# Answers

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## Number

### Integers, decimals and symbols

1 $\frac{1}{0.01}$ 0.1 $(0.1)^2$ $\frac{1}{1000}$ $(-1)^3$
2 **a** 35 **b** 0.01285 **c** –270 **d** 40
3 **a** 4644 **b** 4644 **c** 86 **d** 540
4 **a** $12.56 \times 3.45 = 0.1256 \times 345$
**b** $(-8)^2 > -64$ **c** $6 - 12 = 8 - 14$
**d** $(-7) \times (0) < (-7) \times (-3)$

### Addition, subtraction, multiplication and division

1 **a** 76.765 **b** 201.646 **c** 91.33 **d** 10.564
2 **a** 1176 **c** 44.62 **e** 27
**b** 2166 **d** 0.6572 **f** 63
3 **a** 1156 **b** 7.5 **c** 5.76

### Using fractions

1 $\frac{2}{5} = \frac{16}{\mathbf{40}} = \frac{\mathbf{30}}{75} = \frac{50}{\mathbf{125}}$
2 **a** $5\frac{1}{3}$ **b** $9\frac{7}{13}$
3 **a** $7\frac{1}{12}$ **b** $7\frac{1}{2}$ **c** $2\frac{9}{20}$
4 $\frac{5}{56}$ 5 $\frac{1}{2}$ $\frac{7}{12}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{7}{8}$

### Different types of number

1 **a** 7 **b** 49 **c** 2 **d** 6 **e** 6
2 **a** $3^2 \times 7 \times 11$ **b** 63 **c** 10 395
3 441 4 5 minutes

### Listing strategies

1 210 seconds 3 1100 students
2 5 friends 4 15 pairs

### The order of operations in calculations

1 **a** Ravi has worked out the expression from left to right, instead of using BIDMAS. He should have performed the division and multiplication before the addition.
**b** Correct answer: 40
2 **a** 122 **b** −3 **c** 40
3 **a** 6 **b** 14 **c** 8

### Indices

1 **a** $10^6$ **b** $10^8$ **c** $10^6$ **d** $10^3$
2 **a** 1 **b** $\frac{1}{9}$ **c** 2 **d** 7
3 **a** $\frac{3}{2}$ **b** 16 **c** $\frac{1}{6}$ **d** 64
4 $x = 1.5$

### Surds

1 **a** 5 **b** 30 **c** 18
2 $\frac{5\sqrt{3}}{4}$
3 $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 2\sqrt{3} + 2\sqrt{3} - 3 = 1$
4 $a = 30$
5 $-\sqrt{5} - 7$
6
$$\frac{1}{\sqrt{2}} + \frac{1}{4} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{1}{4}$$
$$= \frac{\sqrt{2}}{2} + \frac{1}{4}$$
$$= \frac{2\sqrt{2}}{4} + \frac{1}{4}$$
$$= \frac{1 + 2\sqrt{2}}{4}$$

7
$$\frac{2}{1 - \frac{1}{\sqrt{2}}} = \frac{2}{\frac{\sqrt{2}}{\sqrt{2}}\ \frac{1}{\sqrt{2}}}$$
$$= \frac{2}{\frac{\sqrt{2} - 1}{\sqrt{2}}}$$
$$= \frac{2\sqrt{2}}{\sqrt{2} - 1}$$
$$= \frac{2\sqrt{2}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$
$$= \frac{4 + 2\sqrt{2}}{2 - 1}$$
$$= 4 + 2\sqrt{2}$$

8
$$\frac{3}{\sqrt{3}} + \sqrt{75} + (\sqrt{2} \times \sqrt{6}) = \frac{3\sqrt{3}}{3} + \sqrt{3 \times 25} + \sqrt{12}$$
$$= \sqrt{3} + 5\sqrt{3} + \sqrt{3 \times 4}$$
$$= \sqrt{3} + 5\sqrt{3} + 2\sqrt{3}$$
$$= 8\sqrt{3}$$

### Standard form

1 **a** $2.55 \times 10^{-3}$ **b** $1.006 \times 10^{10}$ **c** $8.9 \times 10^{-8}$
2 **a** $6 \times 10^{14}$ **c** $2 \times 10^2$ **e** $9 \times 10^{-3}$
**b** $1.1 \times 10^6$ **d** $1 \times 10^{-2}$
3 2680 4 $a = 3.3$

### Converting between fractions and decimals

1 **a** 0.55 **b** 0.375
2 **a** terminating **b** recurring **c** recurring
3
$$\text{Let } x = 0.\dot{4}0\dot{2} = 0.402402402\ldots$$
$$1000x = 402.402402\ldots$$
$$1000x - x = 402.402402\ldots - 0.402402402\ldots$$
$$999x = 402$$
$$x = \frac{402}{999} = \frac{134}{333}$$
Hence $0.\dot{4}0\dot{2} = \frac{134}{333}$
4 $\frac{323}{495}$

### Converting between fractions and percentages

1 **a** $\frac{7}{20}$ **b** $\frac{7}{100}$ **c** $\frac{19}{25}$ **d** $\frac{1}{8}$
2 **a** 20% **b** 68% **c** 250% **d** 17.5%
3 53.33% (to 2 d.p.)
4 $\frac{66}{90} = \frac{66}{90} = 73.3\%$ (to 1 d.p.)
Jake did better in chemistry.

### Fractions and percentages as operators

1 £34.79 4 **a** £14 400 5 $\frac{14}{33}$
2 48 **b** £320
3 7040

### Standard measurement units

1 175 000 cm 2 17
3 1286 (to nearest whole number)
4 **a** $1.99 \times 10^{-23}$ g (to 3 s.f.) **b** $1.99 \times 10^{-26}$ kg (to 3 s.f.)
5 $7.20 \times 10^{-26}$ g (to 3 s.f.)

### Rounding numbers

1 **a** 35 **c** 0 **e** 2
**b** 101 **d** 0
2 **a** 34.88 **b** 34.877
3 **a** 12 800 **b** 0.011 **c** $7 \times 10^{-5}$
4 **a** −0.00993 **b** 34.4 **c** 12 300

### Estimation

1 200 3 0.16 5 10.6
2 **a** 236.2298627 4 5 6 4
**b** 240
7 **a** $5 \times 10^{-28}$ kg
**b** This will be an underestimate, as the mass of one electron has been rounded down.

### Upper and lower bounds

1 $2.335 \leq l < 2.345$ kg
2 **a** **i** 2.472 **ii** 2.451 **b** 2.5 (to 2 s.f.)
3 34

## Algebra

### Simple algebraic techniques

1 **a** formula **c** expression **e** formula
**b** identity **d** identity

2 $x + 6x^2$

3 $y^3 - y = (1)^3 - 1 = 0$ so $y = 1$ is correct.
$y^3 - y = (-1)^3 - (-1) = -1 + 1 = 0$ so $y = -1$ is correct.

4 **a** $10x$ **b** $4x^2 - 6x$ **c** $18x^2$

5 **a** 2 **b** $\frac{7}{8}$ **c** $-\frac{3}{2}$

### Removing brackets

1 **a** $24x - 56$ **b** $-6x + 12$

2 **a** $3x + 9$ **c** $10a^2b - 5ab^2$
**b** $8xy + 6x - 2y$ **d** $2x^3y^3 + 3x^2y^4$

3 **a** $m^2 + 5m - 24$ **c** $9x^2 - 6x + 1$
**b** $8x^2 + 26x - 7$ **d** $6x^2 + xy - y^2$

4 **a** $x^2 + 7x + 10$ **c** $x^2 - 6x - 7$
**b** $x^2 - 16$ **d** $15x^2 + 14x + 3$

5 **a** $x^3 + 6x^2 + 5x - 12$ **b** $18x^3 - 63x^2 + 37x + 20$

### Factorising

1 **a** $5x(5x - y)$ **b** $2\pi(2r^2 + 3x)$ **c** $6ab^2(a^2 + 2)$

2 **a** $(3x + 1)(3x - 1)$ **b** $4(2x + 1)(2x - 1)$

3 **a** $(a + 4)(a + 8)$ **b** $(p - 6)(p - 4)$

4 **a** $a(a + 12)$ **b** $(b + 3)(b - 3)$ **c** $(x - 5)(x - 6)$

5 **a** $(3x + 8)(x + 4)$ **b** $(3x + 13)(x - 1)$ **c** $(2x - 5)(x + 2)$

6 $\frac{2}{(x - 3)}$ 7 $\frac{2x - 1}{4x + 1}$

### Changing the subject of a formula

1 $T = \frac{PV}{nR}$ 3 $a = \frac{v - u}{t}$ 5 $v = \sqrt{\frac{2E}{m}}$

2 $y = \frac{1 - 4x}{2}$ 4 $x = 5(y + m)$

6 **a** $r = \sqrt{\frac{3V}{\pi h}}$ **b** 3.45 cm (to 2 d.p.)

7 **a** $x = \frac{y + 9}{3}$ **b** 4

8 $x = \frac{3y - 2}{a + 1}$

9 **a** $c = \frac{b}{a}$ **b** upper bound for $c = 1.18$ (to 3 s.f.)
lower bound for $c = 1.11$ (to 3 s.f.)

### Solving linear equations

1 **a** $x = 7$ **d** $x = 32$ **g** $x = -2$
**b** $x = 5$ **e** $x = 25$ **h** $x = 84$
**c** $x = 4$ **f** $x = -9$

2 $x = \frac{2}{3}$

3 **a** $x = \frac{1}{2}$ **b** $x = -\frac{8}{5}$

### Solving quadratic equations using factorisation

1 **a** $(x - 3)(x - 4)$ **b** $x = 3$ or $x = 4$

2 **a** $(2x - 1)(x + 3)$ **b** $x = \frac{1}{2}$ or $x = -3$

3 $x = -2$ or $x = 6$

4 **a** $x(x - 8) - 7 = x(5 - x)$
$x^2 - 8x - 7 = 5x - x^2$
$2x^2 - 13x - 7 = 0$
**b** $x = -\frac{1}{2}$ or $x = 7$

5 $x = 2$ cm

### Solving quadratic equations using the formula

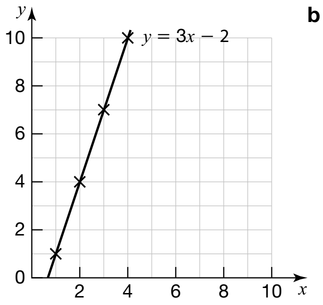
1 **a** $\frac{3}{x + 7} = \frac{2 - x}{x + 1}$
$3(x + 1) = (2 - x)(x + 7)$
$3x + 3 = 2x + 14 - x^2 - 7x$
$3x + 3 = -x^2 - 5x + 14$
$x^2 + 8x - 11 = 0$
**b** $x = 1.20$ or $-9.20$ (to 2 d.p.)

2 $x = 2.78$ cm (to 2 d.p.) 3 $x = 3.30$ or $-0.30$ (to 2 d.p.)

### Solving simultaneous equations

1 $x = 2$ and $y = 3$

2 **a**



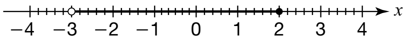
**b** $x = 3, y = 7$

3 $x = 0, y = -3$ or $x = 3, y = 0$

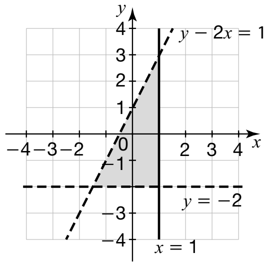
### Solving inequalities

1 **a** $x \geq -9$ **b** $x < -12$

2



3 **a**



**b** (1, 2), (1, 1), (0, 0), (1, 0), (0, −1), (1, −1)

4 $-3 \leq x \leq 1$ 5 $x < -3$ and $x > 5$

### Problem solving using algebra

1 42 m²

2 cost of adult ticket = £7.50
cost of child ticket = £4

3 **a** 16 years **b** 9 years

### Use of functions

1 **a** 19 **b** $x = -1$

2 **a** $(x - 6)^2$ **b** $x^2 - 6$

3 **a** $\pm 3$ **b** $2x + 5$

4 $f^{-1}(x) = \sqrt{\frac{x - 3}{5}}$

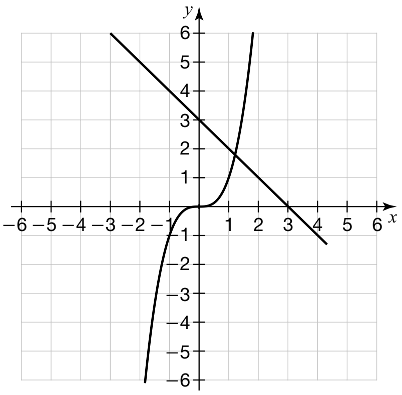
### Iterative methods

1 Let $f(x) = 2x^3 - 2x + 1$
$f(-1) = 2(-1)^3 - 2(-1) + 1 = 1$
$f(-1.5) = 2(-1.5)^3 - 2(-1.5) + 1 = -2.75$
There is a sign change of $f(x)$, so there is a solution between $x = -1$ and $x = -1.5$.

2 $x_1 = 0.1121111111$
$x_2 = 0.1125202246$
$x_3 = 0.1125357073$

3 **a** $x_4 = 1.5213705 \approx 1.521$ (to 3 d.p.)
**b** Checking value of $x^3 - x - 2$ for $x = 1.5205, 1.5215$:
When $x = 1.5205$ $f(1.5205) = -0.0052$
$x = 1.5215$ $f(1.5215) = 0.0007$
Since there is a change of sign, the root is 1.521 correct to 3 decimal places.

4 **a** **i**



**ii** There is a root of $x^3 + x - 3 = 0$ where the graphs of $y = x^3$ and $y = 3 - x$ intersect. The graphs intersect once so there is one real root of the equation $x^3 + x - 3 = 0$.

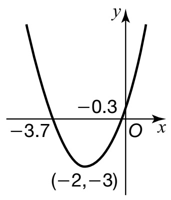
**b** $x_1 = 1.216440399$
$x_2 = 1.212725591$
$x_3 = 1.213566964$
$x_4 = 1.213376503$
$x_5 = 1.213419623$
$x_6 = 1.213409861 = 1.2134$ (to 4 d.p.)

## Equation of a straight line

1 A
2 **a** $-\frac{4}{3}$ **b** $y = -\frac{1}{2}x + \frac{7}{2}$ **c** $y = 2x + 1$
3 (3.8, 11.4) (to 1 d.p.)

## Quadratic graphs

1 **a** $x = -0.3$ or $-3.7$ (to 1 d.p.)
**b**



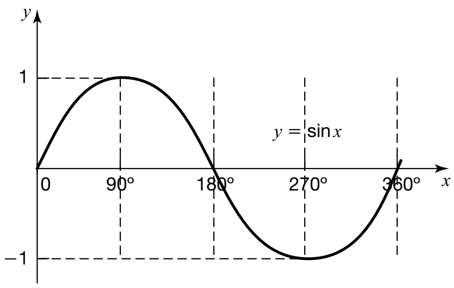
2 $a = 5$, $b = -2$ and $c = -10$
3 $a = 2$, $b = 3$ and $c = -15$

## Recognising and sketching graphs of functions

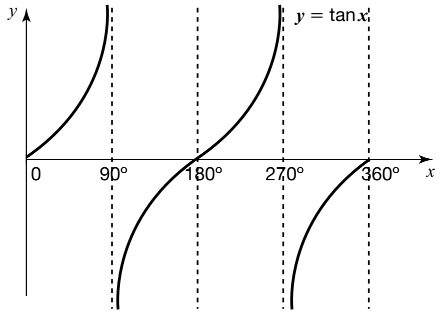
1

| Equation | Graph |
|---|---|
| $y = x^2$ | B |
| $y = 2^x$ | D |
| $y = \sin x°$ | E |
| $y = x^3$ | C |
| $y = x^2 - 6x + 8$ | A |
| $y = \cos x°$ | F |

2 **a**



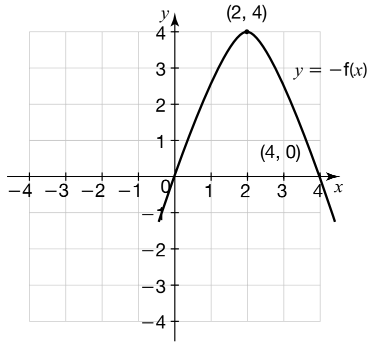
**b**



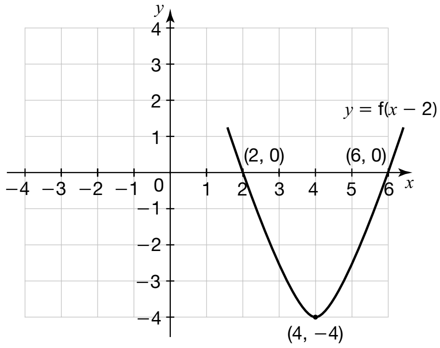
3 $\theta = 70.5°$ or $289.5°$ (to 1 d.p.)

## Translations and reflections of functions

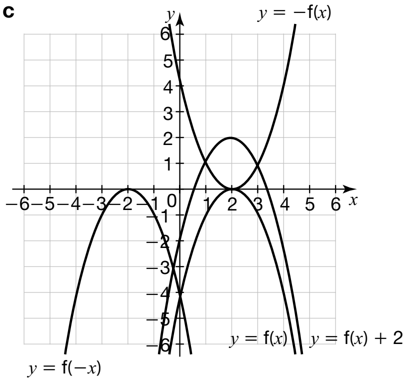
1 **a**



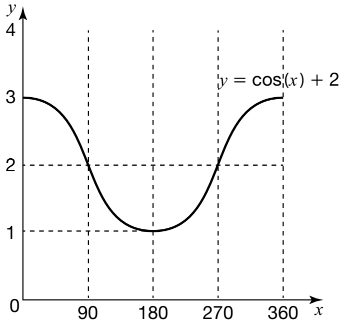
**b**



2 **a, b, c**



3



## Equation of a circle and tangent to a circle

1 **a** 5
**b** 7
**c** 2

2 radius of the circle = $\sqrt{21} = 4.58$
distance of the point (4, 3) from the centre of the circle (0, 0) = $\sqrt{16 + 9} = \sqrt{25} = 5$
This distance is greater than the radius of the circle, so the point lies outside the circle.

3 **a** $\sqrt{74}$
**b** $x^2 + y^2 = 74$ **c** $y = -\frac{5}{7}x + \frac{74}{7}$

### Real-life graphs

1 **a** 1 m/s$^2$
 **b** 225 m
2 **a** The graph is a straight line starting at the origin, so this represents constant acceleration from rest of $\frac{15}{6}$ = 2.5 m/s$^2$.
 **b** The gradient decreases to zero, so the acceleration decreases to zero.
 **c** 118 m (to nearest integer); 117 m is also acceptable
 **d** It will be a slight underestimate, as the curve is always above the straight lines forming the tops of the trapeziums.

### Generating sequences

1 **a** **i** $\frac{1}{2}$ **ii** 243 **iii** 21
 **b** 14, 1
2 −3, −11
3 **a** 25, 36
 **b** 15, 21
 **c** 8, 13

### The $n$th term

1 **a** $n$th term = $4n - 2$
 **b** $n$th term = $4n - 2 = 2(2n - 1)$
 2 is a factor, so the $n$th term is divisible by 2 and therefore is even.
 **c** 236 is not a term in the sequence.
2 **a** 5
 **b** −391
 **c** $n^2$ is always positive, so the largest value $9 - n^2$ can take is 8 when $n = 1$. All values of $n$ above 1 will make $9 - n^2$ smaller than 8. So 10 cannot be a term.
3 $n$th term = $n^2 - 3n + 3$

### Arguments and proofs

1 The only prime number that is not odd is 2, which is the only even prime number.
Hence, statement is false because 2 is a prime number that is not odd.
2 **a** true: $n = 1$ is the smallest positive integer and this would give the smallest value of $2n + 1$ which is 3.
 **b** true: 3 is a factor of $3(n + 1)$ so $3(n + 1)$ must be a multiple of 3.
 **c** false: $2n$ is always even and subtracting 3 will give an odd number.
3 Let first number = $x$ so next number = $x + 1$
Sum of consecutive integers = $x + x + 1 = 2x + 1$
Regardless of whether $x$ is odd or even, $2x$ will always be even as it is divisible by 2.
Hence $2x + 1$ will always be odd.
4 $(2x - 1)^2 - (x - 2)^2$
$= 4x^2 - 4x + 1 - (x^2 - 4x + 4)$
$= 4x^2 - 4x + 1 - x^2 + 4x - 4$
$= 3x^2 - 3$
$= 3(x^2 - 1)$
The 3 outside the brackets shows that the result is a multiple of 3 for all integer values of $x$.
5 Let two consecutive odd numbers be $2n - 1$ and $2n + 1$.
$(2n + 1)^2 - (2n - 1)^2$
$= (4n^2 + 4n + 1) - (4n^2 - 4n + 1)$
$= 8n$
Since 8 is a factor of $8n$, the difference between the squares of two consecutive odd numbers is always a multiple of 8.
(If you used $2n + 1$ and $2n + 3$ for the two consecutive odd numbers, difference of squares = $8n + 8 = 8(n + 1)$.)

## Ratio

### Introduction to ratios

1 30
2 £7500, £8500, £9000
3 210 acres
4 $x = \frac{5}{7}$
5 144

### Scale diagrams and maps

1 5 km
2 **a** 0.92 km **b** 0.12 km
3 1 : 800
4 1 : 200000

### Percentage problems

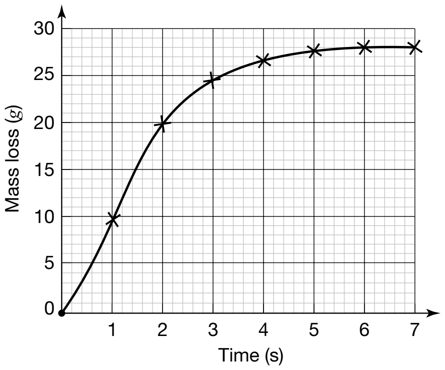
1 10%
2 83.3%
3 £18 000
4 £16 250
5 £896

### Direct and inverse proportion

1 **a** $P = kT$ **b** 74 074 pascals (to nearest whole number)
2 £853 (to nearest whole number)
3 **a** $c = \frac{36}{h}$ **b** 2.4
4 **a** €402.50 **b** £72.07 (to nearest penny) **c** £2.50

### Graphs of direct and inverse proportion and rates of change

1 B
2 B
3 **a**



 **b** **i** 9.8 g/minute **ii** 0.16 g/second (to 2 d.p.)

### Growth and decay

1 **a** 178 652 **b** 5 years
2 £12 594
3 0.1 (to 1 s.f.)

### Ratios of lengths, areas and volumes

1 **a** 3.375 or $\frac{27}{8}$
 **b** 22.5 cm$^2$
 **c** 133 cm$^3$ (to nearest whole number)
2 $h$ = 15 cm (to nearest cm)
3 **a** **i** 9 cm **ii** 4.5 cm
 **b** 4 : 1

### Gradient of a curve and rate of change

1 **a** $\frac{2}{3}$ m/s$^2$
 **b** 0.26 m/s$^2$
 **c** 0.37 m/s$^2$
 **d** 34 s

### Converting units of areas and volumes, and compound units

1 500 N/m$^2$
2 25 000 N/m$^2$
3 107 g (to nearest g)
4 He has worked out the area in m$^2$ by dividing the area in cm$^2$ by 100, which is incorrect.
There are 100 × 100 = 10 000 cm$^2$ in 1 m$^2$, so the area should have been divided by 10 000.
Correct answer:
area in m$^2$ = $\frac{9018}{10\,000}$
= 0.9018
= 0.90 m$^2$ (to 2 d.p.)
5 72 km/h

# Geometry and measures

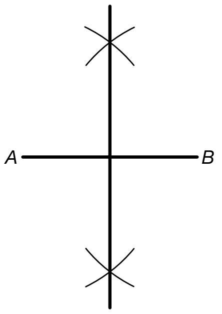
## 2D shapes

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | **a** | true | **c** | true | **e** | true |
| | **b** | false | **d** | true | **f** | false (this would be true only for a regular pentagon) |

| | | | | |
|---|---|---|---|---|
| 2 | **a** | rhombus | **c** | equilateral triangle |
| | **b** | parallelogram | **d** | kite |

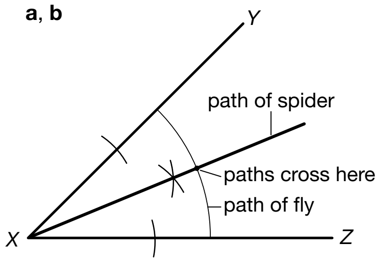
## Constructions and loci

1



2

3 **a**, **b**



## Properties of angles

1 **a** angle *ACB* = angle *BAC* = 30° (base angles of isosceles triangle *ABC*, since *AB* and *BC* are equal sides of a rhombus)

**b** angle *AOB* = 90° (diagonals of a rhombus intersect at right angles)

**c** angle *ABO* = 180 − (90 + 30) = 60° (angle sum of a triangle)

angle *BDC* = angle *ABO* = 60° (alternate angles between parallel lines *AB* and *DC*)

2 angle *BAC* = $\frac{(180 - 36)}{2}$ = 72° (angle sum of a triangle and base angles of an isosceles triangle)

angle *BDC* = 180 − 90 = 90° (angle sum on a straight line)

angle *ABD* = 180 − (90 + 72) = 18° (angle sum of a triangle)

3 **a** $x = 30°$

**b** If lines *AB* and *CD* are parallel, the angles $4x$ and $3x + 30$ would be corresponding angles, and so equal.

$4x = 4 \times 30 = 120°$

$3x + 30 = 3 \times 30 + 30 = 120°$

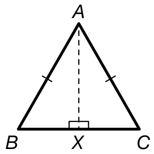
These two angles are equal so lines *AB* and *CD* are parallel.

4 $x = 90 + 72 = 162°$

## Congruent triangles

1 *BD* common to triangles *ABD* and *CDB*
angle *ADB* = angle *CBD* (alternate angles)
angle *ABD* = angle *CDB* (alternate angles)
Therefore triangles *ABD* and *CDB* are congruent (ASA).
Hence angle *BAD* = angle *BCD*

2 Draw the triangle and the perpendicular from *A* to *BC*.
*AX* = *AX* (common)
*AB* = *AC* (triangle *ABC* is isosceles)
angle *AXB* = *AXC* = 90° (given)
Therefore triangles *ABX* and *ACX* are congruent (RHS).
Hence *BX* = *XC*, so *X* bisects *BC*.

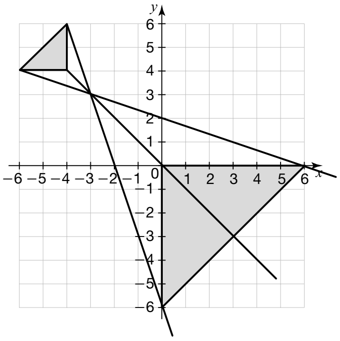


3 *OQC* = 90° (corresponding angles), so *PB* = *OQ* (perpendicular distance between 2 parallel lines)
*AP* = *PB* (given), so *AP* = *OQ*
*PO* = *QC* (*Q* is the midpoint of *BC*)
angle *ABC* = angle *APO* = angle *OQC*
= 90° (*OQ* is parallel to *AB* and *OP* parallel to *BC*)
Therefore triangles *AOP* and *OCQ* are congruent (SAS).

## Transformations

1 translation of $\begin{pmatrix}-7\\-5\end{pmatrix}$

2



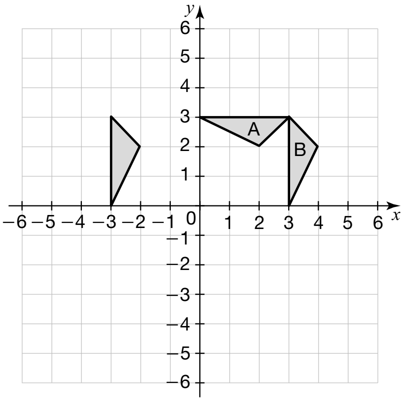
3 **a** translation of $\begin{pmatrix}-7\\0\end{pmatrix}$ **b** reflection in the line $y = 3$

**c** rotation of 90° clockwise about (0, 1)

## Invariance and combined transformations

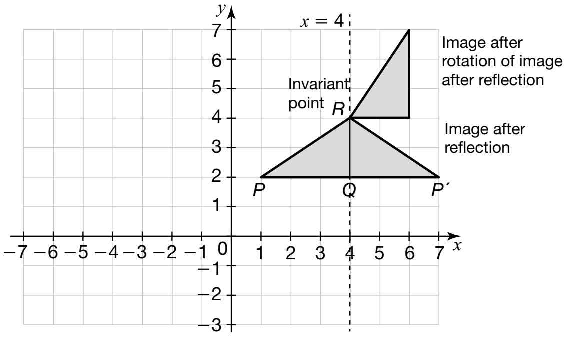
1 **a** 1

**b** **i** invariant point (3, 3)



**ii** rotation 90° anticlockwise about the point (3, 3)

2 **a** The shaded triangle is the image after the two transformations.



**b** invariant point is R (4, 4)

## 3D shapes

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | **a** | G | **c** | A, H | **e** | C |
| | **b** | B | **d** | B | **f** | A, H |

## Parts of a circle

1 **a** radius
**b** diameter
**c** chord
**d** arc

2 **a** minor sector
**b** major segment
**c** major sector
**d** minor segment

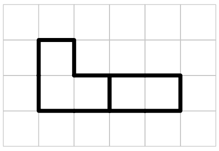
## Circle theorems

1 angle $OTB = 90°$ (angle between tangent and radius)
angle $BOT = 180 - (90 + 28) = 62°$ (angle sum in a triangle)
angle $AOT = 180 - 62 = 118°$ (angle sum on a straight line)
$AO = OT$ (radii), so triangle $AOT$ is isosceles
angle $OAT = \frac{(180 - 118)}{2} = 31°$ (angle sum in a triangle)

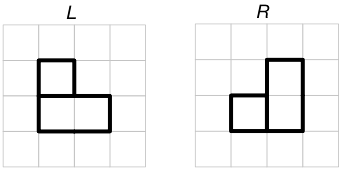
2 **a** angle $ACB = 30°$ (angle at centre twice angle at circumference)
**b** angle $BAC$ = angle $CBX = 70°$ (alternate segment theorem)
**c** $OA = OB$ (radii), so triangle $AOB$ is isosceles
angle $AOB = 60°$, so triangle $AOB$ is equilateral
angle $OAB = 60°$ (angle of equilateral triangle)
angle CAO $= 70 - 60 = 10°$

## Projections

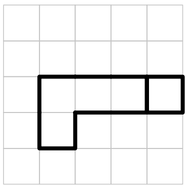
1



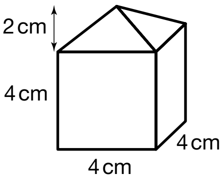
2 **a**



**b**



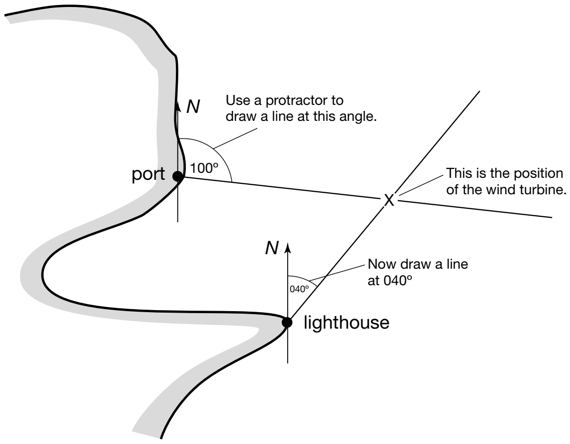
3



## Bearings

1 230°

2



## Pythagoras' theorem

1 76 m (to nearest m)
2 9.8 cm (to 1 d.p.)
3 **a** 9.1 cm (to 2 d.p.)
**b** $48.76\,\text{cm}^2$ (to 2 d.p.)

## Area of 2D shapes

1 **a** **i** 5.66 cm (to 2 d.p.)
**ii** $19.80\,\text{cm}^2$ (to 2 d.p.)
**b** $50.65\,\text{cm}^2$
2 **a** $9\,\text{cm}^2$ **b** $6\,\text{cm}^2$
3 **a** $27\pi\,\text{cm}^2$ **b** $18\pi + 6\,\text{cm}$

## Volume and surface area of 3D shapes

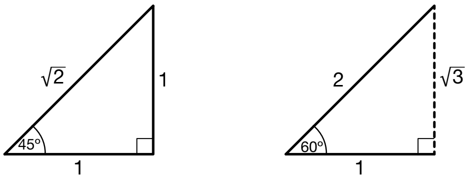
1 **a** $3.975\,\text{m}^2$ **b** $6.36\,\text{m}^3$ (to 2 d.p.)
2 5 glasses
3 **a** 7.4 cm (to 1 d.p.)
**b** 3.8 cm (to 1 d.p.)
4 0.64 cm

## Trigonometric ratios

1 **a** 6.0 cm (to 1 d.p.)
**b** 36.9° (to 1 d.p.)
2 44.4° (to 1 d.p.)
3 9.4 cm (to 1 d.p.)
4 21.8° (to 1 d.p.)

## Exact values of sin, cos and tan

1



$\tan 45° = \frac{1}{1} = 1$ $\cos 60° = \frac{1}{2}$

Hence, $\tan 45° + \cos 60° = 1 + \frac{1}{2} = \frac{3}{2}$

2 **a** **i** $\sin 45° = \frac{1}{\sqrt{2}}$ **ii** $\cos 45° = \frac{1}{\sqrt{2}}$

**b** $\frac{\sin 45°}{\cos 45°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$\tan 45° = 1$

Hence $\frac{\sin 45°}{\cos 45°} = \tan 45°$

3 **a** $\sqrt{5}$

**b** **i** $\sin x = \frac{1}{\sqrt{5}}$ **ii** $\cos x = \frac{2}{\sqrt{5}}$

**c** $(\sin x)^2 = \frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{1}{5}$

$(\cos x)^2 = \frac{2}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{4}{5}$

$(\sin x)^2 + (\cos x)^2 = \frac{1}{5} + \frac{4}{5}$
$= 1$

4 $\tan 30° + \tan 60° + \cos 30°$

$$= \frac{1}{\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{3}}{3} + \sqrt{3} + \frac{\sqrt{3}}{2}$$
$$= \frac{2\sqrt{3} + 6\sqrt{3} + 3\sqrt{3}}{6}$$
$$= \frac{11\sqrt{3}}{6}$$

## Sectors of circles

1 34° (to nearest degree)
2 $364.4\,\text{cm}^2$
3 **a** 1.67 cm (to 3 s.f.)
**b** 1 : 1.04

4 **a** length of arc $AB = \frac{\theta}{360} \times 2\pi r$

$$5.4 = \frac{\theta}{360} \times 2\pi \times 6$$

$$\theta = 51.5662$$

Area of sector $AOB = \frac{\theta}{360} \times 2\pi r^2$

$$= \frac{51.5662}{360} \times \pi \times 6^2$$

$$= 16.2 \text{ cm}^2$$

Note that both $a$ and $b$ are equal to the radius $r$ of the circle.

**b** area of triangle $AOB$

$$= \frac{1}{2} a\, b \sin c$$

$$= \frac{1}{2} \times 6 \times 6 \sin 51.5662$$

$$= 14.09988 \text{ cm}^2$$

area of shaded segment

= area of sector − area of triangle

$= 16.2 - 14.1$

$= 2.1\text{cm}^2$ (correct to 1 decimal place)

## Sine and cosine rules

1 **a** $225\,\text{cm}^2$

**b** $\frac{4}{5}$

**c** 18.0 cm (to 3 s.f.)

2 **a** 18.6 cm (to 3 s.f.)

**b** $92.4\,\text{cm}^2$ (to 3 s.f.)

3 $\frac{2 + 6\sqrt{2}}{17}$

## Vectors

1 **a** $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ **b** $\begin{pmatrix} -22 \\ 9 \end{pmatrix}$

2 **a** $\mathbf{b} - \mathbf{a}$ **b** $\frac{3}{5}(\mathbf{b} - \mathbf{a})$

**c** $\overrightarrow{OQ} = \frac{2}{5}\overrightarrow{OA} = \frac{2}{5}\mathbf{a}$

$$\overrightarrow{QP} = \overrightarrow{QA} + \overrightarrow{AP}$$

$$= \frac{3}{5}\mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$$

$$= \frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{3}{5}\mathbf{a}$$

$$= \frac{3}{5}\mathbf{b}$$

As $\overrightarrow{QP} = \frac{3}{5}\mathbf{b}$ and $\overrightarrow{OB} = \mathbf{b}$ they both have the same vector part and so are parallel.

3 **a** $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$$= -3\mathbf{b} + \mathbf{a}$$

$$= \mathbf{a} - 3\mathbf{b}$$

**b** $\overrightarrow{PB} = \frac{1}{3}\overrightarrow{AB} = \mathbf{b}$

$$\overrightarrow{PM} = \overrightarrow{PB} + \overrightarrow{BM}$$

$$= \overrightarrow{PB} + \frac{1}{2}\overrightarrow{BC}$$

$$= \mathbf{b} + \frac{1}{2}(\mathbf{a} - 3\mathbf{b})$$

$$= \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$$

$$= \frac{1}{2}(a - b)$$

$$\overrightarrow{MD} = \overrightarrow{MC} + \overrightarrow{CD}$$

$$= \frac{1}{2}\overrightarrow{BC} + \overrightarrow{CD}$$

$$= \frac{1}{2}(\mathbf{a} - 3\mathbf{b}) + \mathbf{a}$$

$$= \frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}$$

$$= \frac{3}{2}(\mathbf{a} - \mathbf{b})$$

Both $\overrightarrow{PM}$ and $\overrightarrow{MD}$ have the same vector part $(\mathbf{a} - \mathbf{b})$ so they are parallel. Since they both pass through $M$, they are parts of the same line, so $PMD$ is a straight line.

# Probability

## The basics of probability

1 **a**

**Dice 1**

**Dice 2**

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

**b** $\frac{1}{36}$ **c** $\frac{5}{12}$ **d** 7

2 **a** 21 chocolates **b** $\frac{3}{7}$

3

**Bethany**

**Amy**

| | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| **1** | 1, 1 | 1, 2 | 1, 3 | 1, 4 | 1, 5 | 1, 6 |
| **2** | 2, 1 | 2, 2 | 2, 3 | 2, 4 | 2, 5 | 2, 6 |
| **3** | 3, 1 | 3, 2 | 3, 3 | 3, 4 | 3, 5 | 3, 6 |
| **4** | 4, 1 | 4, 2 | 4, 3 | 4, 4 | 4, 5 | 4, 6 |
| **5** | 5, 1 | 5, 2 | 5, 3 | 5, 4 | 5, 5 | 5, 6 |
| **6** | 6, 1 | 6, 2 | 6, 3 | 6, 4 | 6, 5 | 6, 6 |

**a** $\frac{1}{6}$ **b** $\frac{5}{12}$

## Probability experiments

1 **a** He is wrong because 100 spins is a very small number of trials. To approach the theoretical probability you would have to spin many more times. Only when the number of spins is extremely large will the frequencies start to become similar.

**b** $\frac{11}{50}$ **c** 95

2 **a** $x = 0.08$ **b** 0.24 **c** 16

## The AND and OR rules

1 **a** $\frac{9}{169}$ **b** $\frac{3}{169}$ **c** $\frac{1}{52} \times \frac{1}{52} = \frac{1}{2704}$

2 **a** When an event has no effect on another event, they are said to be independent events. Here the colour of the first marble has no effect on the colour of the second marble.

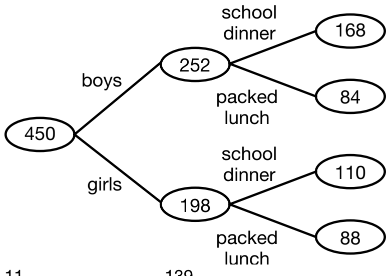
**b** $\frac{9}{100}$ **c** $\frac{3}{10}$

3 **a** $\frac{9}{140}$ **b** $\frac{6}{35}$

## Tree diagrams

1 **a** $\frac{2}{15}$ **b** $\frac{8}{15}$

2 **a**



**b** $\frac{11}{45}$ **c** $\frac{139}{225}$

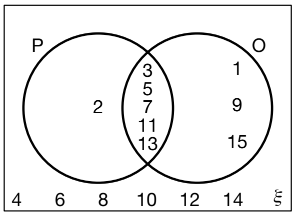
3 **a** 0.179 (to 3 d.p.) **b** 0.238 (to 3 d.p.) **c** 0.131 (to 3 d.p.)

## Venn diagrams and probability

1 **a** 9, 8 **c** 1, 3, 4, 10, 12, 15

**b** 1, 2, 3, 5, 7, 8, 9, 12, 15 **d** 4, 10

2 **a**



**b** $\frac{1}{3}$

3 **a** $\frac{17}{20}$ **b** $\frac{18}{73}$

# Statistics

## Sampling

1 **a** Ling's, as he has a larger sample so it is more likely to represent the whole population (i.e. students at the school).

**b** 22

2 The sample should be taken randomly, with each member of the population having an equal chance of being chosen.

The sample size should be large enough to represent the population, since the larger the sample size, the more accurate the results.

3 **a** 173

**b** 25

## Two-way tables and pie charts

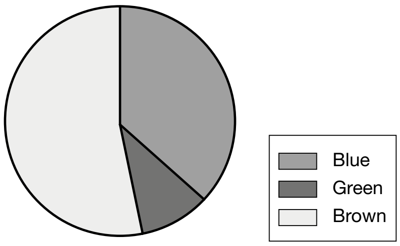
1 **a**

| | Coronation Street | EastEnders | Emmerdale | Total |
|---|---|---|---|---|
| **Boys** | 12 | 31 | 20 | 63 |
| **Girls** | 18 | 12 | 7 | 37 |
| **Total** | 30 | 43 | 27 | 100 |

**b** $\frac{27}{100}$

**c** $\frac{12}{37}$

2



3 **a** 11 students

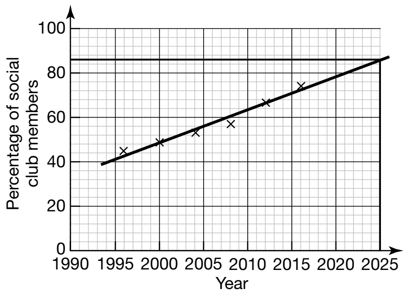
**b** 23 students

## Line graphs for time series data

1 **a** An upward trend, rising slowly at first, then rising quickly, then continuing to rise more slowly.

**b** Mode, because it shows that June is the month when the environment offers the largest sample of insects to study. The median would give May, when there are also a lot of insects.

2 **a**



**b** An upward trend – people are more likely to assume it should be an option.

**c** 86% (or a close value)

**d** Future data may change so that the line of best fit is no longer accurate. Also, the relationship may not be best represented by a straight line, but by a curve.

## Averages and spread

1 **a** 32

**b** 2.7 (to 1 d.p.)

**c** 3

2 66.5 (to 1 d.p.)

3 **a**

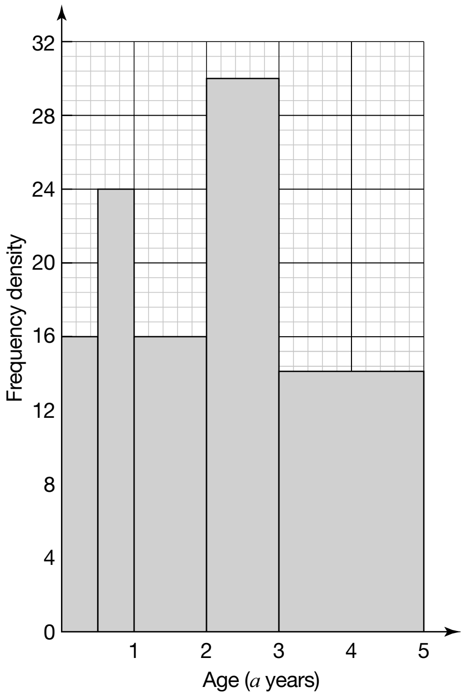
| Age ($t$ years) | Frequency | Mid-interval value | Frequency × mid-interval value |
|---|---|---|---|
| $0 < t \le 4$ | 8 | 2 | 16 |
| $4 < t \le 8$ | 10 | 6 | 60 |
| $8 < t \le 12$ | 16 | 10 | 160 |
| $12 < t \le 14$ | 1 | 13 | 13 |

**b** 35

**c** 7.1 years (to 2 s.f.)

## Histograms

1



2

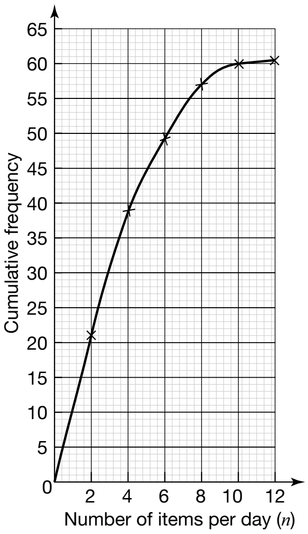
| Wingspan ($w$ cm) | Frequency |
|---|---|
| $0 < w \le 5$ | 2 |
| $5 < w \le 10$ | 6 |
| $10 < w \le 15$ | 24 |
| $15 < w \le 25$ | 30 |
| $25 < w \le 40$ | 9 |

## Cumulative frequency graphs

1 **a**

| Number of items of junk mail per day ($n$) | Frequency | Cumulative frequency |
|---|---|---|
| $0 < n \le 2$ | 21 | 21 |
| $2 < n \le 4$ | 18 | 39 |
| $4 < n \le 6$ | 10 | 49 |
| $6 < n \le 8$ | 8 | 57 |
| $8 < n \le 10$ | 3 | 60 |
| $10 < n \le 12$ | 1 | 61 |

**b**



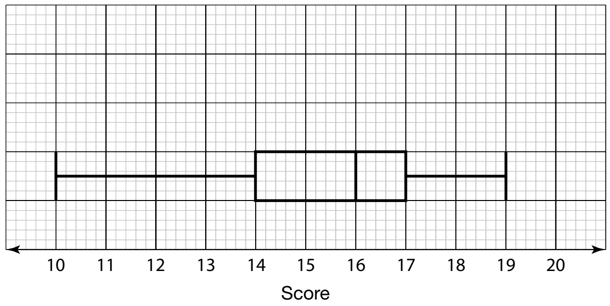
**c** median = value at frequency of 30.5 = 3

**2** **a** 6.5 kg

**b** 32 penguins

## Comparing sets of data

**1** **a**



**b** Sasha's median score is higher (17 compared to 16). The IQR for Sasha is 2 compared to Chloe's 3. The range for Sasha is 5 compared to Chloe's 9. Both these are measures of spread, which means that Sasha's scores are less spread out (i.e. more consistent).

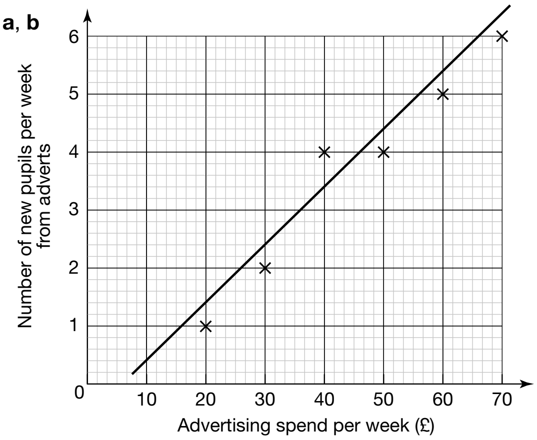
**2** **a** **i** 138 minutes **ii** 22 minutes

**b** On average the men were faster as the median is higher for the men.

The variation in times was greater for the men as their range was greater, although the spread of the middle half of the data (the interquartile range) was slightly greater for the women.

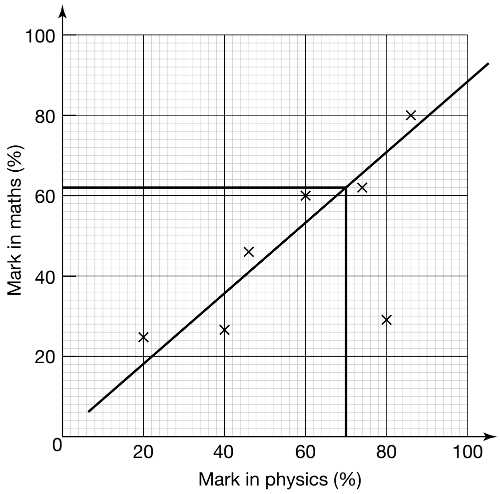
## Scatter graphs

**1** **a, b**



**c** positive correlation

**2** **a**



**b** 62%

**c** The data only goes up to a score of 86% in physics, and the score at 80% in physics is an outlier, so the line may not be accurate for higher scores in physics.

For answers to the practice papers, visit:
www.scholastic.co.uk/gcse