Practice paper (calculator 2): full worked solutions

1 $85\% = £258$

$1\% = \frac{£258}{85}$

$100\% = \frac{£258}{85} \times 100$

$= £303.53$

2 $(0.45 \times 0.78)^2 \approx (0.5 \times 0.8)^2 = 0.4^2 = 0.16$

3 **a** $\frac{2x-5}{11} = 3$

$2x - 5 = 33$

$2x = 38$

$x = 19$

b $x^2 - x - 42 = 0$

$(x + 6)(x - 7) = 0$

$x = -6$ or 7

4 **a** gradient $= \frac{k-2}{-2-3} = \frac{k-2}{-5}$

$\frac{k-2}{-5} = \frac{4}{5}$

$5(k - 2) = -20$

$5k - 10 = -20$

$5k = -10$

$k = -2$

b $y - 2 = \frac{4}{5}(x-3)$

$5y - 10 = 4x - 12$

$5y = 4x - 2$ or $y = \frac{4}{5}x - \frac{2}{5}$

5 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives:

$= \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times -6}}{2 \times 1}$

$= \frac{3 \pm \sqrt{9 + 24}}{2}$

$= \frac{3 \pm \sqrt{33}}{2}$

$x = \frac{3 + \sqrt{33}}{2}$ or $\frac{3 - \sqrt{33}}{2}$

$x = 4.37$ or -1.37 (to 2 d.p.)

6 **a** Completing the square:

$y = x^2 - 2x - 3 = (x-1)^2 - 1 - 3 = (x - 1)^2 - 4$

Coordinates of turning point are $(1, -4)$.

b To find where the curve intersects the x-axis:

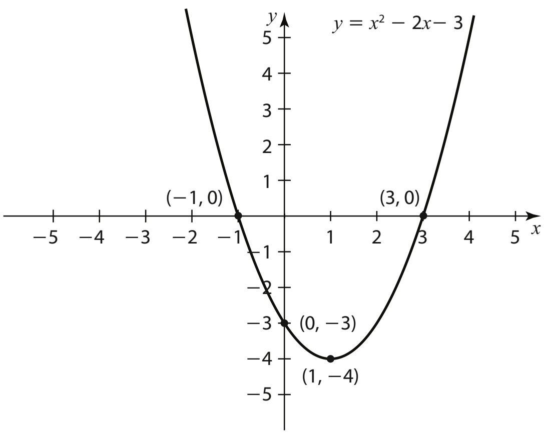
$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x = 3$ or -1

To find where the curve intersects the y-axis:

when $x = 0$, $y = (0)^2 - 2 \times 0 - 3 = -3$



7 **a** Equating expressions for y:

$10x^2 - 5x - 2 = 2x - 3$

$10x^2 - 7x + 1 = 0$

$(5x - 1)(2x - 1) = 0$

$x = \frac{1}{5}$ or $x = \frac{1}{2}$

Substituting $x = \frac{1}{5}$ into $y = 2x - 3$ gives $y = -2\frac{3}{5}$

Substituting $x = \frac{1}{2}$ into $y = 2x - 3$ gives $y = -2$

$x = \frac{1}{5}$ and $y = -2\frac{3}{5}$ or $x = \frac{1}{2}$ and $y = -2$

b the two points where the curve and line intersect

8 $\frac{3\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \frac{(3\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})}$

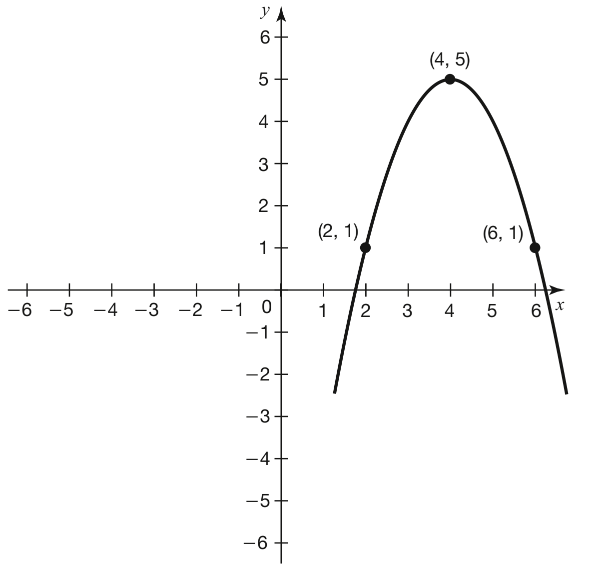
$= \frac{9 + 3\sqrt{6} - \sqrt{6} - 2}{3 + \sqrt{6} - \sqrt{6} - 2} = \frac{7 + 2\sqrt{6}}{1} = 7 + 2\sqrt{6}$

9 $y = f(x)$ to $y = f(x - 1) + 1$ is a translation of 1 unit to the right and 1 unit up (i.e. a translation of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$):

(1, 0) will become (2, 1)

(5, 0) will become (6, 1)

(3, 4) will become (4, 5)



10 **a** $f(-2) = 5 \times (-2)^2 + 4 = 24$

b Let $y = 5x^2 + 4$

$\frac{y-4}{5} = x^2$

$x = \sqrt{\frac{y-4}{5}}$

$f^{-1}(x) = \sqrt{\frac{x-4}{5}}$

c $fg(x) = f(x + 1)$

$= 5(x + 1)^2 + 4$

11 **a** Angles VRP and VRQ are right angles because V is directly above R.

Using Pythagoras' theorem:

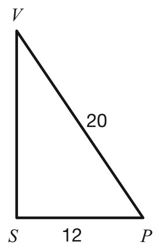
$20^2 = VR^2 + 16^2$

$400 = VR^2 + 256$

$VR^2 = 144$

$VR = 12\,\text{cm}$

b



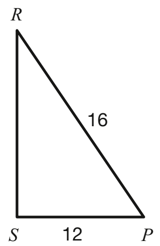
Using Pythagoras' theorem:

$20^2 = VS^2 + (24 \div 2)^2$

$400 = VS^2 + 144$

$VS^2 = 256$

$VS = 16\,\text{cm}$



Using Pythagoras' theorem:

$16^2 = RS^2 + 12^2$

$256 = RS^2 + 144$

$RS^2 = 112$

$RS = 10.6\,\text{cm}$ (to 3 s.f.)

c $\cos\theta = \frac{10.6}{16}$

$\theta = \cos^{-1}\left(\frac{10.6}{16}\right)$

$= 48.5°$ (to 3 s.f.)

12 Let Amy's age $= x$ years.

Amy's mother's age $= 3x$ years.

In 12 years time, Amy will be $x + 12$ and her mother will be $3x + 12$

$x + 12 = \frac{3x + 12}{2}$

$2x + 24 = 3x + 12$

$x = 12$

Amy's mother is 36 years old.

13 a $l = \frac{\theta}{360} \times 2\pi r = \frac{35}{360} \times 2\pi \times 8 = 4.89\,\text{cm}$ (to 2 d.p.)

b $A = \frac{\theta}{360}\pi r^2 = \frac{35}{360}\pi \times 8^2 = 19.55\,\text{cm}^2$ (to 2 d.p.)

14 $\frac{1}{3x^2 + 5x - 2} \div \frac{1}{9x^2 - 1} = \frac{1}{3x^2 + 5x - 2} \times \frac{9x^2 - 1}{1}$

$= \frac{1}{(3x - 1)(x + 2)} \times (3x - 1)(3x + 1)$

$= \frac{3x + 1}{x + 2}$

$a = 3, b = 1, c = 1, d = 2$

15 multiples of 12: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144 ...

multiples of 16: 16, 32, 48, 64, 80, 96, 112, 128, 144 ...

multiples of 18: 18, 36, 54, 72, 90, 108, 126, 144 ...

a The first number to appear in 2 lists is 36.

36 days

b The first number to appear in all 3 lists is 144.

144 days

16 a i $\overrightarrow{AM} = \overrightarrow{AO} + \frac{1}{2}\overrightarrow{OB}$

$= -\mathbf{a} + \frac{\mathbf{b}}{2}$

$= \frac{\mathbf{b}}{2} - \mathbf{a}$

ii $\overrightarrow{AR} = \frac{2}{3}\overrightarrow{AM}$

$= \frac{2}{3}\left(\frac{\mathbf{b}}{2} - \mathbf{a}\right)$

$= \frac{\mathbf{b}}{3} - \frac{2\mathbf{a}}{3}$

b $\overrightarrow{BP} = \overrightarrow{BO} + \overrightarrow{OP} = -\mathbf{b} + \frac{\mathbf{a}}{2} = \frac{\mathbf{a}}{2} - \mathbf{b} = \frac{1}{2}(\mathbf{a} - 2\mathbf{b})$

$\overrightarrow{BR} = \overrightarrow{BA} + \overrightarrow{AR} = \mathbf{a} - \mathbf{b} + \frac{\mathbf{b}}{3} - \frac{2\mathbf{a}}{3} = \frac{\mathbf{a}}{3} - \frac{2\mathbf{b}}{3} = \frac{1}{3}(\mathbf{a} - 2\mathbf{b})$

$\overrightarrow{BP}$ and $\overrightarrow{BR}$ have the same vector part ($\mathbf{a} - 2\mathbf{b}$) and are therefore parallel. As they both pass through point B, points P and R lie on the same straight line.

17 $2.535 \le T < 2.545$

$9.75 \le g < 9.85$

Upper bound for h will be when T has its upper bound and g has its upper bound.

$h = \frac{gT^2}{2} = \frac{9.85 \times 2.545^2}{2} = 31.89935 = 31.90\,\text{m}$ (to 4 s.f.)

Lower bound for h will be when T has its lower bound and g has its lower bound.

$h = \frac{gT^2}{2} = \frac{9.75 \times 2.535^2}{2} = 31.32785 = 31.33\,\text{m}$ (to 4 s.f.)

Using the upper bounds gives an answer of 32 m (to 2 s.f.) but using the lower bounds gives 31 (to 2 s.f.), so h lies between these values. You can say that $31 < h < 32$, or that $h = 30$ (to 1 s.f.)

18 a Let $f(x) = x^3 - 4x + 2$

$f(1) = 1^3 - 4(1) + 2 = -1$

$f(0) = 0^3 - 4(0) + 2 = 2$

As there is a sign change, the root lies between 0 and 1.

b $x^3 - 4x + 2 = 0$

$4x = x^3 + 2$

$x = \frac{x^3 + 2}{4}$

$x = \frac{x^3}{4} + \frac{1}{2}$

c $x_1 = 0.53125$

$x_2 = 0.53748$ (to 5 d.p.)

$x_3 = 0.53882$ (to 5 d.p.)

$x_4 = 0.53911$ (to 5 d.p.) ≈ 0.539 (to 3 d.p.)

19 a 9 parts = 36 so 1 part = 4

number of red counters $= 4 \times 4 = 16$

b i P(1st counter is red) $= \frac{16}{36} = \frac{4}{9}$

P(2nd counter is red) $= \frac{15}{35} = \frac{3}{7}$

P(both counters are red) $= \frac{4}{9} \times \frac{3}{7} = \frac{12}{63} = \frac{4}{21}$

ii P(different colours) = P(red then blue) + P(blue then red)

$= \left(\frac{4}{9} \times \frac{4}{7}\right) + \left(\frac{5}{9} \times \frac{16}{35}\right)$

$= \frac{32}{63}$