Higher Mathematics Revision Guide for All Exam Boards

Full worked solutions

Number

Integers, decimals and symbols

1 a $23 \times 8.7 = 200.1$
  b $2.3 \times 0.87 = 2.001$
  c $\frac{2.001}{0.87} = 2.3$
  d $\frac{2001}{23} = 87$

2 a $4.86 \times 29 = 140.94$
  b $0.486 \times 2.9 = 1.4094$
  c $14094 \div 48.6 = 290$
  d $140.94 \div 29 = 4.86$

3 Ascending order means going up in size.
  $-0.5 \quad 0 \quad 0.012 \quad 0.12 \quad 12$

4 a $\frac{5}{0.5} = 10$
  b $1\frac{5}{9} > \frac{4}{3}$
  c $-3 < -1$

5 a $-2 + 7 = 5$
  b $-3 - 5 = -8$
  c $3 \times -5 = -15$
  d $-12 \div -2 = 6$
  e $-1 + 7 - 10 = -4$

Addition, subtraction, multiplication and division

1 a $1083 + 478 = 1561$
  b $2445 + 89 + 513 = 3047$
  c $66.55 + 3.38 = 69.93$
  d $7.08 + 4.5 + 12.343 = 23.923$

2 a $4556 - 1737 = 2819$
  b $674 - 387 = 287$
  c $12.935 - 4.75 = 8.185$
  d $5.77 - 0.369 = 5.401$

3 a $634 \times 47 = 29\,798$
  b $7.7 \times 3.8 = 29.26$
  c $8.32 \times 4.9 = 40.768$

4 a $1058 \div 23 = 46$
  b $617.4 \div 1.8 = 343$
  c $88.5 \div 2.5 = 35.4$

Using fractions

1 a $\frac{16}{5} = 3\frac{1}{5}$
  b $1\frac{1}{5} = \frac{6}{5}$
  c In ascending order: $\frac{5}{8} \quad \frac{3}{4} \quad \frac{9}{10} \quad 1\frac{1}{5} \quad \frac{16}{5}$
  d $1\frac{1}{5} + \frac{16}{5} = 1\frac{1}{5} + 3\frac{1}{5} = 4\frac{2}{5}$
  e $\frac{16}{5} - \frac{5}{8} = \frac{128}{40} - \frac{25}{40} = \frac{103}{40} = 2\frac{23}{40}$

  $45 \div 15 = 3$

  $12 \div 4 = 3$

  $48 \div 16 = 3$

2 $\frac{15}{45}, \frac{4}{12}, \frac{16}{48}$

3 a $3\frac{1}{3} \times 2\frac{1}{5} = \frac{10}{3} \times \frac{11}{5} = \frac{22}{3} = 7\frac{1}{3}$
  b $1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1} = \frac{7}{2} = 3\frac{1}{2}$

4 Fraction of weekly wage spent $= \frac{1}{3} + \frac{1}{5} + \frac{1}{4}$
  $= \frac{20 + 12 + 15}{60} = \frac{47}{60}$
  Ravi has $1 - \frac{47}{60} = \mathbf{\frac{13}{60}}$ of his wage left.

Different types of number

1 a **16** is one more than 15, which is a multiple of 5.
  b **5** is one less than 6, which is a factor of 12.
  c **16** is not a prime number as it has factors 1, 2, 4, 8 and 16.

2 $300 = 30 \times 10 = 15 \times 2 \times 5 \times 2 = 5 \times 3 \times 2 \times 5 \times 2$
  $= 2 \times 2 \times 3 \times 5 \times 5$

  It is best to write the product of prime factors in ascending order (i.e. smallest number first).

3 Find the multiples of each number.
  12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156 ...
  16, 32, 48, 64, 80, 96, 112, 128, 144, 160 ...
  18, 36, 54, 72, 90, 108, 126, 144, 162 ...
  Now look for the lowest number that appears in all three lists: 144.
  This means that all three grandchildren will call on the same day every **144 days**. (The lowest number to appear in only two lists is 36, so the earliest that only two grandchildren will call on the same day is after 36 days.)

4 a $756 = 252 \times 3 = 126 \times 2 \times 3 = 63 \times 2 \times 2 \times 3$
$= 7 \times 9 \times 2 \times 2 \times 3$
$= 7 \times 3 \times 3 \times 2 \times 2 \times 3 = 2^2 \times 3^3 \times 7$

b Comparing the two products and looking for the factors that are common we see that $2^2 \times 3^2 = 36$
Hence **36** is the highest common factor.

Listing strategies

1 There are 2 possible outcomes to tossing a coin.
There are 6 possible outcomes to rolling a six-sided dice.
So $2 \times 6 = 12$ different possible outcomes.

2 As the first number cannot be zero, it could be any number from 1 to 9 (i.e. any of 9 numbers). The second number could be any number from 0 to 9 (i.e. any of 10 numbers). For the whole 3-digit number to be divisible by 5 the last number must be 0 or 5 (i.e. 2 numbers).
So total number of 3-digit numbers that could be picked $= 9 \times 10 \times 2 = 180$.

The order of operations in calculations

1 a $10 + 4 \times 2 = 10 + 8 = 18$

b $5 \times 3 - 4 \div 2 = 15 - 2 = 13$

c $(7 - 4)^2 + (8 \div 2)^2 = 3^2 + 4^2 = 9 + 16 = 25$

2 a $2 + 3 \div \frac{1}{3} - 1 = 2 + 9 - 1 = 10$

Note that the division must be performed first and that $3 \div \frac{1}{3} = 3 \times \frac{3}{1} = 9$

b $15 - (4 - 6)^3 = 15 - (-2)^3 = 15 - (-8) = 15 + 8 = 23$

Brackets are worked out first.

c $\sqrt{4 - 3 \times (-7)} = \sqrt{4 + 21} = \sqrt{25} = 5$ or -5

Note that the whole calculation inside the square root sign should be completed first.

Indices

1 a $7^7 \times 7^3 = 7^{7+3} = 7^{10}$

b $3^{-2} \div 3^4 = 3^{(-2-4)} = 3^{-6}$

c $(5^4)^5 = 5^{4\times5} = 5^{20}$

2 a $\frac{5^4 \times 5^6}{5^3} = 5^{4+6-3} = 5^7$

b $6^{(-2)\times3} \div 6^{-3} = 6^{-6} \div 6^{-3} = 6^{-6-(-3)} = 6^{-3}$

c $2^8 \times 2^2 \times 5^4 \times 5^{-7} = 2^{10} \times 5^{-3}$

d $\frac{7^4 \times 7^6 \times 11^3}{11^4} = 7^{4+6} \times 11^{3-4} = 7^{10} \times 11^{-1}$

3 a $16^0 = 1$

b $100^{\frac{1}{2}} = \sqrt{100} = 10$ or -10

c $64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$

d $25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$ (or 0.2) or $-\frac{1}{5}$ (or -0.2)

e $27^{\frac{1}{3}} \times 36^{\frac{1}{2}} = \sqrt[3]{27} \times \sqrt{36} = 3 \times 6 = 18$ or 3×-6
$= -18$

4 $(3^{2x})^2 = 81$
$3^{4x} = 3^4$
The powers on each side must be the same, so
$4x = 4$
$x = 1$

Surds

1 a $\sqrt{3} \times \sqrt{2} = \sqrt{6}$

b $(\sqrt{5})^2 = 5$

c $2 \times 3 \times \sqrt{3} \times \sqrt{3} = 18$

d $(2\sqrt{5})^2 = 2 \times 2 \times \sqrt{5} \times \sqrt{5} = 20$

2 $\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$, hence $a = 2$

3 a $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

b $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$

4 a $\frac{16}{3\sqrt{2}} = \frac{16 \times \sqrt{2}}{3 \times \sqrt{2} \times \sqrt{2}} = \frac{16\sqrt{2}}{3 \times 2} = \frac{8\sqrt{2}}{3}$

b $\frac{18}{4 + \sqrt{7}} \times \frac{4 - \sqrt{7}}{4 - \sqrt{7}} = \frac{72 - 18\sqrt{7}}{16 - 7} = 8 - 2\sqrt{7}$

5 a $(1 + \sqrt{5})(1 - \sqrt{5}) = 1 - \sqrt{5} + \sqrt{5} - 5 = -4$

b $(2 + \sqrt{3})^2 = (2 + \sqrt{3})(2 + \sqrt{3}) = 4 + 2\sqrt{3} + 2\sqrt{3} + 3$
$= 7 + 4\sqrt{3}$

c $(1 + \sqrt{3})(2 + \sqrt{3}) = 2 + \sqrt{3} + 2\sqrt{3} + 3 = 5 + 3\sqrt{3}$

Standard form

1 a $5 \times 10^{-3} = 0.005$

b $5.65 \times 10^5 = 565\,000$

2 a $25\,000 = 2.5 \times 10^4$

b $0.00125 = 1.25 \times 10^{-3}$

c $0.05 \times 10^4 = 5 \times 10^2$

d $14 \times 10^{-3} = 1.4 \times 10^{-2}$

3 a $(3 \times 10^{-2})^2 = 9 \times 10^{-4}$

b $8 \times 10^4 \times 3 \times 10^{-2} = 24 \times 10^2 = 2.4 \times 10^3$

c $8 \times 10^4 \div 4 \times 10^2 = 2 \times 10^2$

d $8 \times 10^4 + 4 \times 10^2 = 10^2 (8 \times 10^2 + 4) = 10^2 \times 804$
$= 8.04 \times 10^4$

4 a 13.3 billion $= 1.33 \times 10^{10}$ pounds

b Number of people $= \frac{1.33 \times 10^{10}}{500} = 1.33 \times 2 \times 10^7$
$= 26\,600\,000$ people

5 a $15\,500 = 1.55 \times 10^4$

b $6.55 \times 10^5 = 655\,000$

c $\frac{1.25 \times 10^5}{2.5 \times 10^2} = \frac{12.5 \times 10^4}{2.5 \times 10^2} = 5 \times 10^2$

d $\sqrt{1.6 \times 10^7} = (1.6 \times 10^7)^{\frac{1}{2}} = 4 \times 10^3$

Converting between fractions and decimals

1 a $\frac{43}{100} = 0.43$

b $\frac{3}{8} = 0.375$

c $\frac{11}{20} = 0.55$

2 a $0.8 = \frac{8}{10} = \frac{4}{5}$

b $0.45 = \frac{45}{100} = \frac{9}{20}$

c $0.584 = \frac{584}{1000} = \frac{73}{125}$

3 a Let $x = 0.\dot{7} = 0.777777...$ and $10x = 7.777777...$
To eliminate the block of recurring digits after the decimal point we take x from $10x$.

$10x - x = 7.777777\ldots - 0.777777\ldots$

$9x = 7$

$x = \frac{7}{9}$

b Let $x = 0.0\dot{4} = 0.0444444\ldots$

So $10x = 0.444444$ and $100x = 4.444444$

Subtract: $100x - 10x = 4.444444\ldots - 0.444444\ldots$

$90x = 4$

$x = \frac{4}{90} = \frac{2}{45}$

c Let $x = 0.\dot{9}5\dot{4} = 0.954545454\ldots$

So $10x = 9.54545454\ldots$ and $1000x = 954.545454\ldots$

Subtract: $1000x - 10x = 954.545454\ldots - 9.54545454\ldots$

$990x = 945$

$x = \frac{945}{990}$

Dividing both parts of fraction by 5 and then by 9 gives $= \frac{21}{22}$

4 a Let $x = 0.\dot{5}1\dot{8} = 0.518518\ldots$

$1000x = 518.518518\ldots$

$1000x - x = 518.518518\ldots - 0.518518\ldots$

$999x = 518$

$x = \frac{518}{999}$

Dividing both parts of fraction by 37 gives $x = \frac{14}{27}$

b $0.76 = \frac{76}{100} = \frac{19}{25}$

5 $0.\dot{7} = \frac{7}{9}$

So $0.\dot{7} + \frac{2}{9} = \frac{7}{9} + \frac{2}{9}$

$= \frac{9}{9}$

$= 1$

6 $\frac{7}{8} = 0.875$

$\frac{4}{5} = 0.8$

$\frac{7}{10} = 0.7$

Putting the decimals in ascending order:

$-0.9, 0.7, 0.8, 0.85, 0.875$

Rewriting using the numbers in their original form:

$-0.9, \frac{7}{10}, \frac{4}{5}, 0.85, \frac{7}{8}$

Converting between fractions and percentages

1 a $25\% = \frac{1}{4}$

b $85\% = \frac{17}{20}$

c $68\% = \frac{17}{25}$

2 Maths: $\frac{65}{80} \times 100 = 81.25\%$

$81.25 > 80$

So Charlie did better at maths.

3 a $\frac{3}{10} = 30\%$

b $\frac{4}{25} = 16\%$

c $\frac{3}{7} = 42.9\%$ (to 3 s.f.)

Fractions and percentages as operators

1 a $\frac{3}{4}$ of £640 $= \frac{3}{4} \times 640 = 3 \times 160 = £480$

b 15% of £30 $= \frac{15}{100} \times 30 = £4.50$

c 195% of 80 kg $= \frac{195}{100} \times 80$

$= 156$ kg

2 Number of boys in School A = 56% of 600

$= \frac{56}{100} \times 600 = 336$

Percentage of boys in School B $= 100 - 35 = 65\%$

Number of boys in School B = 65% of 700

$= \frac{65}{100} \times 700 = 455$

Standard measurement units

1 a $9.7\,\text{kg} = 9.7 \times 1000 = 9700\,\text{g}$

b $850\,\text{cm}^3 = 850\,\text{ml} = 850 \div 1000 = 0.85$ litres

c $2.05\,\text{km} = 2.05 \times 1000\,\text{m} = 2050\,\text{m}$

$= 2050 \times 100\,\text{cm}$

$= 205\,000\,\text{cm}$

2 1 day = 24 hours $= 24 \times 60$ minutes

$= 24 \times 60 \times 60$ seconds $= 86400$ seconds

$= 8.64 \times 10^4$ seconds

3 $0.34 \times 20 \times 12 = £81.60$

Rounding numbers

1 a 1259 = 1260 (correct to 3 s.f.)

b 14.919 = 14.9 (correct to 3 s.f.)

c 0.0003079 = 0.000308 (correct to 3 s.f.)

Remember, you don't start counting the digits for significant figures until the first non-zero digit.

d 9 084 097 = 9 080 000 (correct to 3 s.f.)

e $1.8099 \times 10^{-4} = 1.81 \times 10^{-4}$ (correct to 3 s.f.)

2 a 10.565 = 10.6 (correct to 1 decimal place)

b 123.9765 = 123.977 (correct to 3 decimal places)

c 0.02557 = 0.03 (correct to 2 decimal places)

d 3.9707 = 3.971 (correct to 3 decimal places)

e 0.00195 = 0.002 (correct to 3 decimal places)

f 4.098 = 4.10 (correct to 2 decimal places)

3 a 1989 = 2000 (correct to 1 significant figure)

b 1989 = 2000 (correct to 2 significant figures)

c 1989 = 1990 (correct to 3 significant figures)

4 $3.755 \times 10^{-4} = 0.0003755 = 0.0004$ (correct to 4 decimal places)

Estimation

1 a $\frac{5.9 \times 189}{0.54} \approx \frac{6 \times 200}{0.5} \approx \frac{1200}{0.5} \approx 2400$

Write each number to 1 significant figure.

There are twice as many halves as units in 1200.

b $\sqrt{4.65 + 28.9 \div 6} \approx \sqrt{5 + 30 \div 6} \approx \sqrt{5 + 5} \approx \sqrt{10} \approx 3$

2

	Question	Estimation	Answer
a	$3.45 \times 2.78 \times 0.09$	$\approx 3 \times 3 \times 0.09 \approx 0.8$	A
b	$12.56 \times 1.87 \times 0.45$	$\approx 10 \times 2 \times 0.5 \approx 10$	C
c	$120 \div 0.45$	$\approx 100 \div 0.5 \approx 200$	B
d	$0.01 \times 0.15 \times 109$	$\approx 0.01 \times 0.2 \times 100 \approx 0.2$	B
e	$0.12 \times 300 \times 0.53$	$\approx 0.1 \times 300 \times 0.5 \approx 15$	A
f	$6.07 \times 3.67 \times 0.1$	$\approx 6 \times 4 \times 0.1 \approx 2$	B
g	$20.75 \div 6.98$	$\approx 20 \div 7 \approx 3$	C
h	$0.01 \times 145 \times 35$	$\approx 0.01 \times 100 \times 40 \approx 40$	A
i	$6.5 \times 0.3 \times 0.01$	$\approx 7 \times 0.3 \times 0.01 \approx 0.02$	B
j	$65 \div 1050$	$\approx 70 \div 1000 \approx 0.07$	A

3 a $\sqrt{36} < \sqrt{45} < \sqrt{49}$ so $\sqrt{45}$ is between 6 and 7.
$\sqrt{45} \approx 6.7$ (accept 6.5 to 6.9)

b $\sqrt{100} < \sqrt{104} < \sqrt{121}$ so $\sqrt{104}$ is between 10 and 11.
$\sqrt{104} \approx 10.2$ (accept 10.1 to 10.3)

c $(2)^3 < (2.3)^3 < (3)^3$ so $(2.3)^3$ is between 8 and 27.
$(2.3)^3 \approx 12$ (accept 10 to 14)

d $3^1 < 3^{1.4} < 3^2$ so $3^{1.4}$ is between 3 and 9.
$3^{1.4} \approx 5$ (accept 4 to 6)

Upper and lower bounds

1 a Half of the smallest unit is 0.5 cm
Lower bound = least length = 144.5 cm

b Upper bound = greatest length = 145.5 cm

c $144.5 \le l < 145.5$ cm

Review it!

1 a 24 647.515

b 21.5443469 = 21.5 (1 d.p.)

2 a $8 - 1.5 = 6.5$

b $-1 + 4 + 1 = 4$

c $27 \times 3 = 81$

d $1.65 \times 3.6 = 4 \times 1.65 \times 9 \div 10 = 6.6 \times 9 \div 10 = (66 - 6.6) \div 10 = 59.4 \div 10 = 5.94$

3 a $\frac{64}{12} = \frac{16}{3} = 5\frac{1}{3}$

b $\frac{124}{13} = 9\frac{7}{13}$

4 $4\frac{3}{4} + \frac{1}{2}$

Use the largest integer as the whole number and the next largest as the numerator.

5 a Half the smallest unit is 0.005 cm.
Least width = 4.545 cm and least length = 2.225 cm
Least area = $4.545 \times 2.225 = 10.112625$ cm² = 10.113 cm² (3 d.p.)
Greatest width = 4.555 cm and greatest length = 2.235 cm
Greatest area = $4.555 \times 2.235 = 10.180425$ cm² = 10.180 cm² (3 d.p.)

b The upper and lower bounds for area are the same when written to the nearest cm² (5 and 2).
Area = $5 \times 2 = 10$ cm²

6 a $6.02 \times 10^{23} \div 18 \times 10 = 3.34 \times 10^{23}$ molecules (3 s.f.)

b 18 g = 0.018 kg
$0.018 \div 6.02 \times 10^{23} = 2.99 \times 10^{-26}$ kg (3 s.f.)

7 $(0.45 \times 0.78)^2 \approx (0.5 \times 0.8)^2 \approx 0.4^2 \approx 0.16$

8 a $7^0 = 1$

b $9^{\frac{1}{2}} = \sqrt{9} = 3$ or -3

c $8^{-2} = \frac{1}{8^2} = \frac{1}{64}$

d $64^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$ or $-\frac{1}{4}$

9 Half the smallest unit = $0.1 \div 2 = 0.05$
Upper bound = $5.6 + 0.05 = 5.65$
Lower bound = $5.6 - 0.05 = 5.55$
Error interval is $5.55 \le y < 5.65$

10 $\frac{1-\sqrt{2}}{1+\sqrt{2}} = \frac{1-\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-2\sqrt{2}+2}{1-2} = \frac{3-2\sqrt{2}}{-1} = 2\sqrt{2} - 3$

11 Upper bound = 112.5 cm and lower bound = 111.5 cm
Error interval is $111.5 \le a < 112.5$

12 Let $x = 0.\dot{7}\dot{2} = 0.727272...$
So $100x = 72.727272...$
$100x - x = 72.727272... - 0.727272...$
$99x = 72$
$x = \frac{72}{99} = \frac{8}{11}$

13 a a: upper bound = 0.67545, lower bound = 0.67535
b: upper bound = 2.345, lower bound = 2.335
Greatest value of $c = \frac{\text{Upper bound of } a}{\text{Lower bound of } b} = \frac{0.67545}{2.335}$
= 0.289272
Least value of $c = \frac{\text{Lower bound of } a}{\text{Upper bound of } b} = \frac{0.67535}{2.345}$
= 0.287996
Error interval is $0.287996 \le c < 0.289272$

b The value of c is the same when rounded to 2 significant figures.
So $c = 0.29$ (to 2 s.f.)

14 a $\frac{3}{25} \div \frac{9}{50} = \frac{3}{25} \times \frac{50}{9} = \frac{1}{1} \times \frac{2}{3} = \frac{2}{3}$

b $25 \div \frac{5}{16} = \frac{25}{1} \times \frac{16}{5} = \frac{5}{1} \times \frac{16}{1} = 80$

15 a $0.00000045 = 4.5 \times 10^{-7}$

b 12 million = 12 000 000 = 1.2×10^7

c $5640 = 5.64 \times 10^3$

16 $(8 \times 10^{-5}) \times (4 \times 10^3) = 8 \times 4 \times 10^{-5} \times 10^3 = 32 \times 10^{-2} = 3.2 \times 10^{-1}$

17 a Factors of 64: 1, 2, 4, 8, 16, 32, 64

b Factors of 100: 1, 2, 4, 5, 10, 20, 25, 50, 100
Highest common factor = 4

Algebra

Simple algebraic techniques

1 a Formula

b Identity

c Expression

d Identity

e Equation

2 **a** $15x^2 - 4x + x^2 + 9x - x - 6x^2 = 10x^2 + 4x$

b $7a + 5b - b - 4a - 5b = 3a - b$

c $8yx + 5x^2 + 2xy - 8x^2 = -3x^2 + 10xy$ (or $10xy - 3x^2$)

d $x^3 + 3x - 5 + 2x^3 - 4x = 3x^3 - x - 5$

3 $P = I^2R = \left(\frac{2}{3}\right)^2 \times 36 = \frac{4}{9} \times 36 = 16$

4 $v = u + at = 20 + (-8)(2) = 20 - 16 = 4$

Removing brackets

1 **a** $2x + 8$

b $63x + 21$

c $-1 + x$ or $x - 1$

d $3x^2 - x$

e $3x^2 + 3x$

f $20x^2 - 8x$

2 **a** $2(x + 3) + 3(x + 2) = 2x + 6 + 3x + 6 = 5x + 12$

b $6(x + 4) - 3(x - 7) = 6x + 24 - 3x + 21 = 3x + 45$

c $3x^2 + x + x^2 + x = 4x^2 + 2x$

d $3x^2 - 4x - 6x + 8 = 3x^2 - 10x + 8$

3 **a** $p^{-\frac{4}{3}} \times p^{\frac{8}{3}} = p^{-\frac{4}{3} + \frac{8}{3}} = p^{\frac{4}{3}}$

b $\frac{r^5 \times r^{-2}}{r^4} = \frac{r^{5-2}}{r^4}$

$= \frac{r^3}{r^2}$

$= r^{3-2}$

$= r^{-1}$

c $\left(b^{\frac{3}{2}}\right)^{-4} = b^{\frac{3}{2} \times (-4)}$

$= b^{-6}$

4 **a** $(t + 3)(t + 5) = t^2 + 5t + 3t + 15 = t^2 + 8t + 15$

b $(x - 3)(x + 3) = x^2 + 3x - 3x - 9 = x^2 - 9$

c $(2y + 9)(3y + 7) = 6y^2 + 14y + 27y + 63$

$= 6y^2 + 41y + 63$

d $(2x - 1)^2 = (2x - 1)(2x - 1) = 4x^2 - 2x - 2x + 1$

$= 4x^2 - 4x + 1$

5 **a** $(x + 7)(x + 2)(2x + 3) = (x^2 + 9x + 14)(2x + 3)$

$= 2x^3 + 21x^2 + 55x + 42$

b $(2x - 1)(3x - 2)(4x - 3) = (6x^2 - 7x + 2)(4x - 3)$

$= 24x^3 - 28x^2 + 8x - 18x^2 + 21x - 6$

$= 24x^3 - 46x^2 + 29x - 6$

Factorising

1 **a** $24t + 18 = 6(4t + 3)$

b $9a - 2ab = a(9 - 2b)$

c $5xy + 15yz = 5y(x + 3z)$

d $24x^3y^2 + 6xy^2 = 6xy^2(4x^2 + 1)$

2 **a** $(x + 7)(x + 3)$

b $(x + 5)(x - 3)$

c To get 6, use factors 2 and 3, and to get 10 use factors 2 and 5. This gives $2x \times 2 = 4x$ and $3x \times 5 = 15x$, total $19x$; so solution is $(2x + 5)(3x + 2)$

d Difference of two squares. Factorises to $(2x + 7)(2x - 7)$

e $25x^2 - 7 = (5x + \sqrt{7})(5x - \sqrt{7})$

3 $\frac{2x^2 + 7x - 4}{x^2 + 7x + 12} = \frac{(2x - 1)(x + 4)}{(x + 3)(x + 4)}$

$= \frac{2x - 1}{x + 3}$

4 $\frac{1}{x - 7} - \frac{x + 10}{2x^2 - 11x - 21} = \frac{1}{x - 7} - \frac{x + 10}{(2x + 3)(x - 7)}$

Factorise the denominator of the 2nd fraction.

Make both denominators the same.

$= \frac{2x + 3}{(x - 7)(2x + 3)} - \frac{x + 10}{(2x + 3)(x - 7)}$

Combine into one fraction and simplify

$= \frac{2x + 3 - x - 10}{(x - 7)(2x + 3)}$

$= \frac{x - 7}{(x - 7)(2x + 3)}$

$= \frac{1}{2x + 3}$

Changing the subject of a formula

1 **a** $A = \pi r^2$ $\quad \frac{A}{\pi} = r^2$

$r = \sqrt{\frac{A}{\pi}}$

b $A = 4\pi r^2$

$\frac{A}{4\pi} = r^2$

$r = \sqrt{\frac{A}{4\pi}}$

c $V = \frac{4}{3}\pi r^3$

$3V = 4\pi r^3$

$\frac{3V}{4\pi} = r^3$

$r = \sqrt[3]{\frac{3V}{4\pi}}$

2 **a** $y = mx + c$ $\quad (c)$ — Subtract mx from both sides.

$c = y - mx$

b $v = u + at$ $\quad (a)$ — Subtract u from both sides.

$v - u = at$ — Divide both sides by t.

$a = \frac{v - u}{t}$

c $v^2 = 2as$ $\quad (s)$ — Divide both sides by $2a$.

$s = \frac{v^2}{2a}$

d $v^2 = u^2 + 2as$ $\quad (u)$ — Subtract $2as$ from both sides.

$v^2 - 2as = u^2$ — Square root both sides.

$u = \sqrt{v^2 - 2as}$

e $s = \frac{1}{2}(u + v)t$ $\quad (t)$ — Multiply both sides by 2.

$2s = (u + v)t$ — Divide both sides by $(u + v)$.

$t = \frac{2s}{u + v}$

f $\frac{1}{x} = \frac{1}{2p} + \frac{1}{q}$

$2pq = xq + 2px$

$x(2p + q) = 2pq$

$x = \frac{2pq}{2p + q}$

Solving linear equations

1 **a** $x - 7 = -4$

$x = -4 + 7 = 3$

b $9x = 27$

$x = 27 \div 9 = 3$

c $\frac{x}{5} = 4$

$x = 4 \times 5 = 20$

2 a $3x + 1 = 16$

$3x = 15$

$x = 5$

b $\frac{2x}{3} = 12$

$2x = 36$

$x = 18$

c $\frac{3x}{5} + 4 = 16$

$\frac{3x}{5} = 12$

$3x = 60$

$x = 20$

3 a $5(1 - x) = 15$

$5 - 5x = 15$

$-5x = 10$

$x = -2$

b $2m - 4 = m - 3$

$m - 4 = -3$

$m = 1$

c $9(4x - 3) = 3(2x + 3)$

$36x - 27 = 6x + 9$

$30x - 27 = 9$

$30x = 36$

$x = \frac{36}{30} = \frac{6}{5}, 1\frac{1}{5}$ or 1.2

Always cancel fractions so that they are in their lowest terms. Here both top and bottom can be divided by 6.

Solving quadratic equations using factorisation

1 a $(x + 3)(x + 2) = 0$ giving $x = -2$ or -3

b $(x + 3)(x - 4) = 0$ giving $x = -3$ or 4

c $(2x + 7)(x + 5) = 0$ giving $x = -\frac{7}{2}$ or -5

2 a Area of triangle = $\frac{1}{2} \times$ base $\times$ height

$\frac{1}{2}(2x + 3)(x + 4) = 9$

$2x^2 + 11x + 12 = 18$

$2x^2 + 11x - 6 = 0$

b $2x^2 + 11x - 6 = 0$

$(2x - 1)(x + 6) = 0$

So $x = \frac{1}{2}$ or $x = -6$

Since x represents a height, only the positive value is valid.

$x = \frac{1}{2}$

c $x = 0.5$, so base is $2 \times 0.5 + 3 = 4$ cm and height is $0.5 + 4 = 4.5$ cm

3 By Pythagoras' theorem

$(x + 1)^2 + (x + 8)^2 = 13^2$

$x^2 + 2x + 1 + x^2 + 16x + 64 = 169$

$2x^2 + 18x - 104 = 0$

Dividing by 2 gives

$x^2 + 9x - 52 = 0$

$(x - 4)(x + 13) = 0$

so $x = 4$ or $x = -13$

(Disregard $x = -13$ as x is a length.)

Hence, $x = 4$ cm

(This also means the sides of the triangle are 5, 12 and 13 cm.)

Solving quadratic equations using the formula

1 Comparing the equation given, with $ax^2 + bx + c$ gives $a = 2$, $b = -1$ and $c = -7$.

Substituting these values into the quadratic equation formula gives:

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-7)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{57}}{4}$$

$$= \frac{1 + 7.550}{4} \text{ or } \frac{1 - 7.550}{4} \text{ (to 4 s.f.)}$$

$x = 2.14$ or -1.64 (to 3 s.f.)

2 a $\frac{2x + 3}{x + 2} = 3x + 1$

$2x + 3 = (x + 2)(3x + 1)$

$2x + 3 = 3x^2 + x + 6x + 2$

$0 = 3x^2 + 5x - 1$

or $3x^2 + 5x - 1 = 0$

b Comparing the equation given, with $ax^2 + bx + c = 0$ gives $a = 3$, $b = 5$ and $c = -1$

Substituting these values into the quadratic equation formula gives:

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-5 \pm \sqrt{25 + 12}}{6} = \frac{-5 \pm \sqrt{37}}{6} = \frac{-5 + \sqrt{37}}{6} \text{ or } \frac{-5 - \sqrt{37}}{6}$$

Hence $x = 0.18$ or -1.85 (2 d.p.)

Solving simultaneous equations

1 a Firstly write the second equation so that in both equations the x value and the numerical value are aligned.

$y = 3x - 7$ (1)

$y = -2x + 3$ (2)

Notice that the coefficient of y (the number multiplying y, i.e. 1) is the same for both equations. We can eliminate y by subtracting equation (2) from equation (1).

Subtracting (1) − (2) we obtain

$0 = 5x - 10$

$5x = 10$

$x = 2$

Substituting $x = 2$ into equation (1) we obtain

$y = 3(2) - 7$

$= 6 - 7$

$= -1$

Checking by substituting $x = 2$ into equation (2) we obtain

$y = -2x + 3$

$= -2(2) + 3$

$= -1$

Hence solutions are $x = 2$ and $y = -1$.

b $y = 2x - 6$ (1)

$y = -3x + 14$ (2)

Subtracting (1) − (2) we obtain

$0 = 5x - 20$

$x = 4$

$y = 2 \times 4 - 6$ (1)

$y = 2.$

2 Equating expressions for y gives

$10x^2 - 5x - 2 = 2x - 3$

$10x^2 - 7x + 1 = 0$

Factorising this quadratic gives

$(5x - 1)(2x - 1) = 0$

Hence $x = \frac{1}{5}$ or $x = \frac{1}{2}$

Substituting $x = \frac{1}{5}$ into $y = 2x - 3$ gives

$y = -2\frac{3}{5}$

Substituting $x = \frac{1}{2}$ into $y = 2x - 3$ gives

$y = -2$

Hence $x = \frac{1}{5}$ and $y = -2\frac{3}{5}$ or $x = \frac{1}{2}$ and $y = -2$

3 Equating the y values gives

$x^2 + 5x - 4 = 6x + 2$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or -2

When $x = 3$, $y = 6 \times 3 + 2 = 20$

When $x = -2$, $y = 6 \times (-2) + 2 = -10$

Points are (3, 20) and (−2, −10)

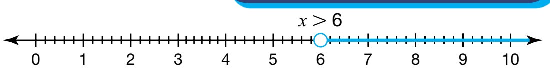
Solving inequalities

1 a $1 - 2x < -11$

$-2x < -12$

$x > 6$

Subtract 1 from both sides. Divide both sides by −2 and reverse sign.



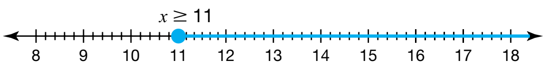
$\{x: x > 6\}$

b $2x - 7 \geq 15$

$2x \geq 22$

$x \geq 11$

Add 7 to both sides. Divide both sides by 2.



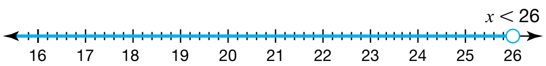
$\{x: x \geq 11\}$

c $\frac{x-5}{3} < 7$

$x - 5 < 21$

$x < 26$

Multiply both sides by 3. Add 5 to both sides.



$\{x: x < 26\}$

2 a $2x - 4 > x + 6$

$x - 4 > 6$

$x > 10$

b $4 + x < 6 - 4x$

$4 < 6 - 5x$

$-2 < -5x$

$\frac{-2}{-5} > x$

$x < 0.4$ or $\frac{2}{5}$

c $2x + 9 \geq 5(x - 3)$

$2x + 9 \geq 5x - 15$

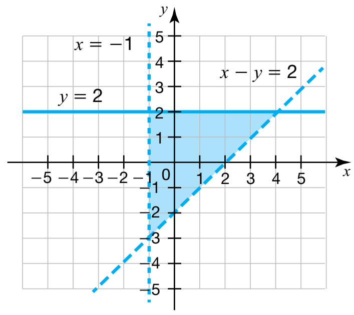
$9 \geq 3x - 15$

$24 \geq 3x$

$8 \geq x$

or $x \leq 8$

3 a



b (0, 2), (0,1), (0, 0), (0, −1), (1, 2), (1, 1), (1, 0), (2, 2), (2, 1), (3, 2)

4 $x^2 > 3x + 10$

$x^2 - 3x - 10 > 0$

$(x - 5)(x + 2) > 0$

$x < -2$ and $x > 5$

Problem solving using algebra

1 Let the larger number $= x$ and the smaller number $= y$.

$x + y = 77$

$x - y = 25$

Adding gives $2x = 102$ which gives $x = 51$ so y must be 26.

2 Let the number added $= x$

$\frac{15 + x}{31 + x} = \frac{5}{6}$

$6(15 + x) = 5(31 + x)$

$90 + 6x = 155 + 5x$

$x = 65$

Check the answer $\frac{15 + 65}{31 + 65} = \frac{80}{96} = \frac{5}{6}$

3 Perimeter: $2x + 2y = 24$ so $x + y = 12$ (1)

Area: $xy = 27$ (2)

From equation (1) $y = 12 - x$

Substitute into equation (2):

$x(12 - x) = 27$

So, $12x - x^2 = 27$

Hence, $x^2 - 12x + 27 = 0$

Factorising gives $(x - 3)(x - 9) = 0$

So $x = 3$ or $x = 9$

Substituting each of these values into equation (1) we have

$3 + y = 12$ or $9 + y = 12$, giving $y = 9$ or $y = 3$.

Hence, length = 9 cm and width = 3 cm.

Use of functions

1 **a** Divide by 5, then subtract 4: $\frac{x}{5} - 4$

b Add 4, then multiply by 5: $5(x + 4)$

2 **a** **i** $3 - 1 = 2$; reciprocal of $2 = \frac{1}{2}$

ii $1\frac{1}{4} - 1 = \frac{1}{4}$; reciprocal of $\frac{1}{4} = 4$

b reciprocal of $-3 = \frac{-1}{3}$; $\frac{-1}{3} + 1 = \frac{2}{3}$

c Subtract 1, then find the reciprocal: $\frac{1}{x - 1}$

3 **a** $f(0) = \frac{1}{0 - 1} = -1$

b $f(-\frac{1}{2}) = \frac{1}{-\frac{1}{2} - 1} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$

c Let $y = \frac{1}{x - 1}$

$y(x - 1) = 1$

$xy - y = 1$

$xy = y + 1$

$x = \frac{y + 1}{y}$

$f^{-1}(x) = \frac{x + 1}{x}$

4 **a** $fg(x) = \sqrt{((x + 4)^2 - 9)}$

$= \sqrt{x^2 + 8x + 16 - 9}$

$= \sqrt{x^2 + 8x + 7}$

b $gf(x) = \sqrt{(x^2 - 9)} + 4$

c $gf(3) = \sqrt{(3^2 - 9)} + 4$

$= 4$

Iterative methods

1

x	$3x^3 - 2x^2 - 3$
0	−3
0.1	−2.977
0.2	−2.896
0.3	−2.739
0.4	−2.488
0.5	−2.125
0.6	−1.632
0.7	−0.991
0.8	−0.184
0.9	0.807
0.81	−0.093
0.82	−0.001
0.83	0.093

$x = 0.8$

or

x	$3x^3 - 2x^2 - 3$
0	−3
1	2
0.5	−2.125
0.75	−0.609
0.85	0.287

$x = 0.8$

2 $x_0 = 1.5$

$x_1 = 1.5182945$

$x_2 = 1.5209353$

$x_3 = 1.5213157$

$x_4 = 1.5213705 \approx 1.521$ (correct to three decimal places)

Check value of $x^3 - x - 2$ for $x = 1.5205, 1.5215$

x	$f(x)$
1.5205	−0.005225
1.5215	0.0007151

Since there is a change of sign, $a = 1.521$ is correct to three decimal places.

Equation of a straight line

1 **a** $2y = 4x - 5$

$y = 2x - \frac{5}{2}$

Comparing this to $y = mx + c$ we have gradient $m = 2$

b Gradient $= -\frac{1}{m} = -\frac{1}{2}$

c $y = -\frac{1}{2}x + 5$ (or $2y = -x + 10$)

2 $y - y_1 = m(x - x_1)$ where $m = 3$ and $(x_1, y_1) = (2, 3)$.

$y - 3 = 3(x - 2)$

$y - 3 = 3x - 6$

$y = 3x - 3$

3 $y - y_1 = m(x - x_1)$ where $m = 2$ and $(x_1, y_1) = (-1, 0)$

$y - 0 = 2(x - (-1))$

$y = 2(x + 1)$

$y = 2x + 2$

$-y + 2x + 2 = 0$ (or $2x - y + 2 = 0$)

4 **a** Gradient $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{6 - (-2)} = \frac{4}{8} = \frac{1}{2}$

b $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 6}{2}, \frac{0 + 4}{2}\right) = (2, 2)$

c **i** Gradient $= -2$ (i.e invert $\frac{1}{2}$ and change the sign)

ii $y - y_1 = m(x - x_1)$

$y - 2 = -2(x - 2)$

$y - 2 = -2x + 4$

$y = -2x + 6$

Quadratic graphs

1 **a** $2x^2 - 12x + 1 = 2\left[x^2 - 6x + \frac{1}{2}\right]$

$= 2\left[(x - 3)^2 - 9 + \frac{1}{2}\right]$

$= 2\left[(x - 3)^2 - \frac{17}{2}\right]$

$= 2(x - 3)^2 - 17$

b **i** Turning point is at $(3, -17)$

ii At the roots,

$2(x - 3)^2 - 17 = 0$

$2(x - 3)^2 = 17$

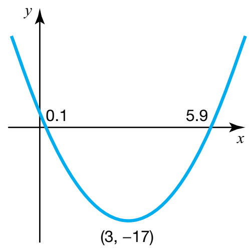
$(x - 3)^2 = \frac{17}{2}$

$x - 3 = \sqrt{\frac{17}{2}}$

$x = \sqrt{\frac{17}{2}} + 3$

Roots are $x = 0.1$ and $x = 5.9$ (1 d.p.)

c



2 a $y = (x + 1)(x - 5)$ or $y = x^2 - 4x - 5$

b $y = -(x - 2)(x - 7)$ or $y = -x^2 + 9x - 14$

3 a $x^2 + 12x - 16 = (x + 6)^2 - 36 - 16$

$= (x + 6)^2 - 52$

b Turning point is at $(-6, -52)$

Recognising and sketching graphs of functions

1 a B

b F

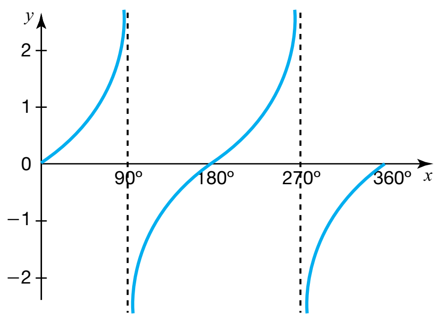
c E

d A

e D

f C

2 a



b Read up from 60° to the graph, then read across until you hit the graph again.

$x = 240°$

3 a A

b G

c F

d E

Translations and reflections of functions

1 a (3, 5) (i.e. a movement of one unit to the right)

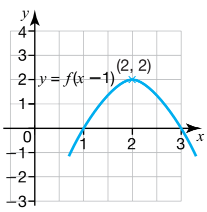
b (−1, 5) (i.e. a movement of three units to the left)

c (2, −5) (i.e. a reflection in the x-axis)

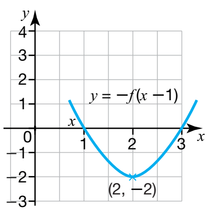
d (−2, 5) (i.e. a reflection in the y-axis)

2

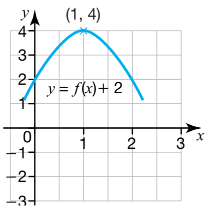
a



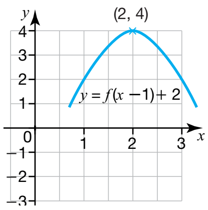
c



b



d



Equation of a circle and tangent to a circle

1 a Centre is (0, 0)

b radius $= \sqrt{49} = 7$

2 a $x^2 + y^2 = 100$

b Gradient of radius to (8, 6) $= \frac{6}{8} = \frac{3}{4}$

Gradient of tangent $= -\frac{4}{3}$

c $y - y_1 = m(x - x_1)$

$y - 6 = -\frac{4}{3}(x - 8)$

$y = -\frac{4}{3}x + 16\frac{2}{3}$ (or $3y = -4x + 50$)

Real-life graphs

1 a 08:00 to 09:00 is 1 hour (h), which is 1 unit on x axis.

Average speed = gradient $= \frac{2.5}{0.5} = 5$ km/h

b 15 mins = 0.25 hours

c Average speed = gradient between 09:30 and 09:45

$= \frac{6}{0.25} = 24$ km/h

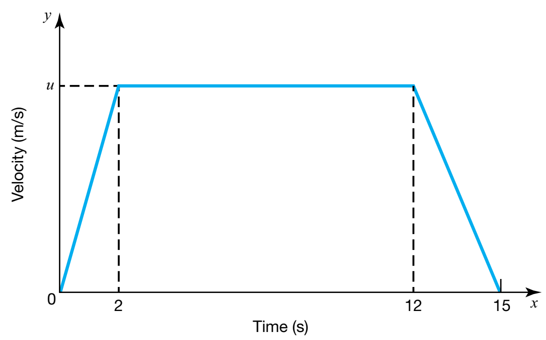
2

NAIL IT!

When drawing a velocity-time graph, ensure that the axes are labelled with quantities and units.

Any values and letters for quantities that need to be found should be labelled on the graph.

a



b Total distance travelled = Area under the velocity-time graph

Use the formula for the area of a trapezium:

Distance $= \frac{1}{2}(15 + 10) \times u$

$= 12.5u$

Use the formula for the area of a trapezium.

The total distance travelled = 50 m

Hence $50 = 12.5u$

$u = 4$ m/s

c Velocity = 4 m/s and time for deceleration = 3 s

Deceleration = gradient $= \frac{4}{3} = 1.33$ m/s²

Since deceleration is negative acceleration, a positive answer is appropriate.

Generating sequences

1 **a** 17: sequence goes up by 3

b 3.0: sequence goes up by 0.2

c −12: sequence goes down by 3

d 432: last term is multiplied by 6

e $\frac{1}{48}$: last term is multiplied by $\frac{1}{2}$

f $-\frac{1}{16}$: last term is multiplied by $-\frac{1}{2}$

2 Second term is $(-4)^2 + 1 = 17$ and third term is $17^2 + 1 = 290$

Second term is 17, third term is 290.

3 Reverse the process: to find the preceding term, subtract 1 and halve.

Second term is $(12 - 1) \div 2 = \frac{11}{2} = 5.5$

First term is $(5.5 - 1) \div 2 = \frac{4.5}{2} = 2.25$

First term is 2.25, second term is 5.5

The nth term

1 **a** When $n = 1$, $50 - 3(1) = 47$

When $n = 2$, $50 - 3(2) = 44$

When $n = 3$, $50 - 3(3) = 41$

First three terms are 47, 44, 41

b Use the nth term formula to find the value of n when the nth term = 34

$50 - 3n = 34$

$3n = 16$

$n = 16 \div 3$

The value of n is not an integer so 34 is not a number in the sequence.

c Use the nth term formula to find the value of n when the nth term is less than zero (i.e. negative).

$50 - 3n < 0$ (subtracting 50 from both sides)

$-3n < -50$ (dividing both sides and reversing the inequality sign)

$n > \frac{50}{3}$

$n > 16\frac{2}{3}$

As n has to be an integer, its lowest possible value is $n = 17$.

Check that you get a negative term when $n = 17$ is put back into the nth term formula.

17th term $= 50 - 3 \times 17 = 50 - 51 = -1$

2 **a** The first four terms are: $2 \times 3^1, 2 \times 3^2, 2 \times 3^3, 2 \times 3^4$

$= 6, 18, 54, 162$

b As the nth term formula is 2×3^n both 2 and 3 are factors, so 6 must also be a factor.

3 **a** By inspection: nth term $= \frac{1}{n}$

b By inspection: nth term $= \frac{1}{(\sqrt{2})^n}$ or $\left(\frac{1}{\sqrt{2}}\right)^n$

4 **a** Common difference between terms = 2 so formula will start with $2n$.

When $n = 1$, you need to subtract 3 from $2n$ to get an answer of −1.

Therefore nth term $= 2n - 3$

b $59 = 2x - 3$

$2x = 62$

$x = 31$

5

	4,		17,		38,		67
First differences		13		21		29	
Second differences			8		8		

As a second difference is needed before a constant difference is found, there is an n^2 term in the nth term. The number in front of this n^2 will be $\frac{8}{2} = 4$.

So first part of the nth term will be $4n^2$.

n	1	2	3	4
Term	4	17	38	67
$4n^2$	4	16	36	64
Term − $4n^2$	0	1	2	3

Use this set of information to work out the linear part of the sequence (the part with an n term and a number).

Difference 1 1 1

This means that the linear sequence will start with n.

When $n = 2$, 'Term − $4n^2$' is 1, not 2, so if n is in the term you also need to subtract 1.

This makes the linear part of the sequence $n - 1$.

Check it with a different value of n. When $n = 3$, $n - 1$ equals 2. This is the correct value for 'Term − $4n^2$'.

Combining the terms gives nth term $= 4n^2 + n - 1$

Arguments and proofs

1 **a** $2n$ is always even as it has 2 as a factor. Adding 1 to an even number always gives an odd number. The statement is true.

b $x^2 - 9 = 0$ so $x^2 = 9$ and $x = \sqrt{9} = \pm 3$

The statement is false.

c n could be a decimal such as 4.25 so squaring it would not give an integer.

The statement is false.

d If n was 1, or a fraction smaller than 1, this would not be true.

The statement is false.

2 Let the consecutive integers be n, $n + 1$, $n + 2$ and $n + 3$, where n is an integer that can be either odd or even.

Sum of the integers $= n + n + 1 + n + 2 + n + 3$

$= 4n + 6 = 2(2n + 3)$

As 2 is a factor of this expression, the sum of four consecutive integers must be a multiple of 2, and therefore even.

3 Let the consecutive integers be x, $x + 1$ and $x + 2$, where x is an integer that can be either odd or even.

Sum of the integers $= x + x + 1 + x + 2 = 3x + 3$

$= 3(x + 1)$

As 3 is a factor of this expression, the sum of three consecutive integers must be a multiple of 3.

4 **a** The numerator is larger than the denominator so the fraction will always be greater than 1. The statement is false.

b As a is larger than b, squaring a will result in a larger number than squaring b. Hence $a^2 > b^2$ so the statement is false.

c The square root of a number can have two values, one positive and the other negative so, this statement is false.

Review it!

1 **a** $-3(3x - 4) = -9x + 12$

b $4x + 3(x + 2) - (x + 2) = 4x + 3x + 6 - x - 2$

$= 6x + 4$

c $(x + 3)(2x - 1)(3x + 5) = (2x^2 + 5x - 3)(3x + 5)$

$= 6x^3 + 25x^2 + 16x - 15$

2 **a** $2x^2 + 7x - 4 = (2x - 1)(x + 4)$

b $2x^2 + 7x - 4 = 0$

$x = \frac{1}{2}$ or $x = -4$

3 **a** $(2x^2y)^3 = 8x^6y^3$

b $2x^{-3} \times 3x^4 = 6x$

c $\frac{15a^3b}{3a^3b^2} = \frac{5}{b}$

4 $3x + 2y = 8$ (1)

$5x + y = 11$ (2)

(2) × 2: $10x + 2y = 22$ (3)

(3) − (1): $7x = 14$

$x = 2$

Substitute into (2) to find y

$5 \times 2 + y = 11$

$y = 11 - 10$

$y = 1$

5 **a** $\frac{3}{x + 7} = \frac{2 - x}{x + 1}$

$3(x + 1) = (2 - x)(x + 7)$

$3x + 3 = 2x + 14 - x^2 - 7x$

$3x + 3 = -x^2 - 5x + 14$

$x^2 + 8x - 11 = 0$

b $x^2 + 8x - 11 = 0$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4 \times 1 \times (-11)}}{2 \times 1}$

$x = \frac{-8 \pm \sqrt{108}}{2}$

$x = \frac{-8 \pm 6\sqrt{3}}{2}$

$x = -4 + 3\sqrt{3}$ or $x = -4 - 3\sqrt{3}$

So $x = 1.20$ or $x = -9.20$ (to 2 d.p.)

6 $\frac{3y - x}{z} = ax + 2$ $\quad (x)$

$3y - x = z(ax + 2)$

$3y - x = axz + 2z$

$3y - 2z = axz + x$

$3y - 2z = x(az + 1)$

$x = \frac{3y - 2z}{az + 1}$

7 **a** $y = \frac{x}{3} + 5$

$3y = x + 15$

$x = 3y - 15$

Now replace x with $f^{-1}(x)$ and y with x.

$f^{-1}(x) = 3x - 15$ or $f^{-1}(x) = 3(x - 5)$

b $fg(x) = \frac{(2(x)^2 + k)}{3} + 5$

So $fg(2) = \frac{(8 + k)}{3} + 5$

We know that $fg(2) = 10$

So $\frac{(8 + k)}{3} + 5 = 10$

$(8 + k) + 15 = 30$

$8 + k = 15$

$k = 7$

8 **a** Let $n = 1$: $30 - 4 \times 1 = 26$

Let $n = 2$: $30 - 4 \times 2 = 22$

Let $n = 3$: $30 - 4 \times 3 = 18$

First three terms are 26, 22, 18.

b $30 - 4n < 0$

$-4n < -30$

$n > \frac{-30}{-4}$

$n > 7.5$

n must be an integer, so the lowest possible value of n is $n = 8$

Therefore the first negative term of the sequence is: $30 - 4 \times 8 = -2$

9 $x = 4, y = 3$

$(4)^2 + (3)^2 = 16 + 9 = 25$

So $x^2 + y^2 > 21$

Hence the point (4, 3) lies outside the circle.

10 $(\sqrt{x} + \sqrt{9y})(\sqrt{x} - 3\sqrt{y})$

Simplify terms inside the brackets if possible

$(\sqrt{x} + 3\sqrt{y})(\sqrt{x} - 3\sqrt{y}) = x + 3\sqrt{xy} - 3\sqrt{xy} - 9y$

$= x - 9y$

11 **a** $2x^2 + 8x + 1 = 2(x^2 + 4x) + 1$

$= 2(x + 2)^2 - 8 + 1$

$= 2(x + 2)^2 - 7$

b **i** Turning point is $(-2, -7)$.

ii For $2x^2 + 8x + 1 = 0$

$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 2 \times 1}}{2 \times 2}$

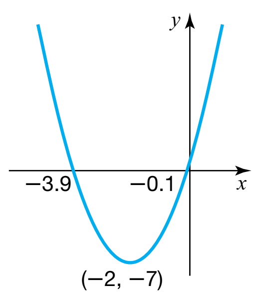
$x = \frac{-8 \pm \sqrt{56}}{4}$

$x = \frac{-8 \pm 2\sqrt{14}}{4}$

$x = \frac{-4 \pm \sqrt{14}}{2}$

So roots are at $x = -3.9$ and $x = -0.1$ (1 d.p.)

c



12 Perimeter of *ABCD* $= 2 \times (4x + (2x - 3)) = 12x - 6$

Perimeter of *EFG* $= 2x - 1 + x + 9 + 5x - 2 = 8x + 6$

Equate the perimeters to find x

$12x - 6 = 8x + 6$

$4x = 12$

$x = 3$

The height of the triangle, $EF = 2 \times 3 - 1 = 5$ cm

The base of the triangle, $EG = 3 + 9 = 12$ cm

Area of the triangle $= \frac{1}{2} \times 12 \times 5 = 30\,\text{cm}^2$

13 Side AB is parallel to side CD, so $k = 5$.

Gradient of $BD = \frac{5-(-2)}{-1-(-2)} = \frac{7}{1} = 7$

Using point B $(-2, -2)$

$y - (-2) = 7(x - (-2))$

$y + 2 = 7x + 14$

Equation of BD is $y = 7x + 12$

14 $x^2 + y^2 = 4$ (1)

$2y - x = 2$ (2)

Rearrange (2) for y: $y = \frac{1}{2}x + 1$ (3)

Substitute (3) into (1):

$x^2 + \left(\frac{1}{2}x + 1\right)^2 = 4$

$x^2 + \frac{x^2}{4} + x + 1 = 4$

$5x^2 + 4x + 4 = 16$

$5x^2 + 4x - 12 = 0$

$(5x - 6)(x + 2) = 0$

$x = \frac{6}{5} = 1.2$ or $x = -2$

Substitute into (2) to find y

So $x = \frac{6}{5}, y = \frac{8}{5}$ or $x = -2, y = 0$

Ratio, proportion and rates of change

Introduction to ratios

1 **a** 2 : 6 = 1: 3 (divide both sides by 2)

b 25 : 60 = 5 : 12 (divide both sides by 5)

c 1.6 : 3.6 = 4 : 9 (divide both sides by 0.4, or multiply by ten then divide by four)

2 **a** 250 g : 2 kg = 250 g : 2000 g = 250 : 2000 = 1 : 8

b 25 m : 250 mm = 25 000 mm : 250 mm
= 25 000 : 250 = 100 : 1

c 2 cl : 1 l = 2 cl : 100 cl = 2 : 100 = 1 : 50

3 Ratio = 3.5 : 2.1 = 35 : 21 = 5 : 3

Total shares = 5 + 3 = 8

1 share = £$\frac{400}{8}$ = £50

5 shares = 5 × £50 = £250

3 shares = 3 × £50 = £150

4 4 parts = 180

1 part = 45 (Dividing both sides by 4)

3 parts = 3 × 45

= 135

Hence there are 180 + 135 = 315 members of the gym.

5 The ratio is 21 : 25 : 29

Total shares = 21 + 25 + 29 = 75

One share = £$\frac{150\,000}{75}$ = £2000

Youngest daughter receives 21 × 2000 = £42 000

6 Total number of parts in the ratio = 5 + 2 = 7

Now pick a number that is divisible by 7. We will choose 70.

Dividing this into the ratio 5 : 2 gives 50 male guests and 20 female guests.

60% of male guests are under 40 and 70% of female guests are under 40.

60% of 50 = 30 (males under 40)

70% of 20 = 14 (females under 40)

So if there were 70 guests, 44 of them would be under 40 years old. Use this information to work out the correct percentage, whatever the number of guests:

$\frac{44}{70} \times 100 = 62.9\%$

7 There are 2 more parts of the ratio for 20p coins, and 6 more 20p coins.

So 2 parts = 6 coins

1 part = 3 coins.

Hence there are 5 × 3 = 15 10p coins, and 7 × 3 = 21 20p coins

Total amount (£) in the money box = 15 × 0.1 + 21 × 0.2
= 1.5 + 4.2 = £5.70

8 Let x be the number of yellow marbles.

So $5x$ = number of red marbles, and $2 \times 5x = 10x$ = number of blue marbles.

Hence the ratio of blue to red to yellow marbles $= 10x : 5x : x = 10 : 5 : 1$

Scale diagrams and maps

1 First convert 150 km to cm.

150 km = 150 000 m = 15 000 000 cm

500 000 cm is equivalent to 1 cm on the map.

Distance in cm on the map = $\frac{15\,000\,000}{500\,000}$ = 30 cm

2 **a** Distance between the ship and the port = 2 cm.

This is measured using a ruler on the map.

We now need to get the units the same.

10 km = 10 000 m = 1 000 000 cm

The scale is 2 : 1 000 000

Dividing both sides of the ratio by 2 gives

1: 500 000

b Measuring the actual distance between the two ships gives 1.2 cm

Actual distance = 1.2 × 500 000 = 600 000 cm

Divide this by 100 and then 1000 to give the actual distance in km.

600 000 cm = 6 km

Percentage problems

1 $\frac{8}{300} \times 100 = 2.\dot{6} = 2.67\%$ to 2 d.p.

2 Increase = Final earnings − Initial earnings
= 1 100 000 − 600 000 = £500 000

% increase $= \frac{\textit{Increase}}{\textit{original value}} \times 100$

$= \frac{500\,000}{600\,000} \times 100$

= 83.3% to 1 d.p.*

*This answer differs from the one in the Revision Guide due to an error in our first edition. This answer has now been re-checked and corrected.

3 $3.5\% = \frac{3.5}{100} = 0.035$

Add 1 to create a multiplier for the original number: 1.035

New salary = 1.035 × 38 000 = £39 330

4 $18\% = \frac{18}{100} = 0.18$

1 − 0.18 = 0.82

82% of original price = £291.92

1% of original price = $\frac{291.92}{82} = 3.56$

100% of original price = 3.56 × 100 = 356

Original price = £356

5 Amount of interest in one year = 3.5% of £12 000
$= \frac{3.5}{100} \times 12000 = £420$

Total interest paid over 6 years = 6 × £420 = £2520

Direct and inverse proportion

1 Inverse proportion means that if one quantity doubles the other quantity halves.

2 **a** $y = kx$

b $8 = k \times 3$ giving $k = \frac{8}{3}$

$y = \frac{8}{3}x$

When $x = 4$, $\frac{8}{3} \times 4 = \frac{32}{3} = 10.7$ (1 d.p.)

3 **a** Find the equivalent price in £

$€\frac{120}{1.27} = £94.49$

The sunglasses are cheaper in the UK.

b £94.49 − £89 = £5.49 cheaper

4 $V \propto r^3$ so $V = kr^3$

When $V = 33.5$, $r = 2$ so $33.5 = k \times 2^3$

$k = \frac{33.5}{8} = 4.1875$

Substituting this value of k back into the equation gives

$V = 4.1875r^3$

When $r = 4$, $V = 4.1875 \times 4^3 = 268$ cm^3

5 $P \propto \frac{1}{V}$ so $P = \frac{k}{V}$

Hence $100000 = \frac{k}{1}$, giving $k = 100000$

Formula is $P = \frac{100000}{V}$

When $V = 3$,

$P = \frac{100000}{3}$

$= 33333$ (to the nearest whole number)

6 $a = kb^2$

When $a = 96$, $b = 4$ so $96 = k \times 4^2$ giving $k = 6$

Hence $a = 6b^2$ and when $b = 5$, $a = 6 \times 5^2 = 150$

7 **a** A square of side x cm has an area of x^2.

$A = kx^2$

As there are 6 faces to a cube, surface area $= 6x^2$

Hence $A = 6x^2$

So constant of proportionality $k = 6$

b $A = 6x^2$

When $x = 4$ cm

$A = 6 \times 4^2 = 96$ cm^2

Graphs of direct and inverse proportion and rates of change

1 Graph C

2 Graph B

3 As P and V are inversely proportional, $P = \frac{k}{V}$

At point A, when $P = 12$, $V = 3$ so $12 = \frac{k}{3}$ hence $k = 36$

Substituting this value of k back into the equation we have $P = \frac{36}{V}$

When $V = 6$, $P = \frac{36}{6} = 6$

Hence $a = 6$

4 As x and y are inversely proportional, $y \propto \frac{1}{x}$, so $y = \frac{k}{x}$

When $x = 1$, $y = 4$ so $4 = \frac{k}{1}$ so $k = 4$

The equation of the curve is now $y = \frac{4}{x}$

When $x = 4$, $y = \frac{4}{4} = 1$ so $a = 1$

When $y = 0.8$, $0.8 = \frac{4}{x}$ giving $x = 5$ so $b = 5$.

Hence $a = 1$ and $b = 5$.

5 $y = kx^2$

When $x = 2$, $y = 16$ so $16 = k \times 2^2$ giving $k = 4$.

$y = 4x^2$

Hence when $y = 36$,

$36 = 4x^2$ so $x = 3$ or -3

Since a is positive,

$a = 3$

Growth and decay

1 For a, b and c, multiplier is $\frac{100\% + \text{percentage given}}{100}$

a 1.05

b 1.25

c 1.0375

d Multiplier is $\frac{100\% - \text{percentage given}}{100} = 0.79$

2 Multiplier $= 1 - \frac{18}{100} = 0.82$

Value at the end of n years $= A_0 \times (\text{multiplier})^n$ where A_0 is the initial value.

Value at the end of 3 years = 9000 × $(0.82)^3$ = £4962.312
= £4962 (nearest whole number)

3 multiplier for years 1 and 2 = 1.04

multiplier for years 3 and 4 = 1.03

amount after 4 years = 650 × 1.04^2 × 1.03^2 = £745.86 (to nearest penny)

4 Number of restaurants after n years, $R_n = R_0 \times (\text{multiplier})^n$

Multiplier = 1.25, $n = 3$, $R_n = 4000$

So $R_0 = \frac{4000}{(1.25)^3} = 2048$

There were 2048 restaurants 3 years ago.

Ratios of lengths, areas and volumes

1 Scale factor $= \frac{6}{9} = \frac{2}{3}$

$x = \frac{2}{3} \times 10 = \frac{20}{3} = 6\frac{2}{3}$ cm

2 As the shapes are similar $\frac{V_A}{V_B} = \left(\frac{r_A}{r_B}\right)^3$

Cube rooting both sides gives $\frac{r_A}{r_B} = \sqrt[3]{\frac{V_A}{V_B}}$

$\frac{r_A}{r_B} = \sqrt[3]{\frac{27}{64}}$

$= \frac{3}{4}$

$\frac{\text{radius of cylinder A}}{\text{radius of cylinder B}} = \frac{3}{4}$

$\frac{\text{surface area of cylinder A}}{\text{surface area of cylinder B}} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

So surface area of cylinder A = surface area of cylinder B × $\frac{9}{16}$

Surface area of cylinder A = 96 × $\frac{9}{16}$ = 54 cm²

3 **a** As *BE* is parallel to *CD* all the corresponding angles in both triangles are the same so triangles *ABE* and *ACD* are similar.

$\frac{BE}{8} = \frac{5}{10}$ giving *BE* = 4 cm

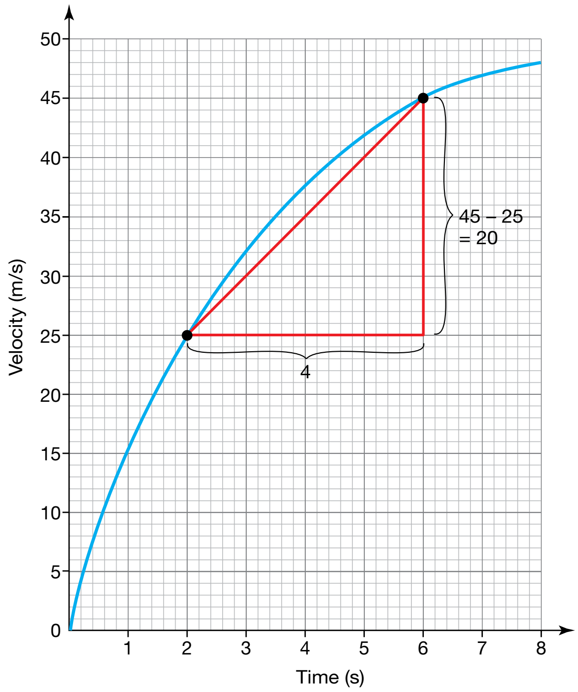
b *BE* = 4 cm and *AB* = 5 cm. Angle *ABE* is a right angle because angle *ACD* is a right angle and the triangles are similar.

Hence area *ABE* = $\frac{1}{2}$ × 4 × 5 = 10 cm²

Gradient of a curve and rate of change

1 **a** The gradient represents the acceleration.

b Find the gradient between when time = 2 s and time = 6 s.



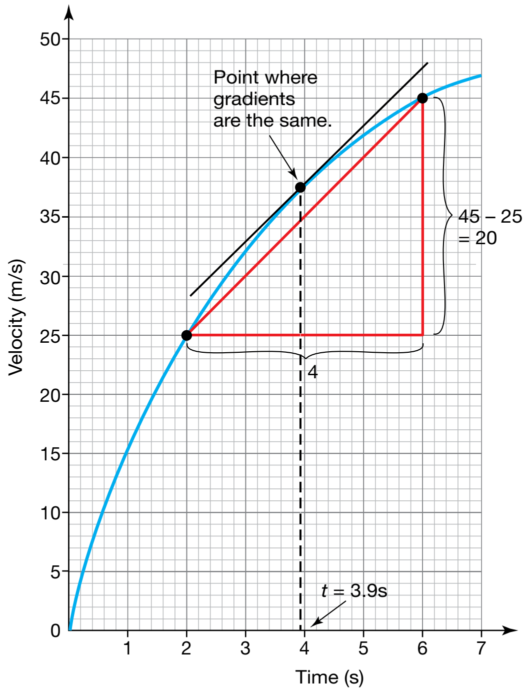
From the graph, acceleration between time = 2 s and time = 6 s

$= \frac{45 - 25}{6 - 2}$

$= \frac{20}{4}$

= 5 m/s².

c We need to find a point on the graph where the gradient equals that of the tangent already drawn (5 m/s²).



Time when the instantaneous acceleration is the same as the average acceleration = 3.9 s

Converting units of areas and volumes, and compound units

1 **a** Surface area = 2 × 4 × 6 + 2 × 4 × 5 + 2 × 6 × 5
= 148 cm²

i 1cm² = 10 × 10 mm² = 100 mm²

148 cm² = 148 × 100 mm² = 14 800 mm²

ii 1 m² = 100 × 100 cm² = 10 000 cm²

148 cm² = 148 ÷ 10 000 m² = 0.0148 m²

b Volume = 6 × 5 × 4 = 120 cm³

1 m³ = 100 × 100 × 100 cm³ = 1 000 000 cm³

120 cm³ = 120 ÷ 1 000 000 = 0.00012 cm³

2 Density = $\frac{\text{mass}}{\text{volume}} = \frac{1159}{600}$ = 1.932 g/cm³ (3 d.p.)

3 First get the units the same.

1m³ = 100 × 100 × 100 cm³ = 1 000 000 cm³

Number of ball bearings = $\frac{1\,000\,000}{0.5}$ = 2 000 000 (i.e. 2 million)

4 50 km/h = 50 000 m/h

1 h = 60 × 60 s = 3600 s

Now as fewer m/s will be covered compared to m/h we divide by 3600 to convert the speed to m/s.

Hence 50 000 m/h = $\frac{50\,000}{3600}$ = 13.89 m/s (2 d.p.)

5 **a** Distance = speed × time = 70 × 4 = 280 km

Mary's speed = 280/5 = 56 km/h

b If Mary's route was longer, her average speed would increase. If her route was shorter, her average speed would decrease.

Review it!

1 80% of the original price = £76.80

10% of the original price = $\frac{76.80}{8}$ = £9.60

100% of the original price = £9.60 × 10 = £96

2 **a** $A_1 = 1.02 \times A_0$

$= 1.02 \times 5$

$= 5.10\ \text{m}^2$ (2 d.p.)

b $A_2 = 1.02 \times A_1 = 1.02 \times 5.1 = 5.202\ \text{m}^2$

$A_3 = 1.02A_2 = 1.02 \times 5.202 = 5.31\ \text{m}^2$ (2 d.p.)

3 $y = \frac{k}{x}$

$4 = \frac{k}{2.5}$

$k = 10$

Hence $y = \frac{10}{x}$

When $x = 5$, $y = \frac{10}{5} = 2$

4 2 parts of the ratio = 54 students, so 1 part = 27 students.

Students in the whole year = 2 + 7 parts = 9 parts.

In Year 11, there are 9 × 27 = 243 students.

5 The price of the house has more than doubled.

Multiplier = 1 + 1.2 = 2.2

So £220 000 × 2.2 = £484 000

The value of the house = £485 000 to the nearest £5000

6 **a** Simple interest: One year = 2000 × 0.025 = £50

50 × 5 = 250

After 5 years, there will be £2250 in the account.

b Compound interest: 2000×1.025^5 = £2262.82

7 **a** 400 × 8.55 = 3420 yuan

b Travel agent: 800 ÷ 8.6 = £93.02

Commission = 93.02 × 0.025 = £2.33

So Tom would get £93.02 − £2.33 = £90.69

From the post office, Tom would get 800 ÷ 8.9 = £89.89

Tom will get a better deal from the travel agent.

8 The difference between Luke's and Amy's payouts was £4000. The difference in parts is 7 − 5 = 2.
So 2 parts = £4000 and 1 part = £2000

The profits are split into 3 + 5 + 7 = 15 parts.

Total profits = 15 × 2000 = £30 000

9 $\frac{\text{Surface area A}}{\text{Surface area B}} = \left(\frac{\text{Length A}}{\text{Length B}}\right)^2$

So $\frac{25}{4} = \frac{5^2}{2^2}$

Similarly, $\frac{\text{Volume A}}{\text{Volume B}} = \left(\frac{\text{Length A}}{\text{Length B}}\right)^3$

$\frac{10}{\text{Volume B}} = \frac{5^3}{2^3}$

So Volume B $= \frac{10 \times 8}{125} = \frac{80}{125} = 0.64\ \text{cm}^3$

10 7 + 4 = 11 parts in the ratio.

Let this be 1100 members.

There would then be 700 male club members: 25% of this is 0.25 × 700 = 175 junior members.

There would be 400 female club members: 10% of this is 0.1 × 400 = 40 junior members.

In total, there would be 175 + 40 = 215 junior members.

As a percentage, this is $\frac{215}{1100} \times 100 = 19.54$
= 20% to the nearest integer.

11 Each cm³ of the alloy contains 9 parts (= 0.9 cm³) copper and 1 part (= 0.1 cm³) tin.

So the mass of copper in 1 cm³ = 8.9 × 0.9 = 8.01 g

The mass of tin in 1 cm³ = 7.3 × 0.1 = 0.73 g

Mass of 1 cm³ of alloy = 8.01 + 0.73 = 8.74 g

So the density of the alloy = 8.7 g/cm³ (1 d.p.)

Geometry and measures

2D shapes

1 **a** True

b False

c True

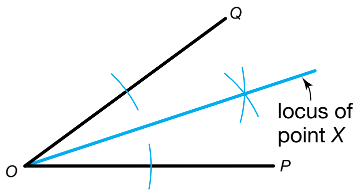
d False

e True

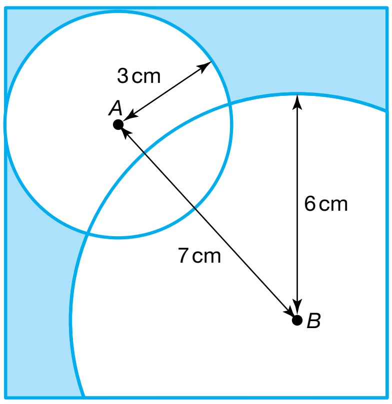
f True

Constructions and loci

1 The line which bisects the angle will be the locus of point *X*.



2 Draw two circles: one with a radius of 3 cm from *A*, another with a radius of 6 cm from *B*. Shade the area outside the two circles. The area outside the two circles shows all possible locations for the wind farm.



Properties of angles

1 Exterior angle $= \frac{360°}{\text{number of sides}} = \frac{360}{10} = 36°$

Interior angle + exterior angle = 180°, so interior angle = 144°

The interior and exterior angle always add up to 180°.

2 a Number of sides $= \frac{360°}{\text{exterior angle}} = \frac{360}{40} = 9$

b Interior angle $= 180° - $ exterior angle
$= 180 - 40 = 140°$

As there are 9 sides, total of interior angles
$= 140 \times 9 = 1260°$

3 Angle $BAC = 50°$ (corresponding angles)

Angle $ABC = 180 - (50 + 60) = 70°$ (angles in a triangle add up to 180°)

$x = 180 - (90 + 70) = 20°$ (angles in triangle ABD add up to 180°)

4 There are several different ways of working the answers out and the method shown is only one of them.

a Angle x is an alternate angle to angle EBC.
Hence $x = 55°$.

b Angle $EHI = 180 - (85 + 55) = 40°$ as angles in a triangle add up to 180°.

Angle DEH is an alternate angle to angle EHI so it is 40°.

5 Opposite angles in a rhombus are equal.

So $2(3a + 5) + 2(5a - 17) = 360$

$6a + 10 + 10a - 34 = 360$

$16a - 24 = 360$

$16a = 384$

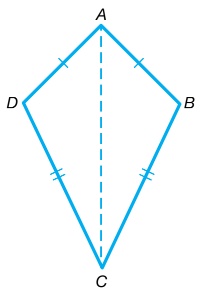
$a = \frac{384}{16}$

$a = 24°$

There are two angles of $3 \times 24 + 5 = 77°$, and two angles of $180 - 77 = 103°$

Congruent triangles

1



There are two triangles with a common side AC.

$AB = AD$ (given in the question)

$BC = CD$ (given in the question)

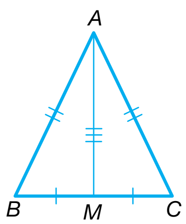
Triangles ACB and ACD are congruent (SSS).

Hence, angle ABC = angle ADC.

2 $AB = AC$ (given in the question)

$BM = MC$ (as M is the midpoint of BC)

$AM = AM$ (common to both triangles)



There are three pairs of equal corresponding sides so the triangles ABM and ACM are congruent (SSS).

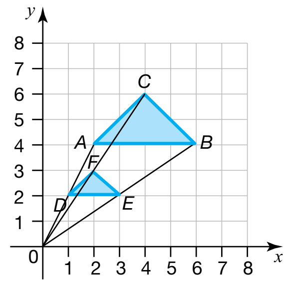
Hence angle AMB = angle AMC.

Angle AMB + angle $AMC = 180°$, (angles on a straight line)

So angle $AMB = \frac{180}{2} = 90°$

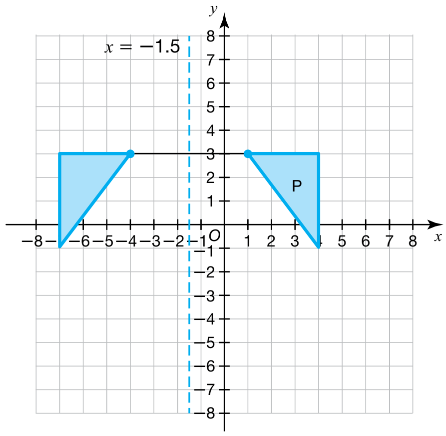
Transformations

1 a and **b**

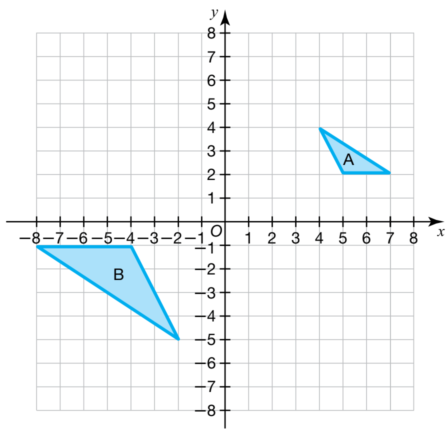


c Enlargement, scale factor 2, centre (0, 0)

2



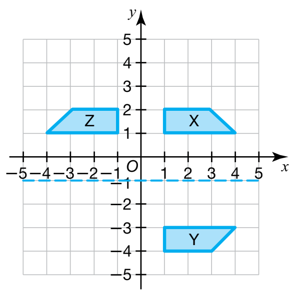
3



4 A translation by $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$

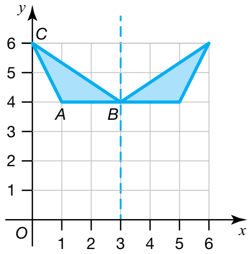
Invariance and combined transformations

1 a, b, c



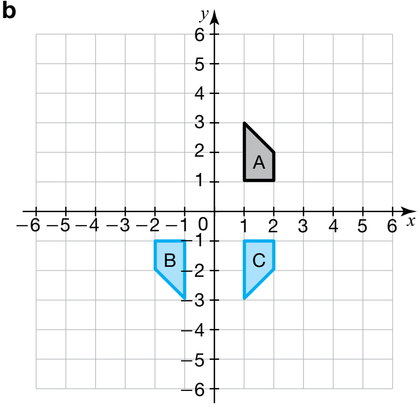
d Reflection in the y-axis, or reflection in the line $x = 0$

2 a, b



c Invariant point is (3, 4)

3 a, b



c A reflection in the x-axis.

3D shapes

1 a

Shape	Number of vertices, *V*	Number of faces, *F*	Number of edges, *E*
Triangular-based pyramid	4	4	6
Cone	1	2	1
Cuboid	8	6	12
Hexagonal prism	12	8	18

b $V = 16, F = 10, E = 24; V + F - E = 16 + 10 - 24 = 2$

Parts of a circle

1 a radius

b chord

c minor arc

d minor segment

e minor sector

Circle theorems

1 a Angle *ACB* = 30° as it is equal to angle *YAB* (alternate segment theorem)

b Angle *ABC* = 90° (angle in a semicircle is always a right angle)

c Angle *ADC* = 90° (angle in a semicircle is always a right angle)

2 a Angle *OAX* = 90° (angle formed by a tangent to a radius or diameter is always a right angle)

b Angle *AOX* = 180 − (90 + 30) = 60° (angles in a triangle add up to 180°)

c Angle *ACB* = 60 ÷ 2 = 30° (angle at the centre is twice the angle at the circumference standing on the same arc)

3 a Angle *ADB* = 40° as it is equal to angle *ACB* (angles at the circumference on the same arc are equal)

b Angle *ABD* = angle *ADE* = 50° (alternate segment theorem)

Angle *EDB* = 90° (i.e. 40° + 50°) which means *BD* must be a diameter, as a tangent and a diameter are at right angles to each other.

As *BD* is a diameter, angle *BAD* is 90° as it is the angle in a semicircle.

4 Angle *ACB* = 46° (angles bounded by the same chord in the same segment are equal)

Angle *ABC* = 90° (angle in a semicircle is a right-angle)

Angle *BAC* = 180 − (90 + 46) = 44° (angles in a triangle add up to 180°)

5 a Angle *ABC* = 60° (angle in an equilateral triangle)

Angle *ADC* = 180 – 60 = 120° (opposite angles in a cyclic quadrilateral add up to 180°)

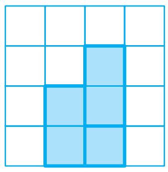
b Angle *BAC* = 60° (angle in an equilateral triangle)

Angle *CBQ* = angle *BAC* (alternate segment theorem)

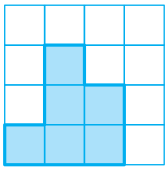
= 60°

Projections

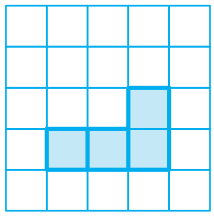
1 a



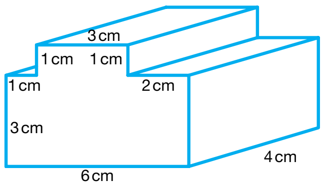
b



c

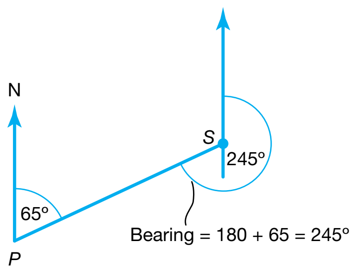


2



Bearings

1



Bearing = 245°

2 a Bearing of P from Q = 180 + 45 = 225°

b Bearing of Q from R = 180 + 140 = 320°

Pythagoras' theorem

1 **a** By Pythagoras' theorem

$x^2 = 5^2 + 9^2 = 106$

$x = \sqrt{106} = 10.3$ cm (1 d.p.)

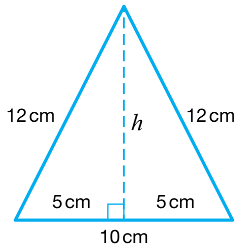
b By Pythagoras' theorem

$13.5^2 = 10.2^2 + x^2$

$x^2 = 182.25 - 104.04 = 78.21$

$x = \sqrt{78.21} = 8.8$ cm (1 d.p.)

2



By Pythagoras' theorem

$12^2 = 5^2 + h^2$

$144 = 25 + h^2$

$h^2 = 119$

$h = \sqrt{119}$

$= 10.9087$ cm (4 d.p.)

Area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 10 \times 10.9087$

$= 54.54$ cm^2 (2 d.p.)

3 $AC^2 = 3^2 + 11^2 = 130$

$x^2 = 14^2 - AC^2 = 196 - 130 = 66$

$x = \sqrt{66} = 8.12$ cm (2 d.p.)

4 Let the perpendicular height of the triangle $= h$

Area of triangle $= \frac{1}{2} \times 14 \times h$

Hence $\frac{1}{2} \times 14 \times h = 90$

$h = \frac{90 \times 2}{14} = \frac{180}{14} = \frac{90}{7}$

$h = 12.8571$ cm (4 d.p.)

By Pythagoras' theorem, $AC^2 = 7^2 + 12.8571^2$

$AC = \sqrt{214.305}$ (3 d.p.)

$AC = 14.6$ cm (3 s.f.)

Area of 2D shapes

1 Right angle and parallel lines show that the office is a trapezium.

Area of a trapezium $= \frac{1}{2} \times (a + b) \times h = \frac{1}{2} \times (9 + 12) \times 4$

Office area $= 42$ m^2

Cost of new floor $=$ £38 $\times$ 42 $=$ £1596

2 Length of side of square $= \frac{\text{perimeter}}{4} = \frac{20}{4} = 5$ cm

Area of square $= 5 \times 5 = 25$ cm^2

Area of small triangle $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 2.5 \times 2.5 = 3.125$ cm^2

Area of large triangle $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 2.5 \times 5 = 6.25$ cm^2

Shaded area $= 25 - (3.125 + 6.25) = 15.625$

Proportion shaded $= \frac{15.625}{25}$

Multiply by 8 to get rid of the decimal:

$= \frac{125}{200}$

Cancel fraction (divide by 25):

$= \frac{5}{8}$ of the square.

3 **a** Area of semicircle $= \frac{1}{2} \times \pi \times x^2 = \frac{\pi x^2}{2}$

Area of rectangle $= 4x \times 2x = 8x^2$

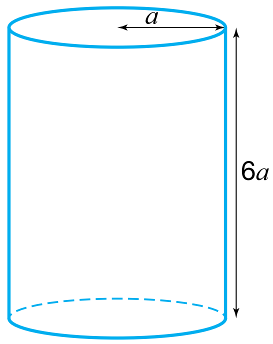
Area of shape $= \frac{\pi x^2}{2} + 8x^2 = x^2(8 + \frac{\pi}{2})$

b Perimeter $= 2x + 4x + 4x + \frac{1}{2} \times 2\pi x = 10x + \pi x = x(10 + \pi)$.

An exact answer is required, so you should leave π in your answer.

Volume and surface area of 3D shapes

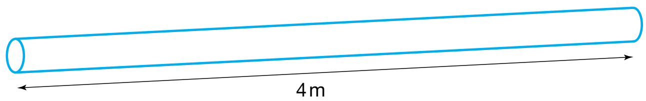
1



Volume of cylinder $=$ cross-sectional area $\times$ length

$= \pi a^2 \times 6a$

$= 6\pi a^3$



Volume of rod $=$ cross-sectional area $\times$ length

$= \pi r^2 \times 4$

$= 4\pi r^2$

The two volumes are the same so we can write

$4\pi r^2 = 6\pi a^3$ (Dividing both sides by π)

$4r^2 = 6a^3$ (Dividing both sides by 2)

$2r^2 = 3a^3$ (Dividing both sides by 2)

$r^2 = \frac{3}{2} a^3$ (Square rooting both sides)

$r = \sqrt{\frac{3}{2} a^3}$

2 **a** Area of trapezium $= \frac{1}{2}(a + b)h = \frac{1}{2}(0.75 + 1.75) \times 10 = 12.5$ m^2

Cross-sectional area $= 12.5$ m^2

b Volume $=$ Cross-sectional area $\times$ length

$= 12.5 \times 5 = 62.5$ m^3

c Total volume per minute of water entering $= 2 \times 0.05 = 0.1$ m^3 per minute

Number of minutes it takes to fill $= \frac{62.5}{0.1}$

$= 625$ mins

Number of hours it takes to fill $= \frac{625}{60}$

$= 10.42$ hours (2 d.p.)

$= 10$ hours (nearest hour)

3 Need to first find the slant height, l, as this is needed for the formula.

By Pythagoras' theorem $l^2 = 4^2 + 15^2 = 241$

$l = 15.524$ m (3 d.p.)

Curved surface area of a cone $= \pi rl = \pi \times 15 \times 15.524$

$= 731.55$ m² (2 d.p.)

Curved surface area of a cylinder $= 2\pi rh$

$= 2 \times \pi \times 15 \times 5$

$= 471.24$ m² (2 d.p.)

Total surface area of the tent $= 731.55 + 471.24$

$= 1202.79$ m²

The tent uses 1200 m² of fabric (to 3 s.f.)

4 Volume = volume of large cone – volume of small cone

$= \frac{1}{3} \times \pi \times 5^2 \times 20 - \frac{1}{3} \times \pi \times 3.5^2 \times 14$

$= \frac{500}{3}\pi - \frac{171.5}{3}\pi$

$= \frac{328.5}{3}\pi$

$= 109.5\pi$ cm³

Trigonometric ratios

1 a $\cos 30° = \frac{15}{x}$

x is the hypotenuse and the 15cm side is the adjacent so we use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.

$x = \frac{15}{\cos 30°} = 17.32$ cm (2 d.p.)

b $\cos 40° = \frac{x}{12}$

x is the adjacent and the 12 cm side is the hypotenuse so we use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.

Hence, $x = 12\cos 40°$

$= 9.19$ cm (2 d.p.)

2 a $\tan\theta = \frac{10}{3}$

$\theta = \tan^{-1}\left(\frac{10}{3}\right)$

$= 73.3°$ (nearest 0.1°)

b $\sin\theta = \frac{10}{13}$

The 13cm side is the hypotenuse and the 10cm side is the opposite, so we use $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$.

$\theta = \sin^{-1}\left(\frac{10}{13}\right)$

$= 50.3°$ (nearest 0.1°)

3 $\sin\theta = \frac{b}{c}$ (i.e. $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$)

and $\cos\theta = \frac{a}{c}$ (i.e. $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$)

Remember that when you divide by fractions you turn the bottom fraction upside down and replace the division by a multiplication.

Hence $\frac{\sin\theta}{\cos\theta} = \frac{\frac{b}{c}}{\frac{a}{c}}$

$= \frac{b}{c} \times \frac{c}{a}$

$= \frac{b}{a}$

Now, $\frac{b}{a} = \frac{\text{opposite}}{\text{adjacent}} = \tan\theta$

Hence, $\frac{\sin\theta}{\cos\theta} = \tan\theta$

4 a By Pythagoras' theorem, $BD^2 = AB^2 + AD^2$

$= 10^2 + 7^2$

$= 149$

$BD = \sqrt{149}$

$= 12.207$ cm (3 d.p.)

Applying Pythagoras' theorem to triangle BCD

$CD^2 = BD^2 + BC^2$

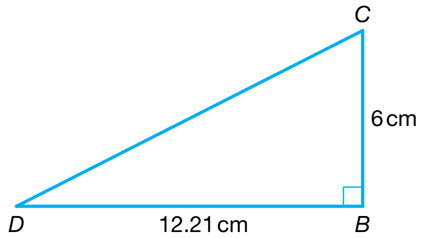
$= 12.207^2 + 6^2$

$= 185.011$ (3 d.p.)

$CD = \sqrt{185.011}$

$= 13.60$ cm (2 d.p.)

b



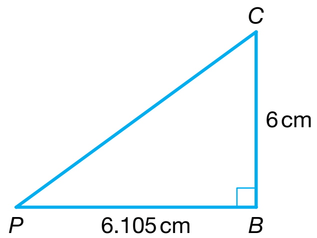
Let angle $BCD = x$

$\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{12.21}{6}$

$x = \tan^{-1}\left(\frac{12.21}{6}\right)$

So $x = 63.8°$ (1 d.p.)

c $BP = \frac{1}{2} \times BD = \frac{1}{2} \times 12.21 = 6.105$ cm

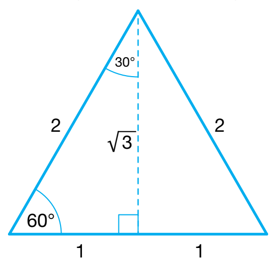


By Pythagoras' theorem, $PC^2 = 6.105^2 + 6^2$

$PC = 8.56$ cm (2 d.p.)

Exact values of sin, cos and tan

1 First draw an equilateral triangle with sides 2 cm and mark on the following sides and angles.



From the diagram $\tan 30° = \frac{1}{\sqrt{3}}$ and $\cos 60° = \frac{1}{2}$

$\sqrt{3}\tan 30° + \cos 60° = \sqrt{3} \times \frac{1}{\sqrt{3}} + \frac{1}{2} = 1 + \frac{1}{2} = \frac{3}{2}$

where $a = 3$ and $b = 2$.

2 $\sin 30 = \frac{1}{2}$ and $\cos 30 = \frac{\sqrt{3}}{2}$

$\sin^2 30 = \frac{1}{4}$ and $\cos^2 30 = \frac{3}{4}$

$\sin^2 30 + \cos^2 30 = \frac{1}{4} + \frac{3}{4} = 1$

Sectors of circles

1 $l = \frac{\theta}{360} \times 2\pi r$

$10 = \frac{\theta}{360} \times 2\pi \times 12$

$\theta = \frac{10 \times 360}{2\pi \times 12} = \frac{5 \times 30}{\pi}$

$= 47.7°$ (1 d.p.)

2 **a** Perimeter of logo = 2 × arc length of sector + 2 × radius of sector + base of triangle

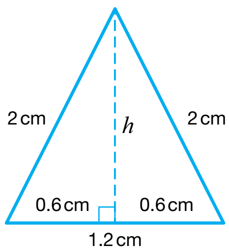
Arc length $= \frac{\theta}{360} \times 2\pi r = \frac{40}{360} \times 2 \times \pi \times 2 = 1.396$ cm

So perimeter $= 2 \times 1.396 + 2 \times 2 + 1.2$
$= 7.99$ cm (3 s.f.)

b Area of logo = 2 × area of sector + area of triangle

Area of sector $= \frac{\theta}{360} \times \pi r^2 = \frac{40}{360} \times \pi \times 2^2$
$= 1.396\ \text{cm}^2$ (3 d.p.)

Find the height of the isosceles triangle to find its area.



Use Pythagoras' theorem

$2^2 = 0.6^2 + h^2$

So $h = 1.9079$ cm (4 d.p.)

Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 1.2 \times 1.9079 = 1.1447\ \text{cm}^2$

Area of logo $= 2 \times 1.396 + 1.1447 = 3.9367$
$= 3.94\ \text{cm}^2$ (3 s.f.)

3 Area of sector $= \frac{85}{360} \times \pi \times r^2$

Area $= 25\pi$ so $\frac{85}{360} \times \pi \times r^2 = 25\pi$

$r^2 = \frac{25\pi \times 360}{85\pi} = 105.88$ (2 d.p.)

$r = OA = \sqrt{105.88} = 10.3$ cm (1 d.p.)

4 First find the radius of the circle using the length of arc.

$\frac{85}{360} \times \pi \times d = 10$

$d = \frac{3600}{85\pi}$

$r = \frac{3600}{85\pi} \div 2$

$= 6.7407$ (4 d.p.)

Area of sector $= \frac{275}{360} \times \pi \times 6.7407^2 = 109\ \text{cm}^2$ to the nearest whole number

Sine and cosine rules

1 **a** Area of triangle $ABC = \frac{1}{2} bc \sin A$

$= \frac{1}{2} \times 10 \times 6 \sin 150°$

$= 15\ \text{cm}^2$

b Using the cosine rule

$BC^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \cos 150°$

$BC^2 = 36 + 100 - 120 \cos 150°$

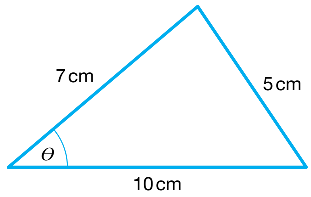
$BC^2 = 136 + 103.92$

$BC = \sqrt{239.92}$

$BC = 15.5$ cm (to 3 s.f.)

2 The smallest angle of any triangle is always opposite the smallest side.

Draw a sketch of the triangle and let the smallest angle be θ.



Use the cosine rule

$5^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \cos \theta$

$25 = 49 + 100 - 140 \cos \theta$

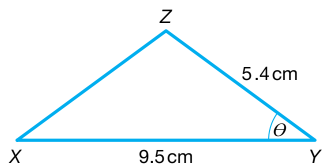
$140 \cos \theta = 124$

$\cos \theta = 0.8857$

$\theta = \cos^{-1}(0.8857)$

$= 27.7°$ (nearest 0.1°)

3 **a**



Let angle $XYZ = \theta$

Area $= \frac{1}{2} \times XY \times YZ \times \sin \theta$

From the question, area = 16 cm²

So $16 = \frac{1}{2} \times 9.5 \times 5.4 \times \sin \theta$

$\sin \theta = \frac{16 \times 2}{9.5 \times 5.4}$

$\theta = \sin^{-1}\left(\frac{32}{51.3}\right)$

So $\theta = 38.6°$ (1 d.p.)

b If angle XZY is not obtuse, then angle XYZ can be obtuse.

So an alternative answer for $\theta = 180 - 38.6$
$= 141.4°$ (1 d.p).

Vectors

1 **a** $\mathbf{b} - \mathbf{a} = \begin{pmatrix}-2\\3\end{pmatrix} - \begin{pmatrix}4\\-5\end{pmatrix} = \begin{pmatrix}-6\\8\end{pmatrix}$

b $3\mathbf{a} + 5\mathbf{b} = 3\begin{pmatrix}4\\-5\end{pmatrix} + 5\begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}12\\-15\end{pmatrix} + \begin{pmatrix}-10\\15\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix}$

2 **a** $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\mathbf{a} + \mathbf{b}$

$= \mathbf{b} - \mathbf{a}$

b $\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AB}$

$= \frac{3}{5}(\mathbf{b} - \mathbf{a})$

c $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$

Multiply out the bracket and then simplify the terms.

$= \mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{3}{5}\mathbf{a}$

$= \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

$= \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$

3 a $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$\overrightarrow{BA} = -\overrightarrow{AB} = -\mathbf{a}$

$\overrightarrow{AC} = \overrightarrow{AP} + \overrightarrow{PC} = -2\mathbf{b} - \mathbf{b} = -3\mathbf{b}$

So $\overrightarrow{BC} = -\mathbf{a} - 3\mathbf{b}$

b If *M* is the midpoint of *BC*, $\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BC}$

$= \frac{1}{2}(-\mathbf{a} - 3\mathbf{b}) = -\frac{1}{2}(\mathbf{a} + 3\mathbf{b})$

If *B* is the midpoint of *AD*, *AB* = *BD* = **a**

So $\overrightarrow{PM} = \overrightarrow{PA} + \overrightarrow{AB} + \overrightarrow{BM}$

$= 2\mathbf{b} + \mathbf{a} - \frac{1}{2}(\mathbf{a} + 3\mathbf{b})$

$= 2\mathbf{b} + \mathbf{a} - \frac{1}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}$

$= \frac{1}{2}(\mathbf{a} + \mathbf{b})$

Similarly, $\overrightarrow{MD} = \overrightarrow{MB} + \overrightarrow{BD}$

$= \frac{1}{2}(\mathbf{a} + 3\mathbf{b}) + \mathbf{a}$

$= \frac{1}{2}\mathbf{a} + \frac{3}{2}\mathbf{b} + \mathbf{a}$

$= \frac{3}{2}(\mathbf{a} + \mathbf{b})$

$\overrightarrow{PM}$ and $\overrightarrow{MD}$ have the same vector part, (**a** + **b**), therefore they are parallel. Both lines pass through point *M* and parallel lines cannot pass through the same point unless they are the same line. Hence *PMD* is a straight line.

Review it!

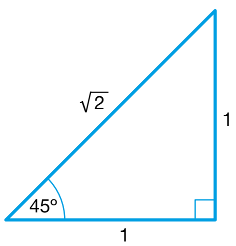
1 Angle *ADE* = $y°$ (alternate segment theorem)

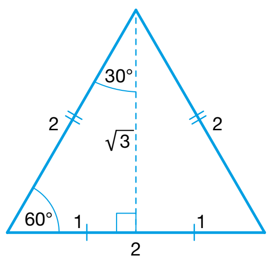
Angle *ADC* = $(180 - x)°$ (opposite angles in cyclic quadrilateral add up to 180°)

EDF is a tangent, so it is a straight line.

So angle *CDF* = 180 − angle *ADE* − angle *ADC*
$= 180 - y - (180 - x) = (x - y)°$

2 Draw the following triangles and mark on the angles.





$\cos 45° = \frac{1}{\sqrt{2}}$ $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 45° + \sin 60° = \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}$

$= \frac{1}{2}(\sqrt{2} + \sqrt{3})$

3 Using Pythagoras' theorem we obtain

$(4x - 3)^2 = (x + 1)^2 + (3x)^2$

$16x^2 - 24x + 9 = x^2 + 2x + 1 + 9x^2$

So $6x^2 - 26x + 8 = 0$

Dividing both sides by 2 we obtain

$3x^2 - 13x + 4 = 0$

$(3x - 1)(x - 4) = 0$

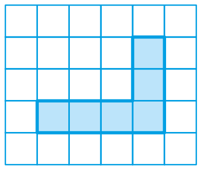
$3x - 1 = 0$ or $x - 4 = 0$

Hence $x = \frac{1}{3}$ or $x = 4$.

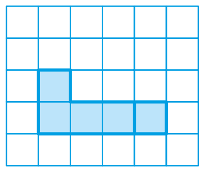
We need to put both values into each of the expressions for the sides to see if any of the sides end up negative. When $x = \frac{1}{3}$ is substituted into $4x - 3$ the result is negative. As you cannot have a negative length for a side, $x = \frac{1}{3}$ is disregarded. Hence $x = 4$.

When x is 4 the sides are 5, 12 and 13.

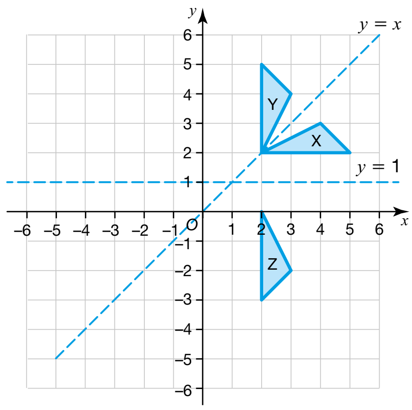
4 a



b



5 a, b i and **c**



b ii Invariant point (vertex) is (2, 2)

d Rotation 90° clockwise about (1, 1)

6 a Work out angle *ABC* in the triangle.

The angle between *AB* and due south at *B* is 70° (alternate with 70° marked at *A*).

The angle between due south at *B* and *BC* is 30° (angles on a straight line add up to 180°).

So angle *ABC* = 100°

Using the Cosine Rule:

$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos B$

$AC^2 = 15^2 + 10^2 - 2 \times 15 \times 10 \times \cos 100°$

$AC^2 = 325 - (-52.094)$

So $AC = \sqrt{377.094} = 19.4$ km (3 s.f.)

b Let angle = θ

Using the Sine Rule, $\frac{19.4}{\sin 100} = \frac{10}{\sin \theta}$

So $\sin \theta = \frac{10 \sin 100}{19.4} = \frac{9.84}{19.4}$

$\theta = \sin^{-1}\left(\frac{9.84}{19.4}\right)$

$\theta = 30.48°$

Bearing of C from $A = 70 + 30.48 = 100.48°$

So bearing of A from $C = 100.48 + 180 = 280°$ to the nearest degree.

Probability

The basics of probability

1 There are two possible even numbers (i.e. 2, 4)

Probability of landing on an even number

$= \frac{\text{Number of ways something can happen}}{\text{Total of number of possible outcomes}} = \frac{2}{5}$

Probability of getting an even number on each of three spins $= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$

2 a As there are no white counters, the probability of picking one = 0

b Probability of picking a black counter $= \frac{4}{20} = \frac{1}{5}$

c Probability of picking a green counter $= \frac{9}{20}$

Prob of picking a green counter + prob of not picking a green counter = 1

Prob of not picking a green counter

$= 1 - \frac{9}{20} = \frac{11}{20}$

Probability experiments

1 a Relative frequency for a score of 3 $= \frac{25}{120}$

$= 0.21$ (2 d.p.)

b Relative frequency for a score of 6 $= \frac{17}{120}$

$= 0.14$ (2 d.p.)

c Sean is wrong. 120 spins is a small number of spins and it is only over a very large number of spins that the relative frequencies may start to be nearly the same.

2 a Estimated probability = relative frequency

$= \frac{\text{Frequency}}{\text{Total Frequency}} = \frac{20}{500} = 0.04$

b Number of cans containing less than 330 ml $= 0.04 \times 15\,000 = 600$

Another way to do this is to see how many times 500 divides into 15 000. This is 30. So there will be $30 \times 20 =$ **600 cans** containing less than 30 ml.

3 Expected frequency = Probability of the event × number of events

$= \frac{3}{40} \times 600$

= 45 apples

The AND and OR rules

1 a Independent events are events where the probability of one event does not influence the probability of another event occurring. Here it means that the probability of the first set of traffic lights being red does not affect the probability of the second set being red.

b P(A AND B) = P(A) × P(B)

P(stopped at first AND stopped at second)
= P(stopped first) × P(stopped second)

$= 0.2 \times 0.3$

$= 0.06$

c $P = 0.8 \times 0.7 = 0.56$

> You can work out the probability of lights not being on red by subtracting the probability of being red from 1. So the probability of them not being red at the first set is 1 – 0.2 = 0.8 and for the second set it is 1 – 0.3 = 0.7

2 a Probability of all events taking place $= \frac{1}{4} \times \frac{2}{3} \times \frac{7}{8}$

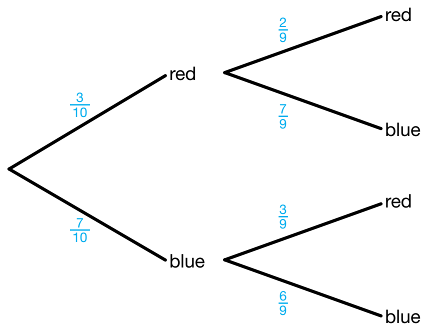
$= \frac{2 \times 7}{12 \times 8} = \frac{7}{48}$

b Probability of none of the events taking place

$= \frac{3}{4} \times \frac{1}{3} \times \frac{1}{8} = \frac{3}{3 \times 32} = \frac{1}{32}$

Tree diagrams

1 a The following tree diagram is drawn.



P(red AND red) $= \frac{3}{10} \times \frac{2}{9} = \frac{6}{90} = \frac{1}{15}$

b P(red AND blue) = P(RB) + P(BR)

> Note that red and blue does not specify an order. There are two paths that need to be considered on the tree diagram.

> Remember to fully cancel fractions. Use your calculator to help you.

$= \frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{3}{9}$

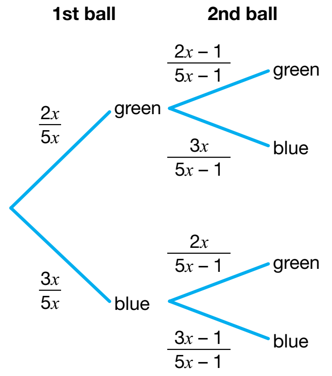
$= \frac{7}{30} + \frac{7}{30}$

$= \frac{14}{30}$

$= \frac{7}{15}$

2 Let the number of green balls in the bag be $2x$. Let the number of blue balls be $3x$. So the total number of balls in the bag is $5x$.

Put these values into a tree diagram:



P(blue AND blue) $= \frac{3x}{5x} \times \frac{3x-1}{5x-1} = \frac{33}{95}$

Divide both sides by $\frac{3}{5}$ and cancel the x top and bottom on the left.

So $\frac{3x-1}{5x-1} = \frac{11}{19}$

$19(3x - 1) = 11(5x - 1)$

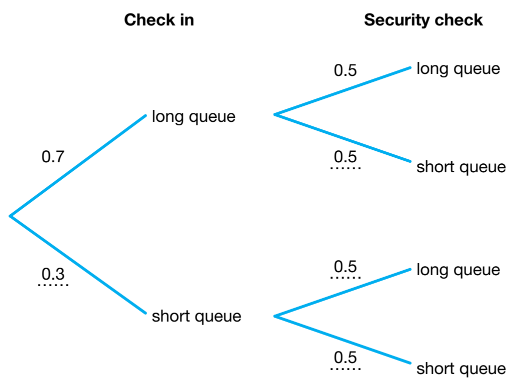
$57x - 19 = 55x - 11$

$2x = 8$

$x = 4$

Hannah put $5x = 5 \times 4 = 20$ balls into the bag.

3 **a**



b Probability $= 0.3 \times 0.5 = 0.15$

c Probability $= 1 -$ probability of short queue at both $= 1 - 0.15 = 0.85$

4 **a** Total number of students in school $= 450 + 500$ $= 950$

Number of male students in upper school

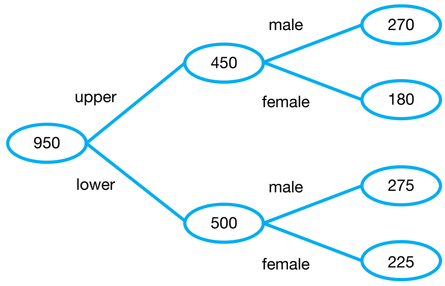
$= 0.6 \times 450 = 270$

Number of female students in upper school $= 450 - 270 = 180$

Number of male students in lower school $= 0.55 \times 500 = 275$

Number of female students in lower school $= 500 - 275 = 225$

This information is added to the frequency tree.



b Total males $= 270 + 275$

$= 545$

P(student is male) $= \frac{545}{950}$

$= \frac{109}{190}$

$= 0.57$

Venn diagrams and probability

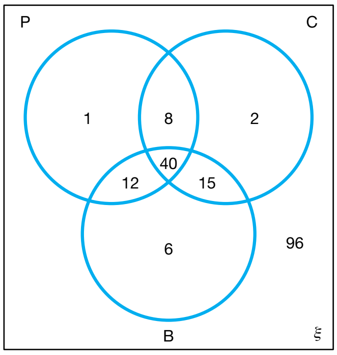
1 **a** **i** $A \cup B = \{1, 3, 4, 5, 8, 9, 10, 11\}$

ii $A \cap B = \{8, 9\}$

iii $A' = \{2, 5, 6, 10, 13\}$

b $P(B') = \frac{7}{11}$

2 Complete the diagram, using the information you are given to work out the unknown areas.

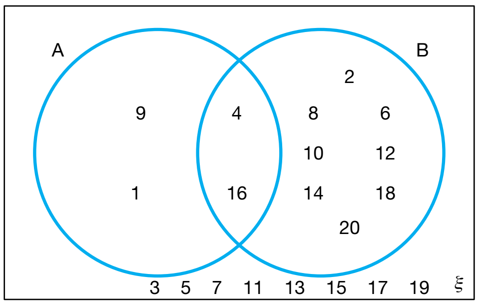


a P(all 3 sciences) $= \frac{40}{84} = \frac{10}{21}$

b P(only one science) $= \frac{(1+2+6)}{84} = \frac{9}{84} = \frac{3}{28}$

c P(chemistry if study physics) $= \frac{(8+40)}{(8+40+12+1)} = \frac{48}{61}$

3 **a**



b $P(A \cup B)' = \frac{8}{20} = \frac{2}{5}$

4 a $P(A \cap B) = P(A \mid B) \times P(B)$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{48}}{\frac{1}{4}}$

$= \frac{4}{48}$

$= \frac{1}{12}$

b $P(A \cap B) = P(B \mid A) \times P(A)$

$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$

$= \frac{\frac{1}{48}}{\frac{1}{10}}$

$= \frac{10}{48}$

$= \frac{5}{24}$

5 a $165 + 74 + 235 + 236 = 710$

b A = 165 + 235 claims

B = 165 + 74 claims

$165 = A \cap B$

There is a value for $A \cap B$: the two groups intersect. So A and B are not independent.

Or:

$P(B) = \frac{165 + 74}{710}$

$= \frac{239}{710}$

$= 0.34$

$P(B \text{ given } A) = P(B|A) = \frac{P(A \cap B)}{P(A)}$

$= \frac{\frac{165}{710}}{\frac{165 + 235}{710}}$

$= \frac{165}{400}$

$= \frac{33}{80}$

$= 0.41$

As P(B given A) $\neq$ P(B) the two events are not independent.

6 a $P(P \cap Q) = P(P \mid Q) \times P(Q)$

$P(P \mid Q) = \frac{P(P \cap Q)}{P(Q)}$

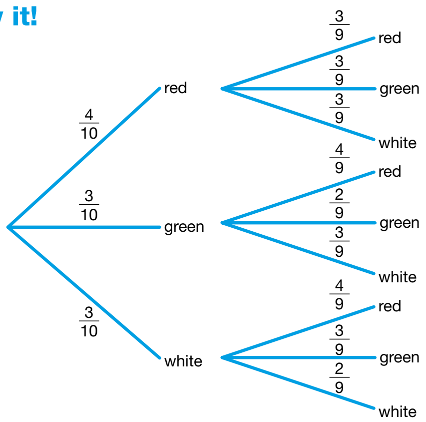
$= \frac{0.3}{0.3 + 0.4} = \frac{0.3}{0.7} = \frac{3}{7}$ (or $0.\dot{4}2857\dot{1}$)

b $P(Q \mid P) = \frac{P(P \cap Q)}{P(P)}$

$= \frac{0.3}{0.2 + 0.3} = \frac{0.3}{0.5} = \frac{3}{5}$ (or 0.6)

Review it!

1 a



b Probability the same colour

= P (2 red) + P(2 green) + P(2 white)

$= \left(\frac{4}{10} \times \frac{3}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right)$

$= \frac{12}{90} + \frac{6}{90} + \frac{6}{90}$

$= \frac{24}{90}$

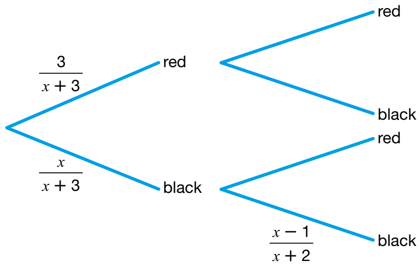
$= \frac{4}{15}$

c Probability different colours = 1 − probability of the same colour

$= 1 - \frac{4}{15}$

$= \frac{11}{15}$

2 a



Probability two black balls chosen $= \left(\frac{x}{x+3}\right) \times \left(\frac{x-1}{x+2}\right)$

Also, Probability two black balls chosen $= \frac{7}{15}$

$\left(\frac{x}{x+3}\right) \times \left(\frac{x-1}{x+2}\right) = \frac{7}{15}$

$15x(x - 1) = 7(x + 3)(x + 2)$

$15x^2 - 15x = 7x^2 + 35x + 42$

$8x^2 - 50x - 42 = 0$

$4x^2 - 25x - 21 = 0$

b Solving the quadratic equation

$4x^2 - 25x - 21 = 0$

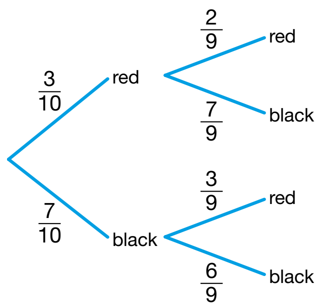
$(4x + 3)(x - 7) = 0$

$x = -\frac{3}{4}$ (which is impossible as x has to be a positive integer) or $x = 7$.

Hence $x = 7$

Total number of balls in the bag = 3 + 7 = 10

c Producing a new tree diagram now that x is known:



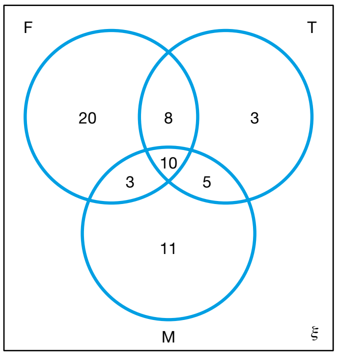
Probability of two different colours

$= \left(\frac{3}{10} \times \frac{7}{9}\right) + \left(\frac{7}{10} \times \frac{3}{9}\right)$

$= \frac{21}{90} + \frac{21}{90}$

$= \frac{7}{15}$

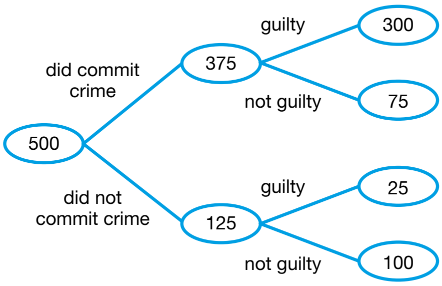
3



a P(only liked motor racing) $= \frac{11}{60}$

b P(student who liked motor racing also liked tennis) $= \frac{15}{29}$

4 a



b P(random defendant found guilty) $= \frac{300 + 25}{500}$
$= \frac{13}{20}$

c P(defendant who did not commit crime found guilty) $= \frac{25}{125} = \frac{1}{5}$

Statistics

Sampling

1 Find the % of males in the youth club

$= \frac{\text{Number of males}}{\text{Total number of members}} \times 100$

The total number of members is 185 + 165 = 350

$= \frac{185}{350} \times 100$
$= 52.86\%$

Now find this same % of the sample = 52.86% of 50 = 0.5286 × 50 = 26 (nearest whole number)

A quicker way to do this would be to notice that 50 = 350 ÷ 7, so:

Males chosen $= \frac{185}{7} = 26$ to nearest whole number.

Two-way tables and pie charts

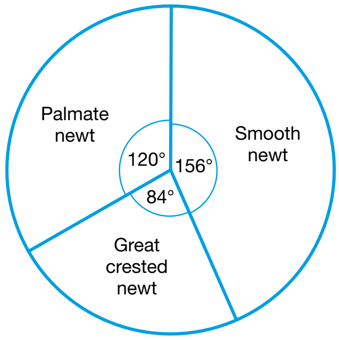
1 a Total frequency = 13 + 7 + 10 = 30

30 newts = 360° so 1 newt represents $\frac{360}{30} = 12°$

Smooth newt angle = 13 × 12 = 156°

Great crested newt angle = 7 × 12 = 84°

Palmate newt angle = 10 × 12 = 120°

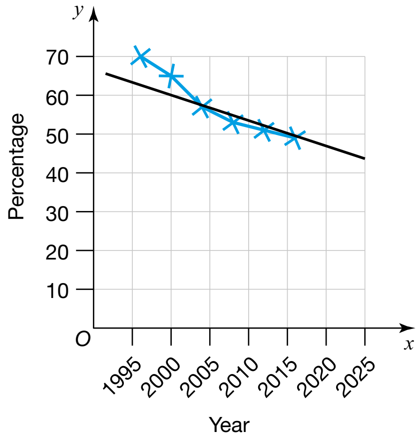


b No. The other pond might have had more newts in total. The proportion of smooth newts in the second pond is lower, but there may be more newts.

Line graphs for time series data

1 As these values are decreasing with time the trend is decreasing sales.

2 a, b and **c**



b The percentage of people visiting the local shop is decreasing.

c Drawing a trend line gives 44%

d There are no points, so you would be trying to predict the future. There may be a change of ownership or a refurbishment, making it more popular. It could even close down before then.

Averages and spread

1 Mean $= \frac{\text{Total age of members}}{\text{Number of members}}$

Rearranging, we have:

Total age of members = Mean × Number of members

Total age for boys = 13 × 10 = 130

Total age for girls = 14 × 12 = 168

Total age for boys and girls = 130 + 168 = 298

Total number of boys and girls in club = 10 + 12 = 22

Mean $= \frac{\text{Total age of members}}{\text{Number of members}} = \frac{298}{22} = 13.5$ years

2 Mean $= \frac{\text{Total number of marks}}{\text{Number of students}}$

Total mark for boys = 50 × 10 = 500

Total mark for girls = 62 × 15 = 930

Total mark for the whole class of 25 = 500 + 930
= 1430

Mean for whole class $= \frac{\text{Total number of marks}}{\text{Number of students}} = \frac{1430}{25}$
$= 57.2\%$

Joshua is wrong, because he didn't take account of the fact that there were different numbers of boys and girls.

3 a

Cost (£C)	Frequency	Mid-interval value	Frequency × mid-interval value
$0 < C \leq 4$	12	2	24
$4 < C \leq 8$	8	6	48
$8 < C \leq 12$	10	10	100
$12 < C \leq 16$	5	14	70
$16 < C \leq 20$	2	18	36

b Estimated mean $= \frac{24 + 48 + 100 + 70 + 36}{37} =$ £7.51 (to nearest penny)

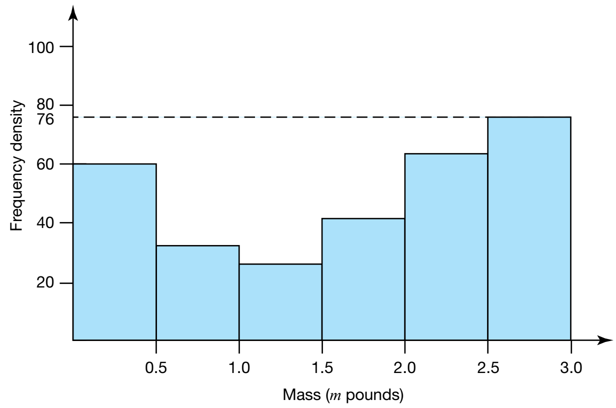
Histograms

1 Notice there is no scale for frequency density on the histogram. It is necessary to use a bar on the histogram where there is a known frequency so the frequency density can be found which will then enable the scale to be found.

Take the class $2.5 < m \leq 3.0$ which has a high frequency of 38.

frequency density $= \frac{\text{frequency}}{\text{class width}} = \frac{38}{0.5} = 76$

This bar has a height of 76 and is also the highest bar so the scale can be worked out.



For the class $0.5 < m \leq 1.0$, the frequency density can be obtained from the graph (i.e. 32).

frequency = frequency density × class width

$= 32 \times 0.5 = 16$

For the class $2.0 < m \leq 2.5$, the frequency density from the graph is 64.

frequency = frequency density × class width

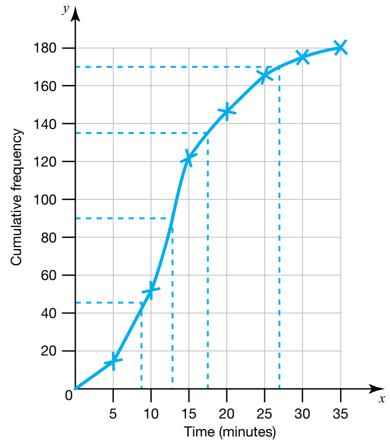
$= 64 \times 0.5 = 32$

The table can now be completed with the new frequencies.

Mass (m pounds)	Frequency
$0.0 < m \leq 0.5$	30
$0.5 < m \leq 1.0$	**16**
$1.0 < m \leq 1.5$	13
$1.5 < m \leq 2.0$	21
$2.0 < m \leq 2.5$	**32**
$2.5 < m \leq 3.0$	38

Cumulative frequency graphs

1 a



b To read off the median find half-way through the cumulative frequency and then draw a horizontal line and where it meets the curve draw a vertical line down. The median is the value where this line cuts the horizontal axis.

Hence median = 13 minutes

c i To find the upper quartile read off $\frac{3}{4}$ of the way through the cumulative frequency to the curve and read off the value on the horizontal axis.
Upper quartile = 17.5 minutes

ii To find the lower quartile read off $\frac{1}{4}$ of the way through the cumulative frequency to the curve and read off the value on the horizontal axis.
Lower quartile = 9 minutes

iii Interquartile range = upper quartile − lower quartile = 17.5 − 9 = 8.5 minutes

d Read vertically up from 27 minutes to the curve and then across and read off the cumulative frequency. This gives 170 which means 180 − 170 = 10 waited more than 27 minutes.

$\frac{\text{Number of people waiting 27 minutes or more}}{\text{Total number of people}} \times 100 = \frac{10}{180} \times 100 = 5.6\%$ (1 d.p.)

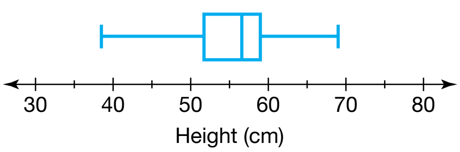
So 5.6% of passengers waited for 27 minutes or more at security.

Comparing sets of data

1

	Height (cm)
Lowest height	38
Lower quartile	**52**
Median	**57**
Upper quartile	**59**
Highest height	69

Measurements from the scale on the box plot are transferred to the table.



The lowest height and the highest height are added to complete the box plot.

2 **a** **i** Range = 120 − 0 = 120 marks

ii Median = 65 marks

iii Upper quartile = 75 marks

iv Lower quartile = 51 marks

v Interquartile range = 75 − 51 = 24 marks

b For the girls:

You need to find the median and upper and lower quartiles from the graph for the girls.

Median mark = 74 marks

Upper quartile = 89 marks

Lower quartile = 58 marks

Interquartile range = 89 − 58 = 31 marks

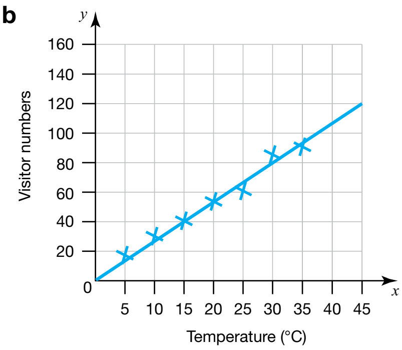
Comparison:

The median mark for the girls is higher.

The interquartile range is lower for the boys showing that their marks are less spread out for the middle half of the marks.

Scatter graphs

1 **a, b**



c 71 visitors

d **i** 120 visitors

ii There are no points near this temperature so you cannot assume the trend continues. As the temperature gets extremely high people might choose to stay away from the beach.

Review it!

1 **a** Number of students in class = 6 + 15 + 9 + 6 = 36

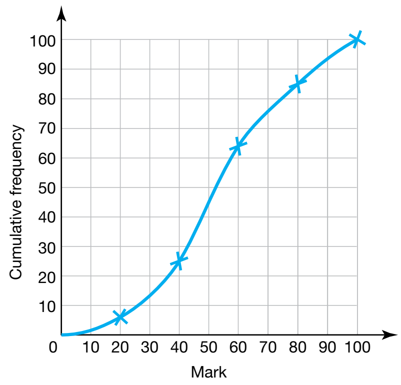
b

Amount of pocket money (£a per week)	Frequency	Mid-interval value	Frequency × mid-interval value
$2 < a \le 4$	6	3	6 × 3 = 18
$4 < a \le 6$	15	5	15 × 5 = 75
$6 < a \le 8$	9	7	9 × 7 = 63
$8 < a \le 10$	6	9	6 × 9 = 54

Estimate for mean = $\frac{18 + 75 + 63 + 54}{36}$ = £5.83 (to nearest penny)

c Modal class interval is $4 < a \le 6$

2 **a**



b Median = 54 (i.e. the mark corresponding to half-way through the cumulative frequency)

c The top 75% would pass so the bottom 25% would fail. This 25% corresponds to a cumulative frequency of 25 which gives a pass mark of 40.

3 **a** D (the median mark is furthest to the right)

b C (the largest gap between the quartiles)

c D (the median mark is the highest and the interquartile range is small, which means 50% of pupils got near to the median mark).

4 **a** The sample needs to be representative of the people living on the street, so they should take a stratified sample that includes different groups such as males and females, adults and children, in the right proportions.

b $\frac{15}{40} \times 320 = 120$

They should buy lemonade for 120 people.