AQA Higher Mathematics Exam Practice Book

Full worked solutions

Number

Integers, decimals and symbols

1 $(-1)^3 = -1$

$(0.1)^2 = 0.01$

$\frac{1}{1000} = 0.001$

0.1

$\frac{1}{0.01} = 100$

Descending order: $\frac{1}{0.01}$ 0.1 $(0.1)^2$ $\frac{1}{1000}$ $(-1)^3$

2 **a** $0.035 \times 1000 = 35$

b $12.85 \div 1000 = 0.01285$

c $(-3) \times 0.09 \times 1000 = -0.27 \times 1000 = -270$

d $(-1) \times (-0.4) \times 100 = 0.4 \times 100 = 40$

3 **a** $86 \times 54 = (0.86 \times 100) \times 54 = 46.44 \times 100 = 4644$

b $8.6 \times 540 = (0.86 \times 10) \times (54 \times 10) = 46.44 \times 100 = 4644$

c $\frac{4644}{54} = \frac{46.44 \times 100}{54} = 0.86 \times 100 = 86$

d $\frac{46.44}{0.086} = \frac{46.44}{0.86 \div 10} = 54 \times 10 = 540$

4 **a** $12.56 \times 3.45 = 0.1256 \times 345$

b $(-8)^2 = 64$, so $(-8)^2 > -64$

c $6 - 12 = -6$ and $8 - 14 = -6$, so $6 - 12 = 8 - 14$

d $(-7) \times (0) = 0$ and $(-7) \times (-3) = 21$,

so $(-7) \times (0) < (-7) \times (-3)$

Addition, subtraction, multiplication and division

1 **a** $67.78 + 8.985 = 76.765$

b $124.706 + 76.9 + 0.04 = 201.646$

c $93.1 - 1.77 = 91.33$

d $23.7 + 8.94 - 22.076 = 32.64 - 22.076 = 10.564$

2 **a** $147 \times 8 = 1176$

b $57 \times 38 = 2166$

c $9.7 \times 4.6 = 44.62$

d $1.24 \times 0.53 = 0.6572$

e $486 \div 18 = 27$

f $94.5 \div 1.5 = 945 \div 15 = 63$

3 **a** $34^2 = 34 \times 34 = 1156$

b $\frac{1.5 \times 2.5}{0.5} = 3 \times 2.5 = 7.5$

c $2.4^2 = 2.4 \times 2.4 = 5.76$

Using fractions

1 $\frac{2}{5} = \frac{16}{40} = \frac{30}{75} = \frac{50}{125}$

2 **a** $\frac{64}{12} = 5\frac{4}{12} = 5\frac{1}{3}$

b $\frac{124}{13} = 9\frac{7}{13}$

3 **a** $4\frac{1}{4} \times 1\frac{2}{3} = \frac{17}{4} \times \frac{5}{3} = \frac{85}{12} = 7\frac{1}{12}$

b $1\frac{7}{8} \div \frac{1}{4} = \frac{15}{8} \times \frac{4}{1} = \frac{60}{8} = \frac{15}{2} = 7\frac{1}{2}$

c $3\frac{1}{5} - \frac{3}{4} = 3\frac{4}{20} - \frac{15}{20} = 2\frac{24}{20} - \frac{15}{20} = 2\frac{9}{20}$

4 $1 - \left(\frac{2}{7} + \frac{3}{8} + \frac{1}{4}\right) = 1 - \frac{2 \times 8 + 3 \times 7 + 1 \times 14}{56} = 1 - \frac{51}{56} = \frac{5}{56}$

$\frac{5}{56}$ of the students travel by car.

5 The lowest common multiple of all the denominators is 24.

$\frac{2}{3} = \frac{16}{24}$ $\frac{3}{4} = \frac{18}{24}$ $\frac{7}{8} = \frac{21}{24}$ $\frac{1}{2} = \frac{12}{24}$ $\frac{7}{12} = \frac{14}{24}$

Order with the smallest first:

$\frac{1}{2}$ $\frac{7}{12}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{7}{8}$

Different types of number

1 **a** 7

b 49

c 2

d 6

e 6

2 **a** $693 = 3^2 \times 7 \times 11$

b $945 = 3^3 \times 5 \times 7$

3^2 and 7 are common to both lists.

Highest common factor $= 3^2 \times 7 = 63$

c Lowest common multiple $= 3^3 \times 5 \times 7 \times 11 = 10\,395$

3 $49 = 7^2$

$63 = 9 \times 7 = 3 \times 3 \times 7 = 3^2 \times 7$

Lowest common multiple $= 3^2 \times 7^2 = 441$

4 $15 = 3 \times 5$

$20 = 4 \times 5 = 2^2 \times 5$

$25 = 5^2$

Lowest common multiple $= 2^2 \times 3 \times 5^2 = 300$ seconds $= 5$ minutes

They will make a note together after 5 minutes.

Listing strategies

1 Multiples of 14:

14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154, 168, 182, 196, 210, …

Multiples of 15:

15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, 195, 210, …

210 is common to both lists, so they will sound together after 210 seconds.

2 Multiples of 6:

6, 12, 18, 24, 30, 36, 42, …

Multiples of 6 add 1:

7, 13, 19, 25, 31, 37, 43, …

Multiples of 7:

7, 14, 21, 28, 35, 42, 49, 56, …

Multiples of 7 minus 4:

3, 10, 17, 24, 31, 38, 45, 52

31 is common to both lists. This means 6 chocolates could be given to 5 friends with one chocolate left over.

Hence number of friends = 5

3 List all the ratios that are equivalent to 5 : 6, find the difference between them and look for a difference of 100.

Since this is a college, the numbers are likely to be large.

Male	Female	Difference
50	60	10
500	600	100

There are a total of $500 + 600 = 1100$ students.

4 Call the students A, B, C, D, E and F.

Possible pairs beginning with A:

AB, AC, AD, AE, AF

Possible pairs beginning with B (excluding pairs in first list):

BC, BD, BE, BF

Possible pairs beginning with C (excluding pairs in previous lists):

CD, CE, CF

Possible pairs beginning with D (excluding pairs in previous lists):

DE, DF

Possible pairs beginning with E (excluding pairs in previous lists):

EF

Total number of pairs = 5 + 4 + 3 + 2 + 1 = 15

The order of operations in calculations

1 **a** Ravi has worked out the expression from left to right, instead of using BIDMAS. He should have performed the division and multiplication before the addition.

b $24 \div 3 + 8 \times 4 = 8 + 32$

$= 40$

2 **a** $9 \times 7 \times 2 - 24 \div 6 = 126 - 4 = 122$

b $2 - (-27) \div (-3) + 4 = 2 - 9 + 4 = -3$

c $(4 - 16)^2 \div 4 + 32 \div 8 = (-12)^2 \div 4 + 4 = 144 \div 4 + 4$
$= 36 + 4 = 40$

3 **a** $1 + 4 \div \frac{1}{2} - 3 = 1 + 8 - 3 = 6$

b $15 - (1 - 2)^2 = 15 - (-1)^2 = 15 - 1 = 14$

c $\sqrt{4 \times 12 - 2 \times (-8)} = \sqrt{48 + 16} = \sqrt{64} = 8$

Indices

1 **a** $10^5 \times 10 = 10^{5+1} = 10^6$

b $(10^4)^2 = 10^{4 \times 2} = 10^8$

c $\frac{10^5 \times 10^3}{10^2} = 10^{5+3-2} = 10^6$

d $(10^6)^{\frac{1}{2}} = 10^{6 \times \frac{1}{2}} = 10^3$

2 **a** $5^0 = 1$

b $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

c $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

d $49^{\frac{1}{2}} = \sqrt{49} = 7$

3 **a** $\left(\frac{8}{27}\right)^{-\frac{1}{3}} = \left(\frac{27}{8}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2}$

b $\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2 = 4^2 = 16$

c $36^{-\frac{1}{2}} = \frac{1}{\sqrt{36}} = \frac{1}{6}$

d $16^{\frac{3}{2}} = \left(16^{\frac{1}{2}}\right)^3 = (\sqrt{16})^3 = 4^3 = 64$

4 $125 = 5^3$, so $5^{2x} = 5^3$

$2x = 3$ giving $x = 1.5$

Surds

1 **a** $(\sqrt{5})^2 = \sqrt{5} \times \sqrt{5} = 5$

b $3\sqrt{2} \times 5\sqrt{2} = 3 \times 5 \times \sqrt{2} \times \sqrt{2} = 15 \times 2 = 30$

c $(3\sqrt{2})^2 = 3^2 \times (\sqrt{2})^2 = 9 \times 2 = 18$

2 $\frac{15}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{15 \times \sqrt{3}}{4 \times 3} = \frac{5\sqrt{3}}{4}$

3 $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 2\sqrt{3} + 2\sqrt{3} - 3 = 1$

4 $\sqrt{3}\,(3\sqrt{24} + 4\sqrt{6}) = 3\sqrt{72} + 4\sqrt{18}$

$= 3\sqrt{36 \times 2} + 4\sqrt{9 \times 2}$

$= 3 \times 6\sqrt{2} + 4 \times 3\sqrt{2}$

$= 18\sqrt{2} + 12\sqrt{2}$

$= 30\sqrt{2}$

$a = 30$

5 $(1 - \sqrt{5})(3 + 2\sqrt{5}) = 3 + 2\sqrt{5} - 3\sqrt{5} - 10 = -\sqrt{5} - 7$

6 $\frac{1}{\sqrt{2}} + \frac{1}{4} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{1}{4} = \frac{\sqrt{2}}{2} + \frac{1}{4} = \frac{2\sqrt{2}}{4} + \frac{1}{4} = \frac{1 + 2\sqrt{2}}{4}$

7 $\frac{2}{1 - 1/\sqrt{2}} = \frac{2}{\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}}}$

$= \frac{2}{\frac{\sqrt{2} - 1}{\sqrt{2}}}$

$= \frac{2\sqrt{2}}{\sqrt{2} - 1}$

$= \frac{2\sqrt{2}}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$

$= \frac{4 + 2\sqrt{2}}{2 - 1}$

$= 4 + 2\sqrt{2}$

8 $\frac{3}{\sqrt{3}} + \sqrt{75} + (\sqrt{2} \times \sqrt{6}) = \frac{3\sqrt{3}}{3} + \sqrt{3 \times 25} + \sqrt{12}$

$= \sqrt{3} + 5\sqrt{3} + \sqrt{3 + 4}$

$= \sqrt{3} + 5\sqrt{3} + 2\sqrt{3}$

$= 8\sqrt{3}$

Standard form

1 **a** 2.55×10^{-3}

b 1.006×10^{10}

c 8.9×10^{-8}

2 **a** $3 \times 10^6 \times 2 \times 10^8 = 3 \times 2 \times 10^{6+8} = 6 \times 10^{14}$

b $5.5 \times 10^{-3} \times 2 \times 10^8 = 5.5 \times 2 \times 10^{-3+8} = 11 \times 10^5$
$= 1.1 \times 10^6$

c $\frac{8 \times 10^5}{4 \times 10^3} = \frac{8}{4} \times 10^{5-3} = 2 \times 10^2$

d $\frac{5 \times 10^{-3}}{0.5} = \frac{5}{0.5} \times 10^{-3} = 10 \times 10^{-3} = 1 \times 10^{-2}$

e $\frac{4.5 \times 10^{-6}}{0.5 \times 10^{-3}} = \frac{4.5}{0.5} \times \frac{10^{-6}}{10^{-3}} = 9 \times 10^{-6-(-3)} = 9 \times 10^{-3}$

3 $2.4 \times 10^3 + 2.8 \times 10^2 = 2400 + 280 = 2680$

4 $3.3 \times 10^4 = 33\,000$ and $3.3 \times 10^2 = 330$

$3.3 \times 10^4 + 3.3 \times 10^2 = 33\,330$

$a = 3.3$

Converting between fractions and decimals

1 **a** $\frac{11}{20} = \frac{55}{100} = 0.55$

b $\frac{3}{8} = 3 \div 8 = 0.375$

2 **a** $16 = 2 \times 2 \times 2 \times 2$

All the prime factors in the denominator are 2, so it will produce a terminating fraction.

b The prime factors of the denominators are 7 and 1 so this will produce a recurring decimal.

c Prime factors of 35 are 7 and 5 so this will produce a recurring decimal.

3 Let $x = 0.\dot{4}0\dot{2} = 0.402402402\ldots$

$1000x = 402.402402\ldots$

$1000x - x = 402.402402\ldots - 0.402402402\ldots$

$999x = 402$

$x = \frac{402}{999} = \frac{134}{333}$

Hence $0.\dot{4}0\dot{2} = \frac{134}{333}$

4 Let $x = 0.6\dot{5}\dot{2} = 0.652525252\ldots$

$10x = 6.52525252\ldots$

$1000x = 652.52525252\ldots$

$1000x - 10x = 652.52525252\ldots - 6.52525252\ldots$

$990x = 646$

$x = \frac{646}{990} = \frac{323}{495}$

Hence $0.6\dot{5}\dot{2} = \frac{323}{495}$

Converting between fractions and percentages

1 a $35\% = \frac{35}{100} = \frac{7}{20}$

b $7\% = \frac{7}{100}$

c $76\% = \frac{76}{100} = \frac{19}{25}$

d $12.5\% = \frac{12.5}{100} = \frac{125}{1000} = \frac{1}{8}$

2 a $\frac{1}{5} = \frac{1 \times 20}{5 \times 20} = \frac{20}{100} = 20\%$

b $\frac{17}{25} = \frac{17 \times 4}{25 \times 4} = \frac{68}{100} = 68\%$

c $\frac{150}{60} = \frac{50}{20} = \frac{250}{100} = 250\%$

d $\frac{7}{40} = 7 \div 40 = 0.7 \div 4 = 0.175 = 17.5\%$

3 $\frac{8}{15} = \frac{8}{15} \times 100\% = 53.33\%$ (to 2 d.p.)

4 $\frac{66}{90} = \frac{66}{90} \times 100\% = 73.3\%$ (to 1 d.p.)

Jake did better in chemistry.

Fractions and percentages as operators

1 70% of £49.70 = 0.7 × 49.70 = £34.79

2 8% of 600 = $\frac{8}{100} \times 600 = 48$

48 apples are bad.

3 12% of 8000 = 0.12 × 8000 = 960 components were rejected.

8000 − 960 = 7040 components were accepted.

4 a VAT = 20% of £12 000 = 0.2 × 12 000 = £2400

Total cost = 12 000 + 2400 = £14 400

b Deposit = 20% of £14 400 = 0.2 × 14 400 = £2880

Remainder = 14 400 − 2880 = £11 520

Monthly payment = $\frac{11\,520}{36}$ = £320

5 $\frac{2}{3}$ of $\frac{7}{11} = \frac{2}{3} \times \frac{7}{11} = \frac{14}{33}$

Standard measurement units

1 1.75 km = 1.75 × 1000 m = 1750 m

1 m = 100 cm, so 1750 m = 1750 × 100 = 175 000 cm

Hence 1.75 km = 175 000 cm

2 3 litres = 3000 ml

Number of glasses = $\frac{3000}{175} = 17.14$

Number of complete glasses = 17

3 900 litres = 900 × 1000 = 900 000 cm^3

Number of containers needed to fill pool = $\frac{900\,000}{700} = 1286$ (to nearest whole number)

4 a Mass of one atom = $\frac{12}{6.02 \times 10^{23}} = 1.99 \times 10^{-23}$ g (to 3 s.f.)

b Mass of one atom = $\frac{12 \times 10^{-3}}{6.02 \times 10^{23}} = 1.99 \times 10^{-26}$ kg (to 3 s.f.)

5 $79 \times 9.11 \times 10^{-31}$ kg $= 79 \times 9.11 \times 10^{-31} \times 10^3$ g $= 7.20 \times 10^{-26}$ g (to 3 s.f.)

Rounding numbers

1 a 35 b 101 c 0 d 0 e 2

2 a 34.88 b 34.877

3 a 12 800

b 0.011

c 7×10^{-5}

4 a $(0.18 \times 0.046)^2 - 0.01 = -0.009\,93$ (to 3 s.f.)

b $\frac{1200 \times 1.865}{2.6 \times 25} = 34.4$ (to 3 s.f.)

c $\frac{36}{0.07} \times 12 \div \frac{1}{2} = 12\,300$ (to 3 s.f.)

Estimation

1 $4.6 \times 9.8 \times 3.1 \approx 5 \times 10 \times 3 = 150 \approx 200$ (to 1 s.f.)

2 a 236.229 8627

b $19.87^2 - \sqrt{404} \times 7.89 \approx 20^2 - \sqrt{400} \times 8$

$= 400 - 20 \times 8 = 240$

3 $(0.52 \times 0.83)^2 \approx (0.5 \times 0.8)^2 = (0.4)^2 = 0.16$

4 $\sqrt{5.08 + 4.10 \times 5.45} \approx \sqrt{5 + 4 \times 5} = \sqrt{25} = 5$

5 $\sqrt{100} = 10$ and $\sqrt{121} = 11$

$\sqrt{112}$ is slightly over halfway between the two roots.

Hence $\sqrt{112} = 10.6$ (to 1 d.p.) is a reasonable estimate.

6 $\frac{0.89 \times 7.51 \times 19.76}{2.08 \times 5.44 \times 3.78} \approx \frac{1 \times 8 \times 20}{2 \times 5 \times 4} = \frac{160}{40} = 4$

7 a mass $\approx 9 \times 10^{-31} \times 6 \times 100 = 5.4 \times 10^{-28} \approx 5 \times 10^{-28}$ kg

b This will be an underestimate, as the mass of one electron has been rounded down.

Upper and lower bounds

1 upper bound = 2.345 kg

lower bound = 2.335 kg

$2.335 \le l < 2.345$ kg

2 a i $V = \frac{P}{I}$

The upper bound for V will be when P has its upper bound and I has its lower bound.

upper bound for $P = 3.0525$

lower bound for $I = 1.235$

upper bound for $V = \frac{3.0525}{1.235} = 2.472$

ii The lower bound for V will be when P has its lower bound and I has its upper bound.

lower bound of $P = 3.0515$

upper bound of $I = 1.245$

lower bound for $V = \frac{3.0515}{1.245} = 2.451$

b The upper bound and the lower bound are the same when the numbers are given to 2 significant figures.

$V = 2.5$ (to 2 s.f.)

3 lower bound for shelf length = 1.05 m = 105 cm

upper bound for book thickness = 3.05 cm

number of books on shelf = $\frac{\text{length of shelf}}{\text{thickness of book}}$

number of books that will definitely fit onto shelf =

minimum value for $\frac{\text{length of shelf}}{\text{thickness of book}}$

minimum value for $\frac{\text{length of shelf}}{\text{thickness of book}}$ is when shelf length is at its lower bound and book thickness is at its upper bound.

$\frac{\text{minimum length of shelf}}{\text{maximum thickness of book}} = \frac{105}{3.05} = 34.4$

The answer must be a whole number, so 34 books will definitely fit on the shelf.

Algebra

Simple algebraic techniques

1 a formula c expression e formula
  b identity d identity

2 $4x + 3x \times 2x - 3x = 4x + 6x^2 - 3x = x + 6x^2$

3 $y^3 - y = (1)^3 - 1 = 0$ so $y = 1$ is correct.
  $y^3 - y = (-1)^3 - (-1) = -1 + 1 = 0$ so $y = -1$ is correct.

4 a $6x - (-4x) = 6x + 4x = 10x$
  b $x^2 - 2x - 4x + 3x^2 = 4x^2 - 6x$
  c $(-2x)^2 + 6x \times 3x - 4x^2 = 4x^2 + 18x^2 - 4x^2 = 18x^2$

5 a $s = \frac{3^2 - 1^2}{2 \times 2} = \frac{8}{4} = 2$
  b $s = \frac{(-4)^2 - 3^2}{2 \times 4} = \frac{7}{8}$
  c $s = \frac{5^2 - (-2)^2}{2 \times (-7)} = \frac{21}{-14} = -\frac{3}{2}$

Removing brackets

1 a $8(3x - 7) = 8 \times 3x - 8 \times 7$
  $= 24x - 56$
  b $-3(2x - 4) = -3 \times 2x - 3 \times (-4)$
  $= -6x + 12$

2 a $3(2x - 1) - 3(x - 4) = 6x - 3 - 3x + 12$
  $= 3x + 9$
  b $4y(2x + 1) + 6(x - y) = 8xy + 4y + 6x - 6y$
  $= 8xy + 6x - 2y$
  c $5ab(2a - b) = 10a^2b - 5ab^2$
  d $x^2y^3(2x + 3y) = 2x^3y^3 + 3x^2y^4$

3 a $(m - 3)(m + 8) = m^2 + 8m - 3m - 24$
  $= m^2 + 5m - 24$
  b $(4x - 1)(2x + 7) = 8x^2 + 28x - 2x - 7$
  $= 8x^2 + 26x - 7$
  c $(3x - 1)^2 = (3x - 1)(3x - 1)$
  $= 9x^2 - 3x - 3x + 1$
  $= 9x^2 - 6x + 1$
  d $(2x + y)(3x - y) = 6x^2 - 2xy + 3xy - y^2$
  $= 6x^2 + xy - y^2$

4 a $(x + 5)(x + 2) = x^2 + 2x + 5x + 10$
  $= x^2 + 7x + 10$
  b $(x + 4)(x - 4) = x^2 - 4x + 4x - 16$
  $= x^2 - 16$
  c $(x - 7)(x + 1) = x^2 + x - 7x - 7$
  $= x^2 - 6x - 7$
  d $(3x + 1)(5x + 3) = 15x^2 + 9x + 5x + 3$
  $= 15x^2 + 14x + 3$

5 a $(x + 3)(x - 1)(x + 4) = (x^2 - x + 3x - 3)(x + 4)$
  $= (x^2 + 2x - 3)(x + 4)$
  $= x^3 + 4x^2 + 2x^2 + 8x - 3x - 12$
  $= x^3 + 6x^2 + 5x - 12$
  b $(3x - 4)(2x - 5)(3x + 1) = (6x^2 - 15x - 8x + 20)(3x + 1)$
  $= (6x^2 - 23x + 20)(3x + 1)$
  $= 18x^3 + 6x^2 - 69x^2 - 23x + 60x + 20$
  $= 18x^3 - 63x^2 + 37x + 20$

Factorising

1 a $25x^2 - 5xy = 5x(5x - y)$
  b $4\pi r^2 + 6\pi x = 2\pi(2r^2 + 3x)$
  c $6a^3b^2 + 12ab^2 = 6ab^2(a^2 + 2)$

2 a $9x^2 - 1 = (3x + 1)(3x - 1)$
  b $16x^2 - 4 = (4x + 2)(4x - 2)$
  $= 4(2x + 1)(2x - 1)$

3 a $a^2 + 12a + 32 = (a + 4)(a + 8)$
  b $p^2 - 10p + 24 = (p - 6)(p - 4)$

4 a $a^2 + 12a = a(a + 12)$
  b $b^2 - 9 = (b + 3)(b - 3)$
  c $x^2 - 11x + 30 = (x - 5)(x - 6)$

5 a $3x^2 + 20x + 32 = (3x + 8)(x + 4)$
  b $3x^2 + 10x - 13 = (3x + 13)(x - 1)$
  c $2x^2 - x - 10 = (2x - 5)(x + 2)$

6 $\frac{x + 15}{2x^2 - 3x - 9} + \frac{3}{2x + 3} = \frac{x + 15}{(2x + 3)(x - 3)} + \frac{3}{(2x + 3)}$
  $= \frac{x + 15 + 3(x - 3)}{(2x + 3)(x - 3)}$
  $= \frac{4x + 6}{(2x + 3)(x - 3)}$
  $= \frac{2(2x + 3)}{(2x + 3)(x - 3)}$
  $= \frac{2}{(x - 3)}$

7 $\frac{1}{8x^2 - 2x - 1} \div \frac{1}{4x^2 - 4x + 1} = \frac{1}{8x^2 - 2x - 1} \times (4x^2 - 4x + 1)$
  $= \frac{1}{(4x + 1)(2x - 1)} \times (2x - 1)(2x - 1)$
  $= \frac{2x - 1}{4x + 1}$

Changing the subject of a formula

1 $PV = nRT$
  $T = \frac{PV}{nR}$

2 $2y + 4x - 1 = 0$
  $2y = 1 - 4x$
  $y = \frac{1 - 4x}{2}$

3 $v = u + at$
  $at = v - u$
  $a = \frac{(v - u)}{t}$

4 $y = \frac{x}{5} - m$
  $\frac{x}{5} = y + m$
  $x = 5(y + m)$

5 $E = \frac{1}{2}mv^2$
  $v^2 = \frac{2E}{m}$
  $v = \sqrt{\frac{2E}{m}}$

6 a $V = \frac{1}{3}\pi r^2 h$
  $r^2 = \frac{3V}{\pi h}$
  $r = \sqrt{\frac{3V}{\pi h}}$
  b $r = \sqrt{\frac{3 \times 100}{\pi \times 8}}$
  $= 3.45$ cm (to 2 d.p.)

7 **a** $y = 3x - 9$

$3x = y + 9$

$x = \frac{y+9}{3}$

b $x = \frac{3+9}{4}$

$= 4$

8 $3y - x = ax + 2$

$3y - x - 2 = ax$

$3y - 2 = ax + x$

$3y - 2 = x(a + 1)$

$x = \frac{3y-2}{a+1}$

9 **a** $c^2 = \frac{(16a^2b^4c^2)^{\frac{1}{2}}}{4a^2b}$

$c^2 = \frac{4ab^2c}{4a^2b}$

$c^2 = \frac{bc}{a}$

$c = \frac{b}{a}$

b upper bound of $a = 2.85$ lower bound of $a = 2.75$

upper bound of $b = 3.25$ lower bound of $b = 3.15$

upper bound for $c = \frac{\text{upper bound for } b}{\text{lower bound for } a} = \frac{3.25}{2.75} = 1.18$ (to 3 s.f.)

lower bound for $c = \frac{\text{lower bound for } b}{\text{upper bound for } a} = \frac{3.15}{2.85} = 1.11 =$ (to 3 s.f.)

Solving linear equations

1 **a** $2x + 11 = 25$

$2x = 14$

$x = 7$

b $3x - 5 = 10$

$3x = 15$

$x = 5$

c $15x = 60$

$x = 4$

d $\frac{x}{4} = 8$

$x = 32$

e $\frac{4x}{5} = 20$

$4x = 100$

$x = 25$

f $\frac{2x}{3} = -6$

$2x = -18$

$x = -9$

g $5 - x = 7$

$5 = 7 + x$

$-2 = x$

$x = -2$

h $\frac{x}{7} - 9 = 3$

$\frac{x}{7} = 12$

$x = 84$

2 $5x - 1 = 2x + 1$

$5x = 2x + 2$

$3x = 2$

$x = \frac{2}{3}$

3 **a** $\frac{1}{4}(2x-1) = 3(2x-1)$

$2x - 1 = 12(2x - 1)$

$2x - 1 = 24x - 12$

$11 = 22x$

$x = \frac{11}{22}$

$x = \frac{1}{2}$

b $5(3x + 1) = 2(5x - 3) + 3$

$15x + 5 = 10x - 6 + 3$

$15x + 5 = 10x - 3$

$5x = -8$

$x = -\frac{8}{5}$

Solving quadratic equations using factorisation

1 **a** $x^2 - 7x + 12 = (x - 3)(x - 4)$

b $x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

So $x - 3 = 0$ or $x - 4 = 0$, giving $x = 3$ or $x = 4$

2 **a** $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

b $2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

So $2x - 1 = 0$ or $x + 3 = 0$, giving $x = \frac{1}{2}$ or $x = -3$

3 $x^2 - 3x - 20 = x - 8$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

So $x + 2 = 0$ or $x - 6 = 0$, giving $x = -2$ or $x = 6$

4 **a** $x(x - 8) - 7 = x(5 - x)$

$x^2 - 8x - 7 = 5x - x^2$

$2x^2 - 13x - 7 = 0$

b $2x^2 - 13x - 7 = 0$

$(2x + 1)(x - 7) = 0$

So $2x + 1 = 0$ or $x - 7 = 0$, giving $x = -\frac{1}{2}$ or $x = 7$

5 area of trapezium $= \frac{1}{2}(a + b)h$

$= \frac{1}{2}(x + 4 + x + 8)x$

$= \frac{1}{2}(2x + 12)x$

$= (x + 6)x$

$= x^2 + 6x$

area $= 16\,\text{cm}^2$ so $x^2 + 6x = 16$

$x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$

Solving gives $x = -8$ or $x = 2$

$x = -8$ is impossible as x is the height and so cannot be negative.

Hence $x = 2\,\text{cm}$

Solving quadratic equations using the formula

1 **a** $\frac{3}{x+7} = \frac{2-x}{x+1}$

$3(x + 1) = (2 - x)(x + 7)$

$3x + 3 = 2x + 14 - x^2 - 7x$

$3x + 3 = -x^2 - 5x + 14$

$x^2 + 8x - 11 = 0$

b $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times (-11)}}{2 \times 1}$

$= \frac{-8 \pm \sqrt{64 + 44}}{2}$

$= \frac{-8 \pm \sqrt{108}}{2}$

$= \frac{-8 + \sqrt{108}}{2}$ or $\frac{-8 - \sqrt{108}}{2}$

$= 1.1962$ or -9.1962

$x = 1.20$ or -9.20 (to 2 d.p.)

2 area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2}(3x + 1)(2x + 3)$

$= \frac{1}{2}(6x^2 + 9x + 2x + 3)$

$= \frac{1}{2}(6x^2 + 11x + 3)$

$= 3x^2 + 5.5x + 1.5$

area $= 40$, so $3x^2 + 5.5x + 1.5 = 40$

Hence, $3x^2 + 5.5x - 38.5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-5.5 \pm \sqrt{5.5^2 - 4 \times 3 \times (-38.5)}}{2 \times 3}$$
$$= \frac{-5.5 \pm \sqrt{492.25}}{6}$$
$$= \frac{-5.5 + \sqrt{492.25}}{6} = \text{or } \frac{-5.5 - \sqrt{492.25}}{6}$$

$= 2.78$ or -4.61 (to 2 d.p.)

$x = -4.61$ would give negative lengths, which are impossible.

$x = 2.78$ cm (to 2 d.p.)

3 $x^2 - 2x - 9 = x - 8$

$x^2 - 3x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$
$$= \frac{3 \pm \sqrt{9 + 4}}{2}$$
$$= \frac{3 \pm \sqrt{13}}{2}$$
$$= \frac{3 + \sqrt{13}}{2} = \text{or } \frac{3 - \sqrt{13}}{2}$$

$x = 3.30$ or -0.30 (to 2 d.p.)

Solving simultaneous equations

1 $2x - 3y = -5$ (1)

$5x + 2y = 16$ (2)

Equation (1) × 2 and equation (2) × 3 gives:

$4x - 6y = -10$ (3)

$15x + 6y = 48$ (4)

Equation (3) + equation (4) gives:

$19x = 38$

$x = 2$

Substituting $x = 2$ into equation (1):

$2 \times 2 - 3y = -5$

$4 - 3y = -5$

$3y = 9$

$y = 3$

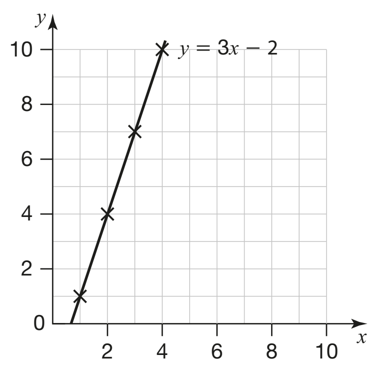
Checking by substituting $x = 2$ and $y = 3$ into equation (2) gives:

$5 \times 2 + 2 \times 3 = 10 + 6 = 16$

$x = 2$ and $y = 3$

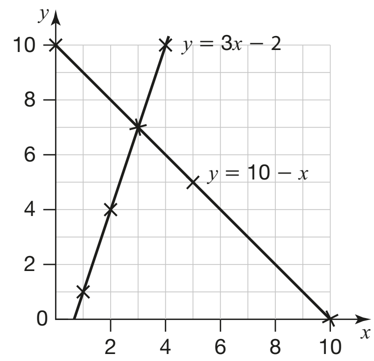
2 **a** Table of values for plotting graph of $y = 3x - 2$:

x	1	2	3	4
y	1	4	7	10



b Table of values for plotting graph of $y = 10 - x$:

x	0	5	10
y	10	5	0



The graphs intersect at (3, 7).

$x = 3$, $y = 7$

3 $x - y = 3$ (1)

$x^2 + y^2 = 9$ (2)

Rearrange equation (1) as $y = x - 3$.

Substitute in equation (2):

$x^2 + (x - 3)^2 = 9$

$x^2 + x^2 - 6x + 9 = 9$

$2x^2 - 6x = 0$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0$ or $x = 3$

Substituting into equation (1) gives:

$x = 0, y = -3$ or $x = 3, y = 0$

Solving inequalities

1 **a** $\frac{x + 5}{4} \geq -1$

$x + 5 \geq -4$

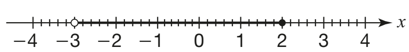
$x \geq -9$

b $3x - 4 > 4x + 8$

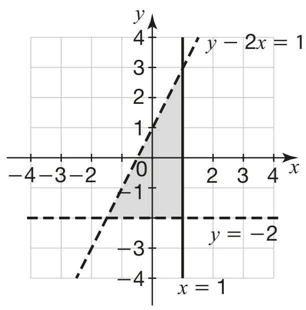
$-x - 4 > 8$

$-x > 12$

$x < -12$

2 

3 **a**



b Coordinates of points that lie in the shaded region or on the solid line:

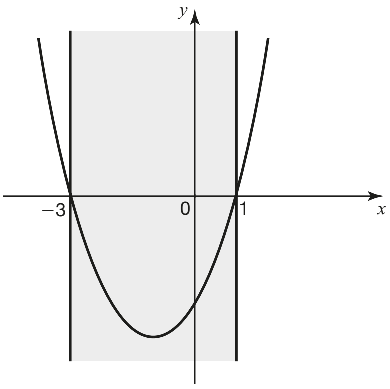
(1, 2), (1, 1), (1, 0), (1, −1), (0, 0), (0, −1), 4 $x^2 + 2x \leq 3$

$x^2 + 2x - 3 \leq 0$

Solving $x^2 + 2x - 3 = 0$:

$(x - 1)(x + 3) = 0$, giving $x = 1$ and $x = -3$

Sketch of the curve $y = x^2 + 2x - 3$:



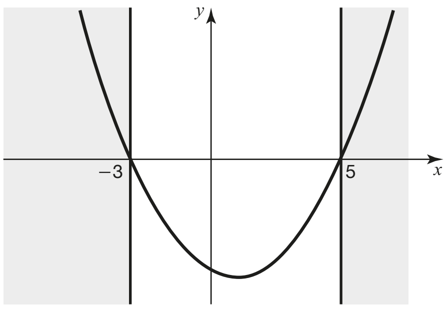
From graph, $x^2 + 2x - 3 \leq 0$ when:

$-3 \leq x \leq 1$

5 Solving $x^2 - 2x - 15 = 0$:

$(x - 5)(x + 3) = 0$, giving $x = 5$ and $x = -3$

Sketch of the curve $y = x^2 - 2x - 15$:



From graph, $x^2 - 2x - 15 > 0$ when:

$x < -3$ and $5 < x$

Problem solving using algebra

1 Let width $= x$

length $= x + 1$

perimeter $= x + x + 1 + x + x + 1 = 4x + 2$

perimeter $= 26$ so

$4x + 2 = 26$

$4x = 24$

$x = 6$

width $= 6$ m and length $= 7$ m

area $= 6 \times 7 = 42\,\text{m}^2$

2 Let cost of adult ticket $= x$ and cost of child ticket $= y$

$2x + 5y = 35$ (1)

$3x + 4y = 38.5$ (2)

Equation (1) $\times$ 3 and equation (2) $\times$ 2 gives:

$6x + 15y = 105$ (3)

$6x + 8y = 77$ (4)

Equation (3) $-$ equation (4) gives:

$7y = 28$

$y = 4$

Substituting $y = 4$ into equation (1) gives:

$2x + 20 = 35$

$2x = 15$

$x = 7.5$

cost of adult ticket = £7.50

cost of child ticket = £4

3 **a** Let Rachel be x years and Hannah be y years.

$xy = 63$

$(x + 2)(y + 2) = 99$

$xy + 2x + 2y + 4 = 99$

Substitute for xy:

$63 + 2x + 2y + 4 = 99$

$2x + 2y = 32$

$x + y = 16$

The sum of their ages is 16 years.

b $y = x - 2$

$x + x - 2 = x + y$

$2x - 2 = 16$

$2x = 18$

$x = 9$

Rachel is 9 years old.

Use of functions

1 **a** $f(3) = 5 \times 3 + 4 = 19$

b Set $f(x) = -1$

$5x + 4 = -1$

$5x = -5$

$x = -1$

2 **a** $fg(x) = f(g(x)) = f(x - 6) = (x - 6)^2$

b $gf(x) = g(f(x)) = g(x^2) = x^2 - 6$

3 **a** $f(5) = \sqrt{5 + 4} = \sqrt{9} = 3$ or -3

b $gf(x) = 2(\sqrt{x + 4})^2 - 3$

$= 2(x + 4) - 3$

$= 2x + 5$

4 Let $y = 5x^2 + 3$

$x = \sqrt{\frac{y - 3}{5}}$

$f^{-1}(x) = \sqrt{\frac{x - 3}{5}}$

Iterative methods

1 Let $f(x) = 2x^3 - 2x + 1$

$f(-1) = 2(-1)^3 - 2(-1) + 1 = 1$

$f(-1.5) = 2(-1.5)^3 - 2(-1.5) + 1 = -2.75$

There is a sign change of $f(x)$, so there is a solution between $x = -1$ and $x = -1.5$.

2 $x_1 = (x_0)^3 + \frac{1}{9} = (0.1)^3 + \frac{1}{9} = 0.1121111111$

$x_2 = (x_1)^3 + \frac{1}{9} = (0.1121111111)^3 + \frac{1}{9} = 0.1125202246$

$x_3 = (x_2)^3 + \frac{1}{9} = (0.1125202246)^3 + \frac{1}{9} = 0.1125357073$

3 **a** $x_1 = 1.5182945$

$x_2 = 1.5209353$

$x_3 = 1.5213157$

$x_4 = 1.5213705 \approx 1.521$ (to 3 d.p.)

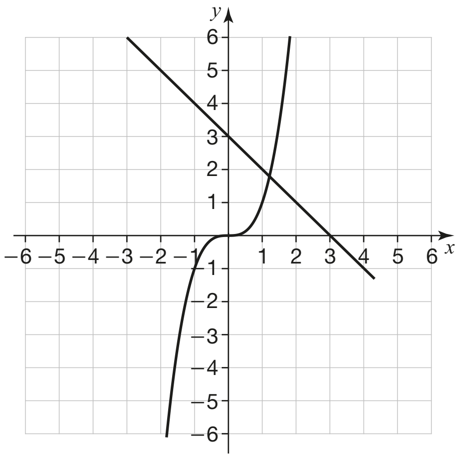
b Checking value of $x^3 - x - 2$ for $x = 1.5205, 1.5215$:

When $x = 1.5205$ $\quad$ $f(1.5205) = -0.0052$

$x = 1.5215$ $\quad$ $f(1.5215) = 0.0007$

Since there is a change of sign, the root is 1.521 correct to 3 decimal places.

4 a i



ii There is a real root of $x^3 + x - 3 = 0$ where the graphs of $y = x^3$ and $y = 3 - x$ intersect. The graphs intersect once so there is one real root of the equation $x^3 + x - 3 = 0$.

b $x_1 = 1.216440399$

$x_2 = 1.212725591$

$x_3 = 1.213566964$

$x_4 = 1.213376503$

$x_5 = 1.213419623$

$x_6 = 1.213409861 = 1.2134$ (to 4 d.p.)

Equation of a straight line

1 Comparing the equation with the equation of a straight line, $y = mx + c$:

$y = -2x + 3$

gradient of line, $m = -2$ (line has a negative gradient).

intercept on the y-axis, $c = 3$

Correct line is A.

2 a gradient, $m = \dfrac{\text{change in } y\text{−values}}{\text{change in } x\text{−values}} = -\dfrac{4}{3}$

b gradient of CD, $m = \dfrac{1-5}{5-(-3)} = \dfrac{-4}{8} = -\dfrac{1}{2}$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 1 = -\frac{1}{2}(x - 5)$

$y = -\frac{1}{2}x + \frac{7}{2}$ or $x + 2y = 7$

c M is at $\left(\dfrac{-3+5}{2}, \dfrac{5+1}{2}\right) = (1, 3)$

gradient of perpendicular to $CD = \dfrac{-1}{-\frac{1}{2}} = 2$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 3 = 2(x - 1)$

$y = 2x + 1$

3 P is the point (x, y).

gradient of line $OP = \frac{y}{x} = 3$, so $y = 3x$

Using Pythagoras' theorem:

$OP^2 = x^2 + y^2$

$12^2 = x^2 + y^2$

$12^2 = x^2 + (3x)^2$

$144 = x^2 + 9x^2$

$144 = 10x^2$

$14.4 = x^2$

$x = 3.8$ (to 1 d.p.)

$y = 3x$

$= 3 \times 3.8$

$= 11.4$

P is the point (3.8, 11.4) (to 1 d.p.).

Quadratic graphs

1 a $x^2 + 4x + 1 = 0$

$(x + 2)^2 - 4 + 1 = 0$

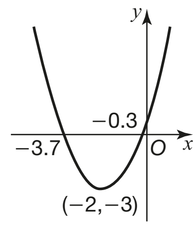
$(x + 2)^2 = 3$

$x + 2 = \pm\sqrt{3}$

$x = \sqrt{3} - 2$ or $-\sqrt{3} - 2$

$x = -0.3$ or -3.7 (to 1 d.p.)

b $x^2 + 4x + 1 = (x + 2)^2 - 3$, so turning point is at $(-2, -3)$.



2 $5x^2 - 20x + 10 = 5[x^2 - 4x + 2]$

$= 5[(x - 2)^2 - 4 + 2]$

$= 5(x - 2)^2 - 10$

$a = 5$, $b = -2$ and $c = -10$

3 $2x^2 + 12x + 3 = 2[x^2 + 6x + \frac{3}{2}]$

$= 2[(x + 3)^2 - 9 + \frac{3}{2}]$

$= 2[(x + 3)^2 - \frac{15}{2}]$

$= 2(x + 3)^2 - 15$

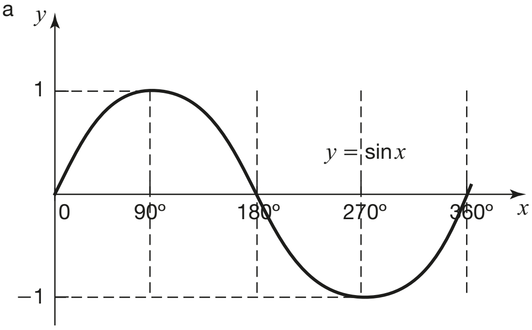
$a = 2$, $b = 3$ and $c = -15$

Recognising and sketching graphs of functions

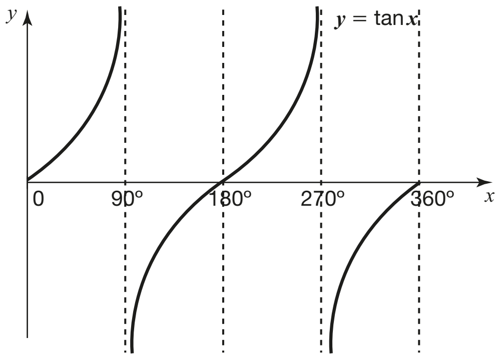
1

Equation	Graph
$y = x^2$	B
$y = 2^x$	D
$y = \sin x°$	E
$y = x^3$	C
$y = x^2 - 6x + 8$	A
$y = \cos x°$	F

2 a

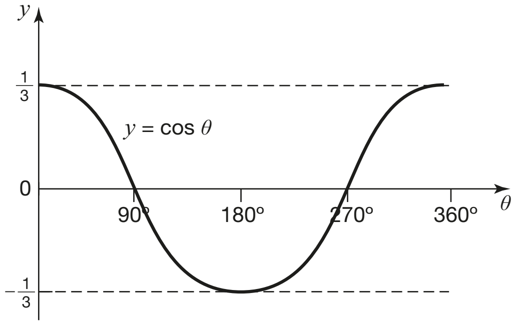


b



3 $3\cos\theta = 1$

$\cos\theta = \frac{1}{3}$



$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.5°$ (to 1 d.p.)

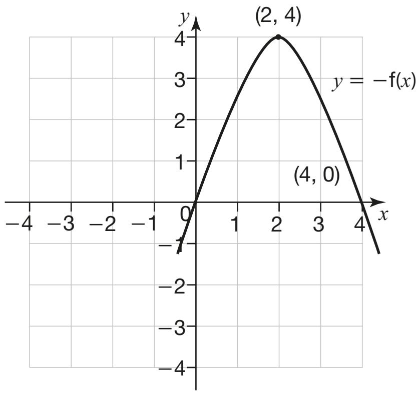
From the graph, $\cos\theta$ is also $\frac{1}{3}$ when $\theta = 360 - 70.5 = 289.5°$

$\theta = 70.5°$ or $289.5°$ (to 1 d.p.)

Translations and reflections of functions

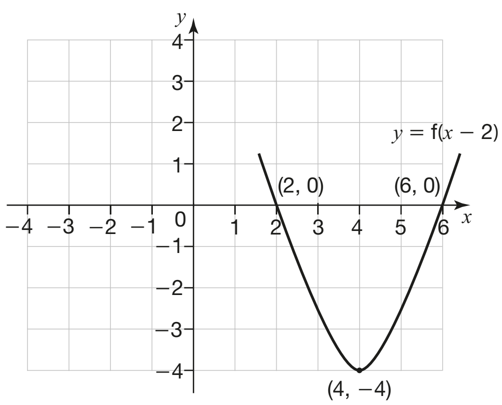
1 **a** $y = -\text{f}(x)$ is a reflection in the $x-$axis of the graph $y = \text{f}(x)$

The points on the $x-$axis stay in the same place and the turning point at (2, −4) is reflected to become a turning point at (2, 4).



b $y = \text{f}(x - 2)$ is a translation by $\begin{pmatrix}2\\0\end{pmatrix}$ of the graph $y = \text{f}(x)$.

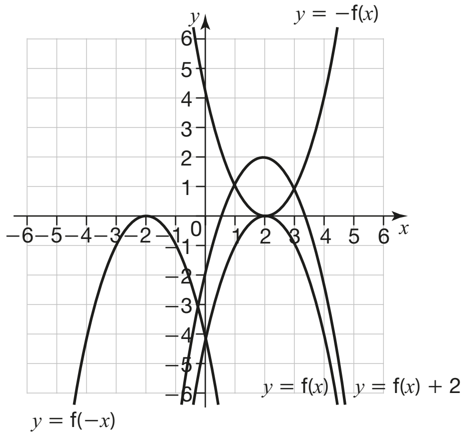
The y-coordinates stay the same but the x-coordinates are shifted to the right by 2 units.



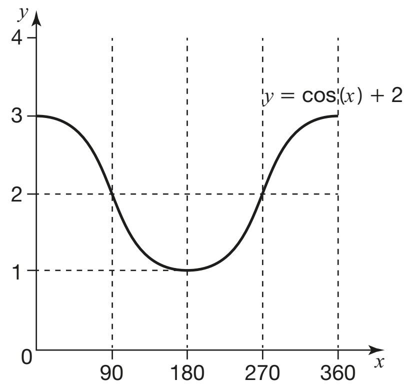
2 **a** $y = -\text{f}(x)$: reflection in the $x-$axis.

b $y = \text{f}(x) + 2$: translation of 2 units vertically upwards.

c $y = \text{f}(-x)$: reflection in the y-axis.



3 The cosine graph is shifted two units in the vertical direction, i.e. a translation of $\begin{pmatrix}0\\2\end{pmatrix}$.



Equation of a circle and tangent to a circle

1 **a** $x^2 + y^2 = 25$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{25} = 5$

b $x^2 + y^2 - 49 = 0$

$x^2 + y^2 = 49$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{49} = 7$

c $4x^2 + 4y^2 = 16$

$x^2 + y^2 = 4$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{4} = 2$

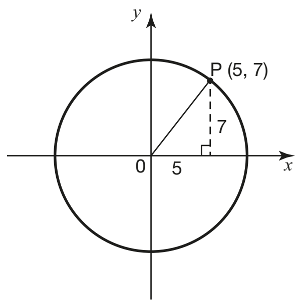
2 Radius of the circle $= \sqrt{21} = 4.58$

Distance of the point (4, 3) from the centre of the circle (0, 0)

$= \sqrt{16 + 9} = \sqrt{25} = 5$

This distance is greater than the radius of the circle, so the point lies outside the circle.

3 **a**



Using Pythagoras' theorem:

$OP^2 = 5^2 + 7^2$

$= 25 + 49$

$= 74$

OP = radius of the circle = $\sqrt{74}$

b equation of a circle, radius r, centre the origin: $x^2 + y^2 = r^2$

$x^2 + y^2 = 74$

c gradient of line $OP = \frac{7}{5}$

gradient of the tangent at $P = -\frac{5}{7}$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 7 = -\frac{5}{7}(x - 5)$

$7y - 49 = -5x + 25$

$7y = -5x + 74$

$y = -\frac{5}{7}x + \frac{74}{7}$

Real-life graphs

1 a acceleration = gradient of line = $\frac{10}{10} = 1\,\text{m/s}^2$

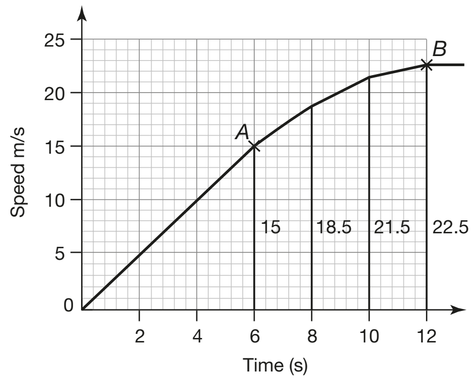
b total distance travelled = area under the velocity–time graph

$= \frac{1}{2}(30 + 15) \times 10 = 225$ m

2 a The graph is a straight line starting at the origin, so this represents constant acceleration from rest of $\frac{15}{6} = 2.5\,\text{m/s}^2$.

b The gradient decreases to zero, so the acceleration decreases to zero.

c area = area of 3 trapeziums



area = $\frac{1}{2}(15 + 18.5) \times 2 + \frac{1}{2}(18.5+21.5) \times 2 +$

$\frac{1}{2}(21.5 + 22.5) \times 2$

$= 117.5$

distance travelled between A and B = 118 m (to nearest integer)

d It will be a slight underestimate, as the curve is always above the straight lines forming the tops of the trapeziums.

Generating sequences

1 a i $\frac{1}{2}$ (term-to-term rule is divide by 2)

ii 243 (term-to-term rule is multiply by 3)

iii 21 (term-to-term rule is add 4)

b 4th term − 1st term = −12 − 27 = −39

common difference = −39 ÷ 3 = −13

missing terms are 14, 1

2 3rd term = 2 × 1 − 5 = −3

4th term = 2 × −3 − 5 = −11

3 a 25, 36 (square numbers)

b 15, 21 (triangular numbers)

c 8, 13 (Fibonacci numbers)

The *n*th term

1 a

n	1		2		3		4
Term	2		6		10		14
Difference		4		4		4	

nth term starts with $4n$

$4n$	4	8	12	16
Term − $4n$	−2	−2	−2	−2

nth term = $4n - 2$

b nth term = $4n - 2 = 2(2n - 1)$

2 is a factor, so the nth term is divisible by 2 and therefore is even.

c Let nth term = 236

$4n - 2 = 236$

$4n = 238$

$n = 59.5$

n is not an integer, so 236 is not a term in the sequence.

2 a 2nd term = $9 - 2^2 = 5$

b 20th term = $9 - 20^2 = 9 - 400 = -391$

c n^2 is always positive, so the largest value value $9 - n^2$ can take is 8 when $n = 1$. All values of n above 1 will make $9 - n^2$ smaller than 8. So 10 cannot be a term.

3

Term	1		1		3		7		13
First difference		0		2		4		6	
Second difference			2		2		2		

The formula starts n^2.

n	1		2		3		4		
Term	1		1		3		7		13
n^2	1		4		9		16		25
Term − n^2	0		−3		−6		−9		−12
Difference		−3		−3		−3		−3	

The linear part of the sequence starts with $-3n$.

$-3n$	−3	−6	−9	−12	−15
Linear term − $(-3n)$	3	3	3	3	3

nth term = $n^2 - 3n + 3$

Checking:

When $n = 1$, term is $1^2 - 3 \times 1 + 3 = 1$

$n = 2$, term is $2^2 - 3 \times 2 + 3 = 1$

$n = 3$, term is $3^2 - 3 \times 3 + 3 = 3$

Arguments and proofs

1 The only even prime number is 2.

Hence, statement is false because 2 is a prime number that is not odd.

2 a true: $n = 1$ is the smallest positive integer and this would give the smallest value of $2n + 1$, which is 3.

b true: 3 is a factor of $3(n+1)$ so $3(n+1)$ must be a multiple of 3.

c false: $2n$ is always even and subtracting 3 will give an odd number.

3 Let first number $= x$ so next number $= x + 1$

Sum of consecutive integers $= x + x + 1 = 2x + 1$

Regardless of whether x is odd or even, $2x$ will always be even as it is divisible by 2.

Hence $2x + 1$ will always be odd.

4 $(2x - 1)^2 - (x - 2)^2 = 4x^2 - 4x + 1 - (x^2 - 4x + 4)$

$= 4x^2 - 4x + 1 - x^2 + 4x - 4$

$= 3x^2 - 3$

$= 3(x^2 - 1)$

The 3 outside the brackets shows that the result is a multiple of 3 for all integer values of x.

5 Let two consecutive odd numbers be $2n - 1$ and $2n + 1$.

$(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1)$

$= 8n$

Since 8 is a factor of $8n$, the difference between the squares of two consecutive odd numbers is always a multiple of 8.

(If you used $2n + 1$ and $2n + 3$ for the two consecutive odd numbers, difference of squares $= 8n + 8 = 8(n + 1)$.)

Ratio, proportion and rates of change

Introduction to ratios

1 3 parts = 18 black balls so 1 part = 6 black balls

There are $3 + 2 = 5$ parts in total, so the total number of balls

$= 5 \times 6 = 30$.

2 total shares $= 15 + 17 + 18 = 50$

50 shares = £25 000

1 share $= \frac{£25\,000}{50} = £500$

15 year old will get $15 \times £500 = £7500$.

17 year old will get $17 \times £500 = £8500$.

18 year old will get $18 \times £500 = £9000$.

3 40% of 800 = = 320 acres

remainder of land area $= 800 - 320 = 480$ acres

total shares $= 9 + 7 = 16$

1 share $= \frac{480}{16} = 30$

area devoted to sheep = 7 shares $= 7 \times 30 = 210$ acres

4 $\frac{3x + 1}{x + 4} = \frac{2}{3}$

$3(3x + 1) = 2(x + 4)$

$9x + 3 = 2x + 8$

$7x = 5$

$x = \frac{5}{7}$

5 For 2 oak trees there are 3 ash trees, so if there are 8 oak trees there would be 12 ash trees.

pine : oak: ash = 5 : 8 : 12

There are a total of $5 + 8 + 12 = 25$ parts.

1 part $= \frac{300}{25} = 12$

number of ash trees $= 12 \times 12 = 144$

Scale diagrams and maps

1 10 cm on map $= 10 \times 50\,000$ cm $= 500\,000$ cm actual distance

500 000 cm = 5000 m = 5 km

Towns are 5 km apart.

2 **a** length of road $= 2.3 \times 40\,000$ cm

$= 92\,000$ cm

$= 920$ m

$= 0.92$ km

b length of road $= 3 \times 40\,000$ mm

$= 120\,000$ mm

$= 12\,000$ cm

$= 120$ m

$= 0.12$ km

3 5 cm : 40 m = 5 cm : 4000 cm

= 5 : 4000

= 1 : 800

scale of drawing = 1 : 800

4 length measured on map = 6 cm

Scale is:

6 cm : 12 km $= 6 : 12 \times 1000 \times 100$

= 6 : 1 200 000

= 1 : 200 000

Percentage problems

1 increase = £35

percentage increase $= \frac{\text{increase}}{\text{original price}} \times 100$

$= \frac{35}{350} \times 100$

$= 10\%$

2 increase = final earnings − initial earnings

$= 1\,100\,000 - 600\,000$

$= £500\,000$

% increase $= \frac{\text{increase}}{\text{original value}} \times 100$

$= \frac{500\,000}{600\,000} \times 100$

$= 83.3\%$

3 multiplier $= 1 - 0.28 = 0.72$

value of car $= 0.72 \times 25\,000$

$= £18\,000$

4 88% of the original price = £14 300

1% of the original price $= \frac{14\,300}{88}$

100% of the original price $= \frac{14\,300}{88} \times 100 = 16\,250$

original price = £16 250

5 interest earned $= \frac{2.8}{100} \times 8000 \times 4$

$= £896$

Direct and inverse proportion

1 **a** $P = kT$

b $P = kT$ so $k = \frac{P}{T} = \frac{200\,000}{540} = 370.370$

$P = 370.370T = 370.370 \times 200 = 74\,074$ pascals (to nearest whole number)

2 $C \propto r^2$

$C = kr^2$

$480 = k \times 3^2$

$k = 53.33$

$C = 53.33r^2$

cost $= 53.33 \times 4^2 = £853.28 = £853$ (to nearest whole number)

3 **a** $c \propto \frac{1}{h}$ so $c = \frac{k}{h}$

$3 = \frac{k}{12}$, so $k = 36$

$c = \frac{36}{h}$

b $c = \frac{36}{h}$

$= \frac{36}{15}$

$= 2.4$

4 **a** $350 \times 1.15 = €402.50$

b $80 \div 1.11 = £72.07$ (to nearest penny)

c The €80 she had left cost her $80 \div 1.15 = £69.57$ before her holiday.

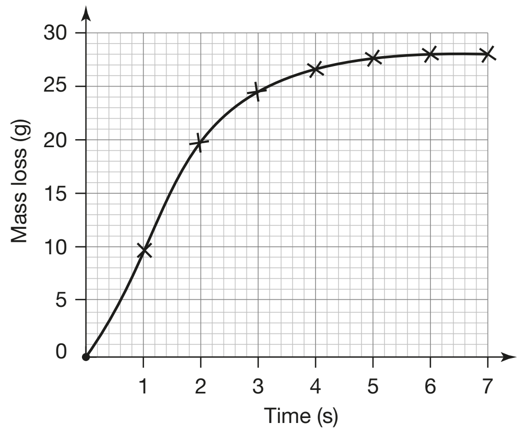
If she hadn't changed this amount, she would have saved $72.07 - 69.57 = £2.50$

Graphs of direct and inverse proportion and rates of change

1 straight line through the origin: B

2 curve that gets close to but does not cross either axis: B

3 **a**



b **i** initial rate of change = gradient over first 2 minutes

$= \frac{19.6}{2}$

$= 9.8$ g/minute

ii initial rate of change $= \frac{9.8}{60} = 0.16$ g/second (to 2 d.p.)

Growth and decay

1 **a** multiplier $= 1 + \frac{\text{\% increase}}{100} = 1 + \frac{6}{100} = 1.06$

population after n years $= A_0 \times (\text{multiplier})^n$,

where A_0 = initial population and n = number of years

population after 3 years $= 150\,000 \times 1.06^3 = 178\,652$

b Try $n = 5$ years: population $= 150\,000 \times 1.06^5 = 200\,733$

Try $n = 4$ years: population $= 150\,000 \times 1.06^4 = 189\,372$

After 5 years, the population will have risen to over 200 000.

2 multiplier $= 1 - \frac{\text{\% decrease}}{100} = 1 - \frac{12}{100} = 0.88$

value of car after 4 years $= 21\,000 \times 0.88^4 = £12\,594$

3 multiplier $= 1 - \frac{\text{\% decrease for each time unit}}{100} = 1 - \frac{50}{100} = 0.5$

2 minutes = 120 seconds

number of time intervals, $n = \frac{120}{12} = 10$

amount at the end of n time intervals $= A_0 \times (\text{multiplier})^n$

$= 100 \times 0.5^{10}$

$= 0.1$ (to 1 s.f.)

Ratios of lengths, areas and volumes

1 **a** scale factor for enlargement $\left(\frac{big}{small}\right)^3 = \left(\frac{12}{8}\right)^3 = 3.375$ or $\frac{27}{8}$

b $\frac{\text{area of triangle of large prism}}{\text{area of triangle of small prism}} = \left(\frac{12}{8}\right)^2$

area of triangle of large prism $= \left(\frac{12}{8}\right)^2 \times 10 = 22.5\,\text{cm}^2$

c $\frac{\text{volume of small prism}}{\text{volume of large prism}} = \left(\frac{8}{12}\right)^3$

volume of small prism $= \left(\frac{8}{12}\right)^3 \times 450$

$= 133.33$

$= 133\,\text{cm}^3$ (to nearest whole number)

2 $\frac{\text{volume of larger cuboid}}{\text{volume of smaller cuboid}} = \left(\frac{h}{12}\right)^3$

$\frac{\text{volume of larger cuboid}}{\text{volume of smaller cuboid}} = 1.953$

$\left(\frac{h}{12}\right)^3 = 1.953$

$h^3 = 3374.784$

$h = 15$ cm (to nearest cm)

3 **a** **i** angle *XYZ* = angle *TUZ* (corresponding angles)

angle *ZXY* = angle *ZTU* (corresponding angles)

angle *XZY* = angle *TZU* (same angle)

Hence triangles *XYZ* and *TUZ* are similar.

$\frac{XY}{TU} = \frac{YZ}{UZ}$

$\frac{XY}{3} = \frac{15}{5}$

$XY = 9$ cm

ii angle *YUT* = angle *YZW* (corresponding angles)

angle *YTU* = angle *YWZ* (corresponding angles)

angle *TYU* = angle *WYZ* (same angle)

Hence triangles *YUT* and *YZW* are similar.

$\frac{WZ}{TU} = \frac{YZ}{YU}$

$\frac{WZ}{3} = \frac{15}{10}$

$WZ = 4.5$ cm

b angle *YTX* = angle *ZTW* (vertically opposite angles)

angle *YXT* = *TZW* (alternate angles)

Two angles of both triangles are equal so the third angles must also be equal.

Hence triangles *XYT* and *ZWT* are similar.

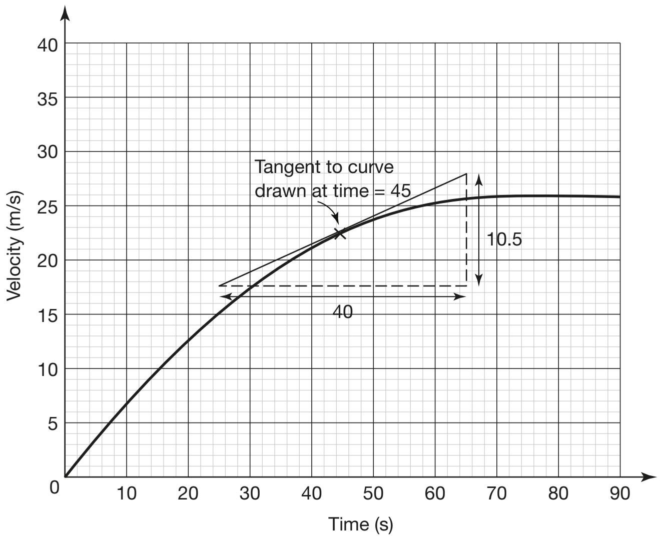
ratio of the areas of *TYX* and *TWZ* $= \left(\frac{XY}{WZ}\right)^2 = \left(\frac{9}{4.5}\right)^2 = 4$

ratio of areas = 4 : 1

Gradient of a curve and rate of change

1 **a** acceleration = gradient of straight line = $\frac{10-0}{15-0} = \frac{2}{3}\,\text{m/s}^2$

b Instantaneous acceleration = gradient of the tangent to the curve at 45 s.



Gradient = $\frac{10.5}{40} = 0.2625$

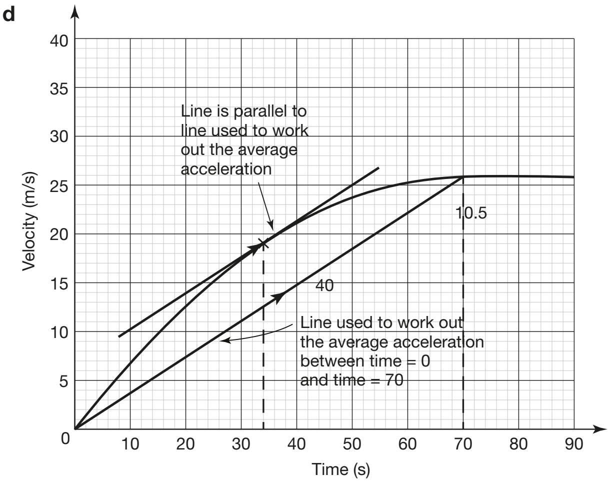
Instantaneous acceleration at 45 s = 0.26 m/s^2 (to 2 d.p.)

c average acceleration over the first 70 s = gradient of the line from (0, 0) to (70, 26)

gradient = $\frac{26-0}{70-0} = 0.3714\ldots$

acceleration = 0.37 m/s^2 (to 2 d.p.)

d



gradients of the two lines are parallel when time = 34 s

Converting units of areas and volumes, and compound units

1 pressure = $\frac{\text{force}}{\text{area}} = \frac{200}{0.4} = 500\,\text{N/m}^2$

2 1 m^2 = 100 × 100 cm^2 = 10 000 cm^2

1 cm^2 = $\frac{1}{10000}$ m^2

200 cm^2 = $\frac{200}{(10\,000)}$ m^2 = 0.02 m^2

pressure = $\frac{\text{force}}{\text{area}} = \frac{500}{0.02} = 25\,000\,\text{N/m}^2$

3 density = $\frac{\text{mass}}{\text{volume}}$

mass = volume × density

= 12 × 8.92

= 107.04 g

mass of wire = 107 g (to nearest g)

4 He has worked out the area in m^2 by dividing the area in cm^2 by 100, which is incorrect.

There are 100 × 100 = 10 000 cm^2 in 1 m^2, so the area should have been divided by 10 000.

Correct answer:

area in m^2 = $\frac{9018}{10\,000}$

= 0.9018

= 0.90 m^2 (to 2 d.p.)

5 distance in first two hours = time × speed

= 2 × 60

= 120 km

distance in next three hours = time × speed

= 3 × 80

= 240 km

total distance = 120 + 240

= 360 km

average speed = $\frac{\text{distance}}{\text{time}}$

= $\frac{360}{5}$

= 72 km/h

Geometry and measures

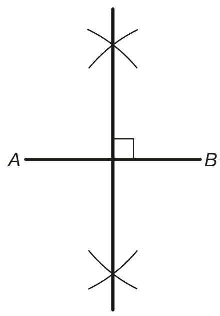
2D shapes

1 **a** true **b** false **c** true **d** true **e** true **f** false (this would be true only for a regular pentagon)

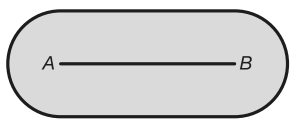
2 **a** rhombus **b** parallelogram **c** equilateral triangle **d** kite

Constructions and loci

1 The locus of the point is the line that bisects and is perpendicular to *AB*.



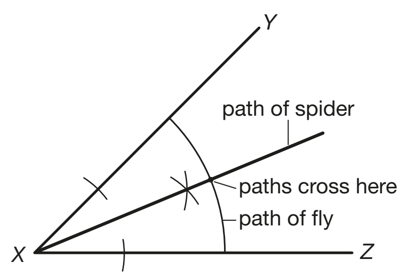
2 The line AB is 6 cm long, with semicircles of radius 1.5 cm at either end, joined by two lines parallel to and 1.5 cm from the line.



Not drawn to scale

3 **a** bisector of angle *YXZ*

b path of fly: arc with centre X of radius 6 cm between the two walls



Properties of angles

1 **a** angle ACB = angle BAC = 30° (base angles of isosceles triangle ABC, since AB and BC are equal sides of a rhombus)

b angle AOB = 90° (diagonals of a rhombus intersect at right angles)

c angle $ABO = 180 - (90 + 30) = 60°$ (angle sum of a triangle)

angle BDC = angle ABO = 60° (alternate angles between parallel lines AB and DC)

2 angle $BAC = \frac{(180 - 36)}{2} = 72°$ (angle sum of a triangle and base angles of an isosceles triangle)

angle $BDC = 180 - 90 = 90°$ (angle sum on a straight line)

angle $ABD = 180 - (90 + 72) = 18°$ (angle sum of a triangle)

3 **a** $2x + 4x = 180$ (angle sum on a straight line)

$6x = 180$

$x = 30°$

b If lines AB and CD are parallel, the angles $4x$ and $3x + 30$ would be corresponding angles, and so equal.

$4x = 4 \times 30 = 120°$

$3x + 30 = 3 \times 30 + 30 = 120°$

These two angles are equal so lines AB and CD are parallel.

4 For the regular pentagon:

exterior angle $= \frac{360°}{\text{number of sides}} = \frac{360}{5} = 72°$

exterior angle of a square = interior angle of a square = 90°

angle $x = 90 + 72 = 162°$

Congruent triangles

1 BD common to triangles ABD and CDB

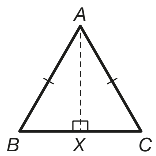
angle ADB = angle CBD (alternate angles)

angle ABD = angle CDB (alternate angles)

Therefore triangles ABD and CDB are congruent (ASA).

Hence angle BAD = angle BCD

2 Draw the triangle and the perpendicular from A to BC.



$AX = AX$ (common)

$AB = AC$ (triangle ABC is isosceles)

angle $AXB = AXC = 90°$ (given)

Therefore triangles ABX and ACX are congruent (RHS).

Hence $BX = XC$, so X bisects BC.

3 $OQC = 90°$ (corresponding angles), so $PB = OQ$ (perpendicular distance between 2 parallel lines).

$AP = PB$ (given), so $AP = OQ$

$PO = QC$ (Q is the midpoint of BC)

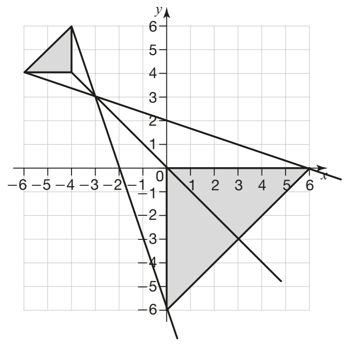
angle ABC = angle APO = angle OQC = 90° (OQ is parallel to AB and OP parallel to BC)

Therefore triangles AOP and OCQ are congruent (SAS).

Transformations

1 translation of $\begin{pmatrix} -7 \\ -5 \end{pmatrix}$

2



3 **a** translation of $\begin{pmatrix} -7 \\ 0 \end{pmatrix}$

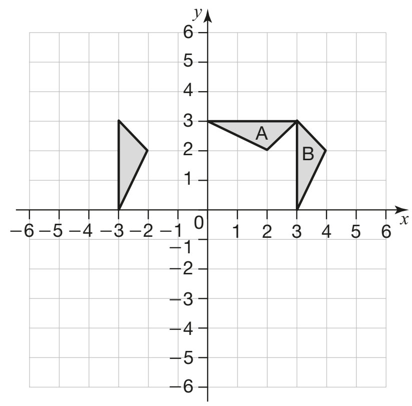
b reflection in the line $y = 3$

c rotation of 90° clockwise about (0, 1)

Invariance and combined transformations

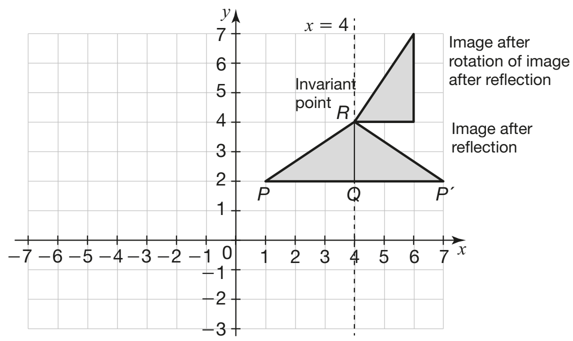
1 **a** 1

b **i** invariant point (3, 3)



ii rotation 90°anticlockwise about (3, 3)

2 **a** The shaded triangle is the image after the two transformations.



b invariant point is R (4, 4)

3D shapes

1 **a** G **c** A, H **e** C

b B **d** B **f** A, H

Parts of a circle

1 **a** radius **c** chord

b diameter **d** arc

2 **a** minor sector **c** major sector

b major segment **d** minor segment

Circle theorems

1 angle $OTB = 90°$ (angle between tangent and radius)

angle $BOT = 180 - (90 + 28) = 62°$ (angle sum in a triangle)

angle $AOT = 180 - 62 = 118°$ (angle sum on a straight line)

$AO = OT$ (radii), so triangle AOT is isosceles

angle $OAT = \frac{(180 - 118)}{2} = 31°$ (angle sum in a triangle)

2 **a** angle $ACB = 30°$ (angle at centre twice angle at circumference)

b angle BAC = angle $CBX = 70°$ (alternate segment theorem)

c $OA = OB$ (radii), so triangle AOB is isosceles

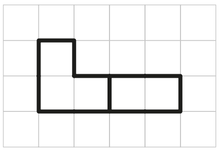
angle $AOB = 60°$, so triangle AOB is equilateral

angle $OAB = 60°$ (angle of equilateral triangle)

angle $CAO = 70 - 60 = 10°$

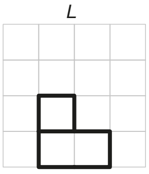
Projections

1

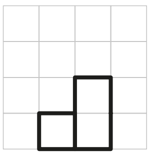


2 **a**

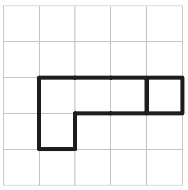
L



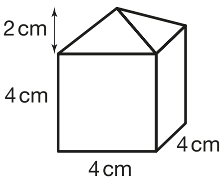
R



b

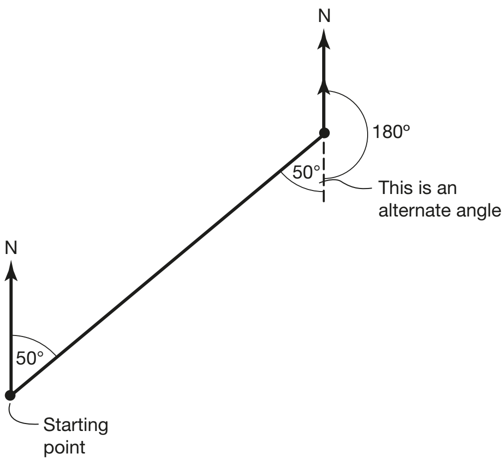


3



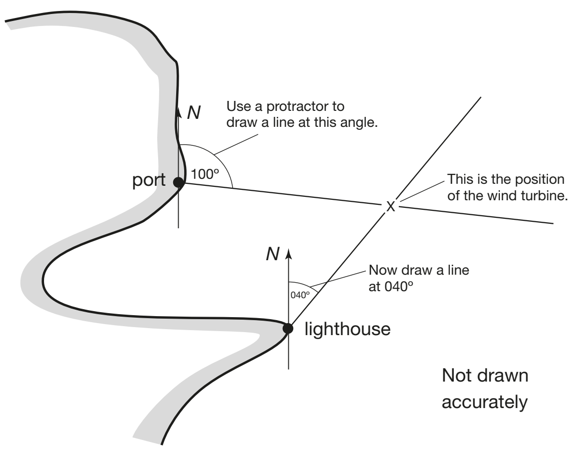
Bearings

1



bearing to return to starting point = 180 + 50 = 230°

2



Pythagoras' theorem

1 $AC^2 = AB^2 + BC^2 = 24^2 + 16^2 = 832$

$AD^2 = AC^2 - CD^2 = 832 - 9^2 = 751$

$AD = 27$ m (to nearest m)

perimeter = 24 + 16 + 9 + 27 = 76 m (to nearest m)

2 In triangle ABC, $AC^2 = AB^2 + BC^2 = 16^2 + 7^2 = 305$

In triangle ACG, $AG^2 = AC^2 + CG^2 = 305 + 5^2 = 330$

$AG = 18.2$ cm (to 1 d.p.)

extra distance = 16 + 7 + 5 − 18.2 = 9.8 cm (to 1 d.p.)

3 **a** angle $ADC = 90°$ (angle in a semicircle)

Using Pythagoras' theorem:

$10.8^2 = 5.8^2 + AD^2$

$AD = 9.11$ cm = (to 2 d.p.)

b area of triangle $ACD = 1/2 \times 5.8 \times 9.11 = 26.419\text{ cm}^2$

angle ABC = 90° (angle in a semicircle)

$\frac{AB}{10.8} = \cos 65°$

$AB = 4.564$ cm

$\frac{BC}{10.8} = \sin 65°$

$BC = 9.788$ cm

area of triangle ABC $= \frac{1}{2} \times 4.564 \times 9.788 = 22.336\text{ cm}^2$

area of quadrilateral $ABCD = 26.419 + 22.336$
$= 48.76\text{ cm}^2$ (to 2 d.p.)

Area of 2D shapes

1 **a** **i** Using Pythagoras' theorem:

$7^2 + AE^2 = 9^2$

$49 + AE^2 = 81$

$AE^2 = 32$

$AE = \sqrt{32} = 5.6569 = 5.66$ cm* (to 2 d.p.)

ii area of triangle $ADE = \frac{1}{2} \times$ base $\times$ height
$= \frac{1}{2} \times 5.6569 \times 7 = 19.80\text{ cm}^2$ (to 2 d.p.)

b Using Pythagoras' theorem:

$7^2 + BF^2 = 8^2$

$49 + BF^2 = 64$

$BF^2 = 15$

$BF = \sqrt{15} = 3.87$ cm (to 2 d.p.)

area of triangle $BCF = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 3.87 \times 7$
$= 13.56\text{ cm}^2$

*This answer differs from that in the Exam Practice book due to an error in our first edition. It has now been re-checked and corrected.

area of trapezium $CDEF$ = area of rectangle − area of ADE − area of BCF

$= 84 - 19.81 - 13.56$

$= 50.65\,\text{cm}^2$

2 **a** perimeter of triangle $= x + 1 + 2x + 1 + x + 2 = 4x + 4$

perimeter of rectangle $= x + 1 + 3 + x + 1 + 3 = 2x + 8$

perimeters are the same:

$4x + 4 = 2x + 8$

$2x = 4$

$x = 2\,\text{cm}$

area of $DEFG = 3 \times (x + 1) = 9\,\text{cm}^2$

b area of triangle $= \frac{1}{2} \times 4 \times 3 = 6\,\text{cm}^2$

3 3 circles in stencil

a area $= 3 \times \pi r^2 = 3 \times \pi \times 3^2 = 27\pi\,\text{cm}^2$

b perimeter = 3 × circumference + 2 × radius

$= 3 \times 2\pi r + 2r$

$= 3 \times 2 \times \pi \times 3 + 2 \times 3$

$= 18\pi + 6\,\text{cm}$

Volume and surface area of 3D shapes

1 **a** area of trapezium $= \frac{1}{2}(a + b)h$

$= \frac{1}{2}(3.5 + 1.8) \times 1.5$

$= 3.975\,\text{m}^2$

b volume of prism = area of cross−section × length

$= 3.975 \times 1.6$

$= 6.36\,\text{m}^3$

2 volume of sphere $= \frac{4}{3}\pi r^3$

volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi \times 4^3$

$= 134.04\,\text{cm}^3$

volume of drink in bottle $= 750\,\text{ml} = 750\,\text{cm}^3$

number of glasses $= \frac{750}{134.04} = 5.6$

A bottle will fill 5 glasses.

3 **a** Using Pythagoras' theorem:

$l^2 = h^2 + r^2$

$= 5.6^2 + 4.8^2$

$= 54.4$

$l = 7.3756\ldots = 7.4\,\text{cm}$ (to 1 d.p.)

b surface area of cone $= \pi rl + \pi r^2$

$= \pi \times 4.8 \times 7.3756 + \pi \times 4.8^2$

$= 183.60\,\text{cm}^2$

surface area of sphere $= 4\pi r^2 = 183.60$

$r = \sqrt{\frac{183.60}{4\pi}} = 3.8\,\text{cm}$ (to 1 d.p.)

4 volume of water in cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 2^2 \times 12$

$= 50.27\,\text{cm}^3$

volume of water in cylinder $= \pi r^2 h = 50.27\,\text{cm}^3$

$\pi \times 5^2 \times h = 50.27$

$h = 0.64\,\text{cm}$

depth of water in the cylinder $= 0.64\,\text{cm}$

Trigonometric ratios

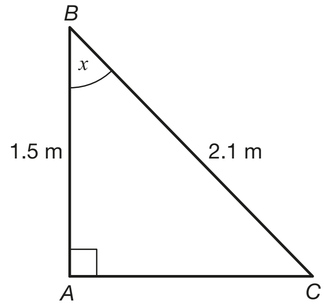
1 **a** $\cos 48° = \frac{x}{9}$

$x = 9\cos 48° = 6.0\,\text{cm}$ (to 1 d.p.)

b $\tan y = \frac{9}{12}$

$y = \tan^{-1}\left(\frac{9}{12}\right) = 36.9°$ (to 1 d.p.)

2



Let angle $ABC = x$

$\cos x = \frac{1.5}{2.1}$

$x = \cos^{-1}\left(\frac{1.5}{2.1}\right) = 44.4°$ (to 2 d.p.)

angle $ABC = 44.4°$ (to 1 d.p.)

3 $\sin 70° = \frac{h}{10}$

$h = 10\sin 70° = 9.40\,\text{cm}$ (to 2 d.p.)

4 In triangle ABC:

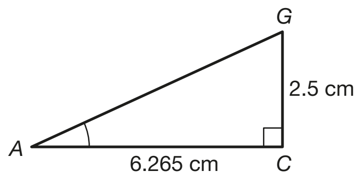
$AC^2 = AB^2 + BC^2$

$= 5.5^2 + 3^2$

$= 39.25$

$AC = \sqrt{39.25} = 6.265\,\text{cm}$

In triangle ACG, you know the lengths AC and CG, and you want to find the angle CAG, so use tan (CG is 'opposite' and AC is 'adjacent').

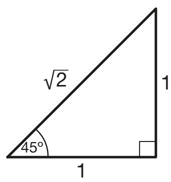


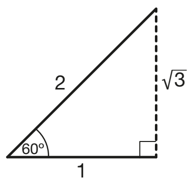
tan (angle CAG) $= \frac{CG}{AC} = \frac{2.5}{6.265} = 0.3990$

angle $CAG = \tan^{-1}(0.3990) = 21.8°$ (to 1 d.p.)

Exact values of sin, cos and tan

1





$\tan 45° = \frac{1}{1} = 1$

$\cos 60° = \frac{1}{2}$

Hence, $\tan 45° + \cos 60° = 1 + \frac{1}{2} = \frac{3}{2}$

2 **a** **i** $\sin 45° = \frac{1}{\sqrt{2}}$

ii $\cos 45° = \frac{1}{\sqrt{2}}$

b $\frac{\sin 45°}{\cos 45°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$\tan 45° = 1$

Hence $\frac{\sin 45°}{\cos 45°} = \tan 45°$

3 **a** Using Pythagoras' theorem:

$AC^2 = 1^2 + 2^2$

$AC = \sqrt{5}$

b **i** $\sin x = \frac{1}{\sqrt{5}}$

ii $\cos x = \frac{2}{\sqrt{5}}$

c $(\sin x)^2 = \frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{1}{5}$

$(\cos x)^2 = \frac{2}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{4}{5}$

$(\sin x)^2 + (\cos x)^2 = \frac{1}{5} + \frac{4}{5}$

$= 1$

4 $\tan 30° + \tan 60° + \cos 30° = \frac{1}{\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{\sqrt{3}\,\sqrt{3}} + \sqrt{3} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{3} + \sqrt{3} + \frac{\sqrt{3}}{2}$

$= \frac{2\sqrt{3} + 6\sqrt{3} + 3\sqrt{3}}{6}$

$= \frac{11\sqrt{3}}{6}$

Sectors of circles

1 $l = \frac{\theta}{360} \times 2\pi r$

$3 = \frac{\theta}{360} \times 2\pi \times 5$

$\theta = 34°$ (to nearest degree)

2 area of sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{290}{360} \times \pi \times 12^2$

$= 364.4\,\text{cm}^2$

3 **a** area of sector $= \frac{\theta}{360} \times \pi r^2$

area of sector A $= \frac{74}{360} \times \pi \times 3^2$

area of sector B $= \frac{360 - 122}{360}\pi r^2 = \frac{238}{360}\pi r^2$

$\frac{238}{360}\pi r^2 = \frac{74}{360} \times \pi \times 3^2$

$r^2 = \frac{74}{238} \times 3^2 = 2.798$

$r = 1.67\,\text{cm}$ (to 3 s.f.)

b perimeter of sector $= \frac{\theta}{360} \times 2\pi r + 2r$

perimeter of sector A $= \frac{74}{360} \times 2 \times \pi \times 3 + 2 \times 3$

$= 9.875\,\text{cm}^2$

perimeter of sector B $= \frac{238}{360} \times 2 \times \pi \times 1.67 + 2 \times 1.67$

$= 10.277\,\text{cm}^2$

perimeter A : perimeter B = 9.875 : 10.277 = 1 : 1.04

4 **a** Arc length $= \frac{\theta}{360} \times 2\pi r$

$5.4 = \frac{\theta}{360} \times 2 \times \pi \times 6$

$\theta = 51.5662°$

area of sector $= \frac{\theta}{360} \times \pi r^2$

Area $= \frac{51.5662}{360} \times \pi \times 6^2$

Area of minor sector = $16.2\,\text{cm}^2$ (to 1 d.p.)

b Area of triangle $= \frac{1}{2} \times 6 \times 6 \times \sin 51.5662° = 14.09988\,\text{cm}^2$

Area of segment = area of sector − area of segment

$= 16.2 - 14.1$

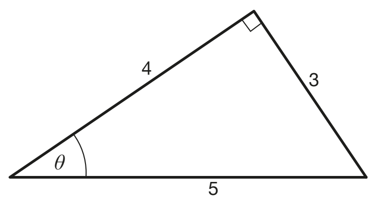
Area of segment $= 2.1\,\text{cm}^2$ (to 1 d.p.)

Sine and cosine rules

1 **a** area $= \frac{1}{2} \times 25 \times 30 \times \frac{3}{5}$

$= 225\,\text{cm}^2$

b Because of the value of $\sin \theta$, this must be a right-angled triangle with sides in the ratio 3 : 4 : 5. Draw the simplest form of this triangle.



adjacent = 4

$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$

c Using the cosine rule:

$AC^2 = 25^2 + 30^2 - 2 \times 25 \times 30 \times \frac{4}{5}$

$= 325$

$AC = 18.0\,\text{cm}$ (to 3 s.f.)

2 **a** Using the sine rule:

$\frac{x}{\sin 84°} = \frac{12}{\sin 40°}$

$x = \frac{12 \sin 84°}{\sin 40°}$

$= 18.5664$

$= 18.6\,\text{cm}$ (to 3 s.f.)

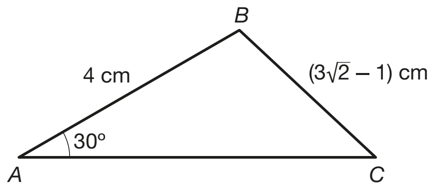
b angle $BAC = 180 - (84 + 40) = 56°$

area $= \frac{1}{2} \times 18.5664 \times 12 \times \sin 56°$

$= 92.4\,\text{cm}^2$ (to 3 s.f.)

If you use your answer from part a (to 3 s.f) you get 92.5, but taking x to 4 d.p. gives the answer of 92.4.

3



Using the sine rule:

$\frac{3\sqrt{2} - 1}{\sin 30°} = \frac{4}{\sin ACB}$

$\sin ACB = \frac{4 \sin 30°}{3\sqrt{2} - 1}$

$= \frac{2}{3\sqrt{2} - 1}$

$= \frac{2(3\sqrt{2} + 1)}{(3\sqrt{2} - 1)(3\sqrt{2} + 1)}$

$= \frac{6\sqrt{2} + 2}{18 - 1}$

$= \frac{2 + 6\sqrt{2}}{17}$

Vectors

1 **a** $\frac{1}{2}(\mathbf{p} + \mathbf{q}) = \frac{1}{2}\left(\begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}6\\-1\end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$

b $2\mathbf{p} - 3\mathbf{q} = 2\begin{pmatrix}-2\\3\end{pmatrix} - 3\begin{pmatrix}6\\-1\end{pmatrix} = \begin{pmatrix}2 \times -2 - 3 \times 6\\2 \times 3 - 3 \times -1\end{pmatrix}$

$= \begin{pmatrix}-22\\9\end{pmatrix}$

2 **a** $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$

b $\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AB} = \frac{3}{5}(\mathbf{b} - \mathbf{a})$

c $\overrightarrow{OQ} = \frac{2}{5}\overrightarrow{OA} = \frac{2}{5}\mathbf{a}$

$\overrightarrow{QP} = \overrightarrow{QA} + \overrightarrow{AP}$

$= \frac{3}{5}\mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$

$= \frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{3}{5}\mathbf{a}$

$= \frac{3}{5}\mathbf{b}$

As $\overrightarrow{QP} = \frac{3}{5}\mathbf{b}$ and $\overrightarrow{OB} = \mathbf{b}$ they both have the same vector part and so are parallel.

3 **a** $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= -3\mathbf{b} + \mathbf{a}$

$= \mathbf{a} - 3\mathbf{b}$

b $\overrightarrow{PB} = \frac{1}{3}\overrightarrow{AB} = \mathbf{b}$

$\overrightarrow{PM} = \overrightarrow{PB} + \overrightarrow{BM}$

$= \overrightarrow{PB} + \frac{1}{2}\overrightarrow{BC}$

$= \mathbf{b} + \frac{1}{2}(\mathbf{a} - 3\mathbf{b})$

$= \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

$= \frac{1}{2}(\mathbf{a} - \mathbf{b})$

$\overrightarrow{MD} = \overrightarrow{MC} + \overrightarrow{CD}$

$= \frac{1}{2}\overrightarrow{BC} + \overrightarrow{CD}$

$= \frac{1}{2}(\mathbf{a} - 3\mathbf{b}) + \mathbf{a}$

$= \frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}$

$= \frac{3}{2}(\mathbf{a} - \mathbf{b})$

Both $\overrightarrow{PM}$ and $\overrightarrow{MD}$ have the same vector part ($\mathbf{a} - \mathbf{b}$) so they are parallel. Since they both pass through M, they are parts of the same line, so PMD is a straight line.

Probability

The basics of probability

1 a

Dice 1

Dice 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b P(prime number) $= \frac{15}{36} = \frac{5}{12}$

c 7, as there are six 7s which is more than all the other scores.

2 a total number of chocolates $= 2x + 1 + x + 2x = 5x + 1$

P(mint) $= \frac{x}{5x + 1} = \frac{4}{21}$

$21x = 20x + 4$

$x = 4$

total number of chocolates $= 5x + 1 = 5 \times 4 + 1 = 21$

b number of truffles $= 2 \times 4 + 1 = 9$

P(truffle) $= \frac{9}{21} = \frac{3}{7}$

3

Beth

Amy	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

a number of possible outcomes $= 36$

number of pairs the same $= 6$

P(scores are equal) $= \frac{6}{36} = \frac{1}{6}$

b The possible scores where Amy's score is higher are:

(2, 1)

(3, 1), (3, 2)

(4, 1), (4, 2), (4, 3)

(5, 1), (5, 2), (5, 3), (5, 4)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5)

P(Amy's score is higher) $= \frac{15}{36} = \frac{5}{12}$

Probability experiments

1 a He is wrong because 100 spins is a very small number of trials. To approach the theoretical probability you would have to spin many more times. Only when the number of spins is extremely large will the frequencies start to become similar.

b relative frequency $= \frac{\text{frequency of event}}{\text{total frequency}} = \frac{22}{100} = \frac{11}{50}$

c Spinning 100 times gives a frequency of 19.

Spinning 500 times gives an estimate for the frequency $= 5 \times 19 = 95$.

2 a $3x + 0.05 + 2x + 0.25 + 0.20 + 0.1 = 5x + 0.6$

The relative frequencies have to add up to 1.

So $5x + 0.6 = 1$

$5x = 0.4$

$x = 0.08$

b relative frequency for a score of 1 $= 3 \times 0.08 = 0.24$

c number of times $= 0.20 \times 80 = 16$

The AND and OR rules

1 a P(picture card) $= \frac{12}{52} = \frac{3}{13}$

P(2 picture cards) $= \frac{3}{13} \times \frac{3}{13} = \frac{9}{169}$

b P(ace) $= \frac{4}{52} = \frac{1}{13}$

P(ace and picture card) $= \frac{1}{13} \times \frac{3}{13} = \frac{3}{169}$

c P(queen of hearts and queen of diamonds) $= \frac{1}{52} \times \frac{1}{52}$

$= \frac{1}{2704}$

2 a When an event has no effect on another event, they are said to be independent events. Here the colour of the first marble has no effect on the colour of the second marble.

b P(red) $= \frac{3}{10}$

P(2 reds) $= \frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$

c P(red then blue) $= \frac{3}{10} \times \frac{5}{10} = \frac{15}{100}$

P(blue then red) $= \frac{5}{10} \times \frac{3}{10} = \frac{15}{100}$

P(red and blue) $= \frac{15}{100} + \frac{15}{100} = \frac{30}{100} = \frac{3}{10}$

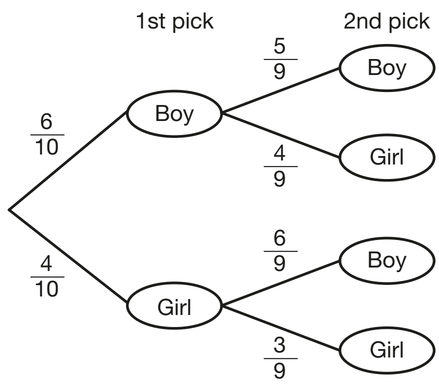
3 a P(homework in all 3 subjects) $= \frac{3}{5} \times \frac{3}{7} \times \frac{1}{4} = \frac{9}{140}$

b P(homework not given in any of the subjects)

$= \frac{2}{5} \times \frac{4}{7} \times \frac{3}{4} = \frac{24}{140} = \frac{6}{35}$

Tree diagrams

1



a P(2 girls) $= \frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$

b P(boy and girl) = P(BG) + P(GB)

$= \left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9}\right)$

$= \frac{8}{15}$

2 **a** number of boys = $0.56 \times 450 = 252$

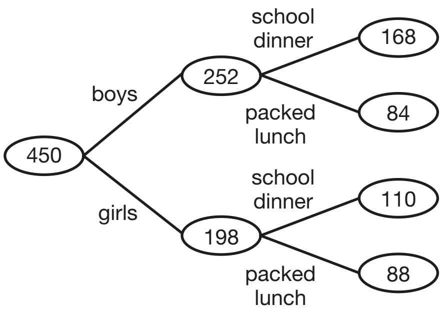
number of girls = $450 - 252 = 198$

number of boys who have a packed lunch = $\frac{1}{3} \times 252 = 84$

number of boys who have a school dinner = $252 - 84 = 168$

number of girls who have a school dinner = $\frac{5}{9} \times 198 = 110$

number of girls who have a packed lunch = $198 - 110 = 88$



b P(girl who has a school dinner) = $\frac{110}{450} = \frac{11}{45}$

c P(boy who has a school dinner) = $\frac{168}{450}$

P(school dinner) = $\frac{110}{450} + \frac{168}{450} = \frac{278}{450} = \frac{139}{225}$

3 **a** Possible ways of one marble of each colour:

RGY RYG GYR GRY YGR YRG

P(RGY) = $\frac{5}{9} \times \frac{3}{8} \times \frac{1}{7} = \frac{15}{504}$

P(RYG) = $\frac{5}{9} \times \frac{1}{8} \times \frac{3}{7} = \frac{15}{504}$

P(one of each colour) = $6 \times \frac{5}{9} \times \frac{3}{8} \times \frac{1}{7} = 6 \times \frac{15}{504} = \frac{90}{504}$

$= \frac{5}{28} = 0.179$ (to 3 d.p.)

b P(no green) = $\frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{120}{504} = \frac{5}{21} = 0.238$ (to 3 d.p.)

c P(all red) = $\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{5}{42}$

P(all green) = $\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} = \frac{1}{84}$

P(same colour) = $\frac{5}{42} + \frac{1}{84} = \frac{11}{84} = 0.131$ (to 3 d.p.)

Venn diagrams and probability

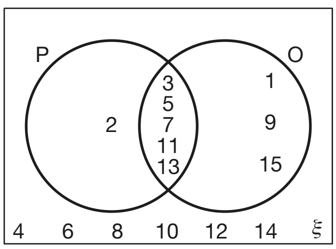
1 **a** 9, 8

b 1, 2, 3, 5, 7, 8, 9, 12, 15

c 1, 3, 4, 10, 12, 15

d 4, 10

2 **a**



b P(number in P ∩ O) = $\frac{5}{15} = \frac{1}{3}$

3 **a** P(team sports) = P(not only individual) = $1 - \frac{15}{100} = \frac{85}{100}$

$= \frac{17}{20}$

Alternatively, you could find the total of all those who play team sports.

b P(student playing large team also plays small team sports)

$= \frac{18}{73}$

Statistics

Sampling

1 **a** Ling's, as he has a larger sample so it is more likely to represent the whole population (i.e. students at the school).

b % of boys in school = $\frac{400}{900} \times 100 = 44.4\%$

number of boys in sample = $\frac{44.4}{100} \times 50 = 22.2 = 22$ (as number has to be an integer)

2 The sample should be taken randomly, with each member of the population having an equal chance of being chosen.

The sample size should be large enough to represent the population, since the larger the sample size, the more accurate the results.

3 **a** 52% males and 48% females.

12% males who smoke and 88% males who do not smoke

10% females who smoke and 90% females who do not smoke

number of females in sample = 48% of 400 = 192

number of females who do not smoke in sample = $0.9 \times 192 = 172.8 = 173$

173 questionnaires needed

b number of males in sample = 52% of 400 = 208

number of males who smoke in sample = $0.12 \times 208 = 24.96 = 25$

25 questionnaires needed

Two-way tables and pie charts

1 **a**

	Coronation Street	EastEnders	Emmerdale	Total
Boys	12	31	20	63
Girls	18	12	7	37
Total	30	43	27	100

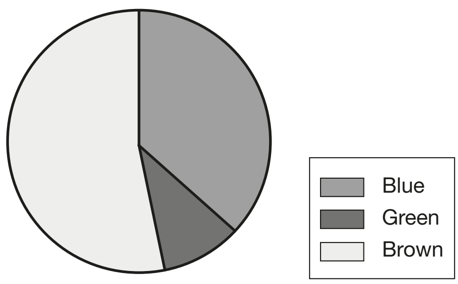
b P(student's favourite is Emmerdale) = $\frac{27}{100}$

c P(girl's favourite is Eastenders) = $\frac{12}{37}$

2 total frequency = 11 + 3 + 16 = 30

angle for one person = $\frac{360}{30} = 12°$

Colour	Frequency	Angle
Blue	11	$11 \times 12 = 132°$
Green	3	$3 \times 12 = 36°$
Brown	16	$16 \times 12 = 192°$



3 **a** The median is the 11th student, working from the smallest value.

median = 11 students

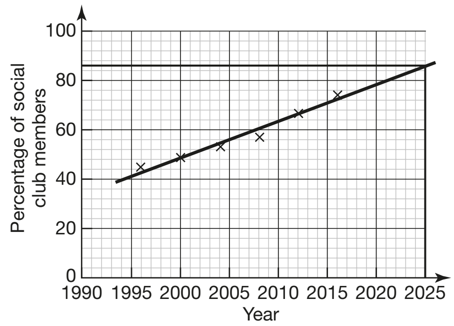
b range = highest value − lowest value = 26 − 3 = 23 students

Line graphs for time series data

1 **a** An upward trend, rising slowly at first, then rising quickly, then continuing to rise more slowly.

b Mode, because it shows that June is the month when the environment offers the largest sample of insects to study. The median would give May, when there are also a lot of insects.

2 **a**



b An upward trend – people are more likely to assume it should be an option.

c Reading from graph: 86% (or a close value).

d Future data may change so that the line of best fit is no longer accurate. Also, the relationship may not be best represented by a straight line, but by a curve.

Averages and spread

1 **a** $5 + 3 + 1 + 4 + 3 + 5 + 0 + 1 + 4 + 1 + 2 + 3 = 32$

b mean $= \frac{32}{12} = 2.7$ (to 1 d.p.)

c Ordering the data gives:

0 1 1 1 2 3 3 3 4 4 5 5

median $= 3$

2 mean mark $= \frac{\text{total marks}}{\text{total number of students}}$

total marks = mean mark × total number of students

For the group of 25 students:

total marks $= 60 \times 25 = 1500$

For the 30 students:

total marks $= 72 \times 30 = 2160$

total marks for both groups $= 1500 + 2160 = 3660$

mean mark for both groups $= \frac{3660}{55} = 66.5$ (to 1 d.p.)

3 **a**

Age (t years)	Frequency	Mid-interval value	Frequency × mid-interval value
$0 < t \le 4$	8	2	16
$4 < t \le 8$	10	6	60
$8 < t \le 12$	16	10	160
$12 < t \le 14$	1	13	13

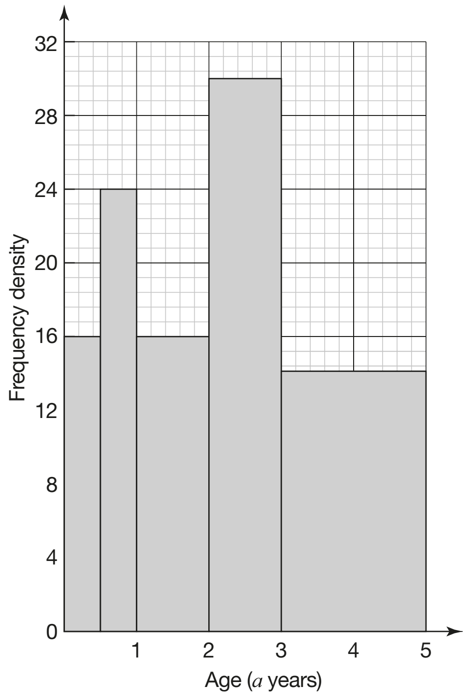
b 35

c estimated mean $= \frac{16 + 60 + 160 + 13}{35} = 7.1$ years (to 2 s.f.)

Histograms

1

Age (a years)	Frequency	Frequency density
$0 < a \le 0.5$	8	$\frac{8}{0.5} = 1$
$0.5 < a \le 1$	12	$\frac{12}{0.5} = 24$
$1 < a \le 2$	16	$\frac{16}{1} = 16$
$2 < a \le 3$	30	$\frac{30}{1} = 30$
$3 < a \le 5$	28	$\frac{28}{2} = 14$



2 The vertical scale of the histogram in your book is incorrect*. It should be marked 0 – 5.

Adding the scale to the vertical axis:

For the class interval $15 < w \le 25$, frequency density $= \frac{\text{frequency}}{\text{class width}} = \frac{30}{10} = 3$

class interval $0 < w \le 5$: frequency $= 5 \times 0.4 = 2$

class interval $25 < w \le 40$: frequency $= 15 \times 0.6 = 9$

Using the axis markings in the book would have given the values 10 and 45 (but these answers would not be correct in relation to the other values in the table).

Wingspan (w cm)	Frequency
$0 < w \le 5$	2
$5 < w \le 10$	6
$10 < w \le 15$	24
$15 < w \le 25$	30
$25 < w \le 40$	9

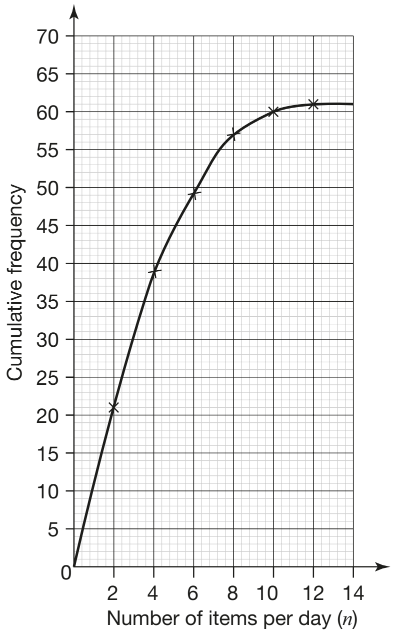
Cumulative frequency graphs

1 **a**

Number of items of junk mail per day (n)	Frequency	Cumulative frequency
$0 < n \le 2$	21	21
$2 < n \le 4$	18	39
$4 < n \le 6$	10	49
$6 < n \le 8$	8	57
$8 < n \le 10$	3	60
$10 < n \le 12$	1	61

*This answer differs from that in the Exam Practice book due to an error in our first edition. It has now been re-checked and corrected.

b



c median = value at frequency of 30.5 = 3 (acceptable values: 2.8 to 3.0)

2 **a** interquartile range = 17.5 − 11 = 6.5 kg

b number of penguins with mass below 10 kg = 8 (from graph)

number of penguins with mass above 10 kg = 40 − 8 = 32

Comparing sets of data

1 **a** Chloe's scores in ascending order:

10 12 12 14 14 15 15 16 17 17 17 17 18 18 19

median = 16

range = 19 − 10 = 9

lower quartile = 14 (halfway between the median and the lowest data value)

upper quartile = 17 (halfway between the median and the highest data value)

interquartile range (IQR) = 17 − 14 = 3

b Sasha's median score is higher (17 compared to 16). The IQR for Sasha is 2 compared to Chloe's 3. The range for Sasha is 5 compared to Chloe's 9. Both these are measures of spread, which means that Sasha's scores are less spread out (i.e. more consistent).

2 **a** **i** Draw a horizontal line, from halfway up the vertical axis across to the curve, then draw a line down to the horizontal axis and read off your result:
median = 138 minutes

ii lower quartile = value at frequency of 25 = 128 minutes

upper quartile = value at frequency of 75 = 150 minutes

interquartile range = 150 − 128 = 22 minutes. (21 or 20 minutes are also acceptable answers, depending on your values for the upper and lower quartiles from the graph.)

b From the box plot for the female runners:

median = 145 minutes

range = 180 − 106 = 74 minutes

lower quartile = 134 minutes

upper quartile = 156 minutes

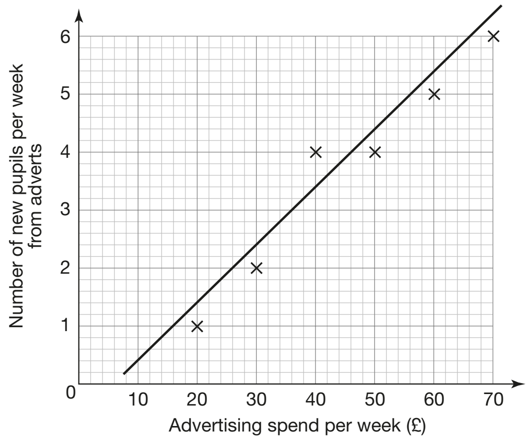
interquartile range = 156 − 134 = 22 minutes

On average the men were faster as the median is lower.

The variation in times were greater for the men as their range was greater, although the spread of the middle half of the data (the interquartile range) was slightly greater for the women.

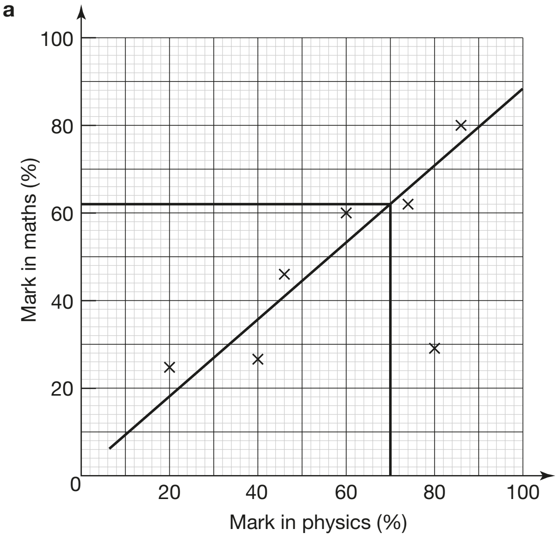
Scatter graphs

1 **a**, **b**



c positive correlation

2 **a**



b Reading from graph: 62%. A suitable range would be 60–63%.

c The data only goes up to a score of 86% in physics, and the score at 80% in physics is an outlier, so the line may not be accurate for higher scores in physics.