Answers

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Number

Integers, decimals and symbols

1 $\frac{1}{0.01}$ 0.1 $(0.1)^2$ $\frac{1}{1000}$ $(-1)^3$
2 **a** 35 **b** 0.01285 **c** –270 **d** 40
3 **a** 4644 **b** 4644 **c** 86 **d** 540
4 **a** $12.56 \times 3.45 = 0.1256 \times 345$
b $(-8)^2 > -64$ **c** $6 - 12 = 8 - 14$
d $(-7) \times (0) < (-7) \times (-3)$

Addition, subtraction, multiplication and division

1 **a** 76.765 **b** 201.646 **c** 91.33 **d** 10.564
2 **a** 1176 **c** 44.62 **e** 27
b 2166 **d** 0.6572 **f** 63
3 **a** 1156 **b** 7.5 **c** 5.76

Using fractions

1 $\frac{2}{5} = \frac{16}{\mathbf{40}} = \frac{\mathbf{30}}{75} = \frac{50}{\mathbf{125}}$
2 **a** $5\frac{1}{3}$ **b** $9\frac{7}{13}$
3 **a** $7\frac{1}{12}$ **b** $7\frac{1}{2}$ **c** $2\frac{9}{20}$
4 $\frac{5}{56}$ 5 $\frac{1}{2}$ $\frac{7}{12}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{7}{8}$

Different types of number

1 **a** 7 **b** 49 **c** 2 **d** 6 **e** 6
2 **a** $3^2 \times 7 \times 11$ **b** 63 **c** 10 395
3 441 4 5 minutes

Listing strategies

1 210 seconds 3 1100 students
2 5 friends 4 15 pairs

The order of operations in calculations

1 **a** Ravi has worked out the expression from left to right, instead of using BIDMAS. He should have performed the division and multiplication before the addition.
b Correct answer: 40
2 **a** 122 **b** −3 **c** 40
3 **a** 6 **b** 14 **c** 8

Indices

1 **a** 10^6 **b** 10^8 **c** 10^6 **d** 10^3
2 **a** 1 **b** $\frac{1}{9}$ **c** 2 **d** 7
3 **a** $\frac{3}{2}$ **b** 16 **c** $\frac{1}{6}$ **d** 64
4 $x = 1.5$

Surds

1 **a** 5 **b** 30 **c** 18
2 $\frac{5\sqrt{3}}{4}$
3 $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 2\sqrt{3} + 2\sqrt{3} - 3 = 1$
4 $a = 30$
5 $-\sqrt{5} - 7$
6
$$\frac{1}{\sqrt{2}} + \frac{1}{4} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{1}{4}$$
$$= \frac{\sqrt{2}}{2} + \frac{1}{4}$$
$$= \frac{2\sqrt{2}}{4} + \frac{1}{4}$$
$$= \frac{1 + 2\sqrt{2}}{4}$$

7
$$\frac{2}{1 - \frac{1}{\sqrt{2}}} = \frac{2}{\frac{\sqrt{2}}{\sqrt{2}}\ \frac{1}{\sqrt{2}}}$$
$$= \frac{2}{\frac{\sqrt{2} - 1}{\sqrt{2}}}$$
$$= \frac{2\sqrt{2}}{\sqrt{2} - 1}$$
$$= \frac{2\sqrt{2}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$
$$= \frac{4 + 2\sqrt{2}}{2 - 1}$$
$$= 4 + 2\sqrt{2}$$

8
$$\frac{3}{\sqrt{3}} + \sqrt{75} + (\sqrt{2} \times \sqrt{6}) = \frac{3\sqrt{3}}{3} + \sqrt{3 \times 25} + \sqrt{12}$$
$$= \sqrt{3} + 5\sqrt{3} + \sqrt{3 \times 4}$$
$$= \sqrt{3} + 5\sqrt{3} + 2\sqrt{3}$$
$$= 8\sqrt{3}$$

Standard form

1 **a** 2.55×10^{-3} **b** 1.006×10^{10} **c** 8.9×10^{-8}
2 **a** 6×10^{14} **c** 2×10^2 **e** 9×10^{-3}
b 1.1×10^6 **d** 1×10^{-2}
3 2680 4 $a = 3.3$

Converting between fractions and decimals

1 **a** 0.55 **b** 0.375
2 **a** terminating **b** recurring **c** recurring
3 Let $x = 0.\dot{4}0\dot{2} = 0.402402402\ldots$
$1000x = 402.402402\ldots$
$1000x - x = 402.402402\ldots - 0.402402402\ldots$
$999x = 402$
$x = \frac{402}{999} = \frac{134}{333}$
Hence $0.\dot{4}0\dot{2} = \frac{134}{333}$
4 $\frac{323}{495}$

Converting between fractions and percentages

1 **a** $\frac{7}{20}$ **b** $\frac{7}{100}$ **c** $\frac{19}{25}$ **d** $\frac{1}{8}$
2 **a** 20% **b** 68% **c** 250% **d** 17.5%
3 53.33% (to 2 d.p.)
4 $\frac{66}{90} = \frac{66}{90} = 73.3\%$ (to 1 d.p.)
Jake did better in chemistry.

Fractions and percentages as operators

1 £34.79 4 **a** £14 400 5 $\frac{14}{33}$
2 48 **b** £320
3 7040

Standard measurement units

1 175 000 cm 2 17
3 1286 (to nearest whole number)
4 **a** 1.99×10^{-23} g (to 3 s.f.) **b** 1.99×10^{-26} kg (to 3 s.f.)
5 7.20×10^{-26} g (to 3 s.f.)

Rounding numbers

1 **a** 35 **c** 0 **e** 2
b 101 **d** 0
2 **a** 34.88 **b** 34.877
3 **a** 12 800 **b** 0.011 **c** 7×10^{-5}
4 **a** −0.00993 **b** 34.4 **c** 12 300

Estimation

1 200 3 0.16 5 10.6
2 **a** 236.2298627 4 5 6 4
b 240
7 **a** 5×10^{-28} kg
b This will be an underestimate, as the mass of one electron has been rounded down.

Upper and lower bounds

1 $2.335 \leq l < 2.345$ kg
2 **a** **i** 2.472 **ii** 2.451 **b** 2.5 (to 2 s.f.)
3 34

Algebra

Simple algebraic techniques

1 **a** formula **c** expression **e** formula
  b identity **d** identity

2 $x + 6x^2$

3 $y^3 - y = (1)^3 - 1 = 0$ so $y = 1$ is correct.
  $y^3 - y = (-1)^3 - (-1) = -1 + 1 = 0$ so $y = -1$ is correct.

4 **a** $10x$ **b** $4x^2 - 6x$ **c** $18x^2$

5 **a** 2 **b** $\frac{7}{8}$ **c** $-\frac{3}{2}$

Removing brackets

1 **a** $24x - 56$ **b** $-6x + 12$

2 **a** $3x + 9$ **c** $10a^2b - 5ab^2$
  b $8xy + 6x - 2y$ **d** $2x^3y^3 + 3x^2y^4$

3 **a** $m^2 + 5m - 24$ **c** $9x^2 - 6x + 1$
  b $8x^2 + 26x - 7$ **d** $6x^2 + xy - y^2$

4 **a** $x^2 + 7x + 10$ **c** $x^2 - 6x - 7$
  b $x^2 - 16$ **d** $15x^2 + 14x + 3$

5 **a** $x^3 + 6x^2 + 5x - 12$ **b** $18x^3 - 63x^2 + 37x + 20$

Factorising

1 **a** $5x(5x - y)$ **b** $2\pi(2r^2 + 3x)$ **c** $6ab^2(a^2 + 2)$

2 **a** $(3x + 1)(3x - 1)$ **b** $4(2x + 1)(2x - 1)$

3 **a** $(a + 4)(a + 8)$ **b** $(p - 6)(p - 4)$

4 **a** $a(a + 12)$ **b** $(b + 3)(b - 3)$ **c** $(x - 5)(x - 6)$

5 **a** $(3x + 8)(x + 4)$ **b** $(3x + 13)(x - 1)$ **c** $(2x - 5)(x + 2)$

6 $\frac{2}{(x - 3)}$ 7 $\frac{2x - 1}{4x + 1}$

Changing the subject of a formula

1 $T = \frac{PV}{nR}$ 3 $a = \frac{v - u}{t}$ 5 $v = \sqrt{\frac{2E}{m}}$

2 $y = \frac{1 - 4x}{2}$ 4 $x = 5(y + m)$

6 **a** $r = \sqrt{\frac{3V}{\pi h}}$ **b** 3.45 cm (to 2 d.p.)

7 **a** $x = \frac{y + 9}{3}$ **b** 4

8 $x = \frac{3y - 2}{a + 1}$

9 **a** $c = \frac{b}{a}$ **b** upper bound for $c = 1.18$ (to 3 s.f.)
  lower bound for $c = 1.11$ (to 3 s.f.)

Solving linear equations

1 **a** $x = 7$ **d** $x = 32$ **g** $x = -2$
  b $x = 5$ **e** $x = 25$ **h** $x = 84$
  c $x = 4$ **f** $x = -9$

2 $x = \frac{2}{3}$

3 **a** $x = \frac{1}{2}$ **b** $x = -\frac{8}{5}$

Solving quadratic equations using factorisation

1 **a** $(x - 3)(x - 4)$ **b** $x = 3$ or $x = 4$

2 **a** $(2x - 1)(x + 3)$ **b** $x = \frac{1}{2}$ or $x = -3$

3 $x = -2$ or $x = 6$

4 **a** $x(x - 8) - 7 = x(5 - x)$ **b** $x = -\frac{1}{2}$ or $x = 7$
  $x^2 - 8x - 7 = 5x - x^2$
  $2x^2 - 13x - 7 = 0$

5 $x = 2$ cm

Solving quadratic equations using the formula

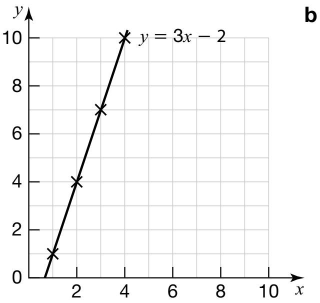
1 **a** $\frac{3}{x + 7} = \frac{2 - x}{x + 1}$ **b** $x = 1.20$ or -9.20 (to 2 d.p.)
  $3(x + 1) = (2 - x)(x + 7)$
  $3x + 3 = 2x + 14 - x^2 - 7x$
  $3x + 3 = -x^2 - 5x + 14$
  $x^2 + 8x - 11 = 0$

2 $x = 2.78$ cm (to 2 d.p.) 3 $x = 3.30$ or -0.30 (to 2 d.p.)

Solving simultaneous equations

1 $x = 2$ and $y = 3$

2 **a**



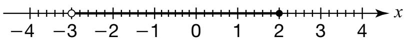
b $x = 3, y = 7$

3 $x = 0, y = -3$ or $x = 3, y = 0$

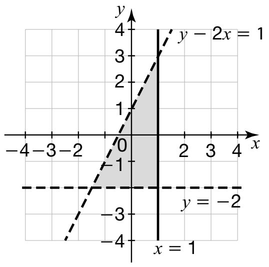
Solving inequalities

1 **a** $x \geq -9$ **b** $x < -12$

2



3 **a**



b (1, 2), (1, 1), (0, 0), (1, 0), (0, −1), (1, −1)

4 $-3 \leq x \leq 1$ 5 $x < -3$ and $x > 5$

Problem solving using algebra

1 42 m^2

2 cost of adult ticket = £7.50
  cost of child ticket = £4

3 **a** 16 years **b** 9 years

Use of functions

1 **a** 19 **b** $x = -1$

2 **a** $(x - 6)^2$ **b** $x^2 - 6$

3 **a** ±3 **b** $2x + 5$

4 $f^{-1}(x) = \sqrt{\frac{x - 3}{5}}$

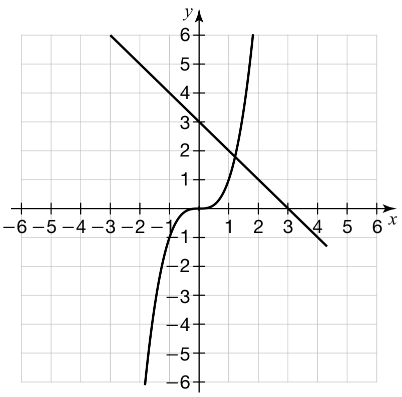
Iterative methods

1 Let $f(x) = 2x^3 - 2x + 1$
  $f(-1) = 2(-1)^3 - 2(-1) + 1 = 1$
  $f(-1.5) = 2(-1.5)^3 - 2(-1.5) + 1 = -2.75$
  There is a sign change of $f(x)$, so there is a solution between $x = -1$ and $x = -1.5$.

2 $x_1 = 0.1121111111$
  $x_2 = 0.1125202246$
  $x_3 = 0.1125357073$

3 **a** $x_4 = 1.5213705 \approx 1.521$ (to 3 d.p.)
  b Checking value of $x^3 - x - 2$ for $x = 1.5205, 1.5215$:
  When $x = 1.5205$ $f(1.5205) = -0.0052$
  $x = 1.5215$ $f(1.5215) = 0.0007$
  Since there is a change of sign, the root is 1.521 correct to 3 decimal places.

4 **a** **i**



ii There is a root of $x^3 + x - 3 = 0$ where the graphs of $y = x^3$ and $y = 3 - x$ intersect. The graphs intersect once so there is one real root of the equation $x^3 + x - 3 = 0$.

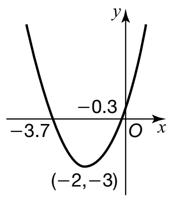
b $x_1 = 1.216440399$
$x_2 = 1.212725591$
$x_3 = 1.213566964$
$x_4 = 1.213376503$
$x_5 = 1.213419623$
$x_6 = 1.213409861 = 1.2134$ (to 4 d.p.)

Equation of a straight line

1 A
2 **a** $-\frac{4}{3}$ **b** $y = -\frac{1}{2}x + \frac{7}{2}$ **c** $y = 2x + 1$
3 (3.8, 11.4) (to 1 d.p.)

Quadratic graphs

1 **a** $x = -0.3$ or -3.7 (to 1 d.p.)
b



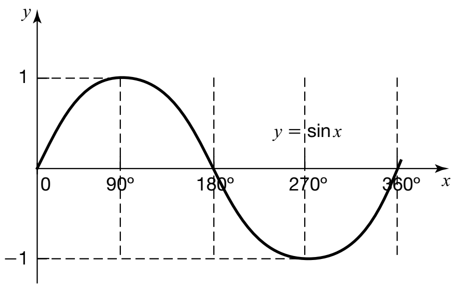
2 $a = 5$, $b = -2$ and $c = -10$
3 $a = 2$, $b = 3$ and $c = -15$

Recognising and sketching graphs of functions

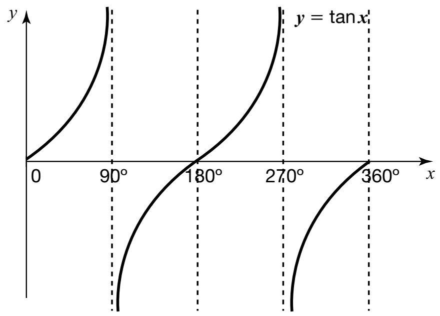
1

Equation	Graph
$y = x^2$	B
$y = 2^x$	D
$y = \sin x°$	E
$y = x^3$	C
$y = x^2 - 6x + 8$	A
$y = \cos x°$	F

2 **a**



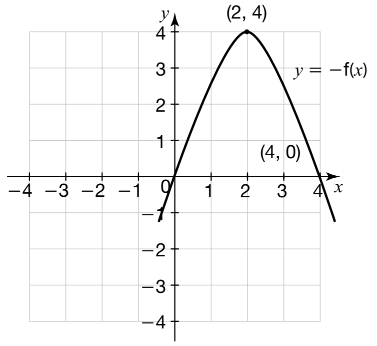
b



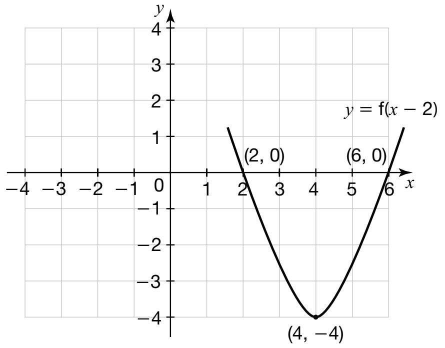
3 $\theta = 70.5°$ or $289.5°$ (to 1 d.p.)

Translations and reflections of functions

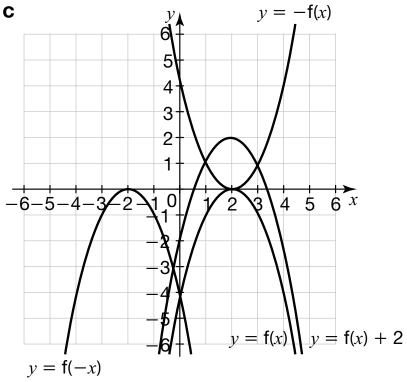
1 **a**



b



2 **a, b, c**



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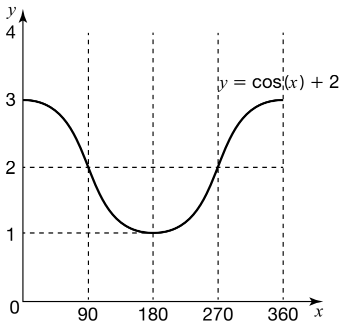


Equation of a circle and tangent to a circle

1 **a** 5
b 7
c 2

2 radius of the circle = $\sqrt{21} = 4.58$

distance of the point (4, 3) from the centre of the circle (0, 0) = $\sqrt{16 + 9} = \sqrt{25} = 5$

This distance is greater than the radius of the circle, so the point lies outside the circle.

3 **a** $\sqrt{74}$
b $x^2 + y^2 = 74$ **c** $y = -\frac{5}{7}x + \frac{74}{7}$

Real-life graphs

1 **a** 1 m/s^2
  b 225 m

2 **a** The graph is a straight line starting at the origin, so this represents constant acceleration from rest of $\frac{15}{6}$ = 2.5 m/s^2.
  b The gradient decreases to zero, so the acceleration decreases to zero.
  c 118 m (to nearest integer); 117 m is also acceptable
  d It will be a slight underestimate, as the curve is always above the straight lines forming the tops of the trapeziums.

Generating sequences

1 **a** **i** $\frac{1}{2}$ **ii** 243 **iii** 21
  b 14, 1

2 −3, −11

3 **a** 25, 36
  b 15, 21
  c 8, 13

The *n*th term

1 **a** nth term = $4n - 2$
  b nth term = $4n - 2 = 2(2n - 1)$
  2 is a factor, so the nth term is divisible by 2 and therefore is even.
  c 236 is not a term in the sequence.

2 **a** 5
  b −391
  c n^2 is always positive, so the largest value $9 - n^2$ can take is 8 when $n = 1$. All values of n above 1 will make $9 - n^2$ smaller than 8. So 10 cannot be a term.

3 nth term = $n^2 - 3n + 3$

Arguments and proofs

1 The only prime number that is not odd is 2, which is the only even prime number.
Hence, statement is false because 2 is a prime number that is not odd.

2 **a** true: $n = 1$ is the smallest positive integer and this would give the smallest value of $2n + 1$ which is 3.
  b true: 3 is a factor of $3(n + 1)$ so $3(n + 1)$ must be a multiple of 3.
  c false: $2n$ is always even and subtracting 3 will give an odd number.

3 Let first number = x so next number = $x + 1$
Sum of consecutive integers = $x + x + 1 = 2x + 1$
Regardless of whether x is odd or even, $2x$ will always be even as it is divisible by 2.
Hence $2x + 1$ will always be odd.

4 $(2x - 1)^2 - (x - 2)^2$
$= 4x^2 - 4x + 1 - (x^2 - 4x + 4)$
$= 4x^2 - 4x + 1 - x^2 + 4x - 4$
$= 3x^2 - 3$
$= 3(x^2 - 1)$
The 3 outside the brackets shows that the result is a multiple of 3 for all integer values of x.

5 Let two consecutive odd numbers be $2n - 1$ and $2n + 1$.
$(2n + 1)^2 - (2n - 1)^2$
$= (4n^2 + 4n + 1) - (4n^2 - 4n + 1)$
$= 8n$
Since 8 is a factor of $8n$, the difference between the squares of two consecutive odd numbers is always a multiple of 8.
(If you used $2n + 1$ and $2n + 3$ for the two consecutive odd numbers, difference of squares = $8n + 8 = 8(n + 1)$.)

Ratio, proportion and rates of change

Introduction to ratios

1 30
2 £7500, £8500, £9000
3 210 acres
4 $x = \frac{5}{7}$
5 144

Scale diagrams and maps

1 5 km
2 **a** 0.92 km **b** 0.12 km
3 1 : 800
4 1 : 200000

Percentage problems

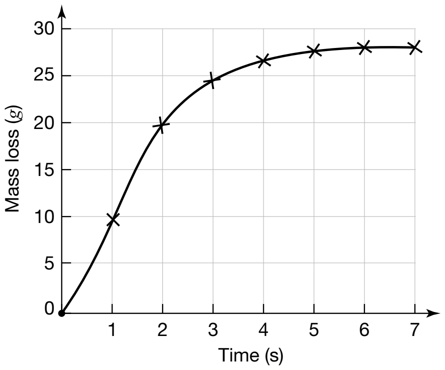
1 10%
2 83.3%
3 £18 000
4 £16 250
5 £896

Direct and inverse proportion

1 **a** $P = kT$ **b** 74 074 Pascals (to nearest whole number)
2 £853 (to nearest whole number)
3 **a** $c = \frac{36}{h}$ **b** 2.4
4 **a** €402.50 **b** £72.07 (to nearest penny) **c** £2.50

Graphs of direct and inverse proportion and rates of change

1 B
2 B
3 **a**



  b **i** 9.8 g/minute **ii** 0.16 g/second (to 2 d.p.)

Growth and decay

1 **a** 178 652 **b** 5 years
2 £12 594
3 0.1 (to 1 s.f.)

Ratios of lengths, areas and volumes

1 **a** 3.375 or $\frac{27}{8}$
  b 22.5 cm^2
  c 133 cm^3 (to nearest whole number)
2 h = 15 cm (to nearest cm)
3 **a** **i** 9 cm **ii** 4.5 cm
  b 4 : 1

Gradient of a curve and rate of change

1 **a** $\frac{2}{3}$ m/s^2
  b 0.26 m/s^2
  c 0.37 m/s^2
  d 34 s

Converting units of areas and volumes, and compound units

1 500 N/m^2
2 25 000 N/m^2
3 107 g (to nearest g)
4 He has worked out the area in m^2 by dividing the area in cm^2 by 100, which is incorrect.
There are 100 × 100 = 10 000 cm^2 in 1 m^2, so the area should have been divided by 10 000.
Correct answer:
area in m^2 $= \frac{9018}{10\,000}$
$= 0.9018$
$= 0.90$ m^2 (to 2 d.p.)
5 72 km/h

Geometry and measures

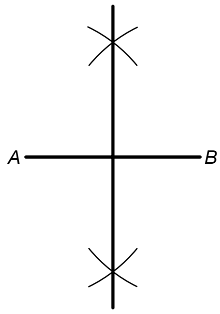
2D shapes

1 **a** true **c** true **e** true
b false **d** true **f** false (this would be true only for a regular pentagon)

2 **a** rhombus **c** equilateral triangle
b parallelogram **d** kite

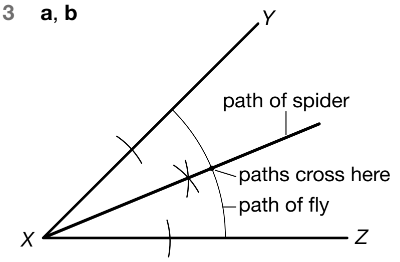
Constructions and loci

1



2

3 **a**, **b**



Properties of angles

1 **a** angle ACB = angle BAC = 30° (base angles of isosceles triangle ABC, since AB and BC are equal sides of a rhombus)

b angle AOB = 90° (diagonals of a rhombus intersect at right angles)

c angle $ABO = 180 - (90 + 30) = 60°$ (angle sum of a triangle)

angle BDC = angle ABO = 60° (alternate angles between parallel lines AB and DC)

2 angle $BAC = \frac{(180 - 36)}{2} = 72°$ (angle sum of a triangle and base angles of an isosceles triangle)

angle $BDC = 180 - 90 = 90°$ (angle sum on a straight line)

angle $ABD = 180 - (90 + 72) = 18°$ (angle sum of a triangle)

3 **a** $x = 30°$

b If lines AB and CD are parallel, the angles $4x$ and $3x + 30$ would be corresponding angles, and so equal.

$4x = 4 \times 30 = 120°$

$3x + 30 = 3 \times 30 + 30 = 120°$

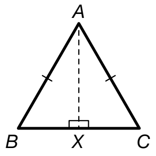
These two angles are equal so lines AB and CD are parallel.

4 $x = 90 + 72 = 162°$

Congruent triangles

1 BD common to triangles ABD and CDB
angle ADB = angle CBD (alternate angles)
angle ABD = angle CDB (alternate angles)
Therefore triangles ABD and CDB are congruent (ASA).
Hence angle BAD = angle BCD

2 Draw the triangle and the perpendicular from A to BC.



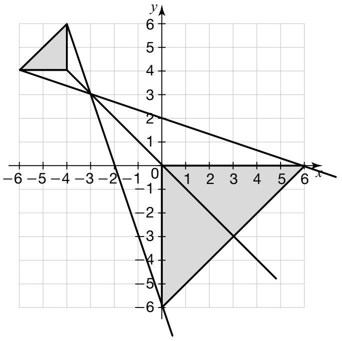
$AX = AX$ (common)
$AB = AC$ (triangle ABC is isosceles)
angle $AXB = AXC = 90°$ (given)
Therefore triangles ABX and ACX are congruent (RHS).
Hence $BX = XC$, so X bisects BC.

3 $OQC = 90°$ (corresponding angles), so $PB = OQ$ (perpendicular distance between 2 parallel lines)
$AP = PB$ (given), so $AP = OQ$
$PO = QC$ (Q is the midpoint of BC)
angle ABC = angle APO = angle OQC
= 90° (OQ is parallel to AB and OP parallel to BC)
Therefore triangles AOP and OCQ are congruent (SAS).

Transformations

1 translation of $\begin{pmatrix}-7\\-5\end{pmatrix}$

2



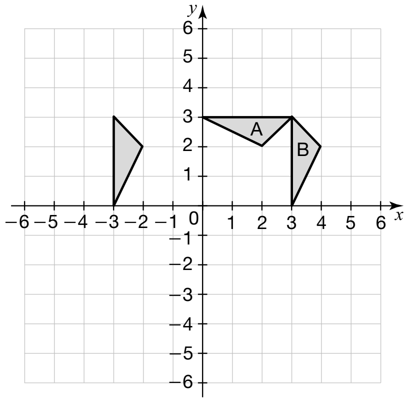
3 **a** translation of $\begin{pmatrix}-7\\0\end{pmatrix}$ **b** reflection in the line $y = 3$

c rotation of 90° clockwise about (0, 1)

Invariance and combined transformations

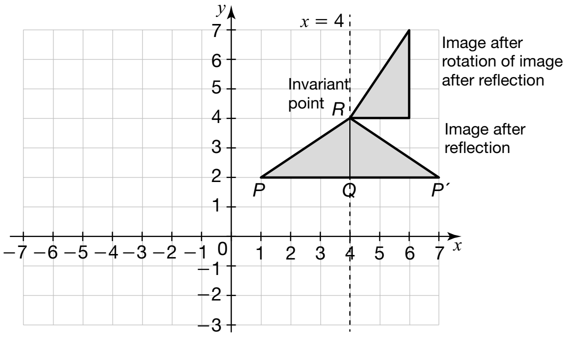
1 **a** 1

b **i** invariant point (3, 3)



ii rotation 90° anticlockwise about the point (3, 3)

2 **a** The shaded triangle is the image after the two transformations.



b invariant point is R (4, 4)

3D shapes

1 **a** G **c** A, H **e** C
b B, D **d** B **f** A, H

Parts of a circle

1 **a** radius **c** chord
  b diameter **d** arc

2 **a** minor sector **c** major sector
  b major segment **d** minor segment

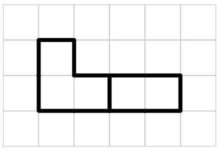
Circle theorems

1 angle $OTB = 90°$ (angle between tangent and radius)
angle $BOT = 180 - (90 + 28) = 62°$
(angle sum in a triangle)
angle $AOT = 180 - 62 = 118°$
(angle sum on a straight line)
$AO = OT$ (radii), so triangle AOT is isosceles
angle $OAT = \frac{(180 - 118)}{2} = 31°$ (angle sum in a triangle)

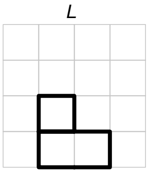
2 **a** angle $ACB = 30°$ (angle at centre twice angle at circumference)
  b angle BAC = angle $CBX = 70°$ (alternate segment theorem)
  c $OA = OB$ (radii), so triangle AOB is isosceles
  angle $AOB = 60°$, so triangle AOB is equilateral
  angle $OAB = 60°$ (angle of equilateral triangle)
  angle $CAO = 70 - 60 = 10°$

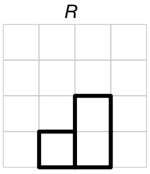
Projections

1

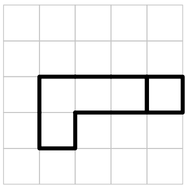


2 **a**

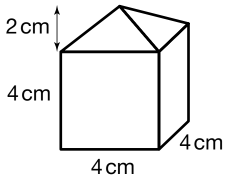




b



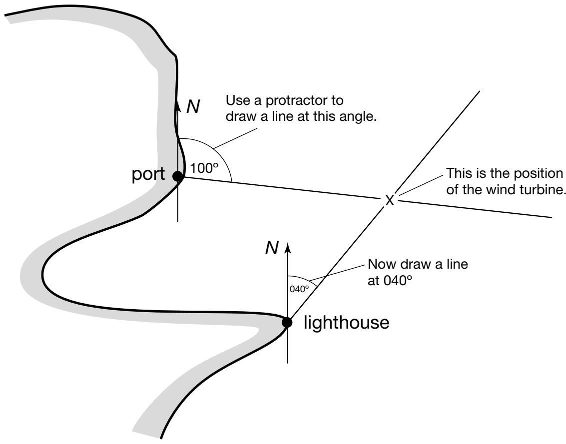
3



Bearings

1 230°

2



Pythagoras' theorem

1 76 m (to nearest m)
2 9.8 cm (to 1 d.p.)
3 **a** 9.1 cm (to 2 d.p.)
  b 48.76 cm^2 (to 2 d.p.)

Area of 2D shapes

1 **a** **i** 5.66cm (to 2 d.p.)
  ii 19.80 cm^2 (to 2 d.p.)
  b 50.65 cm^2
2 **a** 9 cm^2 **b** 6 cm^2
3 **a** $27\pi \text{cm}^2$ **b** $18\pi + 6$ cm

Volume and surface area of 3D shapes

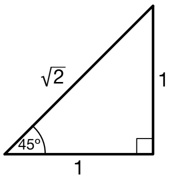
1 **a** 3.975 m^2 **b** 6.36 m^3 (to 2 d.p.)
2 5 glasses
3 **a** 7.4 cm (to 1 d.p.)
  b 3.8 cm (to 1 d.p.)
4 0.64 cm

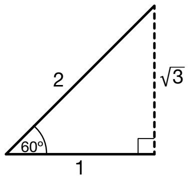
Trigonometric ratios

1 **a** 6.0 cm (to 1 d.p.)
  b 36.9° (to 1 d.p.)
2 44.4° (to 1 d.p.)
3 9.4 cm (to 1 d.p.)
4 21.8° (to 1 d.p.)

Exact values of sin, cos and tan

1





$\tan 45° = \frac{1}{1} = 1 \qquad \cos 60° = \frac{1}{2}$

Hence, $\tan 45° + \cos 60° = 1 + \frac{1}{2} = \frac{3}{2}$

2 **a** **i** $\sin 45° = \frac{1}{\sqrt{2}}$ **ii** $\cos 45° = \frac{1}{\sqrt{2}}$
  b $\frac{\sin 45°}{\cos 45°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$
  $\tan 45° = 1$
  Hence $\frac{\sin 45°}{\cos 45°} = \tan 45°$

3 **a** $\sqrt{5}$
  b **i** $\sin x = \frac{1}{\sqrt{5}}$ **ii** $\cos x = \frac{2}{\sqrt{5}}$
  c $(\sin x)^2 = \frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{1}{5}$
  $(\cos x)^2 = \frac{2}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{4}{5}$
  $(\sin x)^2 + (\cos x)^2 = \frac{1}{5} + \frac{4}{5}$
  $= 1$

4 $\tan 30° + \tan 60° + \cos 30°$

$$= \frac{1}{\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{3}}{3} + \sqrt{3} + \frac{\sqrt{3}}{2}$$
$$= \frac{2\sqrt{3} + 6\sqrt{3} + 3\sqrt{3}}{6}$$
$$= \frac{11\sqrt{3}}{6}$$

Sectors of circles

1 34° (to nearest degree)
2 364.4 cm^2
3 **a** 1.67 cm (to 3 s.f.)
  b 1 : 1.04

4 **a** length of arc $AB = \frac{\theta}{360} \times 2\pi r$

$$5.4 = \frac{\theta}{360} \times 2\pi \times 6$$

$$\theta = 51.5662$$

Area of sector $AOB = \frac{\theta}{360} \times 2\pi r^2$

$$= \frac{51.5662}{360} \times \pi \times 6^2$$

$$= 16.2 \text{ cm}^2$$

Note that both a and b are equal to the radius r of the circle.

b area of triangle AOB

$$= \frac{1}{2} a\, b \sin c$$

$$= \frac{1}{2} \times 6 \times 6 \sin 51.5662$$

$$= 14.09988 \text{ cm}^2$$

area of shaded segment

= area of sector − area of triangle

$= 16.2 - 14.1$

$= 2.1\text{cm}^2$ (correct to 1 decimal place)

Sine and cosine rules

1 **a** $225\,\text{cm}^2$

b $\frac{4}{5}$

c 18.0 cm (to 3 s.f.)

2 **a** 18.6 cm (to 3 s.f.)

b $92.4\,\text{cm}^2$ (to 3 s.f.)

3 $\frac{2 + 6\sqrt{2}}{17}$

Vectors

1 **a** $\binom{2}{1}$ **b** $\binom{-22}{9}$

2 **a** $\mathbf{b} - \mathbf{a}$ **b** $\frac{3}{5}(\mathbf{b} - \mathbf{a})$

c $\overrightarrow{OQ} = \frac{2}{5}\overrightarrow{OA} = \frac{2}{5}\mathbf{a}$

$$\overrightarrow{QP} = \overrightarrow{QA} + \overrightarrow{AP}$$

$$= \frac{3}{5}\mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$$

$$= \frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{3}{5}\mathbf{a}$$

$$= \frac{3}{5}\mathbf{b}$$

As $\overrightarrow{QP} = \frac{3}{5}\mathbf{b}$ and $\overrightarrow{OB} = \mathbf{b}$ they both have the same vector part and so are parallel.

3 **a** $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$$= -3\mathbf{b} + \mathbf{a}$$

$$= \mathbf{a} - 3\mathbf{b}$$

b $\overrightarrow{PB} = \frac{1}{3}\overrightarrow{AB} = \mathbf{b}$

$$\overrightarrow{PM} = \overrightarrow{PB} + \overrightarrow{BM}$$

$$= \overrightarrow{PB} + \frac{1}{2}\overrightarrow{BC}$$

$$= \mathbf{b} + \frac{1}{2}(\mathbf{a} - 3\mathbf{b})$$

$$= \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$$

$$= \frac{1}{2}(a - b)$$

$$\overrightarrow{MD} = \overrightarrow{MC} + \overrightarrow{CD}$$

$$= \frac{1}{2}\overrightarrow{BC} + \overrightarrow{CD}$$

$$= \frac{1}{2}(\mathbf{a} - 3\mathbf{b}) + \mathbf{a}$$

$$= \frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}$$

$$= \frac{3}{2}(\mathbf{a} - \mathbf{b})$$

Both $\overrightarrow{PM}$ and $\overrightarrow{MD}$ have the same vector part ($\mathbf{a} - \mathbf{b}$) so they are parallel. Since they both pass through M, they are parts of the same line, so PMD is a straight line.

Probability

The basics of probability

1 **a**

Dice 1

Dice 2

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b $\frac{1}{36}$ **c** $\frac{5}{12}$ **d** 7

2 **a** 21 chocolates **b** $\frac{3}{7}$

3

Bethany

Amy

	1	**2**	**3**	**4**	**5**	**6**
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

a $\frac{1}{6}$ **b** $\frac{5}{12}$

Probability experiments

1 **a** He is wrong because 100 spins is a very small number of trials. To approach the theoretical probability you would have to spin many more times. Only when the number of spins is extremely large will the frequencies start to become similar.

b $\frac{11}{50}$ **c** 95

2 **a** $x = 0.08$ **b** 0.24 **c** 16

The AND and OR rules

1 **a** $\frac{9}{169}$ **b** $\frac{3}{169}$ **c** $\frac{1}{52} \times \frac{1}{52} = \frac{1}{2704}$

2 **a** When an event has no effect on another event, they are said to be independent events. Here the colour of the first marble has no effect on the colour of the second marble.

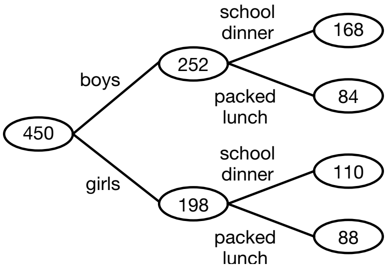
b $\frac{9}{100}$ **c** $\frac{3}{10}$

3 **a** $\frac{9}{140}$ **b** $\frac{6}{35}$

Tree diagrams

1 **a** $\frac{2}{15}$ **b** $\frac{8}{15}$

2 **a**



b $\frac{11}{45}$ **c** $\frac{139}{225}$

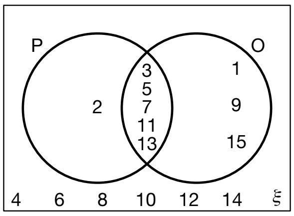
3 **a** 0.179 (to 3 d.p.) **b** 0.238 (to 3 d.p.) **c** 0.131 (to 3 d.p.)

Venn diagrams and probability

1 **a** 9, 8 **c** 1, 3, 4, 10, 12, 15

b 1, 2, 3, 5, 7, 8, 9, 12, 15 **d** 4, 10

2 **a**



b $\frac{1}{3}$

3 **a** $\frac{17}{20}$ **b** $\frac{18}{73}$

Statistics

Sampling

1 **a** Ling's, as he has a larger sample so it is more likely to represent the whole population (i.e. students at the school).

b 22

2 The sample should be taken randomly, with each member of the population having an equal chance of being chosen.

The sample size should be large enough to represent the population, since the larger the sample size, the more accurate the results.

3 **a** 173

b 25

Two-way tables, pie charts and stem-and-leaf diagrams

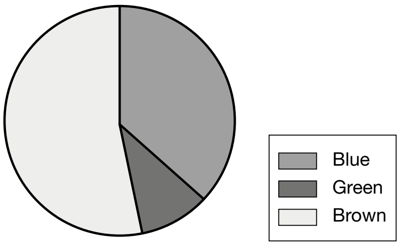
1 **a**

	Coronation Street	EastEnders	Emmerdale	Total
Boys	12	31	20	63
Girls	18	12	7	37
Total	30	43	27	100

b $\frac{27}{100}$

c $\frac{12}{37}$

2



3 **a** Number of students late

0	3 4 4 5 8 8 9
1	0 0 1 1 2 2 2 2 6 7 8
2	0 2 6

Key: 1|2 means 12 students late

b 11 students

c 23 students

Line graphs for time series data

1 **a** January to March: $\frac{12 + 10 + 20}{3} = 14$

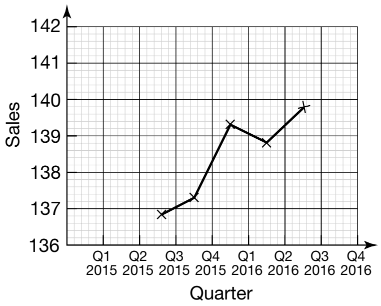
February to April: $\frac{10 + 20 + 54}{3} = 28$

March to May: $\frac{20 + 54 + 87}{3} = 53.7$

April to June: $\frac{54 + 87 + 130}{3} = 90.3$

b increasing sales

2 **a**



b The general trend is that the sales are increasing.

Averages and spread

1 **a** 32

b 2.7 (to 1 d.p.)

c 3

2 66.5 (to 1 d.p.)

3 **a**

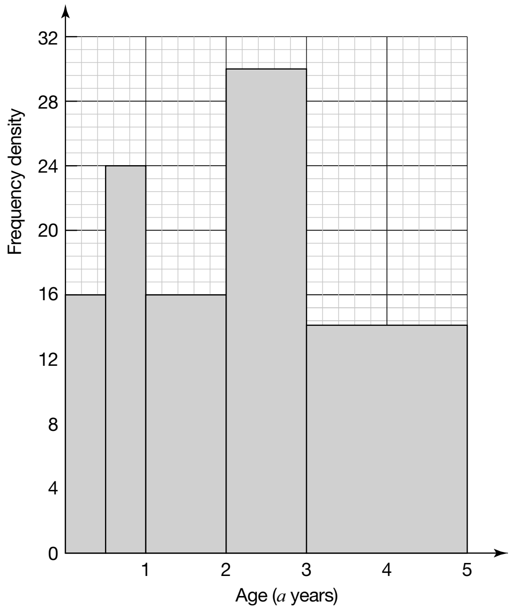
Age (t years)	Frequency	Mid-interval value	Frequency × mid-interval value
$0 < t \leq 4$	8	2	16
$4 < t \leq 8$	10	6	60
$8 < t \leq 12$	16	10	160
$12 < t \leq 14$	1	13	13

b 35

c 7.1 years (to 2 s.f.)

Histograms

1



2

Wingspan (w cm)	Frequency
$0 < w \leq 5$	2
$5 < w \leq 10$	6
$10 < w \leq 15$	24
$15 < w \leq 25$	30
$25 < w \leq 40$	9

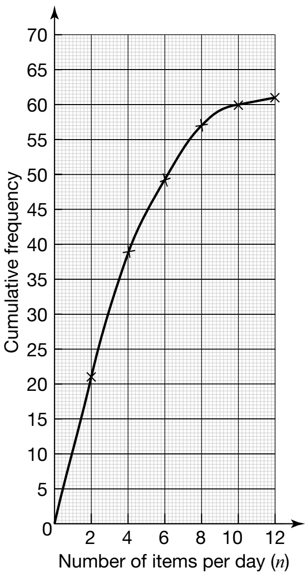
Cumulative frequency graphs

1 **a** $0 < n \leq 2$

b

Number of items of junk mail per day (n)	Frequency	Cumulative frequency
$0 < n \leq 2$	21	21
$2 < n \leq 4$	18	39
$4 < n \leq 6$	10	49
$6 < n \leq 8$	8	57
$8 < n \leq 10$	3	60
$10 < n \leq 12$	1	61

c



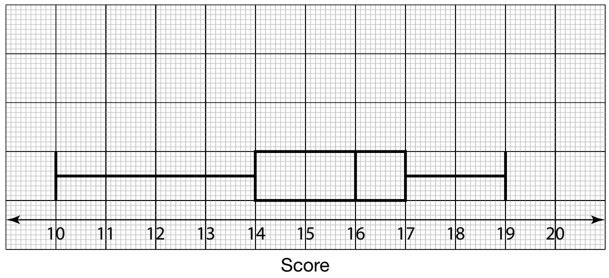
d median = value at frequency of 30.5 = 3

2 **a** **i** 15.8 kg **ii** 6.5 kg

b 32 penguins

Comparing sets of data

1 **a**



b Sasha's median score is higher (17 compared to 16). The IQR for Sasha is 2 compared to Chloe's 3. The range for Sasha is 5 compared to Chloe's 9. Both these are measures of spread, which means that Sasha's scores are less spread out (i.e. more consistent).

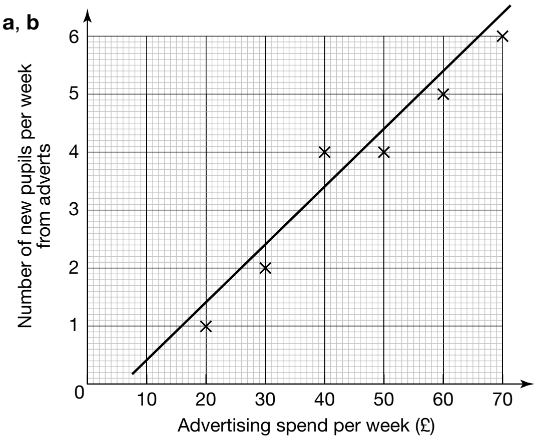
2 **a** **i** 138 minutes **ii** 100 minutes **iii** 22 minutes

b On average the men were faster as the median is lower.

The variation in times was greater for the men as their range was greater, although the spread of the middle half of the data (the interquartile range) was slightly greater for the women.

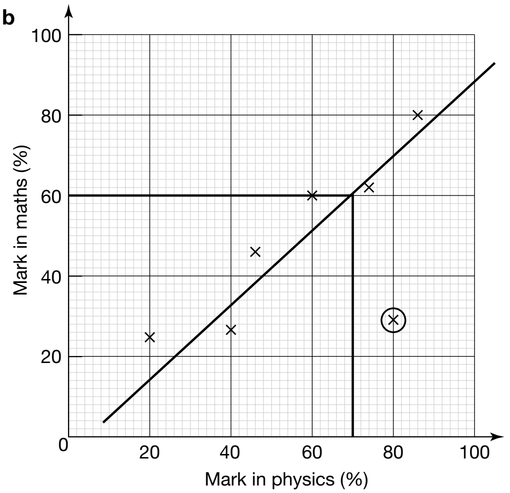
Scatter graphs

1 **a**, **b**



c positive correlation

2 **a**, **b**



c 63%

Practice papers

Non-calculator

1 29.382

2 **a** $3\frac{1}{3}$

b $2\frac{11}{40}$

3 **a** 1.4094

b 4.86

4 $a^2 + 12a + 27$

5 $4(y + 4)(y - 4)$

6 15 : 3 : 1

7 **a** $8x^6y^3$

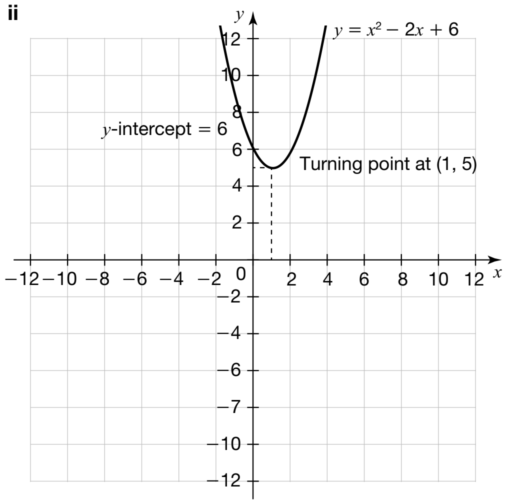
b $6x$

c $\frac{5}{b}$

8 **a** $a = 1$, $b = 5$

b **i** (1, 5)

ii



iii The curve does not intersect the x-axis (i.e. where $y = 0$), so the equation has no roots.

9 **a** 10

b $\frac{16}{25}$

10 $y = \frac{3 - 2c}{2a + 1}$

11 364 cm³ (to nearest integer)

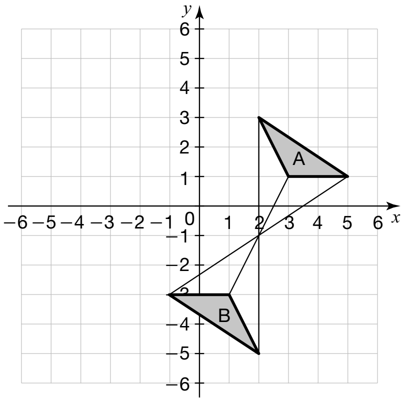
12
$$\frac{\sqrt{3} - 2}{\sqrt{3} + 1} = \frac{\sqrt{3} - 2}{\sqrt{3} + 1} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)}$$
$$= \frac{3 - 3\sqrt{3} + 2}{3 - 1}$$
$$= \frac{5 - 3\sqrt{3}}{2}$$

13 Let $x = 0.1\dot{3}\dot{6} = 0.136363636\ldots$

$$10x = 1.36363636\ldots$$
$$1000x = 136.363636$$
$$1000x - 10x = 136.363636\ldots - 1.36363636\ldots$$
$$990x = 135$$
$$x = \frac{135}{990} = \frac{3}{22}$$

14 **a** (0, −6) **b** (0, 6)

15 **a**



b A rotation of 180° (clockwise or anticlockwise) about the point (2, –1).

16 $x^2 \leq 2x + 15$

$x^2 - 2x + 15 \leq 0$

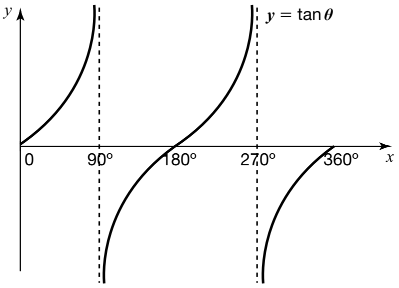
Factorising $x^2 - 2x - 15 = 0$ gives $(x - 5)(x + 3) = 0$: $x = 5$ or -3

As the coefficient of x^2 is positive, the graph of $y = x^2 - 2x - 15$ is ∪-shaped.

$x^2 - 2x - 15 \leq 0$ for the region below the x-axis (i.e. where $y \leq 0$).

$-3 \leq x \leq 5$

17 **a**



b $\theta = 45°$ or $225°$

18 $3x + 4y - 25 = 0$

19 $x = 5$ and $y = 1$

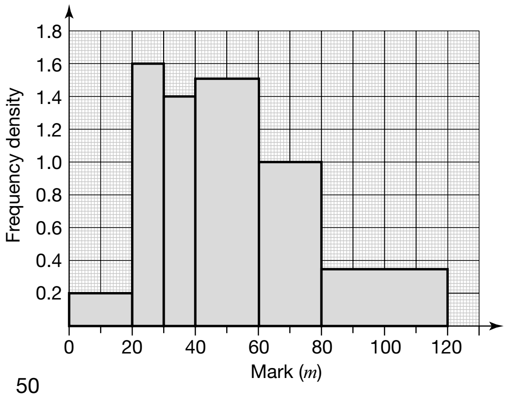
20 $3n^2 - 2n + 4$

21 **a** **i** constant speed of 4 m/s

ii constant acceleration for 3 seconds

b 10 m/s

22 **a**



b 50

23 angle $ABC = 50°$ (base angles of an isosceles triangle equal)
angle $BCW = 50°$ (alternate angles)

24 **a** $x = 2.5$ **b** $n = -2, -1, 0, 1, 2$

Calculator

1 **a** 0.009 cm

b 9×10^{-5} m

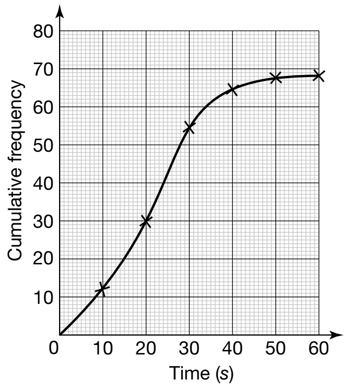
2 **a** When Lois squared both sides to remove the square root sign, she did not square the 2π.

b $l = \frac{gT^2}{4\pi^2}$

3 **a**

Time for call to be answered (t seconds)	Frequency	Cumulative frequency
$0 < t \leq 10$	12	12
$10 < t \leq 20$	18	30
$20 < t \leq 30$	25	55
$30 < t \leq 40$	10	65
$40 < t \leq 50$	2	67
$50 < t \leq 60$	1	68

b



c 21 seconds

d 75% of 68 = 0.75 × 68 = 51

Reading off from 51 on the cumulative frequency axis gives a time of 28 s.

75% of calls are answered within 28 s, so the target is being met.

4 **a** $y < x$

b $x + y < 30$

c $x \leq 2y$

5 $3\frac{3}{7}$

6 **a** There are different numbers of boys and girls so this needs to be taken into account when the mean of the whole class is found.

b 57.2%

7 **a** shaded area = area of large circle − area of small circle

$$= \pi R^2 - \pi r^2$$
$$= \pi(R^2 - r^2)$$
$$= \pi(R + r)(R - r)$$

b 0.0775π m^2

c 2337 kg (to nearest whole number)

8 **a** 84.6 km (to 1 d.p.)

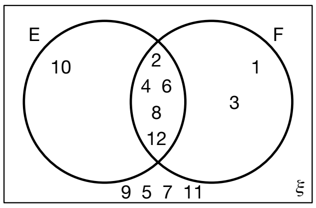
b 202°

9 $2x - 3\sqrt{xy} - 9y$

10 **a** 6.0 cm (to 1 d.p.)

b 36.9° (to 1 d.p.)

11 **a**



b **i** $\frac{5}{12}$ **ii** $\frac{1}{3}$

12 $x = -3$

13 **a** **i** 100°

ii Angles on a straight line sum to 180°.

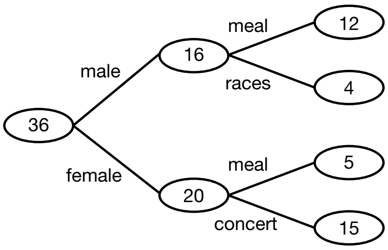
b angle $TSQ = 35°$ (Angle in the alternate segment)

angle $TSR = 60 + 35 = 95°$

angle $TQR = 180 - 95 = 85°$ (opposite angles of a cyclic quadrilateral add up to 180°)

14 **a** 27.7 m^3 (to 3 s.f.) **b** 35 weeks

15 **a**



b $\frac{17}{36}$

16 **a** **i** angle $CDE = 40°$ **ii** alternate angle to angle BAD

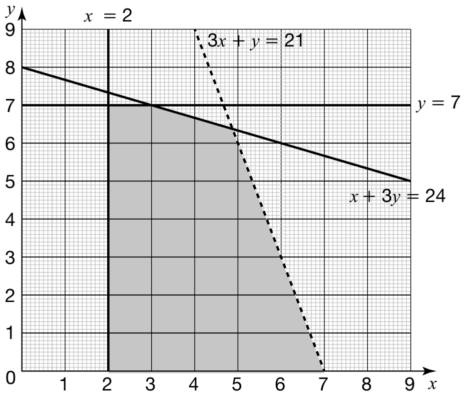
b Angle ABC = angle CED as they are alternate angles.

Angle DCE = angle BCA as vertically opposite angles are equal.

All the equivent angles in both triangles are the same so the triangles are similar.

c $3\frac{1}{3}$ cm

17



18 87 % (to 2 s.f.)

19 Using Pythagoras' theorem in triangle ABC:

$AC^2 = 2^2 + 3^2 = 13$

$AC = \sqrt{13}$ cm

By Pythagoras' theorem in triangle ACD:

$AD^2 = (\sqrt{13})^2 + 3^2 = 13 + 9 = 22$

$AD = \sqrt{22}$ cm