

2nd term = $2^2 + 5 = 9$

3rd term = $3^2 + 5 = 14$

4th term = $4^2 + 5 = 21$

6, 9, 14, 21

6 When $n = 5$:

$3 \times 5^2 - 4 = 3 \times 25 - 4 = 71$

7 1st term = $1^2 + 2 \times 1 = 1 + 2 = 3$

2nd term = $2^2 + 2 \times 2 = 4 + 4 = 8$

3rd term = $3^2 + 2 \times 3 = 9 + 6 = 15$

3, 8, 15

8 **a** 1st term = a

2nd term = b

3rd term = $a + b$

4th term = $b + a + b = a + 2b$

5th term = $a + b + a + 2b = 2a + 3b$

b $b = 5$

$2a + 3b = 23$ (Substitute $b = 5$)

$2a + 3 \times 5 = 23$

$2a + 15 = 23$

$2a = 8$

$a = 4$

Show that...

Stretch it! If n is even, $n - 1$ is odd and $n + 1$ is odd. If you multiply two odd numbers the answer will always be odd. If m is odd, $m - 1$ is even and $m + 1$ is even. If you multiply two even numbers the answer will always be even.
or: $(n + 1)(n - 1) = n^2 - 1$ and $(m + 1)(m - 1) = m^2 - 1$
n^2 will be even × even = even, so $n^2 - 1$ will be odd.
m^2 will be odd × odd = odd, so $m^2 - 1$ will be even.

1 **a** $3 + 5 = 8$ (or any other two primes except 2)

b Mo is not correct.

Let $\frac{n}{2} = x$. If x is even, then $x + 1$ is odd, and $2(x + 1)$ is even. Therefore, $\frac{n}{2}$ is not always even when n is even.

2 **a** LHS = $(x + 2)(x - 2) \equiv x^2 - 2x + 2x - 4 = x^2 - 4$

RHS = $x^2 - 4$

LHS ≡ RHS

So $(x + 2)(x - 2) \equiv x^2 - 4$

b LHS = $(x - 3)^2 \equiv (x - 3)(x - 3)$

$= x^2 - 6x + 9$

RHS = $x^2 - 6x + 9$

LHS ≡ RHS

So $(x - 3)^2 \equiv x^2 - 6x + 9$

c LHS = $(x + 1)^2 + 4 \equiv (x + 1)(x + 1) + 4$

$= x^2 + 2x + 1 + 4 = x^2 + 2x + 5$

RHS = $x^2 + 2x + 5$

LHS ≡ RHS

So $(x + 1)^2 + 4 \equiv x^2 + 2x + 5$

d LHS = $6(a - 3) - 2(2a - 5) + 6$

$= 6a - 18 - 4a + 10 + 6 = 2a - 2$

RHS = $2(a - 1) = 2a - 2$

LHS ≡ RHS

So $6(a - 3) - 2(2a - 5) + 6 \equiv 2(a - 1)$

e LHS = $4(x - 3) + 2(x + 5) = 4x - 12 + 2x + 10$

$= 6x - 2$

RHS = $3(2x - 1) + 1 = 6x - 3 + 1 = 6x - 2$

LHS ≡ RHS

So $4(x - 3) + 2(x + 5) \equiv 3(2x - 1) + 1$

3 $4(ax - 2) + 5(3x + b) \equiv 23x - 3$

LHS = $4ax - 8 + 15x + 5b = (4a + 15)x + (5b - 8)$

Given that RHS ≡ LHS,

$(4a + 15)x \equiv 23x$

so

$4a + 15 = 23$

$4a = 8$

$a = 2$

and $5b - 8 = -3$

$5b = 5$

$b = 1$

4 rod A = n

rod B = $n + 1$

rod C = $n + 2$

rod A + rod C = $n + n + 2$

$= 2n + 2$

$= 2(n + 1)$

This is 2 times the length of rod B.

Functions

1 **a** $y = 2 \times 2 - 4 = 0$

b $y = 1 \times 2 - 4 = -2$

c $y = -4 \times 2 - 4 = -12$

2 **a**

×1 → +2	
x	y
−2	**0**
0	**2**
2	4

b

×2 → −3	
x	y
−2	**−7**
0	−3
2	**1**

3 **a** $y = 3x - 3$

b $10 \times 3 - 3 = 30 - 3 = 27$

c $y = 3x - 3$

$y + 3 = 3x$

$x = \frac{y+3}{3}$

d If $x = y$ then $x = 3x - 3$

$2x = 3$, $x = \frac{3}{2}$ (or 1.5)

Substituting $x = 1.5$ into the function,

$y = 1.5 \times 3 - 3 = 1.5 = \frac{3}{2}$

So x and y can be equal.

Coordinates and midpoints

Stretch it!

Difference in x coordinates of A and B is equal to difference in x coordinates of B and C.

Difference $= 2 - (-1) = 3$

So x coordinate of C is:

$-1 - 3 = -4$

Difference in y coordinates of A and B is equal to difference in y coordinates of B and C.

Difference $= 4 - 3 = 1$

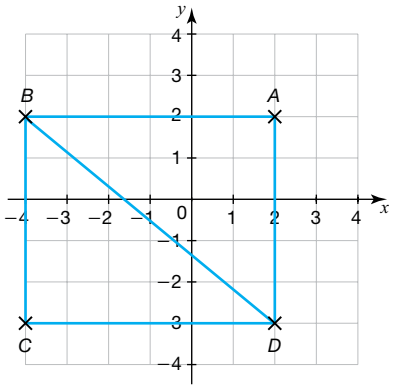
So y coordinate of C is:

$3 - 1 = 2$

C has coordinates $(-4, 2)$.

1 **a** (2, 2)

b, **c** and **d**



d B is $(-4, 2)$, D is $(2, -3)$

x coordinate of midpoint: $\frac{2 + (-4)}{2} = -1$

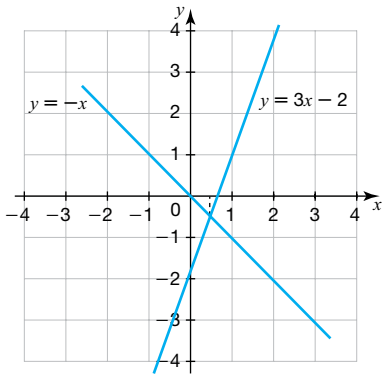
y coordinate of midpoint: $\frac{-3 + 2}{2} = -0.5$

Midpoint is $(-1, -0.5)$

Straight-line graphs

Stretch it!

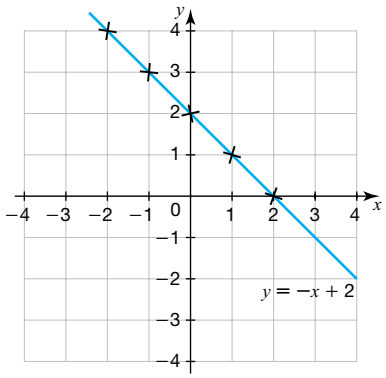
To solve the equation, you need to find where the graph of $y = 3x - 2$ intersects the graph of $y = -x$.



So the solution to $3x - 2 = -x$ is $x = 0.5$.

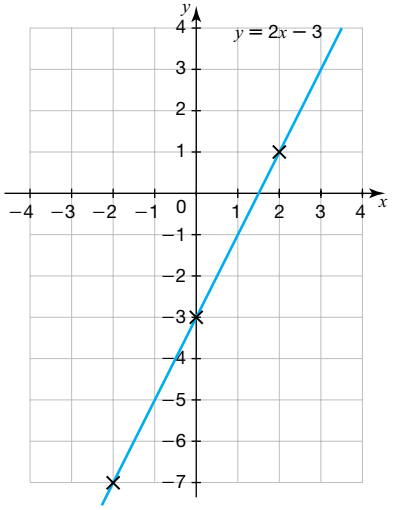
1

x	−2	−1	0	1	2
y	4	3	2	1	0

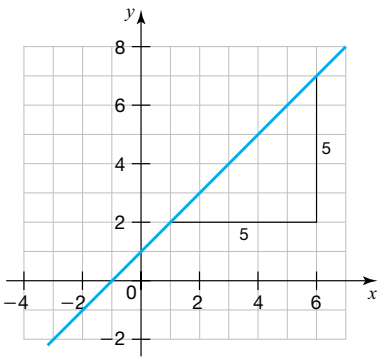


2

x	−2	0	2
y	−7	−3	1



3



Gradient, $m = \frac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}} = \frac{5}{5} = 1$

y-intercept, $c = 1$

Using the general form of the equation of a line $y = mx + c$: the equation of the line is $y = x + 1$

4 Gradient, $m = 2$

$y = mx + c$ (General form of the equation of a line)

$y = 2x + c$

For point $(1, -2)$, $x = 1, y = -2$:

$-2 = 2 \times 1 + c$

$-2 = 2 + c$

$-4 = c$

So the equation of the line is $y = 2x - 4$

5 Gradient $= \frac{\text{difference in } y \text{ coordinate}}{\text{difference in } x \text{ coordinate}} = \frac{4-2}{0-4} = -\frac{2}{4} = -\frac{1}{2}$

Gradient, $m = -\frac{1}{2}$

Given the point $(0, 4)$:

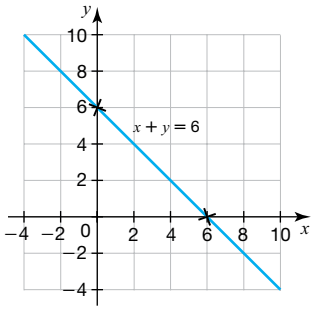
$4 = -\frac{1}{2}(0) + c$

$4 = c$ (y-intercept)

So the equation of the line is $y = -\frac{1}{2}x + 4$.

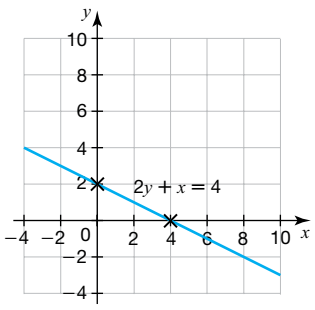
6 a

x	0	6
y	6	0



b

x	0	4
y	2	0



7 A: $y = 4x + 1$

B: $4x + 4y = 4$

$4y = -4x + 4$

$y = -x + 1$

C: $x - 2y = 2$

$-2y = -x + 2$

$y = \frac{x}{2} - 1$

D: $2y = 4 + 8x$

$y = 4x + 2$

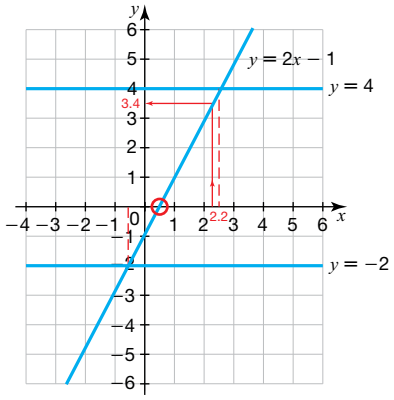
Lines A and D both have the same m value (4) so they are parallel.

8 $2y = x - 4$

$y = \frac{x}{2} - 2$

The y-intercept is $(0, -2)$.

9



a $y = 3.4$ **b** $x = 0.5$

c $x = 2.5$ **d** $x = -0.5$

Solving simultaneous equations

1 $x + y = 16$ (1)

$x - y = 5$ (2)

(1) + (2):

$2x = 21$

$x = \frac{21}{2} = 10\frac{1}{2}$ (or 10.5)

Substitute $x = 10\frac{1}{2}$ into (2):

$10\frac{1}{2} - y = 5$

$y = 10\frac{1}{2} - 5$

$y = 5\frac{1}{2}$ (or 5.5)

Solution: $x = 10.5, y = 5.5$

2 a $2x + y = 4$ (1)

$3x - y = 1$ (2)

(1) + (2): $5x = 5$

$x = 1$

Substitute $x = 1$ in (1)

$2 \times 1 + y = 4$

$2 + y = 4$

$y = 2$

Solution: $x = 1, y = 2$

b $x - y = 5$ (1)

$2x + y = 4$ (2)

(1) + (2): $3x = 9$

$x = 3$

Substitute $x = 3$ into (1)
$3 - y = 5$
$3 = y + 5$
$y = -2$
Solution: $x = 3, y = -2$

c $2x + y = 8$ (1)
$x + y = 2$ (2)
(1) − (2): $x = 6$
Substitute $x = 6$ into (2)
$6 + y = 2$
$y = -4$
Solution: $x = 6, y = -4$

d $4x - y = 10$ (1)
$x + 2y = 7$ (2)
(1) × 2: $8x - 2y = 20$ (3)
(2) + (3): $9x = 27$
$x = 3$
Substitute $x = 3$ into (2):
$3 + 2y = 7$
$2y = 4$
$y = 2$
Solution: $x = 3, y = 2$

e $2x + y = 7$ (1)
$x - 4y = 8$ (2)
(1) × 4: $8x + 4y = 28$ (3)
(2) + (3): $9x = 36$
$x = 4$
Substitute $x = 4$ into (1):
$2 \times 4 + y = 7$
$8 + y = 7$
$y = -1$
Solution: $x = 4, y = -1$

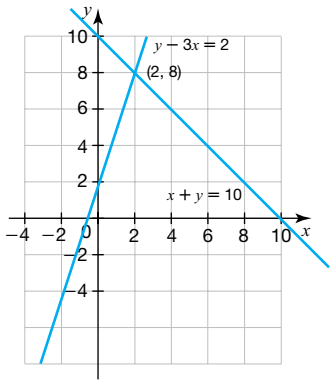
f $2x + 3y = 7$ (1)
$3x - 2y = 4$ (2)
(1) × 2: $4x + 6y = 14$ (3)
(2) × 3: $9x - 6y = 12$ (4)
(3) + (4): $13x = 26$
$x = 2$
Substitute $x = 2$ into (1)
$2 \times 2 + 3y = 7$
$4 + 3y = 7$
$3y = 3$
$y = 1$
Solution: $x = 2, y = 1$

3 $x + y = 21$ (1)
$x - y = 7$ (2)
(1) + (2): $2x = 28$
$x = 14$
Substitute $x = 14$ into (1)
$14 + y = 21$
$y = 7$
The two numbers are 7 and 14.

4 Let b = burger and c = cola.
$3b + 2c = 505$ (1)
$3b + 4c = 725$ (2)
(2) − (1): $2c = 220$
$c = 110$
Substitute $c = 110$ into (1)
$3b + 2 \times 110 = 505$
$3b + 220 = 505$
$3b = 285$
$b = 95$
A burger costs 95p.
A cola costs £1.10.

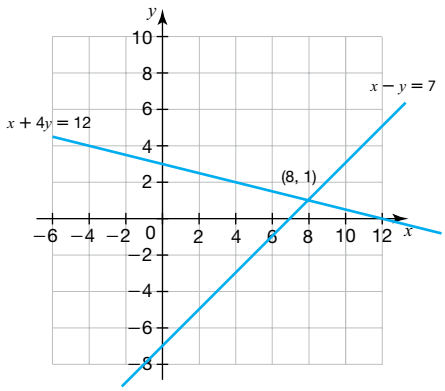
5 $2a + 4c = 60$ (1)
$3a + 3c = 82.50$ (2)
(1) × 3: $6a + 12c = 180$ (3)
(2) × 2: $6a + 6c = 165$ (4)
(3) − (4): $6c = 15$
$c = \frac{15}{6} = 2\frac{3}{6} = 2.5$
Substitute $c = 2.5$ into (1): $2a + 4 \times 2.5 = 60$
$2a + 10 = 60$
$2a = 50$
$a = 25$
Adult ticket: £25 Child ticket: £2.50

6 a



$x = 2, y = 8$

b



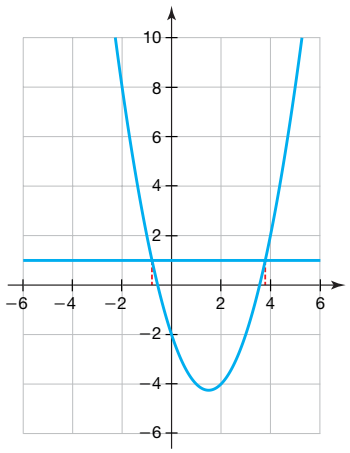
$x = 8, y = 1$

Quadratic graphs

Stretch it!

Rearrange $x^2 - 3x = 3$, to give $x^2 - 3x - 2 = 1$

You can solve this graphically by finding where the lines $y = x^2 - 3x - 2$ and $y = 1$ intersect.



So the solutions to the equation $x^2 - 3x = 3$ are $x = 3.8$ and $x = -0.8$. Acceptable readings from the graph would be in the range 3.6 to 3.9 and -0.6 to -0.9.

Stretch it!

a At points A and B, $y = 0$

Therefore, $(x - 2)(x - 4) = 0$

Either: $x - 2 = 0$

$x = 2$

or: $x - 4 = 0$

$x = 4$

Coordinates are $A(2, 0)$ and $B(4, 0)$

b At point C, $x = 0$

$y = (0 - 2)(0 - 4)$

$= -2 \times -4 = 8$

Coordinates are $C(0, 8)$

c x coordinate of D is the midpoint of the x coordinates of A and B:

$\frac{2 + 4}{2} = 3$

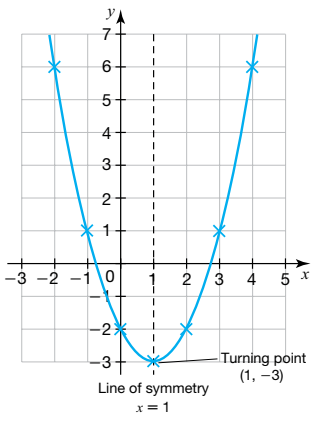
y coordinate $= (3 - 2)(3 - 4)$

$= 1 \times -1 = -1$

Coordinates are $D(3, -1)$

1 a

x	-2	-1	0	1	2	3	4
y	6	1	-2	-3	-2	1	6

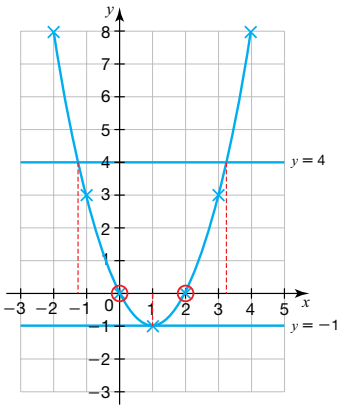


b **i** $x = 1$

ii $(1, -3)$

2

x	-2	-1	0	1	2	3	4
y	8	3	0	-1	0	3	8



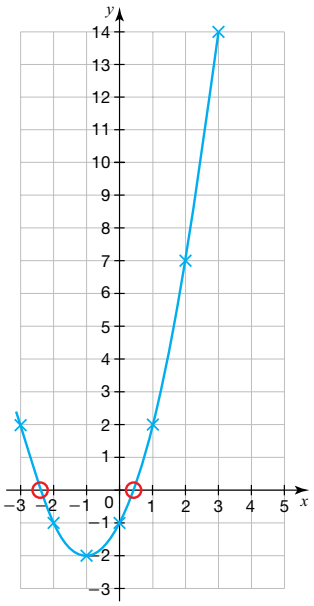
a $x = 0$ and $x = 2$

b $x \approx -1.2$ and $x \approx 3.2$

c $x = 1$

3 a and **b**

x	-3	-2	-1	0	1	2	3
y	2	-1	-2	-1	2	7	14



Read off the values of x where the graph cuts the line $y = 0$ (the x-axis).

$x \approx -2.4$ and $x \approx 0.4$

Solving quadratic equations

Stretch it!

$\frac{x^2}{2} = 8$

$x^2 = 16$

$x = \sqrt{16}$

So $x = 4$ or $x = -4$

$2x^2 = 50$

$x^2 = 25$

$x = \sqrt{25}$

So $x = 5$ or $x = -5$

Stretch it!

$x(x + 6) = 40$

$x^2 + 6x = 40$

$x^2 + 6x - 40 = 0$

$(x + 10)(x - 4) = 0$

Either $x + 10 = 0$ or $x - 4 = 0$

x must be positive as it is a length, therefore, $x = 4$ cm

1 a $x^2 - 4x = 0$

$x(x - 4) = 0$

Either $x = 0$ or $x - 4 = 0$

$x = 4$

So $x = 0$ or $x = 4$

b $x^2 + 7x = 0$

$x(x + 7) = 0$

Either $x = 0$ or $x + 7 = 0$

$x = -7$

So $x = 0$ or $x = -7$

c $x^2 - 16 = 0$ ($x^2 - 16 = x^2 - 4^2$, Factorise)

$(x + 4)(x - 4) = 0$

Either $x + 4 = 0$ or $x - 4 = 0$

$x = -4$ $x = 4$

So $x = -4$ or $x = 4$

d $x^2 + 10x + 9 = 0$

$(x + 1)(x + 9) = 0$

Either $x + 1 = 0$ or $x + 9 = 0$

$x = -1$ $x = -9$

So $x = -1$ or $x = -9$

e $x^2 + x - 12 = 0$

$(x - 3)(x + 4) = 0$

Either $x - 3 = 0$ or $x + 4 = 0$

$x = 3$ $x = -4$

So $x = 3$ or $x = -4$

f $x^2 - 6x - 16 = 0$

$(x + 2)(x - 8) = 0$

Either $x + 2 = 0$ or $x - 8 = 0$

$x = -2$ $x = 8$

So $x = -2$ or $x = 8$

2 a $y = x^2 - 49$ (Set $y = 0$)

$x^2 - 49 = 0$ ($x^2 - 49 = x^2 - 7^2$, Factorise)

$(x + 7)(x - 7) = 0$

Either $x + 7 = 0$ or $x - 7 = 0$

$x = -7$ $x = 7$

So $x = -7$ or $x = 7$

b $y = x^2 - 3x$ (Set $y = 0$)

$x^2 - 3x = 0$

$x(x - 3) = 0$

Either $x = 0$ or $x - 3 = 0$

$x = 3$

So $x = 0$ or $x = 3$

c $y = x^2 + 7x + 6$ (Set $y = 0$)

$x^2 + 7x + 6 = 0$

$(x + 1)(x + 6) = 0$

Either $x + 1 = 0$ or $x + 6 = 0$

$x = -1$ $x = -6$

So $x = -1$ or $x = -6$

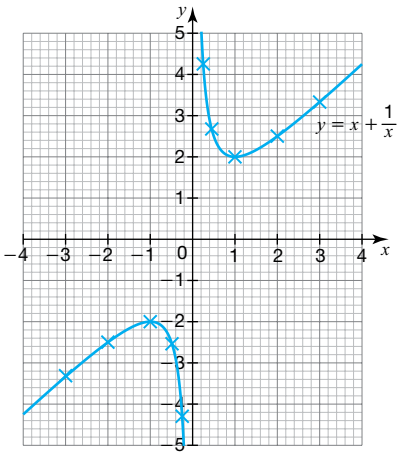
Cubic and reciprocal graphs

Stretch it!

a

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3
y	$-3\frac{1}{3}$	$-2\frac{1}{2}$	-2	$-2\frac{1}{2}$	$-4\frac{1}{4}$	$4\frac{1}{4}$	$2\frac{1}{2}$	2	$2\frac{1}{2}$	$3\frac{1}{3}$

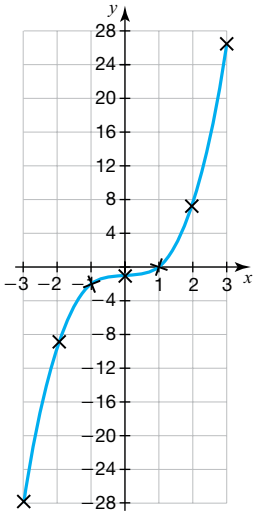
b



1 a

x	-3	-2	-1	0	1	2	3
y	-28	-9	-2	-1	0	7	26

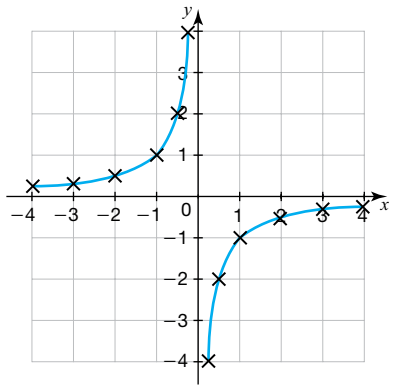
b



2 a

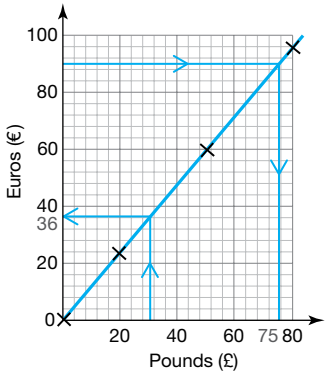
x	-4	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4
y	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	4	-4	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$

b



Drawing and interpreting real-life graphs

1 a



b The graph is a straight line with a positive gradient. As the number of pounds steadily increases, the corresponding number of euros steadily increases. This is direct proportion.

c See lines drawn on graph.

i €36 **ii** £75

d From the graph: £30 = €36

So £90 = €36 × 3 = €108

The ring is cheaper in France.

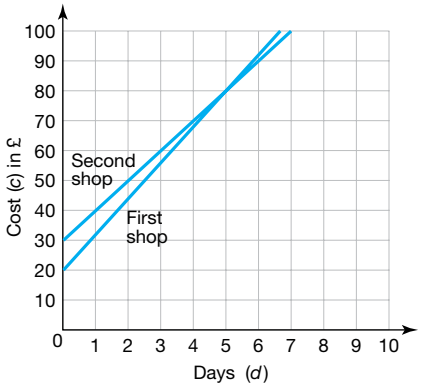
2 a Monthly charge = £10 (cost of 0 minutes from the graph)

b Gradient = $\frac{30}{240} = 0.125$

Charge per minute of calls is 13p.

3 a and **c**

d	0	1	2	3	4
C	20	32	44	56	68



b This is the flat rate that you pay just for hiring the sander, before you pay for the number of days. It is the intercept with the vertical axis:

days (d) = 0

cost (C) = £20

c Using a graphical method: plot the second equation, $C = 10d + 30$, on the same axes. The line for the second shop has a lower gradient, and after the lines cross over (at $d = 5$), the second shop is cheaper. So you would use the second shop.

Alternatively, using an algebraic method:

Let $d = 6$ days (more than 5 days)
First shop: $C = 12 \times 6 + 20 = 92$
Second shop: $C = 10 \times 6 + 30 = 90$
To hire the sander for more than 5 days use the second shop as it is cheaper.

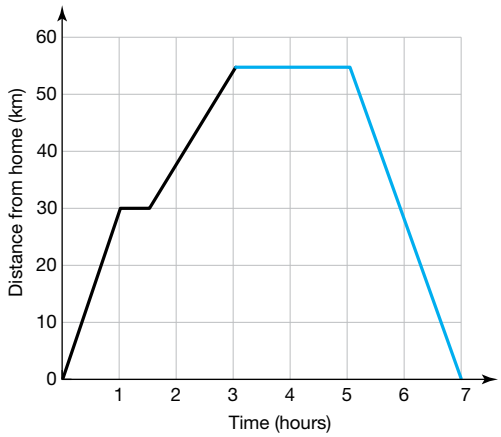
4 a 30 minutes (Horizontal line on graph)

b 55 km

c Speed before break = $\frac{\text{distance (km)}}{\text{time (hours)}} = \frac{30}{1} = 30$ km/hr

Speed after break = $\frac{\text{distance (km)}}{\text{time (hours)}} = \frac{25}{1.5} = 16.7$ km/hr

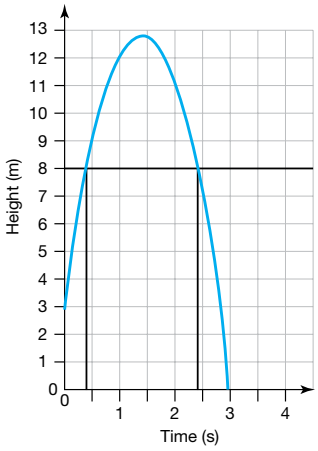
d



5 a Reading off maximum height value from graph: **12.8 m**.

b Reading from the graph, the ball is thrown at time = 0 seconds and returns to the ground at time = **3 seconds**.

c Draw a horizontal line on the graph at height = 8 m.



0.4 seconds and 2.4* seconds

d The ball is thrown from a height of 3 m above the ground.

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

6 **a** 6 m/s

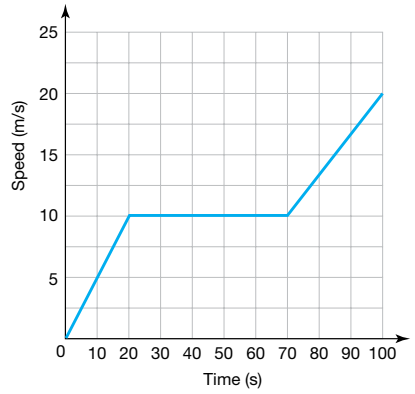
b 4 seconds

c 6 seconds

d acceleration $= \frac{\text{change in speed (m/s)}}{\text{time (s)}} = \frac{6}{4}$

$= 1.5$ m/s^2

7 **a**



b Acceleration $= \frac{\text{change in speed (m/s)}}{\text{time (s)}} = \frac{20-10}{30} = \frac{10}{30}$

$= 0.\dot{3}$ m/s^2

8 **a** The maximum depth of water in the bath before the person got in was 35 cm.

b Between C and D, the person was taking their bath.

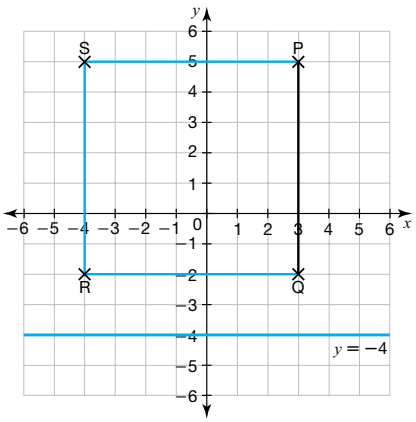
c Between D and E, the person got out of the bath.

d Running water into the bath was quicker. The slope of the line between O and A (filling the bath) is steeper than the slope of the line between E and F (emptying the bath).

Review it!

1 **a** P(3, 5)

b, **c** and **e**



d Q(3, −2), S(−4, 5)

x-coordinate: $-4 + 3 = -1$

$-1 \div 2 = -0.5$

y-coordinate: $5 + (-2) = 3$

$3 \div 2 = 1.5$

Midpoint is $(-0.5, 1.5)$

2 **a** $2x + 8 = 4$

$2x = -4$

$x = -2$

b When $x = 2$ and $y = -4$

A: $\frac{y}{x} = \frac{-4}{2} = -2$

B: $x - y = 2 - (-4) = 6$

C: $xy = 2 \times -4 = -8$

Expression C has the smallest value.

c Millie is correct.

When $x = 4$, $3x^2 = 3 \times 4^2 = 3 \times 16 = 48$

(George has worked out $(3x)^2$ instead.)

3 **a** $7a - (3a + 4) = 7a - 3a - 4 = 4a - 4$

b $4(2x + 3)$

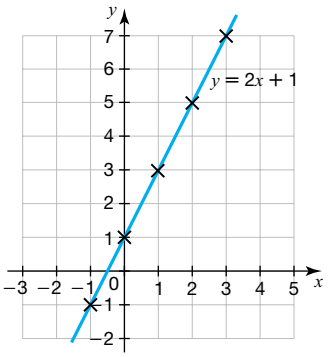
c $m^4 \times m = m^{4+1} = m^5$

d $\frac{x^8}{x^3} = x^{8-3} = x^5$

4 **a**

x	−1	0	1	2	3
y	−1	1	3	5	7

b



c Compare $y = 2x + 1$ with $y = mx + c$ (general form of the equation of a line):

Gradient, $m = 2$

5 $4x + 4 = x + 13$

$3x + 4 = 13$

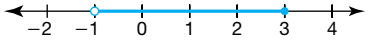
$3x = 9$

$x = 3$

6 **a** 2 is included, and so are all values lower than 2.

$x \leq 2$

b



c 1 can be included, but 4 cannot.

$x = 1, 2, 3$

d $4x + 2 \leq 2x + 5$

$2x + 2 \leq 5$

$2x \leq 3$

$x \leq \frac{3}{2}$

(or $x \leq 1\frac{1}{2}$)

7 **a** $6x$ or $x + 65$

b $6x = x + 65$

$5x = 65$

$x = 13$

Luke is 13 years old.

8 **a** The term-to-term rule is 'add 6'.

$27 + 6 = 33$

b No. The nth term is $6n - 3$.

$6n = 2 \times 3n =$ always even

Because 3 (odd) is always taken away from $6n$, every term in the sequence will be odd. As 44 is even it is not in the sequence.

c When $n = 5$:

$2n^2 - 3 = 2 \times 5^2 - 3 = 2 \times 25 - 3 = 47$

9 $(x + 3)(x + 4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12$

10 Smallest value of $a - b$ is where a is as small as possible and b is as large as possible.

$a > 30$ so its smallest value is 31

$b < 20$ so its largest value is 19

Using $a = 31$ and $b = 19$:

$a - b = 31 - 19 = 12$

11 The opposite sides of a rectangle are equal in length so:

$5x - 8 = 2x + 4$

$3x - 8 = 4$

$3x = 12$

$x = 4$

$14 - 2y = 4y + 2$

$14 = 6y + 2$

$12 = 6y$

$y = 2$

12 a $4x^2 + 6x$

$= x(4x + 6)$

$= 2x\,(2x + 3)$

b $x^2 - 100$

$= (x + 10)(x - 10)$

c $x^2 + 9x + 18 = 0$

$(x + 3)(x + 6) = 0$

Either $x + 3 = 0$ or $x + 6 = 0$

$x = -3$ or $x = -6$

13 $3(ax - 4) + 2(4x + b) \equiv 14x - 6$

$3ax - 12 + 8x + 2b \equiv 14x - 6$

$3ax + 8x - 12 + 2b \equiv 14x - 6$

$3ax + 8x \equiv 14x$

$3a + 8 = 14$

$3a = 6$

$a = 2$

$-12 + 2b \equiv -6$

$2b = 6$

$b = 3$

14 a $12m$

b $3p \times 4p = 3 \times 4 \times p \times p = 12p^2$

c $12x \div 2 = \frac{12x}{2} = 6x$

15 a $5(w - 4) = 35$

$5w - 20 = 35$

$5w = 55$

$w = 11$

b When $a = 7$ and $b = -2$,

$5a + 7b = 5 \times 7 + 7 \times (-2)$

$= 35 + (-14)$

$= 21$

c $5a + 8b$

16 x coordinate of N = x coordinate of L + 6

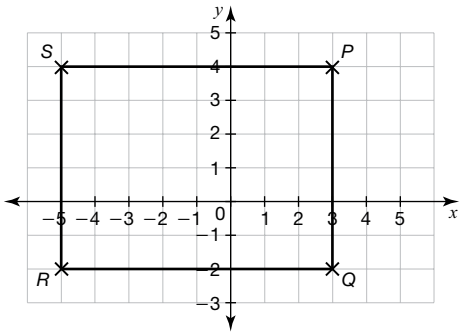
$= 2 + 6 = 8$

y coordinate of N = y coordinate of L − 6

$= 3 - 6 = -3$

Coordinates are $N(8, -3)$

17 a



P and Q are vertically above each other, because they share an x coordinate (3).

R and Q are horizontally aligned, because they share a y coordinate (−2).

As the fourth vertex, S must share an x coordinate with R (−5) and a y coordinate with P (4).

S is the point (−5, 4).

b Length is x coordinate of P or Q minus x coordinate of R or S:

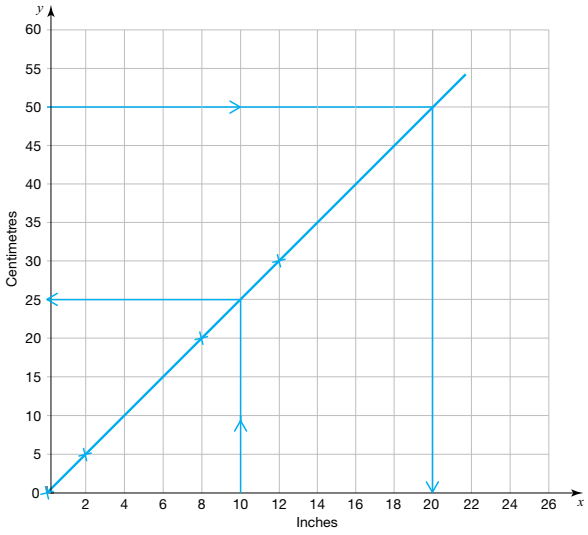
$= 3 - -5 = 8$

Width is y coordinate of S or P minus x coordinate of R or Q:

$= 4 - -2 = 6$

Length is 8 units and width is 6 units.

18 a



b **i** From the graph: 10 inches = 25 cm

ii From the graph: 50 cm = 20 inches

So 50 cm = 10 × 2 = 20 inches

c From the graph: 10 inches = 25 cm

So 60 inches = 25 × 6 = 150 cm

Cost of beading = 150 × 2 = 300p

Cost = £3.00

19 $4 \times 15 = 80 - 8k$

$60 = 80 - 8k$

$8k = 80 - 60$

$8k = 20$

$k = \frac{20}{8} = 2\frac{4}{8} = 2\frac{1}{2}$

20 a $T = 12.50x + 10$

b $72.50 = 12.50x + 10$

$62.50 = 12.50x$

$5 = x$

Suzanne hired the costume for 5 days.

21 Equation of a line: $y = mx + c$

y-intercept $= c = 5$

gradient $= m = \frac{5-0}{0-2} = -\frac{5}{2}$

$y = -2.5x + 5$ **or** $y = -\frac{5}{2}x + 5$

22 a $4x + 2 \leq 8$

$4x \leq 6$

$x \leq \frac{6}{4}$

$x \leq \frac{3}{2}$

(**or** $x \leq 1\frac{1}{2}$ or 1.5)

b $3x - 4 < 17$

$3x < 21$

$x < 7$

$4x + 2 \geq 22$

$4x \geq 20$

$x \geq 5$

If $x < 7$ and $x \geq 5$ then $x = 6$ and $x = 5$ satisfy both.

23 Ollie has squared each term inside the brackets rather than squaring the whole bracket.

Correct working:

$(x + 4)^2 = (x + 4)(x + 4) = x^2 + 4x + 4x + 16 = x^2 + 8x + 16$

24 $P = \frac{Q}{4} + R$

$P - R = \frac{Q}{4}$

$4(P - R) = Q$

25 a $m(m + 8)$

b $(x + 3)(x + 4)$

26 a 2, 5, 8, 11, 14

Common difference $= +3$

$3 \times$ term number: 3, 6, 9, 12, 15

-1 to get each term in the original sequence

So nth term $= 3n - 1$

b $2n - 3 = 112$

$2n = 115$

$n = 57.5$

No, Kadena is incorrect.

112 cannot be term in the sequence because 57.5 is not an integer.

27 a $4(x + 5) - 3(2x - 1) = 4x + 20 - 6x + 3 = -2x + 23$

b $4a^3b^2 \times 5a^2b = 4 \times 5 \times a^3 \times a^2 \times b^2 \times b$

$= 20 \times a^{3+2} \times b^{2+1} = 20a^5b^3$

28 Perimeter $= 3x - 2 + 2x + 1 + 3x + 5 + 2x = 10x + 4$

$10x + 4 = 49$

$10x = 45$

$x = 4.5$

29 A: output $= 6x - 4$

B: output $= 3x + 2$

$6x - 4 = 4(3x + 2)$

$6x - 4 = 12x + 8$

$-4 = 6x + 8$

$-12 = 6x$

$-2 = x$

Input $= -2$

30 1st term: $4 + 2a$

2nd term: $4 + 4a$

3rd term: $4 + 6a$

4th term: $4 + 8a$

5th term: $4 + 10a$

$4 + 10a = 64$

$10a = 60$

$a = 6$

31 a 1st term: a

2nd term: b

3rd term: $a + b$

4th term: $b + a + b = a + 2b$

5th term: $a + b + a + 2b = 2a + 3b$

6th term: $a + 2b + 2a + 3b = 3a + 5b$

7th term: $2a + 3b + 3a + 5b = 5a + 8b$

b $a + b = 5$ (1)

$5a + 8b = 34$ (2)

(1) $\times$ 5: $5a + 5b = 25$ (3)

(2) $-$ (3): $3b = 9$

$b = 3$

Substitute $b = 3$ into (1):

$a + 3 = 5$

$a = 2$

32 Roots 2 and -4 are x-intercepts where the curve cuts the x-axis.

These give factors:

$(x - 2)$ and $(x - -4)$

Equation is $(x - 2)(x + 4) = 0$

Equation C.

Ratio, proportion and rates of change

Units of measure

1 a 3000 m

b 75 mins

c 13 000 cm^2

d 3.52 litres

e 7200 seconds

f 14 kg

2 4.5 $-$ 0.325 $=$ 4.175 kg or 4500 $-$ 325 $=$ 4175 g

3 $5 \div 2.2 = 2.2\dot{7}$ kg

Ratio

Stretch it! 31 + 25 = 56, fraction male = $\frac{31}{56}$

1 **a** 1:4 **b** 1:3:4 **c** 4:5

2 35:5 = 7:1

3 375 ÷ 250 = 1.5.
Allow 1 part cement for 1.5 parts sand.

4 **a** number in evening = 7 parts
number in afternoon = 1 part
7:1

b total parts = 7 + 1 = 8
1 part = 800 ÷ 8 = 100
1 part sold in afternoon
So **100** tickets were sold in the afternoon.

5 **a** There are 5 parts to the ratio. Ratio is:
3 : (5 − 3)
= 3 : 2

b 1 part = 200 ÷ 5 = 40
40 × 3 = 120
120 cats

6 9 = 3 × 3 so multiply the other lengths by 3.
4 × 3 = 12
5 × 3 = 15
12 cm and 15 cm

7 To work out the number of students per teacher (s), you multiply the number of teachers (t) by 20, so: $s = 20t$.

8 **a** There are 5 parts to the ratio.
1 part = 1.5 ÷ 5 = 0.3 kg
2 parts of sugar needed:
2 × 0.3 = 0.6
0.6 kg or 600 g

b 5 − 2 = 3 parts = 60 g more flour
60 ÷ 3 = 20
2 × 20 = 40
40 g of sugar

Scale diagrams and maps

Stretch it! 50 miles on ground = 50 ÷ x or $\frac{50}{x}$ miles on map

1 mile = 1610 m = 161 000 cm
50 miles on ground = $\frac{50}{x}$ × 161 000 cm* on map

1 A, B, F

2 **a** 3 × 12 = 36 km

b 15 ÷ 12 = 1.25 cm

3 12 × 1000 = 12 000 cm = 120 m

4 **a** 2 cm: 2 × 50 000 = 100 000 cm = 1 km
(Any answer within the range of 1 km – 1.1 km is acceptable.)

b 250°

Fractions, percentages and proportion

1 $\frac{20}{3500} = \frac{1}{175}$

2 2 + 3 + 8 = 13 hours
24 − 13 = 11 hours
$\frac{11}{24}$ of the day remaining

3 **a** $\frac{15}{20} = \frac{3}{4}$

b $1 - \frac{3}{4} = \frac{1}{4} = 25\%$

4 1 + 2 + 7 = 10, $\frac{1}{10} = 10\%$

5 School A: 125:145 = 25:29
School B: 100:120 = 5:6
No since the ratios are not equivalent.

6 150 ÷ 100 = 1.5
1.5 × 22 g = 33 g
33 g ÷ 8 = 4.125 g

Direct proportion

Stretch it!

For two values to be in direct proportion, when one is 0 the other must be 0. Here, when distance is 0 miles, the fee is £2.

1 A and E

2 **a** **i** 20 meringues = 2 eggs, divide both by 2 to give:
10 meringues = 1 egg
3 eggs: 3 × 10 = 30 meringues

ii 20 meringues = 120 g of sugar, divide both by 2 to give: 10 meringues = 60 g of sugar. Multiply both by 10 to give 100 meringues

b 20 meringues = 2 eggs, divide both by 2 to give
10 meringues = 1 egg, multiply both by 7 to give
70 meringues = 7 eggs

3 675 ÷ 4.5 = 150 minutes = 2 hours 30 minutes

4 A, D

Inverse proportion

1 D

2 At 60 miles it takes 15 minutes.
60 × $\frac{2}{3}$ = 40
15 ÷ $\frac{2}{3}$ = 22.5 mins

3 2 × 3 = 6 decorators
5 ÷ 3 = 1 $\frac{2}{3}$ of a day

4 **a** 2

b The age of the chicken and the number of eggs it lays are in inverse proportion, this means that as the age of the chicken increases, the number of eggs it lays decreases.

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

Working with percentages

Stretch it! £128

Stretch it! Let percentage rate $= x$

$(1 + \frac{x}{100})^5 \times £100 = £110$

$(1 + \frac{x}{100})^5 = \frac{110}{100}$

$1 + \frac{x}{100} = \sqrt[5]{\frac{110}{100}}$

$1 + \frac{x}{100} = 1.02$

$\frac{x}{100} = 0.02$

$x = 2$

Percentage interest is 2%

1 **a** $1.03 \times 50 = £51.50$

b $2.48 \times 400 = 992$

c $0.195 \times 64 = 12.48$

2 $45 - 40 = 5$, $\frac{5}{40} \times 100 = 12.5\%$

3 $24 \div 115 = 0.209$, $0.209 \times 100 = 20.9°C$

4 $15\,000 \times 1.20^3 = 25\,920$

5 20% is $\frac{1}{5}$ of the price.
$30 \times 5 = £150$

6 $(200 \div 225) \times 100 = 88.9\%$ (to 1 d.p.)
The number of employees in Year 2 is 88.9% of the number in Year 1.

Compound units

Stretch it! $\frac{100}{x}$ mph

1 $29.50 \div 0.18 = 164$ or $2950 \div 18 = 164$ units

2 Time $= \frac{80}{120} = \frac{2}{3}$ hour $= 40$ minutes

3 Density $= \frac{0.72}{3} = 0.24$ g/cm^3

4 Pressure $= \frac{12}{2} = 6$ N/m^2

5 3 m/s $= 3 \times 60$ m/minute $= 3 \times 60 \times 60$ m/hour
$= 10800$ m/hour $= 10.8$ km/hour

6 0.6 litres per second $= 0.6 \times 60$ litres per minute
$= 0.6 \times 60 \times 60$ litres per hour
$= 2160$ litres per hour.

$2160 \div 4.55 = 475$ gallons

475 gallons per hour (to the nearest whole number)

7 Bolt: 100 m in 9.58 seconds $= 10.4$ m/s

Cheetah: 120 km/h $= 120000$ m/hour
$= 120000 \div 60$ m/min
$= 2000$ m/min
$= 2000 \div 60$ m/sec $= 33.3$ m/s

The cheetah is faster.

Review it!

1 **a** $3.2 \times 1000 = 3200$ m

b $9 \times 60 = 540$ seconds

c $0.4 \times 1000 = 400$ ml

2 $4600 \div 1000 = 4.6$ km

3 $2.5 \times 60 = 150$ minutes

4 $1.1 \times 0.32 = 0.352$ m^2 or $110 \times 32 = 3520$ cm^2

5 $3 \times 10000 = 30\,000$ cm^2

6 $\frac{5}{12}$

7 $26:18 = 13:9$

8 $100 - 85 = 15$, $15 \div 3 = 5$ minutes

9 density $= \frac{345}{0.15} = 2300$ kg/m^3

10 $10 - 8 = 2$ km, $\frac{2}{8} \times 100 = 25\%$

11 $25 - 13 = 12$, $\frac{12}{25}$ **or** 48%

12 $15 + 5 + 3 = 23$ mins
$\frac{23}{90}$

13 $20 \div (\frac{4}{5}) = 25$ hours $= 1$ day and 1 hour

14 **a** $50 \div 5 = 10$, Josie: $1 \times 10 = 10$ marbles,
Charlie: $4 \times 10 = 40$ marbles, Charlie has 30 more.

b $C = 4J$

15 $\frac{100}{360} \times 100 = 28\%$

16 $0.8 \times 1200 = £960$
$0.9 \times 960 = £864$

17 1 cm: 50 000 cm
50 000 cm $= 0.5$ km
3 km $\div 0.5 = 6$
6 cm

18 $1.02^3 \times 1500 = £1591.81$

19 $32\,000 \div 4 = 8000$ people

20 $393 \div 125 = 3.144$ hours $= 3$ hours 9 minutes

21 $2.50 + 1.90 + (2 \times 5.30) = £15$
$1.05 \times £15 = £15.75$

22 $37 + 15 + 4 + 19 = 75$
$\frac{15}{75} \times 100 = 20\%$

23 $0.045 \times 3000 = £135$
$3000 + (5 \times 135) = £3675$

24 $30 \div 3 = 10$
boys $= 2 \times 10 = 20$
Girls $= 1 \times 10 = 10$
Boys $= 20 - 2 = 18$
Girls $= 10 + 3 = 13$
18:13

25 Men to women is $7:6 = 35:30$
Ratio of women to children is $15:2 = 30:4$
Ratio of men to women to children is $35:30:4$
$35 + 30 + 4 = 69$
$3450 \div 69 = 50$
$35 \times 50 = 1750$ men

26 No – for two things to be in direct proportion when one is zero the other must be zero; the graph does not go through the origin so this is not the case.

27 Neither, since the time taken to cook increases as the weight increases it is not in indirect proportion. It is not in direct proportion since a graph to illustrate the relationship would not go through the origin.

28 speed $= \frac{\text{distance}}{\text{time}} = \frac{0.05}{17} = \frac{1}{340}$ hours $= \frac{3}{17}$ mins
$= 11$ seconds

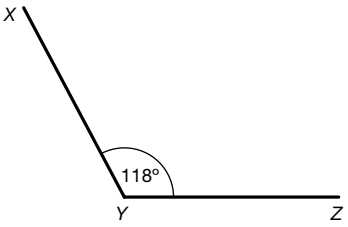
29 She is incorrect since the ratio of females to males must be the same for them to have equivalent proportions: 35:60 is not equivalent to 12:37.

Geometry and measures

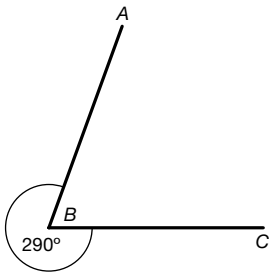
Measuring and drawing angles

1 **a** 43° **b** Acute

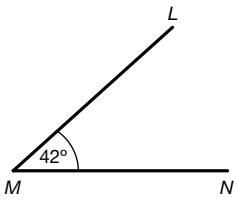
2 **a**



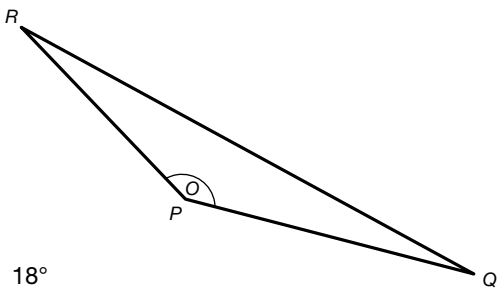
b



c



3 **a**



b 18°

Using the properties of angles

Stretch it!

Angles of triangle are in the ratio 1 : 2 : 3

Total number of parts = 6

1 part = $\frac{180}{6}$ = 30°

Angles in the triangle are 30°, 60° and 90°. It is a right-angled triangle.

1 Angles around a point add up to 360° so:

$a + 112 + 88 + 106 = 360$

$a + 306 = 360$

$a = 54°$

2 **a** **i** $a = (180 - 40) \div 2 = 70°$

ii Base angles of an isosceles triangle are equal.

b Exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices so:

$b = 70 + 40$

$b = 110°$

Or, angles on a straight line add up to 180° so:

$b = 180 - 70 = 110°$

3 Angles around a point add up to 360° so:

$5x + 9x + 108 = 360$

$14x + 108 = 360$

$14x = 252$

$x = 18°$

4 **a** **i** $x = 180 - 126 = 54°$

ii Angles on a straight line add up to 180°.

b Angles in a quadrilateral add up to 360° so:

$y + 135 + 54 + 88 = 360$

$y + 277 = 360$

$y = 83°$

5 **a** Angles on a straight line add up to 180° so:

$x = 180 - 84$

$x = 96°$

b **i** $y = 96°$

ii Use the fact that corresponding angles are equal, then the fact that vertically opposite angles are equal.
Or, use the fact that alternate angles are equal, then use angles on a straight line add up to 180°.

6 **a** Base angles of an isosceles triangle are equal so $a = 58°$.

b Angles in a triangle add up to 180° so:

$b = 180 - 58 - 58$

$b = 64°$

c Alternate angles are equal so $c = 58°$ (since angle a = angle c).

Or, since opposite angles of a parallelogram are equal:

$b + c = 122$

$64 + c = 122$

$c = 58°$

7 Angle *BAD* = 62° (Opposite angles of a parallelogram are equal)

Angle *ADE* = 62° (Alternate angles are equal)

$x = 180 - 62 - 62$ (Base angles of an isosceles triangle are equal)

$x = 56°$

8 Angle *ACB* = 36° (Base angles of an isosceles triangle are equal)

Angle *ABC* = 180 − 36 − 36 (Angles in a triangle add up to 180°)

Angle *ABC* = 108°

$x = 108°$ (Alternate angles are equal)

Using the properties of polygons

Stretch it!

1 The angle sum of a triangle is 180°.

Sum of interior angles of a hexagon = 4 × 180° = 720°.

2

Polygon	Number of sides (n)	Number of triangles formed	Sum of interior angles
Triangle	3	1	180°
Quadrilateral	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°
Heptagon	7	5	900°
Octagon	8	6	1080°
Decagon	10	8	1440°

3 $n - 2$

4 $180 \times (n - 2)$

Stretch it! Exterior angle of a regular hexagon $= 360 \div 6 = 60°$

Interior angle $= 180 - 60 = 120°$

Three hexagons meet at a point, so $120 + 120 + 120 = 360°$

Similarly, interior angle of an octagon $= 180 - (360 \div 8)$
$= 135°$

Interior angle of a square $= 90°$, so $135 + 135 + 90$
$= 360°$.

Regular pentagons have an interior angle of 108°. This does not divide equally into 360°, so these shapes will not fit together at a point in this way.

1 Regular decagon has 10 equal sides.

Exterior angle $= 360° \div 10 = 36°$

2 **a** Number of sides $= 360° \div 15° = 24$

b Angles on a straight line add up to 180° so:

Interior angle + exterior angle = 180

Interior angle + 15 = 180

Interior angle = 165°

Sum of interior angles $= 24 \times 165 = 3960°$

3 Sum of interior angles of regular pentagon $= 180° \times (5 - 2)$

$= 180° \times 3 = 540°$

One interior angle of regular pentagon $= 540° \div 5 = 108°$

If this is a regular pentagon, $AB = AE$ and triangle ABE is isosceles.

In triangle ABE:

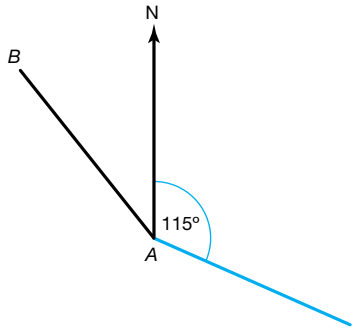
angle ABE = angle AEB $= (180° - 108°) \div 2 = 36°$

so angle $CBE = 108° - 36° = 72°$

Using bearings

1 **a** $360 - 45$ (acute angle) $= 315°$

b



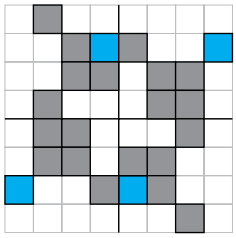
2 Bearing of P from Q $= 180° + 164° = 344°$

3 Kirsty is correct.

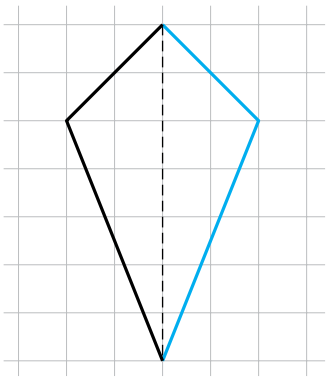
The bearing is 314° (360° − 46°) as it must be measured clockwise from North.

Properties of 2D shapes

Stretch it!

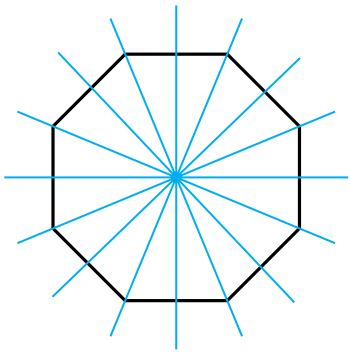


1 **a**



b kite

2 **a** 8 possible lines of symmetry:



b 8

3 **a** A rectangle has rotational symmetry of order **2.**

b A **rhombus** has all sides equal and rotational symmetry of order 2.

c A kite has **1** line of symmetry and **no** rotational symmetry.

d The diagonals of a **square** and a **rhombus** bisect each other at 90°.

Congruent shapes

1 Any accurate copy of shape A, in any orientation.

2 **a** Corresponding angles are equal so $x = 120°$

b Corresponding sides are the same length so $y = 12$ cm

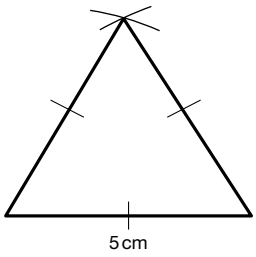
3 **a** SSS (each triangle has equal sides: 3 cm, 3 cm, 2.5 cm)

b ASA (two angles, 70° and 60°, and the included side, 8 cm, are equal)

Constructions

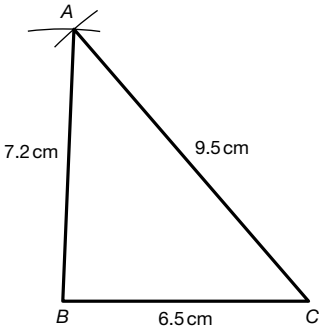
Stretch it!

A triangle with sides of 5cm with constructions lines indicating the use of compasses

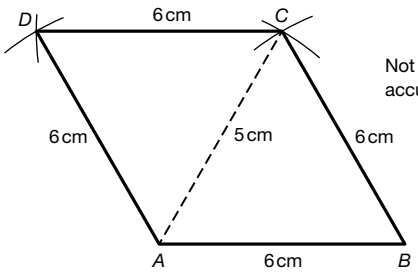


Angle size 60°

1

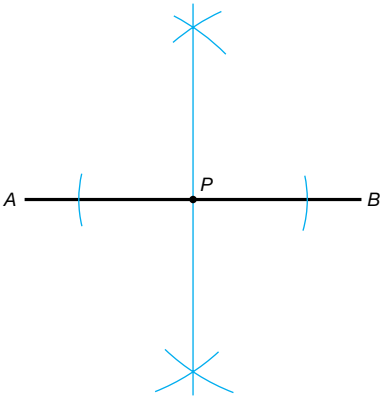


2

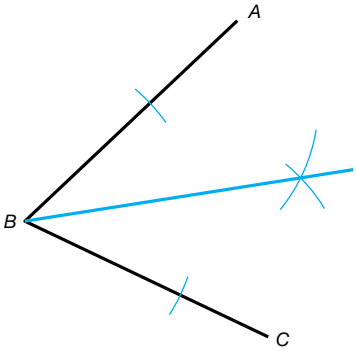


Not drawn accurately

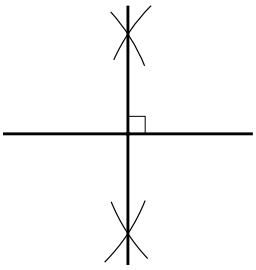
3



4



5



Drawing circles and parts of circles

1 a A **chord** is a straight line that does not pass through the centre of a circle but touches the circumference at each end.

b A **tangent** is a straight line that touches the outside of a circle at one point only.

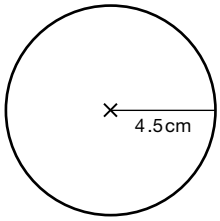
c A **diameter** is a straight line through the centre of a circle that touches the circumference at each end.

d An **arc** is part of the circumference of a circle.

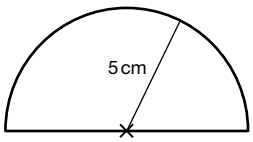
e A **radius** is a straight line from the centre of a circle that is half the length of the diameter.

f The part of a circle that has a chord and an arc as its boundary is called a **segment.**

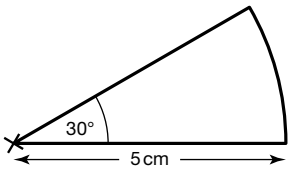
2 a



b

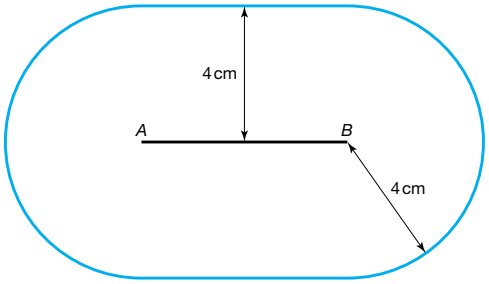


3

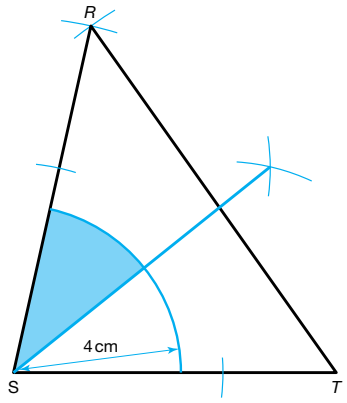


Loci

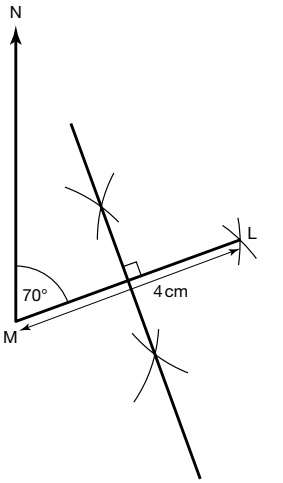
1



2



3 **a** and **b**



Route taken by boat

Perimeter

1 $4 \times 7.2 = 28.8\,\text{cm}$

2 $7 + 9 + 9 + 5 + 5 + 7 = 42\,\text{cm}$

3 Curved edge $= 2\pi r \div 2 = (2 \times \pi \times 4) \div 2 = 4\pi$

Perimeter $= 4\pi + 2 \times r = 4\pi + 8\,\text{cm}$

So $k = 4$ and $b = 8$

4 Perimeter $= (\pi \times 30) + 100 + 100 = 200 + 30\pi$ m

5 Perimeter $= \left(\frac{1}{2} \times \pi \times 32\right) + 32 + 32 = 16\pi + 64\,\text{cm}$

Ribbon $= 16\pi + 64 + 5 = 16\pi + 69\,\text{cm} = 119.3\,\text{cm}$

120 cm must be bought

$12 \times £0.15 = £1.80$

Area

Stretch it! Area of a semicircle $= \frac{\pi r^2}{2}$, area of a quarter circle $= \frac{\pi r^2}{4}$

1 **a** $4.5 \times 2 = 9.0\,\text{cm}^2$

b $3 \times 1.5 = 4.5\,\text{cm}^2$

c $\frac{(5+9)}{2} \times 4 = 28.0\,\text{cm}^2$

d $\frac{1}{2} \times 2 \times 5 = 5.0\,\text{cm}^2$

e $\pi \times 4.5^2 = 63.6\,\text{cm}^2$

2 Length of side $= 12 \div 4 = 3\,\text{cm}$

Area $= 3^2 = 9\,\text{cm}^2$

3 Shaded triangles would fit together to form one triangle with base $10 - 6 = 4$.

So area of shaded triangles $= \frac{1}{2} \times 4 \times 7 = 14\,\text{cm}^2$

Area of trapezium $= \frac{(6+10)}{2} \times 7 = 56\,\text{cm}^2$

Fraction of the shape that is shaded $= \frac{14}{56} = \frac{1}{4}$

4 Area of whole shape $= 6 \times 8 = 48\,\text{cm}^2$

Fraction shaded $= \frac{6}{16} = \frac{3}{8}$

Area shaded $= \left(\frac{3}{8}\right) \times 48 = 18\,\text{cm}^2$

5 Area of square $= 46 \times 46 = 2116\,\text{cm}^2$

Each circle has radius $= 11.5\,\text{cm}$

Area of four circles $= 4 \times \pi \times 11.5^2 = 1661.9\,\text{cm}^2$

Shaded area $= 2116 - 1661.9 = 454.1\,\text{cm}^2$

Sectors

1 Area $= \frac{1}{2} \times \pi \times 5^2 = 39.3\,\text{cm}^2$

Perimeter $= \frac{1}{2} \times \pi \times 10 + 10 = 25.7\,\text{cm}$

2 Area $= \frac{3}{4} \times \pi \times 4^2 = 12\pi\,\text{cm}^2$

3 Area $= \frac{1}{2} \times \pi \times 3^2 = 14.1\,\text{m}^2$

$14.1 \div 2 = 7.05$, so 8 bags needed.

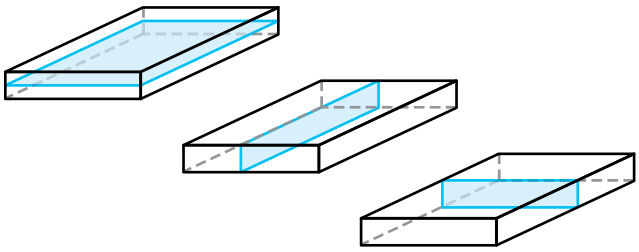
$8 \times 14.99 = £119.92$

3D shapes

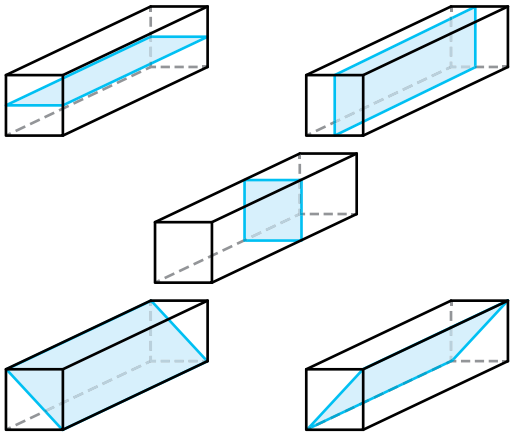
Stretch it!

3D shape	Faces	Edges	Vertices
Cube	6	12	8
Cuboid	6	12	8
Square-based pyramid	5	8	5
Tetrahedron	4	6	4
Triangular prism	5	9	6
Hexagonal prism	8	18	12

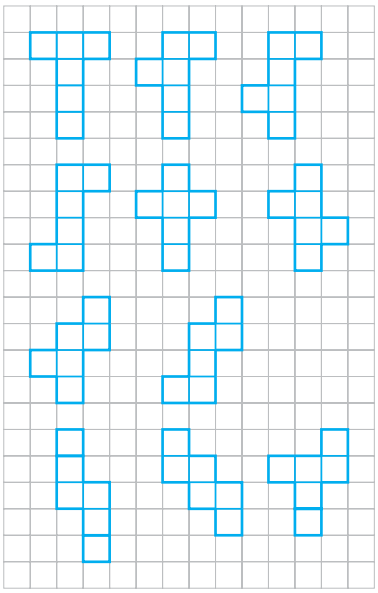
Stretch it! There are three planes of symmetry for the first cuboid:



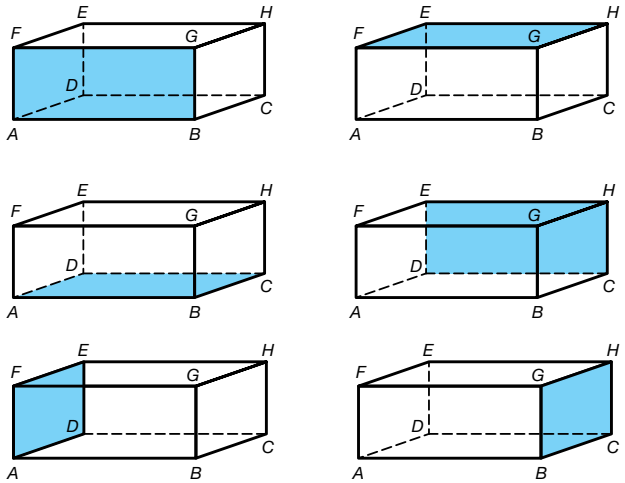
There are 5 planes of symmetry for the second cuboid: the same 3 planes as the first cuboid, plus two more planes along the diagonals of the square faces.



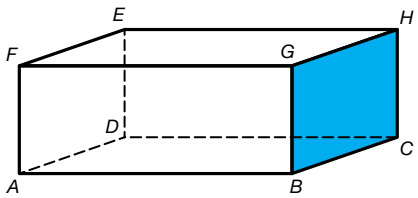
Stretch it!



1 a 6 possible rectangular faces:

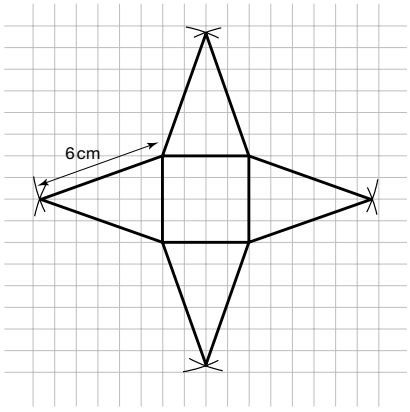


b

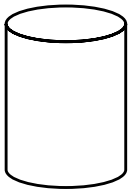


c Kelli has not counted the hidden edges.

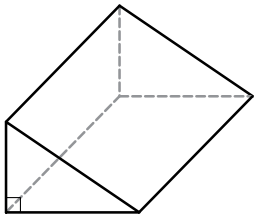
2 Draw a square in the middle with sides of 4 units (1 unit represents 1 cm). Set your compasses to 6 units and draw pairs of intersecting arcs from the corners of the square. These are the apices (top points) of the triangular sides. Draw lines for the sides of the triangles.



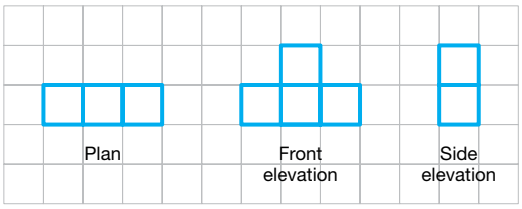
3 a



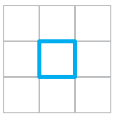
b



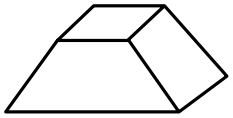
4



5 a



b



Volume

1 $\frac{4}{3} \times \pi \times 4.5^3 = 381.7 = 382\,\text{cm}^3$ (to 3 s.f.)

2 $\pi r^2 h + \frac{1}{3}\pi r^2 h = \pi \times 0.5^2 \times 2 + \frac{1}{3} \times \pi \times 0.5^2 \times 1.5$
$= 0.625\,\pi = 1.96\ \text{m}^3$

3 $\frac{1}{3} \times \pi \times 6^2 \times 22 = \frac{1}{3} \times 792 \times \pi = 264\pi\,\text{cm}^3$
$k = 264$

4 Volume of water $= 18 \times 7 \times 7 = 882\,\text{cm}^3$
$882 = 7 \times 20 \times h$
$882 = 140 \times h$
$h = 6.3\,\text{cm}$

Surface area

1 $6 \times (5 \times 5) = 150\,\text{cm}^2$

2 $4\pi r^2 = 4 \times \pi \times 3^2 = 36\pi\ \text{cm}^2$

3 $18 - 4 = 14\,\text{cm}^2$

4 Sloping surface $= \pi \times 14 \times 45 = 630\pi\ \text{cm}^2$
Base $= \pi \times 14^2 = 196\pi\,\text{cm}^2$
Total surface area $= 196\pi + 630\pi = 826\pi$
Percentage yellow $= \frac{630}{826} \times 100 = 76.3\%$

Using Pythagoras' theorem

1 Using Pythagoras' theorem $c^2 = a^2 + b^2$:
$AC^2 = AB^2 + BC^2$
$15^2 = 11^2 + BC^2$
$BC^2 = 15^2 - 11^2 = 104$
$BC = \sqrt{104}$
$BC = 10.2\,\text{cm}$ (to 3 s.f.)

2 $c^2 = a^2 + b^2$
$c^2 = 2.4^2 + 5.5^2$
$c^2 = 36.01$
$c = \sqrt{36.01}$
$c = 6.0008...$
The ladder is 6 m long, to the nearest metre.

3 Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$XZ^2 = XY^2 + YZ^2$

$15^2 = XY^2 + 9^2$

$XY^2 = 15^2 - 9^2 = 144$

$XY = \sqrt{144}$

$XY = 12\,\text{cm}$

Area $= \frac{1}{2}bh = \frac{1}{2} \times 9 \times 12$

Area $= 54\,\text{cm}^2$

4 If the triangle is right-angled, $PQ^2 = PR^2 + RQ^2$

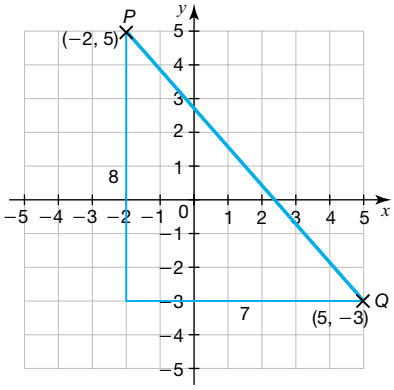
$PQ^2 = 13^2 = 169$

$PR^2 + RQ^2 = 9^2 + 7^2 = 81 + 49 = 130$

$PQ^2 \neq PR^2 + RQ^2$

Claudia is not correct.

5 $P(-2, 5)$, $Q(5, -3)$

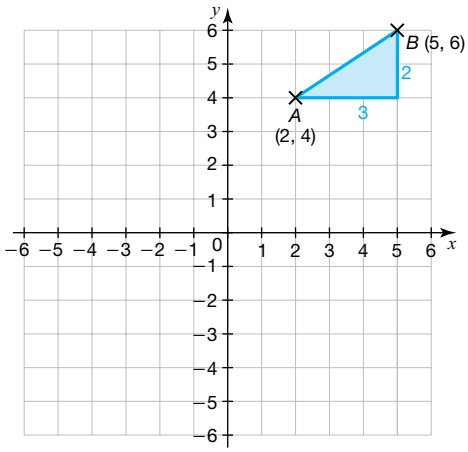


$PQ^2 = 8^2 + 7^2 = 113$

$PQ = \sqrt{113}$

$PQ = 10.63$ (to 2 d.p.)

6



$A(2, 4)$, $B(5, 6)$

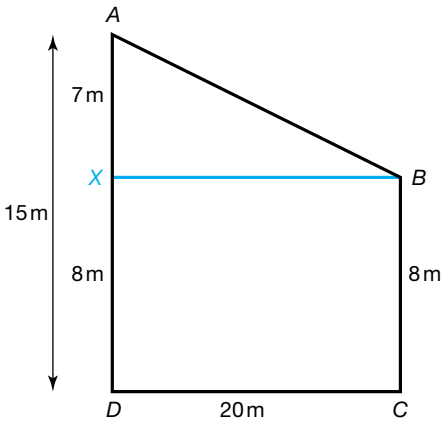
$AB^2 = (6 - 4)^2 + (5 - 2)^2$

$= 2^2 + 3^2$

$= 13$

$AB = \sqrt{13}$

7



Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$AB^2 = AX^2 + BX^2$

$AB^2 = 7^2 + 20^2 = 449$

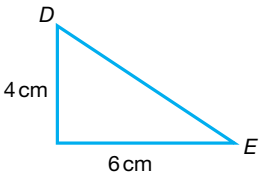
$AB = \sqrt{449}$

$AB = 21.2$ (to 3 s.f.)

Perimeter of field $ABCD = 15 + 20 + 8 + 21.2$
$= 64.2 \approx 65\text{m}$

Cost of fencing $= 65 \times £14 = £910$

8



$DE^2 = 6^2 + 4^2 = 52$

$DE = \sqrt{52}$

$= 7.2$ cm (1 d.p.)

Trigonometry

Stretch it!

Opposite could have been 1 m, hypotenuse could have been 2 m. They could be any lengths that keep opposite and hypotenuse in the ratio 1 : 2.

1 **a** 0.4 **b** 0.6 **c** 1.0
d 26.6 **e** 48.6 **f** 54.7

2 $\text{Cos } 72° = \frac{MN}{15}$ $MN = 15 \cos 72° = 4.6\,\text{cm}$

3 $\text{Tan } ABC = \frac{6}{7}$

$ABC = \tan^{-1}\left(\frac{6}{7}\right)$

$ABC = 40.6°$

4 Let x be the depth of water.

$\sin 15° = \frac{x}{10}$

$x = 10 \sin 15°$

$x = 2.6$ m

Exact trigonometric values

1 **a** 0.5 **b** 0 **c** 0
d $\frac{1}{\sqrt{2}}$ **e** $\sqrt{3}$

2 $\tan 45° = 1 = \frac{\text{opposite}}{\text{adjacent}} = \frac{4}{AC}$

Therefore $AC = 4\,\text{cm}$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{4}{BC}$

$BC = 4\sqrt{2}$

Therefore $BC = 4\sqrt{2}$ cm

3 Since: $\tan 30° = \frac{1}{\sqrt{3}}$ one angle must be 30° and therefore the other is 60°

4 $\sin 30° = \frac{1}{2}$ therefore $ABC = 30°$

5 $\cos 30° = \frac{\sqrt{3}}{2} = 0.866$ (3 d.p.) $\tan 45° = 1$

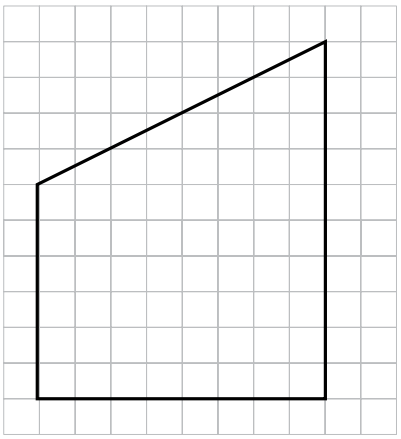
Smallest to largest = 0.5, $\frac{3}{4}$, $\cos 30°$, $\tan 45°$

Transformations

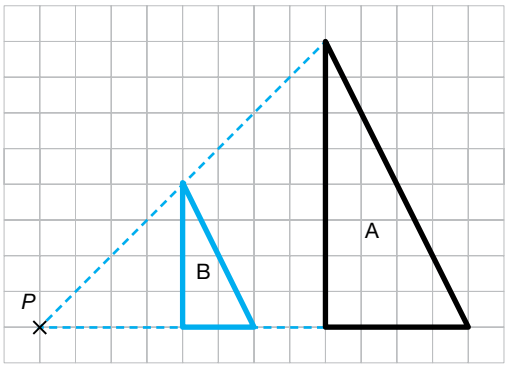
Stretch it!

Yes. Reflection in the x-axis followed by reflection in the y-axis (or vice versa) will always produce a rotation of 180°.

1

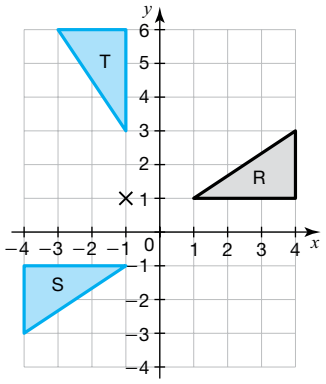


2



3 Translation by vector $\begin{pmatrix}-4\\-2\end{pmatrix}$

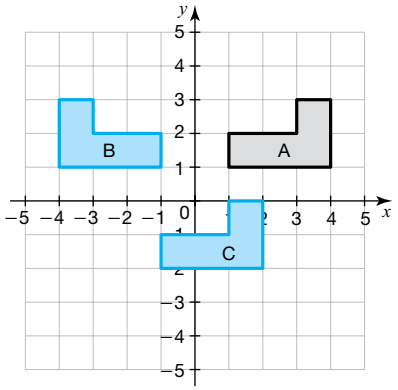
4 **a** and **b**



5 Reflection in the y-axis

6 Enlargement by scale factor $\frac{1}{2}$, centre (3, 3)

7 **a** and **b**



c Rotation of 90° clockwise about (0, 0)

Similar shapes

Stretch it!

Perimeter of $ABC = 3 + 6 + 5 = 14\,\text{cm}$

Perimeter of $DEF = 6 + 12 + 10 = 28\,\text{cm}$

The perimeter of a shape enlarged by scale factor 2 will also be enlarged by scale factor 2.

In general, all lengths on an enlarged shape, including the perimeter, are enlarged by the same scale factor.

Stretch it!

Angle BAC = angle CDE (alternate angles are equal)

Angle ABC = angle CED (alternate angles are equal)

Angle BCA = angle DCE (vertically opposite angles are equal)

All three pairs of angles are equal so triangles ABC and EDC are similar.

1 **a** Angle $DFE = 30°$ (Corresponding angles are the same)

b Scale factor of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{12}{3} = 4$

Length of $EF = 4\,\text{cm} \times 4 = 16\,\text{cm}$

c Length of $AB = 8\,\text{cm} \div 4 = 2\,\text{cm}$

2 **a** Angle $MLO = 80°$ (Corresponding angles are the same: angle MLO = angle QPS)

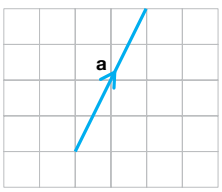
b Scale factor of enlargement $= \frac{\text{enlarged length}}{\text{original length}} = \frac{9}{3} = 3$

Length of $QR = 4.4\,\text{cm} \times 3 = 13.2\,\text{cm}$

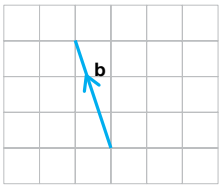
c Length of $LO = 12\,\text{cm} \div 3 = 4\,\text{cm}$

Vectors

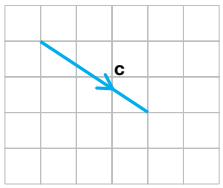
1 **a**



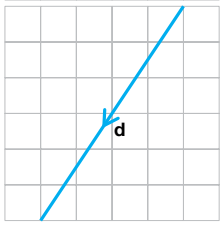
b



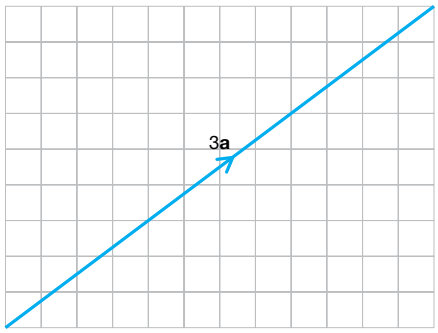
c



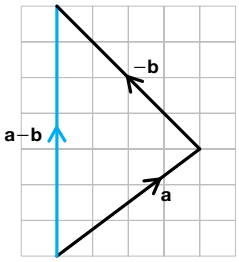
d



2 a



b



c $\mathbf{a} = \begin{pmatrix}4\\3\end{pmatrix}$

So $-2\mathbf{a} = -2 \times \begin{pmatrix}4\\3\end{pmatrix} = \begin{pmatrix}-8\\-6\end{pmatrix}$

3 i $\mathbf{c} = 2\mathbf{a}$

ii $\mathbf{d} = \mathbf{a} - \mathbf{b}$

iii $\mathbf{e} = \mathbf{a} + \mathbf{b}$

4 a $\mathbf{a} + \mathbf{b} = \begin{pmatrix}2\\6\end{pmatrix} + \begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}-3\\9\end{pmatrix}$

b $\mathbf{a} + 2\mathbf{b} = \begin{pmatrix}2\\6\end{pmatrix} + 2 \times \begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix} + \begin{pmatrix}-10\\6\end{pmatrix} = \begin{pmatrix}-8\\12\end{pmatrix}$

c $\mathbf{a} - \mathbf{b} = \begin{pmatrix}2\\6\end{pmatrix} - \begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}7\\3\end{pmatrix}$

d $\mathbf{b} - 2\mathbf{a} = \begin{pmatrix}-5\\3\end{pmatrix} - 2 \times \begin{pmatrix}2\\6\end{pmatrix} = \begin{pmatrix}-5\\3\end{pmatrix} - \begin{pmatrix}4\\12\end{pmatrix} = \begin{pmatrix}-9\\-9\end{pmatrix}$

5 a $= \begin{pmatrix}2\\3\end{pmatrix}$

b $= \begin{pmatrix}4\\-6\end{pmatrix} = 2\begin{pmatrix}2\\-3\end{pmatrix}$

c $= \begin{pmatrix}-2\\-3\end{pmatrix} = -\begin{pmatrix}2\\3\end{pmatrix} = -\mathbf{a}$

d $= \begin{pmatrix}20\\30\end{pmatrix} = 10\begin{pmatrix}2\\3\end{pmatrix} = 10\mathbf{a}$

a, c and **d** are parallel.

6 a $\overrightarrow{PQ} = 4\mathbf{a}$ ($\overrightarrow{PQ}$ and $\overrightarrow{SR}$ are parallel and the same length)

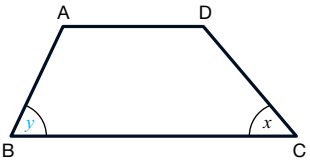
b $\overrightarrow{QR} = -3\mathbf{b}$ ($\overrightarrow{QR}$ and $\overrightarrow{PS}$ are parallel and the same length; $\overrightarrow{PS}$ has opposite direction to $\overrightarrow{SP}$)

c $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$
$= 4\mathbf{a} - 3\mathbf{b}$

d $\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS}$
$= -4\mathbf{a} - 3\mathbf{b}$

Review it

1 a



b $BC = 3.8\,\text{cm}$

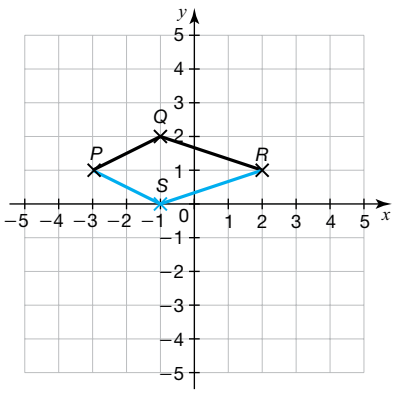
c $x = 50°$

2 a 5 faces

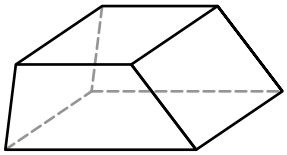
b 6 vertices

3 a (−3, 1)

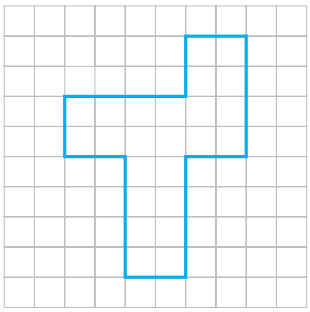
b



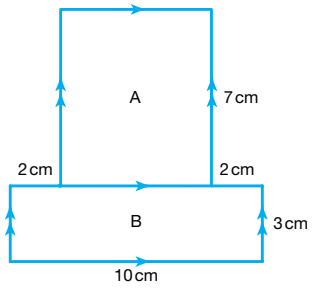
4



5 Any accurate copy of the shape



6 Area A = $7 \times (10 - 2 - 2) = 7 \times 6 = 42\ \text{cm}^2$



Area B = $10 \times 3 = 30\ \text{cm}^2$

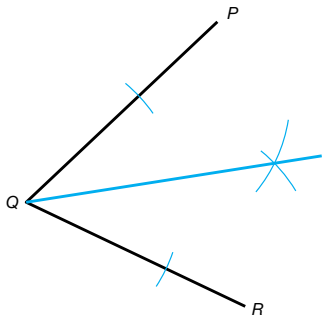
Total area = $42 + 30 = 72\ \text{cm}^2$

7 Area of parallelogram = $3 \times 12 = 36\ \text{cm}^2$

Length of side of square = $\sqrt{36} = 6\ \text{cm}$

Perimeter of square = $4 \times 6 = 24\ \text{cm}$

8



9 Rotation of 180° about (1, 0)

10 Sum of interior angles $= 180(n - 2)$

$n = 8$, therefore sum of interior angles $= 180(8 - 2)$

$= 180 \times 6$

$= 1080$

$x = \frac{1080}{8} = 135$

$x = 135°$

11 Angle *CFE* = 112° (corresponding angles are equal)

Angle *CFG* = 180 − 112 = 68° (angles on a straight line add up to 180°)

Angle *GCF* = angle *CFG* (base angles of an isosceles triangle are equal)

x = (180 − 68 − 68) = 44° (angles in a triangle add up to 180°)

12 Shaded area $= (10 \times 12) - ((\frac{1}{2} \times 12 \times 3) + (\frac{1}{2} \times 8 \times 7) + (\frac{1}{2} \times 10 \times 4))$

$= 120 - (18 + 28 + 20)$

$= 54\ \text{cm}^2$

Proportion $= \frac{54}{120} = \frac{9}{20} = 45\%$

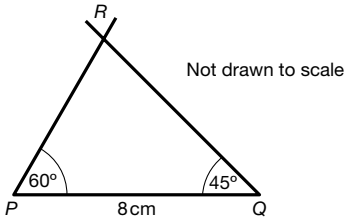
13 If triangle *ABC* is right-angled, $c^2 = a^2 + b^2$

$c^2 = 8^2 = 64$

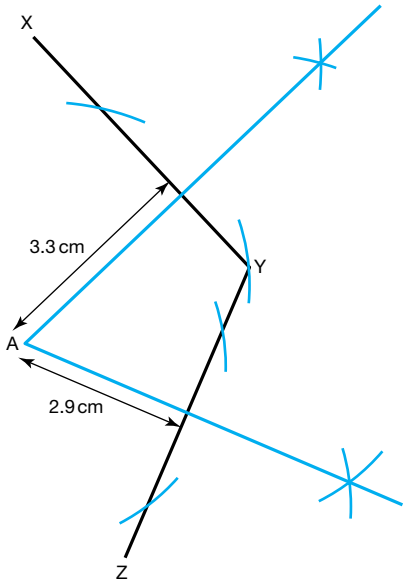
$a^2 + b^2 = 4^2 + 6^2 = 16 + 36 = 52$

$c^2 \neq a^2 + b^2$ so triangle *ABC* is not right-angled.

14



15 a



Scale is 1 : 200

b Distance from *A* to *YZ* = 2.9 cm

$2.9 \times 200 = 580\ \text{cm} = 5.8\ \text{m}$

Distance from *A* to *YX* = 3.3 cm

$3.3 \times 200 = 660\ \text{cm} = 6.6\ \text{m}$

Difference in distance = 6.6 − 5.8 = 0.8 m

16 a $\cos 45° = \frac{1}{\sqrt{2}}$

b Ratio of adjacent to hypotenuse is 1:2

Therefore *AB* = 3 cm

17 a $\mathbf{a} - \mathbf{b} = \begin{pmatrix} 4 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -8 \end{pmatrix}$

b $\mathbf{a} + 2\mathbf{b} = \begin{pmatrix} 4 \\ -5 \end{pmatrix} + 2 \times \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -5 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

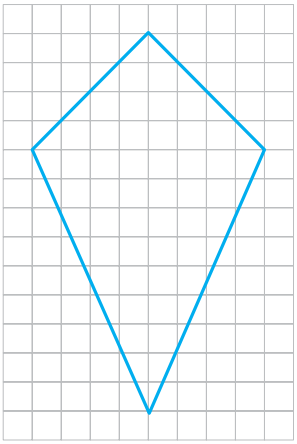
$= \begin{pmatrix} 8 \\ 1 \end{pmatrix}$

c $\mathbf{b} - 2\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2 \times \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 8 \\ -10 \end{pmatrix}$

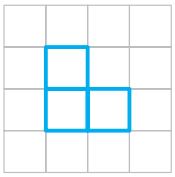
$= \begin{pmatrix} -6 \\ 13 \end{pmatrix}$

18 Any correct answer will have two pairs of equal adjacent sides, two equal angles, and one line of symmetry.



19 Three lines of symmetry and all sides the same length mean it must be an **equilateral triangle**.

20



21 a i 35°

ii Triangle *WYZ* is isosceles, and base angles of an isosceles triangle are equal.

b Angles in a triangle add up to 180° so:

$b = 180 - 35 - 35$

$= 110°$

c Triangle *XYZ* is isosceles, and base angles of an isosceles triangle are equal so:

$c = (180 - 70) \div 2 = 55°$

22 Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$AC^2 = AB^2 + BC^2$

$14^2 = 6^2 + BC^2$

$BC^2 = 14^2 - 6^2 = 160$

$BC = \sqrt{160}$

$BC = 12.6\,\text{cm}$ (to 1 d.p.)

23 Interior angle of a square = 90°

Sum of interior angles of an octagon (with $n = 8$)
$= 180 \times (n - 2) = 180 \times (8 - 2) = 1080°$

Interior angle of a regular octagon $= 1080° \div 8 = 135°$

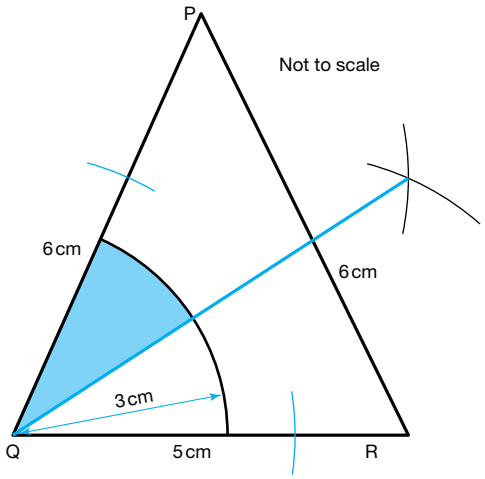
(Or, exterior angle of a regular octagon $= 360° \div 8 = 45°$.
Then interior angle $= 180° - 45° = 135°$)

Angles around a point add up to 360° so:

$x = 360 - 90 - 135$

$x = 135°$

24



25 Divide the trapezium into a rectangle and a triangle.
Draw a line *DX* parallel to *AB*, with *X* on the line *BC*.
$BX = 5\,\text{cm}$, $CX = 7\,\text{cm}$.
Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$DC^2 = CX^2 + DX^2$

$DC^2 = 7^2 + 4^2 = 65$

$DC = \sqrt{65}$

$DC = 8.06$ (to 2 d.p.)

Perimeter of $ABCD = 4 + 5 + 8.06 + 12 = 29.06\,\text{cm}$

26 Length of arc $= \frac{1}{4} \times 2 \times \pi \times 4 = 2\pi$

Perimeter $= 4 + (2 \times 9) + 4 + 2\pi = 32.3\,\text{cm}$

27 Area of square $= 6 \times 6 = 36\,\text{cm}^2$

Area of circle $= \pi \times 3^2 = 9\pi\,\text{cm}^2$

Shaded area $= 36 - 9\pi = 7.7\text{cm}^2$ (1 d.p.)

28 Volume of cylinder $= \pi \times 3^2 \times 15 = 135\pi\,\text{cm}^3$

2 litres $= 2000\,\text{ml} = 2000\,\text{cm}^3$

$2000 \div 135\pi = 4.7$

Glass can be completely filled 4 times.

29 Scale factor of enlargment $= \frac{\text{enlarged length}}{\text{original length}} = \frac{11}{5} = 2.2$

Length $x = 6\,\text{cm} \times 2.2 = 13.2\,\text{cm}$

30 Using Pythagoras' theorem $c^2 = a^2 + b^2$:

$PR^2 = PQ^2 + RQ^2$

$PR^2 = 10^2 + 6^2 = 136$

$PR^2 = PS^2 + SR^2$

$136 = 11^2 + x^2$

$x^2 = 136 - 11^2 = 136 - 121 = 15$

$x = \sqrt{15}$

$x = 3.87$ cm (to 3 s.f.)

31 a Curved surface area $= \pi \times 6 \times 10 = 60\pi\ \text{cm}^2$

Base area $= \pi \times 6^2 = 36\pi\ \text{cm}^2$

Total surface area $= 60\pi + 36\pi = 96\pi = 300\,\text{cm}^2$ to 2 s.f.

b Volume $= \frac{1}{3} \times \pi \times 6^2 \times 8 = 96\pi = 300\,\text{cm}^3$

32 $\tan x = \frac{8}{6}$

$x = \tan^{-1}\left(\frac{8}{6}\right)$

$x = 53.1°$

33 Translation by vector $\begin{pmatrix} -7 \\ -6 \end{pmatrix}$

34 $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= 2\mathbf{a} + 3\mathbf{b} + 3\mathbf{a} - \mathbf{b}$

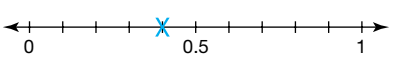
$= 5\mathbf{a} + 2\mathbf{b}$

Probability

Basic probability

Stretch it! No – each time the probability of getting an even number is $\frac{1}{2}$. You would expect to get even numbers approximately 50 times but cannot guarantee it.

1 $\frac{4}{10}$



2 a Total number of sweets $= 12 + 3 + 10 = 25$

$\frac{3}{25}$

b $\frac{(3+10)}{25} = \frac{13}{25}$

3 Pair **a**, because when you flip a coin, you can't get both a head and a tail at the same time. (Prime numbers on a dice are 2, 3, 5 and odd numbers are 1, 3, 5, so events **b** are **not** mutually exclusive because 3 is in both groups.)

4 Pair **b**, because the first sweet chosen is replaced, so the possible outcomes of the second choice remain the same. (If the first sweet chosen is eaten, the possible outcomes of the second choice are altered, and so events **a** are **not** independent.)

5 $P(6) = 1 - (0.1 + 0.15 + 0.1 + 0.02 + 0.2)$
$= 1 - 0.57 = 0.43$

6 $P(\text{green or red}) = 1 - 0.4 = 0.6$

$P(\text{green}) = 2 \times P(\text{red})$

$P(\text{red}) = \frac{0.6}{3} = 0.2$

$P(\text{green}) = 2 \times 0.2 = 0.4$

Two-way tables and sample space diagrams

1

	Chicken	Beef	Vegetarian
Fruit	12	6	4
Cake	5	3	8
Total	17	9	12

a 12 (this is worked out by using the numbers in the 'Total' row, which must add up to 38)

b As shown in the table.

2 **a**

		Dice 1					
		1	2	3	4	5	6
Dice 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b **i** $\frac{2}{36} = \frac{1}{18}$

ii $\frac{3}{36} = \frac{1}{12}$

iii 0

3 To score 6, the player must pick two cards showing 3. To score 2, the player must pick two cards showing 1. Since the probability of getting 3 and 3 is more than 0, and the probability of getting 1 and 1 is more than 0, there must be at least 2 of each of those numbers. So the cards must be 1, 1, 3, 3.

Sets and Venn diagrams

Stretch it!

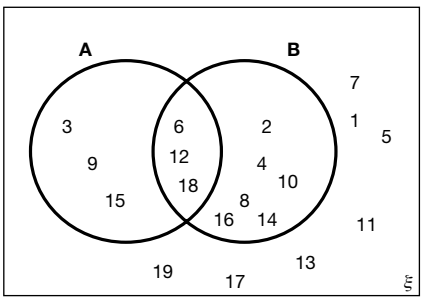
M {−1, 0, 1, 2}

N {2, 3}

2 is in both sets

Stretch it! None

1 **a**



b A ∩ B = {multiples of 6 less than 20} because these numbers are multiples of both 2 and 3.

2 **a** C ∩ T is the set of students who travel by car **and** train

C' ∩ B is the set of students who do **not** travel by car **and** travel by bus.

b **i** $P(C) = \frac{(14 + 11 + 11 + 2)}{(14 + 11 + 11 + 2 + 17 + 19 + 26)} = \frac{38}{100} = \frac{19}{50}$

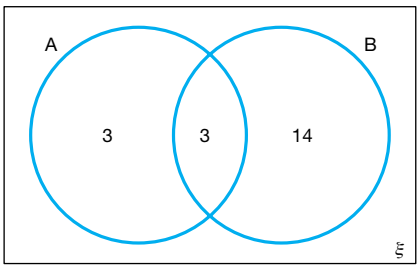
ii $P(B \cup T) = \frac{(19 + 11 + 2 + 0 + 11 + 17)}{100} = \frac{60}{100} = \frac{3}{5}$

iii $P(B' \cap T) = \frac{(11 + 17)}{100} = \frac{28}{100} = \frac{7}{25}$

3 $P(A \cap B) = \frac{3}{20}$ so there must be 3 elements in the intersection.

$P(A) = \frac{3}{10} = \frac{6}{20}$ so there must be a total of 6 elements in A.

The total number of elements must sum to 20.



4 **a** 3, 4, 5, 6

b 1, 2

c 1, 2, 3

Frequency trees and tree diagrams

1 **a** Apple = 123

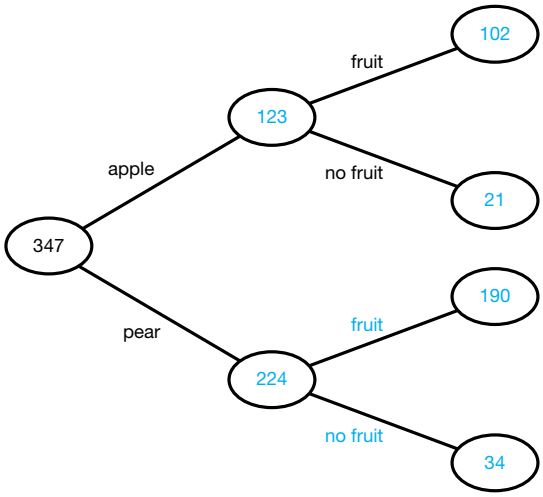
Pear = 347 − 123 = 224

Apple fruiting = 102

Apple not fruiting = 123 − 102 = 21

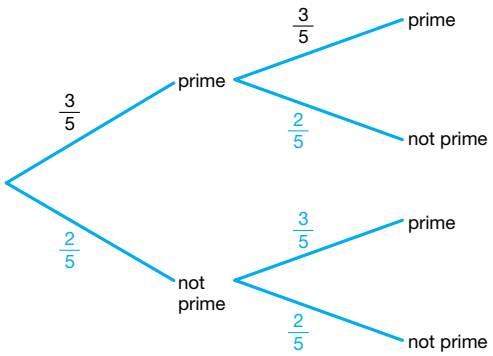
Pear not fruiting = 34

Pear fruiting = 224 − 34 = 190



b $\frac{190}{347}$

2 a



b $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

c $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

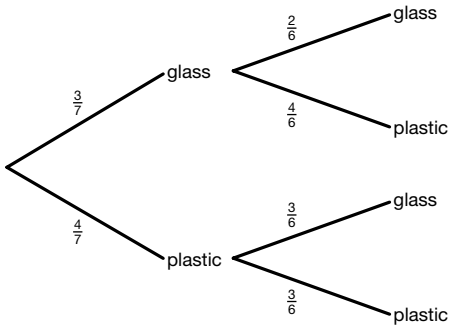
$\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

$\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

P(at least one prime) = 1 − P(no primes)

$= 1 - \frac{2}{5} \times \frac{2}{5} = 1 - \frac{4}{25} = \frac{21}{25}$

3 a



b P(two glass marbles) $= \frac{3}{7} \times \frac{2}{6} = \frac{6}{42}$

P(glass then plastic) $= \frac{3}{7} \times \frac{4}{6} = \frac{12}{42}$

P(plastic then glass) $= \frac{4}{7} \times \frac{3}{6} = \frac{12}{42}$

P(at least one glass) = 1 − P(both plastic)

$= 1 - \frac{4}{7} \times \frac{3}{6} = 1 - \frac{12}{42}$

$= 1 - \frac{2}{7} = \frac{5}{7}$

Expected outcomes and experimental probability

Stretch it! The dice has not been rolled enough times to decide if it is biased. More tests need to be carried out.

1 $0.45 \times 300 = 135$

2 Red $= \frac{2}{10} = \frac{1}{5}$

$\frac{1}{5} \times 100 = 20$ red sweets

3 $\frac{1}{2} \times 100 = 50$ primes

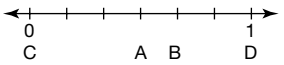
4 a Charlie – he has carried out the most tests.

b $\frac{(112 + 10 + 28)}{(112 + 10 + 28 + 74 + 7 + 19)} \times 10 = 6$

Review it!

1 $0.12 \times 250 = 30$

2



3 $1 - 0.3 = 0.7$

4 B, C

5 a $\frac{3}{5}$ **b** $\left(\frac{1}{5}\right) \times 25 = 5$

6

	Pizza	Pasta	Risotto	Total
Cake	12	6	1	19
Ice Cream	10	11	10	31
Total	22	17	11	50

7 $0.2 + 5x + 0.2 + x = 1$

$6x + 0.4 = 1$

$x = 0.1$

P(white) $= 5x + 0.2 = 0.7$

8 a No, he has not tested his dice enough times.

b $P(2) = \frac{9}{(12 + 9 + 16 + 7 + 6 + 0)} = \frac{9}{50}$

$\frac{9}{50} \times 100 = 18$

9 P(R, R) $= 0.1 \times 0.5 = 0.05$

P(R, G) $= 0.1 \times 0.5 = 0.05$

P(G, R) $= 0.9 \times 0.5 = 0.45$

$0.05 + 0.05 + 0.45 = 0.55$

Or P(at least one red) = 1 − P(green, green)

$= 1 - (0.9 \times 0.5)$

$= 1 - 0.45$

$= 0.55$

10 a

		Dice					
		1	2	3	4	5	6
Coin	Heads	2	4	6	8	10	12
	Tails	3	4	5	6	7	8

b i $\frac{2}{12} = \frac{1}{6}$

ii $\frac{2}{12} = \frac{1}{6}$

11 A{2,3,4}

B{−2,−1, 0, 1, 2, 3}

A∩B = {2, 3}

12 a i 6 **ii** 1 **iii** 5

b $\frac{4}{8} = \frac{1}{2}$

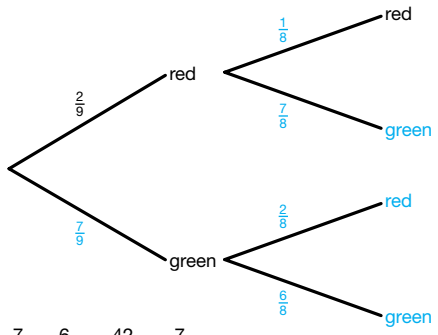
13 a i $P(S) = \frac{9 + 7}{14 + 7 + 9} = \frac{16}{30} = \frac{8}{15}$

ii $P(S \cap P) = \frac{7}{14 + 7 + 9} = \frac{7}{30}$

iii $P(P) = \frac{14 + 7}{14 + 7 + 9} = \frac{21}{30} = \frac{7}{10}$

b No. If you use P(S) + P(P), you will be counting P(S ∩ P) twice.

14 a



b $\frac{7}{9} \times \frac{6}{8} = \frac{42}{72} = \frac{7}{12}$

15 a Possible fractions: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3}, \frac{2}{4}, \frac{3}{4}$

Less than $\frac{1}{2}$ are $\frac{1}{3}$ and $\frac{1}{4}$.

P(less than $\frac{1}{2}$) $= \frac{2}{6} = \frac{1}{3}$

$\frac{1}{3} \times 30 = 10$

b $\frac{1}{3}$ is only a theoretical probability and therefore will not necessarily be accurate in real life.

16 45% of 300 = 135

135 boys and 165 girls.

$\frac{2}{3}$ of 135 = 90

$\frac{4}{5}$ of 165 = 132

Total playing sport = 222

Probability $= \frac{222}{300} = \frac{37}{50} = 0.74$

17 P(hooking a winning duck) $= \frac{5}{20} = 0.25$

If 100 people play, expected number of winners = 0.25 × 100 = 25 people.

The game makes £1 × 100 people = £100.

The money paid out in prizes = 25 winners × £2 = £50

Profit = £100 − £50 = £50

18 a Milo will have the better estimate as he has surveyed a greater number of people.

b Number of left-handed students = 5 + 4 + 7 + 7 = 23

Number of right-handed students = 23 + 18 + 51 + 60 = 152

P(left-handed) $= \frac{23}{23 + 152} = \frac{23}{175}$

$\frac{23}{175} \times 2000 = 262.8$

You would expect to find 263 left-handed students in a school with 2000 students.

Statistics

Data and sampling

Stretch it! A random sample could be taken; you could allocate a number to each pupil and randomly generate the numbers to survey. Any method is acceptable as long as each person in the school has an equally likely chance of being chosen. Alternatively a stratified sample could be taken.

1 Primary source: Recording the data by measuring it yourself.

Secondary source: Any sensible source, e.g. the Meteorological Office, local paper etc.

2 Qualitative data.

3 It is cheaper and quicker than surveying the whole population.

4 a The people working for an animal charity are more likely to be opposed to wearing real fur; every member of the population does not have an equal chance of being chosen.

b Surveying people in the street, a random telephone survey, any sensible method that ensures that any member of the population has an equal chance of being chosen.

5 a $\frac{400}{1600 + 400} = \frac{400}{2000} = \frac{1}{5}$

b $\frac{1}{5} \times 50 = 10$ bottles

6 a $\frac{3}{200} \times 800\,000 = 12\,000$

b The sample is relatively small. The sample is not a random sample as it is taken on one day in a year.

7 a Two of the following: the groups overlap; they are unequal in width; there is no group for anyone with a journey of more than 60 minutes; most people will be in the middle group.

b How long do you spend travelling to school in the morning?

$0 < t \leq 10$

$10 < t \leq 20$

$20 < t \leq 30$

$30 < t \leq 40$

$t > 40$

Frequency tables

1

Number of people on the bus	Frequency
0–9	4
10–19	12
20–29	3
30–39	1

2 a

Number of courgettes	Frequency
0	1
1	0
2	1
3	1
4	9
5	3
6	0

b (0 × 1) + (1 × 0) + (2 × 1) + (3 × 1) + (4 × 9) + (5 × 3) = 56

3 a There are gaps between his groups – times that fall between groups cannot be recorded, e.g. 15.5 hours.

His groups do not have the same width.

b Although one or more of the data values may fall in the $30 \leq h \leq 40$ group, this doesn't mean that those people trained for 40 hours. They could have trained for any length of time between 30 and 40 hours.

Bar charts and pictograms

1 a $15 + 4 + 1 = 20$

b $4 + 1 = 5$

$\frac{5}{20} \times 100 = 25\%$

2 a $11 - 7 = 4$

b Total number of people surveyed $= 18 + 18 + 12 + 3 = 51$

Total number of boys $= 11 + 6 + 3 = 20$

$\frac{20}{51} \times 100 = 39.2\%$

c Proportion of boys who played two sports $= \frac{6}{18} = \frac{1}{3}$

Proportion of boys who played three sports $= \frac{3}{12} = \frac{1}{4}$

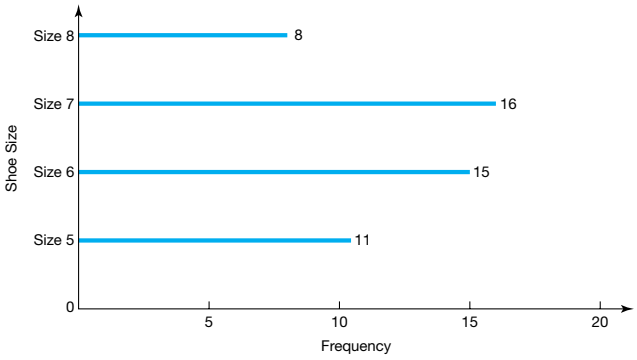
$\frac{1}{3} > \frac{1}{4}$ so the proportion who played two sports is larger.

3 $50 - (11 + 15) = 24$

$24 \div 3 = 8$

Therefore: $2 \times 8 = 16$ size 7 shoes

$1 \times 8 = 8$ size 8 shoes



4 a $90 - 20 = 70$

b Total number of bikes $= 50 + 50 + 20 + 90 = 210$

$\frac{50}{210} = \frac{5}{21}$

Pie charts

Stretch it!

Round appropriately – but check the angles sum to 360°

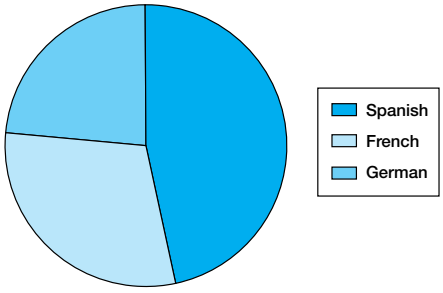
1 $27 + 42 + 21 = 90$

$360° \div 90 = 4°$

French $= 27 \times 4 = 108°$

Spanish $= 42 \times 4 = 168°$

German $= 21 \times 4 = 84°$



2 a $\frac{1.5}{360} \times 240 = 1$ student earned more than £40 000.

b $\frac{288 + 63}{360} \times 100 = 97.5\%$ (**or** $\frac{39}{40}$) of students earned less than £30 000.

3 a $18 + 10 = 28$

b The bar chart, since the frequency is easy to read from the bar chart.

Stem and leaf diagrams

1 a 7

b 4.3 – 4.1 = 0.2 kg (200 g)

2 Age of people using a dentist

2	0 0 0 0 1 1 1
3	2 5 5 7
4	1 2 2 6

Key: 2 | 0 means 20 years old

The leaves were not in ascending order, the spaces between leaves were not regular, and there was no key.

3 a Stem and leaf diagram – you can see the smallest number of passengers was 3; however, on the bar chart you only know it is between 0 and 9.

b Both since the shape of the data is preserved in both.

Measures of central tendency: mode

1 The other three must be 12.2.

2 $1 < t \leq 2$

3 There are three equal 7 'leaves' on the 1 stem. So: 17

4 Max is correct, the modal number of pets is the group with the highest frequency, therefore 2 pets is the mode.

Measures of central tendency: median

1 Ordering the data gives; 2.9, 3.1, 4.3, 6.5, 8.7, 9.2

Median $= \frac{4.3 + 6.5}{2} = 5.4$

2 $29 + 28 + 30 + 3 + 10 = 100$

$\frac{(100 + 1)}{2} = 50.5$ – median term is between the 50th and 51st terms.

Both these lie in the $2 \leq b < 4$ class.

3 a Group A $= \frac{82 + 85}{2} = 83.5$

Group B $= \frac{75 + 79}{2} = 77$

b Group A has a higher median, so they did better on the test.

Measures of central tendency: mean

Stretch it! a mode **b** mean/median **c** mean/median

1 a Total frequency $= 12 + 3 + 5 = 20$

Mean $= \frac{(2 \times 12) + (6 \times 3) + (10 \times 5)}{20} = 4.6$

b You are using the midpoint of the groups as an estimate of the actual value for each group.

2 $\frac{(5 \times 9) + 6}{5 + 1} = 8.5$

3 No – they could be any pair of numbers which sum to 10.

Range

1 9.5 – 0.7 = 8.8

2 **a** Girls = 18 – 15 = 3

b Boys = 18 – 16 = 2

3 Range for Athlete A = 15.2 – 13.0 = 2.2

Range for Athlete B = 15.2 – 14.3 = 0.9

Athlete A has the greatest range.

4 45% – 10 = 35% or 45% + 30 = 75%

Comparing data using measures of central tendency and range

1 **a** **i** Mean $= \frac{(32 + 29 + 18 + 41 + 362 + 19)}{6} = \frac{501}{6}$

$= 83.5$ minutes

ii Ordered data: 18, 19, 29, 32, 41, 362

Median $= \frac{(29+32)}{2} = 30.5$ minutes

b The extreme value (362 mins) affects the mean but not the median.

2 All the data is used to find the mean.

3 Either as long as suitably justified:

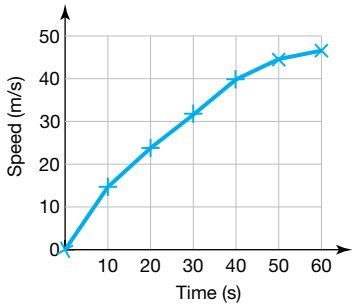
Car A – although the mean time is higher, it is more consistent in performance since the range is smaller.

Car B – the acceleration is quicker on average.

4 **a** and **b** The mode or median since the mean will not be a whole number and therefore not meaningful.

Time series graphs

1



2 **a** 67°C

b Approx. 27°C

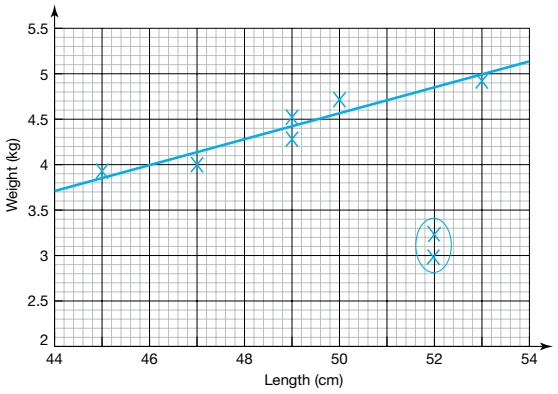
c No, since it is extrapolation (beyond the limits of the data).

3 **a** 17 000 **b** **i** April **ii** August

c The number of tourists peaks in April and again in December. The low seasons are February/March and July/August/September/October*.

Scatter graphs

1 **a** and **b**

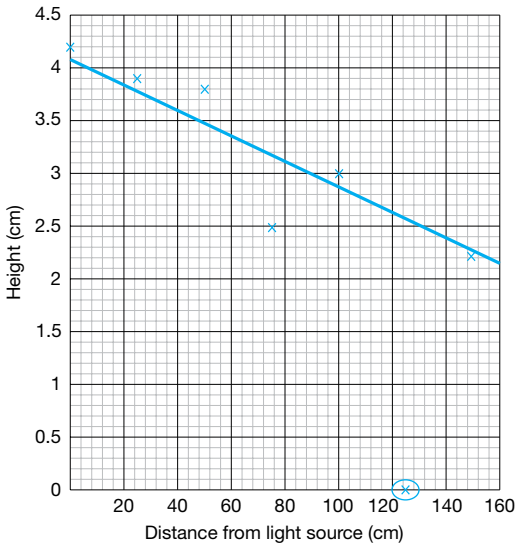


c Positive

d This will vary according to the line of best fit: approximately 4.7 kg. A range of 4.6kg to 4.8kg would be acceptable.

e This is beyond the limits of the data and therefore extrapolation.

2 **a**, **b** and **c**



c The seeds failed to germinate or the seedling died.

d The further the seedling is from the light source the shorter its height.

3 No, although the two things correlate one does not cause another. There may be many reasons why the crime rate is high in the area, perhaps there is poverty and inequality causing social tension.

Graphical misrepresentation

1 The organisation has only shown a small section of the data. This is not enough to understand the overall trend.

2 Chart C correctly shows the information. Chart A has an incorrect vertical axis, suggesting there are more women than there actually are. Chart B has unequal bar widths, also exaggerating the number of women.

Review it!

1 The sample is too small and he only asked his friends. His data is therefore not representative of the population of TV viewers.

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

2 a Margherita

b Total frequency = 11 + 2 + 6 + 1 = 20

$\frac{1}{20} = \frac{5}{100} = 5\%$

c 360° ÷ 20 = 18°

Pepperoni = 1 × 18° = 18°

(or 5% of 360° = 18°)

3 a $\frac{90}{360} = \frac{1}{4}$

b 45° = $\frac{1}{8}$ of 360°

Therefore $\frac{1}{8}$ of the pie chart represents 60 cars.

The whole pie chart = 8 × 60 = 480 cars

c $\left(\frac{105}{360}\right) \times 480 = 140$ cars

4 a The number of people doing their grocery shopping online is increasing.

b Any sensible answer, approximately 75%

c No – it is outside the limits of the data therefore extrapolation.

5 a Outside: **i** Mode = 21 and 31

ii Median = $\frac{28 + 29}{2} = 28.5$

iii Range = 41 − 20 = 21

Greenhouse: **i** Mode = 47

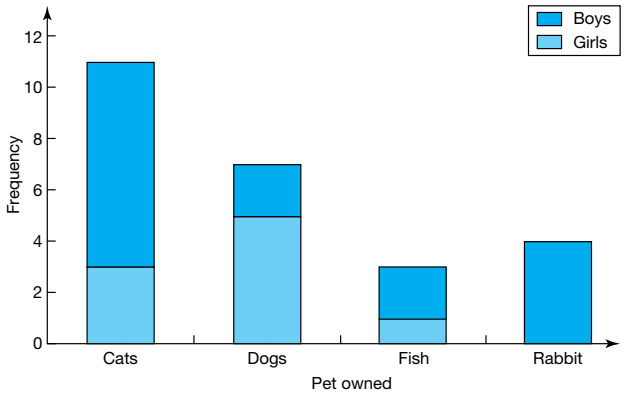
ii Median = $\frac{47 + 47}{2} = 47$

iii Range = 51 − 37 = 14

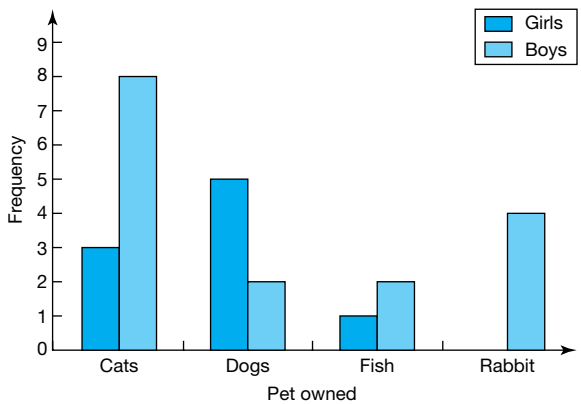
b The seedlings are taller in the greenhouse since both mode and median is larger, the range of data is smaller in the greenhouse so the height the seedlings reach is more consistent.

c Range = 51 − 20 = 31

6 a Comparative bar chart or compound bar chart:



Or:



b Total number of students = (3 + 5 + 1 + 0 + 8 + 2 + 2 + 4) = 25

Number of cats = 3 + 8 = 11

$\frac{11}{25}$

7 a Total frequency = 17 + 2 + 32 + 23 + 9 = 83

Median value = $\frac{(83 + 1)}{2}$ = 42nd term

42nd term is in group 40–59

Median class = 40–59

b The youngest person is between 0 and 19, the youngest may be any age in this range and the oldest is between 80 and 99 therefore any age in this range.

8 a 7

b Size 5

c Mean = $\frac{(3 \times 2) + (4 \times 1) + (5 \times 7) + (6 \times 5) + (7 \times 3)}{2 + 1 + 7 + 5 + 3} = 5.\dot{3}$

d Mode – the mean is not an actual shoe size.

9 a Time for 800 m (seconds)

11	2 2 5 8 9
12	0 1 9
13	1 2

Key: 11|2 = 112 seconds

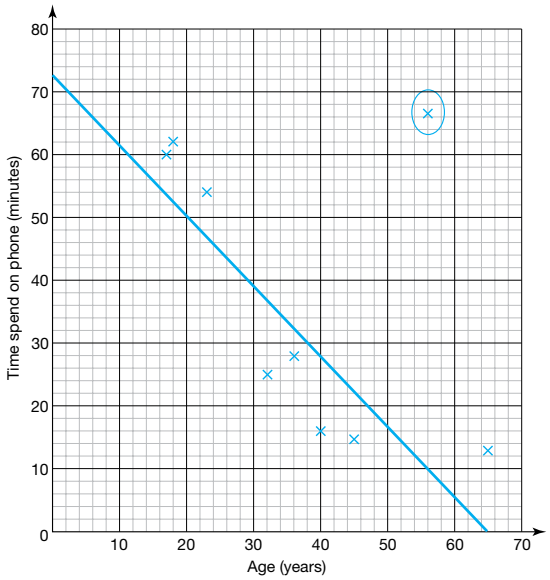
b $\frac{5}{10} = \frac{1}{2}$*

10 a $\frac{50}{150} = \frac{1}{3}$

b 60 − 40 = 20

c Biology

11 a and **c**



b Negative

d Approximately 40 minutes: it depends on line of best fit.

e This is outside the limits of the data and therefore extrapolation.

f As the age of the customer increases the time spent on the phone decreases.

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

12 a $\frac{(65 \times 3) + (75 \times 5) + (85 \times 2)}{3 + 5 + 2} = 74\,\text{kg}$

b The midpoint of the class is used as the age of each of the patients rather than the actual age.

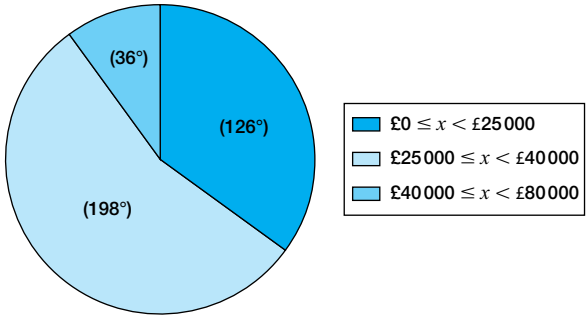
13 Male $< 50 = \left(\frac{12\,201}{36\,579}\right) \times 120 = 40$

Female $< 50 = \left(\frac{10\,678}{36\,579}\right) \times 120 = 35$

Male $\geq 50 = \left(\frac{5699}{36\,579}\right) \times 120 = 19$

Female $\geq 50 = \left(\frac{8001}{36\,579}\right) \times 120 = 26$

14 Annual income for surveyed population



15 Mean $= \frac{(10.3 \times 10) + 9.5}{11} = 10.2$ (1 d.p.)

16 Mean is 3.8 so the sum of the scores is $3.8 \times 5 = 19$

Mode is 3 so she must roll at least two 3s.

Range is 4.

If the range is 4 then the lowest and highest must be either 1 and 5 or 2 and 6.

The numbers are: 2, 3, 3, 5 and 6