Algebra Skills Edexcel Maths Higher GCSE 9–1

Full worked solutions

Revision answers

Simple algebraic techniques p.7

1 a Formula
b Identity
c Expression
d Identity
e Equation

2 a $15x^2 - 4x + x^2 + 9x - x - 6x^2 = 10x^2 + 4x$
b $7a + 5b - b - 4a - 5b = 3a - b$
c $8yx + 5x^2 + 2xy - 8x^2 = -3x^2 + 10xy$ (or $10xy - 3x^2$)
d $x^3 + 3x - 5 + 2x^3 - 4x = 3x^3 - x - 5$

3 $P = I^2R = \left(\frac{2}{3}\right)^2 \times 36 = \frac{4}{9} \times 36 = 16$

4 $v = u + at = 20 + (-8)(2) = 20 - 16 = 4$

Removing brackets p.10

1 a $2x + 8$
b $63x + 21$
c $-1 + x$ or $x - 1$
d $3x^2 - x$
e $3x^2 + 3x$
f $20x^2 - 8x$

2 a $2(x + 3) + 3(x + 2) = 2x + 6 + 3x + 6 = 5x + 12$
b $6(x + 4) - 3(x - 7) = 6x + 24 - 3x + 21 = 3x + 45$
c $3x^2 + x + x^2 + x = 4x^2 + 2x$
d $3x^2 - 4x - 6x + 8 = 3x^2 - 10x + 8$

3 a $(t + 3)(t + 5) = t^2 + 5t + 3t + 15 = t^2 + 8t + 15$
b $(x - 3)(x + 3) = x^2 + 3x - 3x - 9 = x^2 - 9$
c $(2y + 9)(3y + 7) = 6y^2 + 14y + 27y + 63$
$= 6y^2 + 41y + 63$
d $(2x - 1)^2 = (2x - 1)(2x - 1) = 4x^2 - 2x - 2x + 1$
$= 4x^2 - 4x + 1$

4 a $(x + 7)(x + 2)(2x + 3) = (x^2 + 9x + 14)(2x + 3)$
$= 2x^3 + 21x^2 + 55x + 42$
b $(2x - 1)(3x - 2)(4x - 3) = (6x^2 - 7x + 2)(4x - 3)$
$= 24x^3 - 28x^2 + 8x - 18x^2 + 21x - 6$
$= 24x^3 - 46x^2 + 29x - 6$

Factorising p.13

1 a $24t + 18 = 6(4t + 3)$
b $9a - 2ab = a(9 - 2b)$
c $5xy + 15yz = 5y(x + 3z)$
d $24x^3y^2 + 6xy^2 = 6xy^2(4x^2 + 1)$

2 a $(x + 7)(x + 3)$
b $(x + 5)(x - 3)$
c To get 6, use factors 2 and 3, and to get 10 use factors 2 and 5. This gives $2x \times 2 = 4x$ and $3x \times 5 = 15x$, total $19x$; so solution is $(2x + 5)(3x + 2)$
d Difference of two squares. Factorises to $(2x + 7)(2x - 7)$

3 $\frac{1}{x-7} - \frac{x+10}{2x^2 - 11x - 21} = \frac{1}{x-7} - \frac{x+10}{(2x+3)(x-7)}$

Factorise the denominator of the 2nd fraction.

Make both denominators the same.

$= \frac{2x+3}{(x-7)(2x+3)} - \frac{x+10}{(2x+3)(x-7)}$

Combine into one fraction and simplify

$= \frac{2x + 3 - x - 10}{(x-7)(2x+3)}$

$= \frac{x-7}{(x-7)(2x+3)}$

$= \frac{1}{2x+3}$

Changing the subject of a formula p.15

1 a $A = \pi r^2$ $\quad \frac{A}{\pi} = r^2$
$r = \sqrt{\frac{A}{\pi}}$
b $A = 4\pi r^2$
$\frac{A}{4\pi} = r^2$
$r = \sqrt{\frac{A}{4\pi}}$
c $V = \frac{4}{3}\pi r^3$
$3V = 4\pi r^3$
$\frac{3V}{4\pi} = r^3$
$r = \sqrt[3]{\frac{3V}{4\pi}}$

2 a $y = mx + c$ $\quad (c)$ — Subtract mx from both sides.
$c = y - mx$
b $v = u + at$ $\quad (u)$ — Subtract at from both sides.
$u = v - at$
c $v = u + at$ $\quad (a)$ — Subtract u from both sides.
$v - u = at$ — Divide both sides by t.
$a = \frac{v-u}{t}$

d $v^2 = 2as$ (s) ← Divide both sides by $2a$.

$s = \frac{v^2}{2a}$

e $v^2 = u^2 + 2as$ (u) ← Subtract $2as$ from both sides.

$v^2 - 2as = u^2$ ← Square root both sides.

$u = \sqrt{v^2 - 2as}$

f $s = \frac{1}{2}(u + v)t$ (t) ← Multiply both sides by 2.

$2s = (u + v)t$ ← Divide both sides by $(u + v)$.

$t = \frac{2s}{u + v}$

Solving linear equations p.17

1 a $x - 7 = -4$

$x = -4 + 7 = 3$

b $9x = 27$

$x = 27 \div 9 = 3$

c $\frac{x}{5} = 4$

$x = 4 \times 5 = 20$

2 a $3x + 1 = 16$

$3x = 15$

$x = 5$

b $\frac{2x}{3} = 12$

$2x = 36$

$x = 18$

c $\frac{3x}{5} + 4 = 16$

$\frac{3x}{5} = 12$

$3x = 60$

$x = 20$

3 a $5(1 - x) = 15$

$5 - 5x = 15$

$-5x = 10$

$x = -2$

b $2m - 4 = m - 3$

$m - 4 = -3$

$m = 1$

c $9(4x - 3) = 3(2x + 3)$

$36x - 27 = 6x + 9$

$30x - 27 = 9$

$30x = 36$

$x = \frac{36}{30} = \frac{6}{5}, 1\frac{1}{5}$ or 1.2

Always cancel fractions so that they are in their lowest terms. Here both top and bottom can be divided by 6.

Solving quadratic equations using factorisation p.19

1 a $(x + 3)(x + 2) = 0$ giving $x = -2$ or -3

b $(x + 3)(x - 4) = 0$ giving $x = -3$ or 4

c $(2x + 7)(x + 5) = 0$ giving $x = -\frac{7}{2}$ or $x = -5$

2 a Area of triangle = $\frac{1}{2}$ × base × height

$\frac{1}{2}(2x + 3)(x + 4) = 9$

$2x^2 + 11x + 12 = 18$

$2x^2 + 11x - 6 = 0$

b $2x^2 + 11x - 6 = 0$

$(2x - 1)(x + 6) = 0$

So $x = \frac{1}{2}$ or $x = -6$

Since x represents a height, only the positive value is valid.

$x = \frac{1}{2}$

c $x = 0.5$, so base is $2 \times 0.5 + 3 = 4$ cm and height is $0.5 + 4 = 4.5$ cm

3 By Pythagoras' theorem

$(x + 1)^2 + (x + 8)^2 = 13^2$

$x^2 + 2x + 1 + x^2 + 16x + 64 = 169$

$2x^2 + 18x - 104 = 0$

Dividing by 2 gives

$x^2 + 9x - 52 = 0$

$(x - 4)(x + 13) = 0$

so $x = 4$ or $x = -13$

(Disregard $x = -13$ as x is a length.)

Hence, $x = 4$ cm

(This also means the sides of the triangle are 5, 12 and 13 cm.)

Solving quadratic equations using the formula p.21

1 Comparing the equation given, with $ax^2 + bx + c$ gives $a = 2$, $b = -1$ and $c = -7$.

Substituting these values into the quadratic equation formula gives:

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-7)}}{2(2)}$

$= \frac{1 \pm \sqrt{57}}{4}$

$= \frac{1 + 7.550}{4}$ or $\frac{1 - 7.550}{4}$ (to 4 s.f.)

$x = 2.14$ or -1.64 (to 3 s.f.)

2 a $\frac{2x + 3}{x + 2} = 3x + 1$

$2x + 3 = (x + 2)(3x + 1)$

$2x + 3 = 3x^2 + x + 6x + 2$

$0 = 3x^2 + 5x - 1$

or $3x^2 + 5x - 1 = 0$

b Comparing the equation given, with $ax^2 + bx + c = 0$ gives $a = 3$, $b = 5$ and $c = -1$

Substituting these values into the quadratic equation formula gives:

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{-5 \pm \sqrt{25 + 12}}{6} = \frac{-5 \pm \sqrt{37}}{6} = \frac{-5 + \sqrt{37}}{6}$ or $\frac{-5 - \sqrt{37}}{6}$

Hence $x = 0.18$ or -1.85 (2 d.p.)

Solving simultaneous equations p.24

1 a Firstly write the second equation so that in both equations the x value and the numerical value are aligned.

$y = 3x - 7$ (1)

$y = -2x + 3$ (2)

Notice that the coefficient of y (the number multiplying y, i.e. 1) is the same for both equations. We can eliminate y by subtracting equation (2) from equation (1).

Subtracting (1) – (2) we obtain

$0 = 5x - 10$

$5x = 10$

$x = 2$

Substituting $x = 2$ into equation (1) we obtain

$y = 3(2) - 7$

$= 6 - 7$

$= -1$

Checking by substituting $x = 2$ into equation (2) we obtain

$y = -2x + 3$

$= -2(2) + 3$

$= -1$

Hence solutions are $x = 2$ and $y = -1$.

b $y = 2x - 6$ (1)

$y = -3x + 14$ (2)

Subtracting (1) – (2) we obtain

$0 = 5x - 20$

$x = 4$

$y = 2 \times 4 - 6$ (1)

$y = 2.$

2 Equating expressions for y gives

$10x^2 - 5x - 2 = 2x - 3$

$10x^2 - 7x + 1 = 0$

Factorising this quadratic gives

$(5x - 1)(2x - 1) = 0$

Hence $x = \frac{1}{5}$ or $x = \frac{1}{2}$

Substituting $x = \frac{1}{5}$ into $y = 2x - 3$ gives

$y = -2\frac{3}{5}$

Substituting $x = \frac{1}{2}$ into $y = 2x - 3$ gives

$y = -2$

Hence $x = \frac{1}{5}$ and $y = -2\frac{3}{5}$ or $x = \frac{1}{2}$ and $y = -2$

3 Equating the y values gives

$x^2 + 5x - 4 = 6x + 2$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or -2

When $x = 3$, $y = 6 \times 3 + 2 = 20$

When $x = -2$, $y = 6 \times (-2) + 2 = -10$

Points are (3, 20) and (–2, –10)

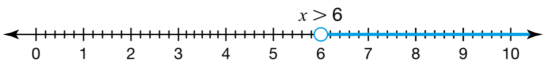
Solving inequalities p.28

1 **a** $1 - 2x < -11$

$-2x < -12$

$x > 6$

Subtract 1 from both sides. Divide both sides by –2 and reverse sign.



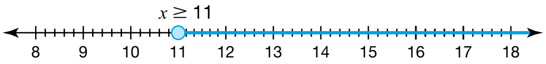
$\{x: x > 6\}$

b $2x - 7 \geq 15$

$2x \geq 22$

$x \geq 11$

Add 7 to both sides. Divide both sides by 2.



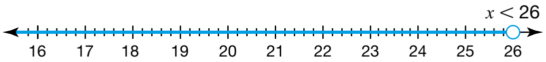
$\{x: x \geq 11\}$

c $\frac{x-5}{3} < 7$

$x - 5 < 21$

$x < 26$

Multiply both sides by 3. Add 5 to both sides.



$\{x: x < 26\}$

2 **a** $2x - 4 > x + 6$

$x - 4 > 6$

$x > 10$

b $4 + x < 6 - 4x$

$4 < 6 - 5x$

$-2 < -5x$

$\frac{-2}{-5} > x$

$x < 0.4$ or $\frac{2}{5}$

c $2x + 9 \geq 5(x - 3)$

$2x + 9 \geq 5x - 15$

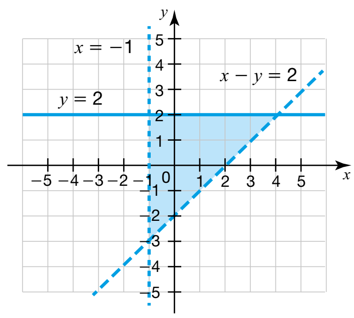
$9 \geq 3x - 15$

$24 \geq 3x$

$8 \geq x$

or $x \leq 8$

3 **a**



b (0, 2), (0,1), (0, 0), (0, –1), (1, 2), (1, 1), (1, 0), (2, 2), (2, 1), (3, 2)

4 $x^2 > 3x + 10$

$x^2 - 3x - 10 > 0$

$(x - 5)(x + 2) > 0$

$x < -2$ and $x > 5$

Problem solving using algebra p.30

1 Let the larger number = x and the smaller number = y.

$x + y = 77$

$x - y = 25$

Adding gives $2x = 102$ which gives $x = 51$ so y must be 26.

2 Let the number added = x

$\frac{15 + x}{31 + x} = \frac{5}{6}$

$6(15 + x) = 5(31 + x)$

$90 + 6x = 155 + 5x$

$x = 65$

Check the answer $\frac{15 + 65}{31 + 65} = \frac{80}{96} = \frac{5}{6}$

3 Perimeter: $2x + 2y = 24$ so $x + y = 12$ (1)

Area: $xy = 27$ (2)

From equation (1) $y = 12 - x$

Substitute into equation (2):

$x(12 - x) = 27$

So, $12x - x^2 = 27$

Hence, $x^2 - 12x + 27 = 0$

Factorising gives $(x - 3)(x - 9) = 0$

So $x = 3$ or $x = 9$

Substituting each of these values into equation (1) we have

$3 + y = 12$ or $9 + y = 12$, giving $y = 9$ or $y = 3$.

Hence, length = 9 cm and width = 3 cm.

Use of functions p.32

1 **a** $f(0) = \frac{1}{0-1} = -1$

b $f(-\frac{1}{2}) = \frac{1}{-\frac{1}{2} - 1} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$

c Let $y = \frac{1}{x-1}$

$y(x - 1) = 1$

$xy - y = 1$

$xy = y + 1$

$x = \frac{y+1}{y}$

$f^{-1}(x) = \frac{x+1}{x}$

2 **a** $fg(x) = \sqrt{((x+4)^2 - 9)}$

$= \sqrt{x^2 + 8x + 16 - 9}$

$= \sqrt{x^2 + 8x + 7}$

b $gf(x) = \sqrt{(x^2 - 9)} + 4$

c $gf(3) = \sqrt{(3^2 - 9)} + 4$

$= 4$

Iterative methods p.34

1 $x_0 = 1.5$

$x_1 = 1.5182945$

$x_2 = 1.5209353$

$x_3 = 1.5213157$

$x_4 = 1.5213705 \approx 1.521$ (correct to three decimal places)

Check value of $x^3 - x - 2$ for $x = 1.5205, 1.5215$

x	$f(x)$
1.5205	−0.005225
1.5215	0.0007151

Since there is a change of sign, $a = 1.521$ is correct to three decimal places.

Equation of a straight line p.38

1 **a** $2y = 4x - 5$

$y = 2x - \frac{5}{2}$

Comparing this to $y = mx + c$ we have gradient, $m = 2$

b Gradient $= -\frac{1}{m} = -\frac{1}{2}$

c $y = -\frac{1}{2}x + 5$ or $2y = -x + 10$

2 $y - y_1 = m(x - x_1)$ where $m = 3$ and $(x_1, y_1) = (2, 3)$.

$y - 3 = 3(x - 2)$

$y - 3 = 3x - 6$

$y = 3x - 3$

3 $y - y_1 = m(x - x_1)$ where $m = 2$ and $(x_1, y_1) = (-1, 0)$

$y - 0 = 2(x - (-1))$

$y = 2(x + 1)$

$y = 2x + 2$

$-y + 2x + 2 = 0$ (or $2x - y + 2 = 0$)

4 **a** Gradient $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{6 - (-2)} = \frac{4}{8} = \frac{1}{2}$

b $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2+6}{2}, \frac{0+4}{2}\right) = (2, 2)$

c **i** Gradient = −2 (i.e invert $\frac{1}{2}$ and change the sign)

ii $y - y_1 = m(x - x_1)$

$y - 2 = -2(x - 2)$

$y - 2 = -2x + 4$

$y = -2x + 6$

Quadratic graphs p.42

1 **a** $2x^2 - 12x + 1 = 2\left[x^2 - 6x + \frac{1}{2}\right]$

$= 2\left[(x-3)^2 - 9 + \frac{1}{2}\right]$

$= 2\left[(x-3)^2 - \frac{17}{2}\right]$

$= 2(x-3)^2 - 17$

b **i** Turning point is at (3, −17)

ii At the roots,

$2(x-3)^2 - 17 = 0$

$2(x-3)^2 = 17$

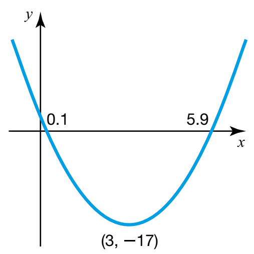
$(x-3)^2 = \frac{17}{2}$

$x - 3 = \sqrt{\frac{17}{2}}$

$x = \sqrt{\frac{17}{2}} + 3$

Roots are $x = 0.1$ and $x = 5.9$ (1 d.p.)

c



2 **a** $y = (x + 1)(x - 5)$ or $y = x^2 - 4x - 5$

b $y = -(x - 2)(x - 7)$ or $y = -x^2 + 9x - 14$

3 **a** $x^2 + 12x - 16 = (x + 6)^2 - 36 - 16$

$= (x + 6)^2 - 52$

b Turning point is at (−6, −52)

Recognising and sketching graphs of functions p.46

1 **a** B

b F

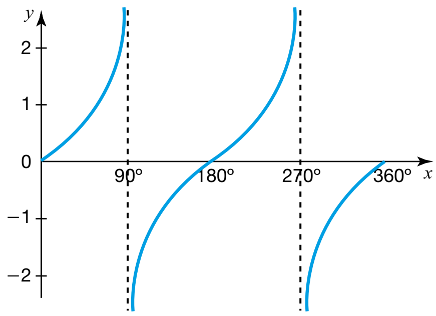
c E

d A

e D

f C

2 **a**



b Read up from 60° to the graph, then read across until you hit the graph again.

$x = 240°$

3 **a** A

b G

c F

d E

Translations and reflections of functions p.49

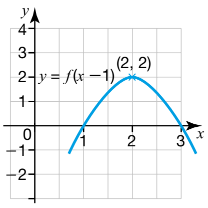
1 **a** (3, 5) (i.e. a movement of one unit to the right)

b (–1, 5) (i.e. a movement of three units to the left)

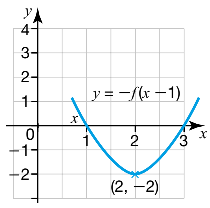
c (2, –5) (i.e. a reflection in the x-axis)

d (–2, 5) (i.e. a reflection in the y-axis)

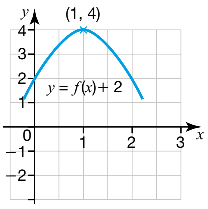
2 **a**



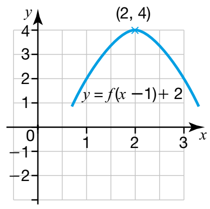
c



b



d



Equation of a circle and tangent to a circle p.51

1 **a** Centre is (0, 0)

b radius = $\sqrt{49} = 7$

2 **a** $x^2 + y^2 = 100$

b Gradient of radius to (8, 6) = $\frac{6}{8} = \frac{3}{4}$

Gradient of tangent = $-\frac{4}{3}$

c $y - y_1 = m(x - x_1)$

$y - 6 = -\frac{4}{3}(x - 8)$

$y = -\frac{4}{3}x + 16\frac{2}{3}$ or $3y = -4x + 50$

Real-life graphs p.54

1 **a** 08:00 to 09:00 is 1 hour (h), which is 1 unit on x axis.

Average speed = gradient = $\frac{2.5}{0.5} = 5$ km/h

b 15 mins = 0.25 hours

c Average speed = gradient between 09:30 and 09:45

$= \frac{6}{0.25} = 24$ km/h

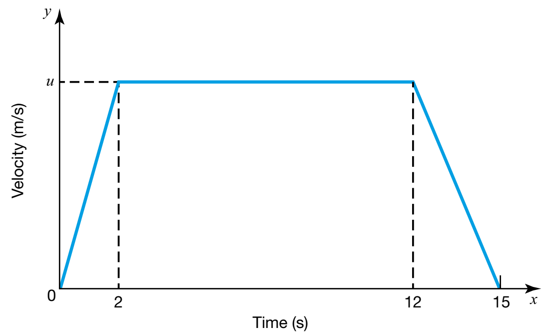
2

NAIL IT!

When drawing a velocity-time graph, ensure that the axes are labelled with quantities and units.

Any values and letters for quantities that need to be found should be labelled on the graph.

a



b Total distance travelled = Area under the velocity-time graph

Use the formula for the area of a trapezium:

Distance = $\frac{1}{2}(15 + 10) \times u$

$= 12.5u$

Use the formula for the area of a trapezium.

The total distance travelled = 50 m

Hence $50 = 12.5u$

$u = 4$ m/s

c Velocity = 4 m/s and time for deceleration = 3 s

Deceleration = gradient = $\frac{4}{3} = 1.33$ m/s^2

Since deceleration is negative acceleration, a positive answer is appropriate.

Generating sequences p.56

1 **a** 17: sequence goes up by 3

b 3.0: sequence goes up by 0.2

c −12: sequence goes down by 3

d 432: last term is multiplied by 6

e $\frac{1}{48}$: last term is multiplied by $\frac{1}{2}$

f $-\frac{1}{16}$: last term is multiplied by $-\frac{1}{2}$

2 Second term is $(-4)^2 + 1 = 17$ and third term is $17^2 + 1 = 290$

Second term is 17, third term is 290.

3 Reverse the process: to find the preceding term, subtract 1 and halve.

Second term is $(12 - 1) \div 2 = \frac{11}{2} = 5.5$

First term is $(5.5 - 1) \div 2 = \frac{4.5}{2} = 2.25$

First term is 2.25, second term is 5.5

The nth term p.58

1 a When $n = 1$, $50 - 3(1) = 47$

When $n = 2$, $50 - 3(2) = 44$

When $n = 3$, $50 - 3(3) = 41$

First three terms are 47, 44, 41

b Use the nth term formula to find the value of n when the nth term = 34

$50 - 3n = 34$

$3n = 16$

$n = 16 \div 3$

The value of n is not an integer so 34 is not a number in the sequence.

c Use the nth term formula to find the value of n when the nth term is less than zero (i.e. negative).

$50 - 3n < 0$ (subtracting 50 from both sides)

$-3n < -50$ (dividing both sides and reversing the inequality sign)

$n > \frac{50}{3}$

$n > 16\frac{2}{3}$

As n has to be an integer, its lowest possible value is $n = 17$.

Check that you get a negative term when $n = 17$ is put back into the nth term formula.

17th term $= 50 - 3 \times 17 = 50 - 51 = -1$

2 a The first four terms are: $2 \times 3^1, 2 \times 3^2, 2 \times 3^3, 2 \times 3^4$

$= 6, 18, 54, 162$

b As the nth term formula is 2×3^n both 2 and 3 are factors, so 6 must also be a factor.

3 a Common difference between terms = 2 so formula will start with $2n$.

When $n = 1$, you need to subtract 3 from $2n$ to get an answer of -1.

Therefore nth term $= 2n - 3$

b $59 = 2x - 3$

$2x = 62$

$x = 31$

4

	4,		17,		38,		67
First differences		13		21		29	
Second differences			8		8		

As a second difference is needed before a constant difference is found, there is an n^2 term in the nth term. The number in front of this n^2 will be $\frac{8}{2} = 4$.

So first part of the nth term will be $4n^2$.

n	1	2	3	4
Term	4	17	38	67
$4n^2$	4	16	36	64
Term – $4n^2$	0	1	2	3

Use this set of information to work out the linear part of the sequence (the part with an n term and a number).

Difference 1 1 1

This means that the linear sequence will start with n.

When $n = 2$, 'Term – $4n^2$' is 1, not 2, so if n is in the term you also need to subtract 1.

This makes the linear part of the sequence $n - 1$.

Check it with a different value of n. When $n = 3$, $n - 1$ equals 2. This is the correct value for 'Term – $4n^2$'.

Combining the terms gives nth term $= 4n^2 + n - 1$

Arguments and proofs p.60

1 a $2n$ is always even as it has 2 as a factor. Adding 1 to an even number always gives an odd number. The statement is true.

b $x^2 - 9 = 0$ so $x^2 = 9$ and $x = \sqrt{9} = \pm 3$

The statement is false.

c n could be a decimal such as 4.25 so squaring it would not give an integer.

The statement is false.

d If n was 1, or a fraction smaller than 1, this would not be true.

The statement is false.

2 Let the consecutive integers be n, $n + 1$, $n + 2$ and $n + 3$, where n is an integer that can be either odd or even.

Sum of the integers $= n + n + 1 + n + 2 + n + 3$

$= 4n + 6 = 2(2n + 3)$

As 2 is a factor of this expression, the sum of four consecutive integers must be a multiple of 2, and therefore even.

3 Let the consecutive integers be x, $x + 1$ and $x + 2$, where x is an integer that can be either odd or even.

Sum of the integers $= x + x + 1 + x + 2 = 3x + 3$

$= 3(x + 1)$

As 3 is a factor of this expression, the sum of three consecutive integers must be a multiple of 3.

4 a The numerator is larger than the denominator so the fraction will always be greater than 1. The statement is false.

b As a is larger than b, squaring a will result in a larger number than squaring b. Hence $a^2 > b^2$ so the statement is false.

c The square root of a number can have two values, one positive and the other negative so, this statement is false.

Review it! p.61

1 a $-3(3x - 4) = -9x + 12$

b $4x + 3(x + 2) - (x + 2) = 4x + 3x + 6 - x - 2$

$= 6x + 4$

c $(x + 3)(2x - 1)(3x + 5) = (2x^2 + 5x - 3)(3x + 5)$

$= 6x^3 + 25x^2 + 16x - 15$

2 a $2x^2 + 7x - 4 = (2x - 1)(x + 4)$

b $2x^2 + 7x - 4 = 0$

$x = \frac{1}{2}$ or $x = -4$

3 a $(2x^2y)^3 = 8x^6y^3$

b $2x^{-3} \times 3x^4 = 6x$

c $\frac{15a^3b}{3a^3b^2} = \frac{5}{b}$

4 $3x + 2y = 8$ (1)

$5x + y = 11$ (2)

(2) × 2: $10x + 2y = 22$ (3)

(3) – (1): $7x = 14$

$x = 2$

Substitute into (2) to find y

$5 \times 2 + y = 11$

$y = 11 - 10$

$y = 1$

5 a $\frac{3}{x+7} = \frac{2-x}{x+1}$

$3(x + 1) = (2 - x)(x + 7)$

$3x + 3 = 2x + 14 - x^2 - 7x$

$3x + 3 = -x^2 - 5x + 14$

$x^2 + 8x - 11 = 0$

b $x^2 + 8x - 11 = 0$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4 \times 1 \times (-11)}}{2 \times 1}$

$x = \frac{-8 \pm \sqrt{108}}{2}$

$x = \frac{-8 \pm 6\sqrt{3}}{2}$

$x = -4 + 3\sqrt{3}$ or $x = -4 - 3\sqrt{3}$

So $x = 1.20$ or $x = -9.20$ (to 2 d.p.)

6 $\frac{3y - x}{z} = ax + 2 \quad (x)$

$3y - x = z(ax + 2)$

$3y - x = axz + 2z$

$3y - 2z = axz + x$

$3y - 2z = x(az + 1)$

$x = \frac{3y - 2z}{az + 1}$

7 a $y = \frac{x}{3} + 5$

$3y = x + 15$

$x = 3y - 15$

Now replace x with $f^{-1}(x)$ and y with x.

$f^{-1}(x) = 3x - 15$ or $f^{-1}(x) = 3(x - 5)$

b $fg(x) = \frac{(2(x)^2 + k)}{3} + 5$

So $fg(2) = \frac{(8 + k)}{3} + 5$

We know that $fg(2) = 10$

So $\frac{(8 + k)}{3} + 5 = 10$

$(8 + k) + 15 = 30$

$8 + k = 15$

$k = 7$

8 a Let $n = 1$: $30 - 4 \times 1 = 26$

Let $n = 2$: $30 - 4 \times 2 = 22$

Let $n = 3$: $30 - 4 \times 3 = 18$

First three terms are 26, 22, 18.

b $30 - 4n < 0$

$-4n < -30$

$n > \frac{-30}{-4}$

$n > 7.5$

n must be an integer, so the lowest possible value of n is $n = 8$

Therefore the first negative term of the sequence is: $30 - 4 \times 8 = -2$

9 $x = 4, y = 3$

$(4)^2 + (3)^2 = 16 + 9 = 25$

So $x^2 + y^2 > 21$

Hence the point (4, 3) lies outside the circle.

10 $(\sqrt{x} + \sqrt{9y})(\sqrt{x} - 3\sqrt{y})$

Simplify terms inside the brackets if possible

$(\sqrt{x} + 3\sqrt{y})(\sqrt{x} - 3\sqrt{y}) = x + 3\sqrt{xy} - 3\sqrt{xy} - 9y$

$= x - 9y$

11 a $2x^2 + 8x + 1 = 2(x^2 + 4x) + 1$

$= 2(x + 2)^2 - 8 + 1$

$= 2(x + 2)^2 - 7$

b i Turning point is (–2, –7).

ii For $2x^2 + 8x + 1 = 0$

$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 2 \times 1}}{2 \times 2}$

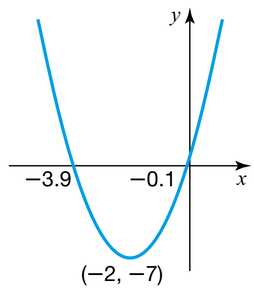
$x = \frac{-8 \pm \sqrt{56}}{4}$

$x = \frac{-8 \pm 2\sqrt{14}}{4}$

$x = \frac{-4 \pm \sqrt{14}}{2}$

So roots are at $x = -3.9$ and $x = -0.1$ (1 d.p.)

c



12 Perimeter of $ABCD = 2 \times (4x + (2x - 3)) = 12x - 6$

Perimeter of $EFG = 2x - 1 + x + 9 + 5x - 2 = 8x + 6$

Equate the perimeters to find x

$12x - 6 = 8x + 6$

$4x = 12$

$x = 3$

The height of the triangle, $EF = 2 \times 3 - 1 = 5$ cm

The base of the triangle, $EG = 3 + 9 = 12$ cm

Area of the triangle $= \frac{1}{2} \times 12 \times 5 = 30\,\text{cm}^2$

13 Side AB is parallel to side CD, so $k = 5$.

Gradient of $BD = \frac{5 - (-2)}{-1 - (-2)} = \frac{7}{1} = 7$

Using point B (–2, –2)

$y - (-2) = 7(x - (-2))$

$y + 2 = 7x + 14$

Equation of BD is $y = 7x + 12$

14 $x^2 + y^2 = 4 \quad (1)$

$2y - x = 2 \quad (2)$

Rearrange (2) for y: $y = \frac{1}{2}x + 1 \quad (3)$

Substitute (3) into (1):

$x^2 + \left(\frac{1}{2}x + 1\right)^2 = 4$

$x^2 + \frac{x^2}{4} + x + 1 = 4$

$5x^2 + 4x + 4 = 16$

$5x^2 + 4x - 12 = 0$

$(5x - 6)(x + 2) = 0$

$x = \frac{6}{5} = 1.2$ or $x = -2$

Substitute into (2) to find y

So $x = \frac{6}{5}, y = \frac{8}{5}$ or $x = -2, y = 0$

Exam practice answers

Simple algebraic techniques p.64

1 **a** formula **c** expression **e** formula
b identity **d** identity

2 $4x + 3x \times 2x - 3x = 4x + 6x^2 - 3x = x + 6x^2$

3 $y^3 - y = (1)^3 - 1 = 0$ so $y = 1$ is correct.
$y^3 - y = (-1)^3 - (-1) = -1 + 1 = 0$ so $y = -1$ is correct.

4 **a** $6x - (-4x) = 6x + 4x = 10x$
b $x^2 - 2x - 4x + 3x^2 = 4x^2 - 6x$
c $(-2x)^2 + 6x \times 3x - 4x^2 = 4x^2 + 18x^2 - 4x^2 = 18x^2$

5 **a** $s = \frac{3^2 - 1^2}{2 \times 2} = \frac{8}{4} = 2$
b $s = \frac{(-4)^2 - 3^2}{2 \times 4} = \frac{7}{8}$
c $s = \frac{5^2 - (-2)^2}{2 \times (-7)} = \frac{21}{-14} = -\frac{3}{2}$

Removing brackets p.65

1 **a** $8(3x - 7) = 8 \times 3x - 8 \times 7$
$= 24x - 56$
b $-3(2x - 4) = -3 \times 2x - 3 \times (-4)$
$= -6x + 12$

2 **a** $3(2x - 1) - 3(x - 4) = 6x - 3 - 3x + 12$
$= 3x + 9$
b $4y(2x + 1) + 6(x - y) = 8xy + 4y + 6x - 6y$
$= 8xy + 6x - 2y$
c $5ab(2a - b) = 10a^2b - 5ab^2$
d $x^2y^3(2x + 3y) = 2x^3y^3 + 3x^2y^4$

3 **a** $(m - 3)(m + 8) = m^2 + 8m - 3m - 24$
$= m^2 + 5m - 24$
b $(4x - 1)(2x + 7) = 8x^2 + 28x - 2x - 7$
$= 8x^2 + 26x - 7$
c $(3x - 1)^2 = (3x - 1)(3x - 1)$
$= 9x^2 - 3x - 3x + 1$
$= 9x^2 - 6x + 1$
d $(2x + y)(3x - y) = 6x^2 - 2xy + 3xy - y^2$
$= 6x^2 + xy - y^2$

4 **a** $(x + 5)(x + 2) = x^2 + 2x + 5x + 10$
$= x^2 + 7x + 10$
b $(x + 4)(x - 4) = x^2 - 4x + 4x - 16$
$= x^2 - 16$
c $(x - 7)(x + 1) = x^2 + x - 7x - 7$
$= x^2 - 6x - 7$
d $(3x + 1)(5x + 3) = 15x^2 + 9x + 5x + 3$
$= 15x^2 + 14x + 3$

5 **a** $(x + 3)(x - 1)(x + 4) = (x^2 - x + 3x - 3)(x + 4)$
$= (x^2 + 2x - 3)(x + 4)$
$= x^3 + 4x^2 + 2x^2 + 8x - 3x - 12$
$= x^3 + 6x^2 + 5x - 12$
b $(3x - 4)(2x - 5)(3x + 1) = (6x^2 - 15x - 8x + 20)(3x + 1)$
$= (6x^2 - 23x + 20)(3x + 1)$
$= 18x^3 + 6x^2 - 69x^2 - 23x + 60x + 20$
$= 18x^3 - 63x^2 + 37x + 20$

Factorising p.66

1 **a** $25x^2 - 5xy = 5x(5x - y)$
b $4\pi r^2 + 6\pi x = 2\pi(2r^2 + 3x)$
c $6a^3b^2 + 12ab^2 = 6ab^2(a^2 + 2)$

2 **a** $9x^2 - 1 = (3x + 1)(3x - 1)$
b $16x^2 - 4 = (4x + 2)(4x - 2)$
$= 4(2x + 1)(2x - 1)$

3 **a** $a^2 + 12a + 32 = (a + 4)(a + 8)$
b $p^2 - 10p + 24 = (p - 6)(p - 4)$

4 **a** $a^2 + 12a = a(a + 12)$
b $b^2 - 9 = (b + 3)(b - 3)$
c $x^2 - 11x + 30 = (x - 5)(x - 6)$

5 **a** $3x^2 + 20x + 32 = (3x + 8)(x + 4)$
b $3x^2 + 10x - 13 = (3x + 13)(x - 1)$
c $2x^2 - x - 10 = (2x - 5)(x + 2)$

6 $\frac{x + 15}{2x^2 - 3x - 9} + \frac{3}{2x + 3} = \frac{x + 15}{(2x + 3)(x - 3)} + \frac{3}{(2x + 3)}$
$= \frac{x + 15 + 3(x - 3)}{(2x + 3)(x - 3)}$
$= \frac{4x + 6}{(2x + 3)(x - 3)}$
$= \frac{2(2x + 3)}{(2x + 3)(x - 3)}$
$= \frac{2}{(x - 3)}$

7 $\frac{1}{8x^2 - 2x - 1} \div \frac{1}{4x^2 - 4x + 1} = \frac{1}{8x^2 - 2x - 1} \times (4x^2 - 4x + 1)$
$= \frac{1}{(4x + 1)(2x - 1)} \times (2x - 1)(2x - 1)$
$= \frac{2x - 1}{4x + 1}$

Changing the subject of a formula p.68

1 $PV = nRT$
$T = \frac{PV}{nR}$

2 $2y + 4x - 1 = 0$
$2y = 1 - 4x$
$y = \frac{1 - 4x}{2}$

3 $v = u + at$
$at = v - u$
$a = \frac{(v - u)}{t}$

4 $y = \frac{x}{5} - m$
$\frac{x}{5} = y + m$
$x = 5(y + m)$

5 $E = \frac{1}{2}mv^2$
$v^2 = \frac{2E}{m}$
$v = \sqrt{\frac{2E}{m}}$

6 **a** $V = \frac{1}{3}\pi r^2 h$
$r^2 = \frac{3V}{\pi h}$
$r = \sqrt{\frac{3V}{\pi h}}$
b $r = \sqrt{\frac{3 \times 100}{\pi \times 8}}$
$= 3.45$ cm (to 2 d.p.)

7 **a** $y = 3x - 9$

$3x = y + 9$

$x = \frac{y+9}{3}$

b $x = \frac{3+9}{4}$

$= 4$

8 $3y - x = ax + 2$

$3y - x - 2 = ax$

$3y - 2 = ax + x$

$3y - 2 = x(a + 1)$

$x = \frac{3y-2}{a+1}$

9 **a** $c^2 = \frac{(16a^2b^4c^2)^{\frac{1}{2}}}{4a^2b}$

$c^2 = \frac{4ab^2c}{4a^2b}$

$c^2 = \frac{bc}{a}$

$c = \frac{b}{a}$

b upper bound of $a = 2.85$ lower bound of $a = 2.75$

upper bound of $b = 3.25$ lower bound of $b = 3.15$

upper bound for $c = \frac{\text{upper bound for } b}{\text{lower bound for } a} = \frac{3.25}{2.75} = 1.18$ (to 3 s.f.)

lower bound for $c = \frac{\text{lower bound for } b}{\text{upper bound for } a} = \frac{3.15}{2.85} = 1.11 =$ (to 3 s.f.)

Solving linear equations p.70

1 **a** $2x + 11 = 25$

$2x = 14$

$x = 7$

b $3x - 5 = 10$

$3x = 15$

$x = 5$

c $15x = 60$

$x = 4$

d $\frac{x}{4} = 8$

$x = 32$

e $\frac{4x}{5} = 20$

$4x = 100$

$x = 25$

f $\frac{2x}{3} = -6$

$2x = -18$

$x = -9$

g $5 - x = 7$

$5 = 7 + x$

$-2 = x$

$x = -2$

h $\frac{x}{7} - 9 = 3$

$\frac{x}{7} = 12$

$x = 84$

2 $5x - 1 = 2x + 1$

$5x = 2x + 2$

$3x = 2$

$x = \frac{2}{3}$

3 **a** $\frac{1}{4}(2x-1) = 3(2x-1)$

$2x - 1 = 12(2x - 1)$

$2x - 1 = 24x - 12$

$11 = 22x$

$x = \frac{11}{22}$

$x = \frac{1}{2}$

b $5(3x + 1) = 2(5x - 3) + 3$

$15x + 5 = 10x - 6 + 3$

$15x + 5 = 10x - 3$

$5x = -8$

$x = -\frac{8}{5}$

Solving quadratic equations using factorisation p.71

1 **a** $x^2 - 7x + 12 = (x - 3)(x - 4)$

b $x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

So $x - 3 = 0$ or $x - 4 = 0$, giving $x = 3$ or $x = 4$

2 **a** $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

b $2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

So $2x - 1 = 0$ or $x + 3 = 0$, giving $x = \frac{1}{2}$ or $x = -3$

3 $x^2 - 3x - 20 = x - 8$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

So $x + 2 = 0$ or $x - 6 = 0$, giving $x = -2$ or $x = 6$

4 **a** $x(x - 8) - 7 = x(5 - x)$

$x^2 - 8x - 7 = 5x - x^2$

$2x^2 - 13x - 7 = 0$

b $2x^2 - 13x - 7 = 0$

$(2x + 1)(x - 7) = 0$

So $2x + 1 = 0$ or $x - 7 = 0$, giving $x = -\frac{1}{2}$ or $x = 7$

5 area of trapezium $= \frac{1}{2}(a + b)h$

$= \frac{1}{2}(x + 4 + x + 8)x$

$= \frac{1}{2}(2x + 12)x$

$= (x + 6)x$

$= x^2 + 6x$

area = 16 cm² so $x^2 + 6x = 16$

$x^2 + 6x - 16 = 0$

$(x + 8)(x - 2) = 0$

Solving gives $x = -8$ or $x = 2$

$x = -8$ is impossible as x is the height and so cannot be negative.

Hence $x = 2$ cm

Solving quadratic equations using the formula p.73

1 **a** $\frac{3}{x+7} = \frac{2-x}{x+1}$

$3(x + 1) = (2 - x)(x + 7)$

$3x + 3 = 2x + 14 - x^2 - 7x$

$3x + 3 = -x^2 - 5x + 14$

$x^2 + 8x - 11 = 0$

b $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times (-11)}}{2 \times 1}$

$= \frac{-8 \pm \sqrt{64 + 44}}{2}$

$= \frac{-8 \pm \sqrt{108}}{2}$

$= \frac{-8 + \sqrt{108}}{2}$ or $\frac{-8 - \sqrt{108}}{2}$

$= 1.1962$ or -9.1962

$x = 1.20$ or -9.20 (to 2 d.p.)

2 area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2}(3x + 1)(2x + 3)$

$= \frac{1}{2}(6x^2 + 9x + 2x + 3)$

$= \frac{1}{2}(6x^2 + 11x + 3)$

$= 3x^2 + 5.5x + 1.5$

area = 40, so $3x^2 + 5.5x + 1.5 = 40$

Hence, $3x^2 + 5.5x - 38.5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-5.5 \pm \sqrt{5.5^2 - 4 \times 3 \times (-38.5)}}{2 \times 3}$

$= \frac{-5.5 \pm \sqrt{492.25}}{6}$

$= \frac{-5.5 + \sqrt{492.25}}{6}$ = or $\frac{-5.5 - \sqrt{492.25}}{6}$

= 2.78 or −4.61 (to 2 d.p.)

$x = -4.61$ would give negative lengths, which are impossible.

$x = 2.78$ cm (to 2 d.p.)

3 $x^2 - 2x - 9 = x - 8$

$x^2 - 3x - 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$

$= \frac{3 \pm \sqrt{9 + 4}}{2}$

$= \frac{3 \pm \sqrt{13}}{2}$

$= \frac{3 + \sqrt{13}}{2}$ = or $\frac{3 - \sqrt{13}}{2}$

$x = 3.30$ or -0.30 (to 2 d.p.)

Solving simultaneous equations p.74

1 $2x - 3y = -5$ (1)

$5x + 2y = 16$ (2)

Equation (1) × 2 and equation (2) × 3 gives:

$4x - 6y = -10$ (3)

$15x + 6y = 48$ (4)

Equation (3) + equation (4) gives:

$19x = 38$

$x = 2$

Substituting $x = 2$ into equation (1):

$2 \times 2 - 3y = -5$

$4 - 3y = -5$

$3y = 9$

$y = 3$

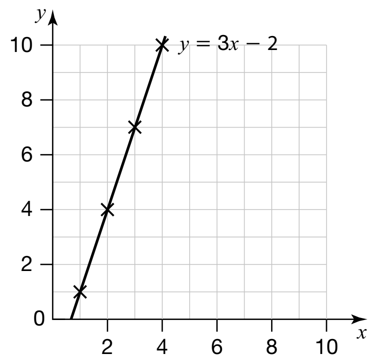
Checking by substituting $x = 2$ and $y = 3$ into equation (2) gives:

$5 \times 2 + 2 \times 3 = 10 + 6 = 16$

$x = 2$ and $y = 3$

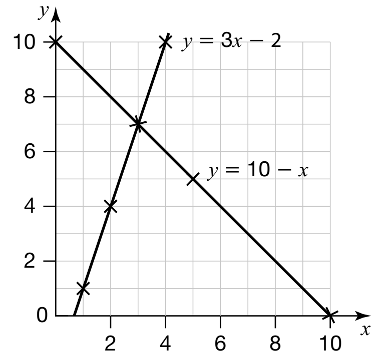
2 **a** Table of values for plotting graph of $y = 3x - 2$:

x	1	2	3	4
y	1	4	7	10



b Table of values for plotting graph of $y = 10 - x$:

x	0	5	10
y	10	5	0



The graphs intersect at (3, 7).

$x = 3$, $y = 7$

3 $x - y = 3$ (1)

$x^2 + y^2 = 9$ (2)

Rearrange equation (1) as $y = x - 3$.

Substitute in equation (2):

$x^2 + (x - 3)^2 = 9$

$x^2 + x^2 - 6x + 9 = 9$

$2x^2 - 6x = 0$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0$ or $x = 3$

Substituting into equation (1) gives:

$x = 0$, $y = -3$ or $x = 3$, $y = 0$

Solving inequalities p.75

1 **a** $\frac{x + 5}{4} \geq -1$

$x + 5 \geq -4$

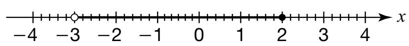
$x \geq -9$

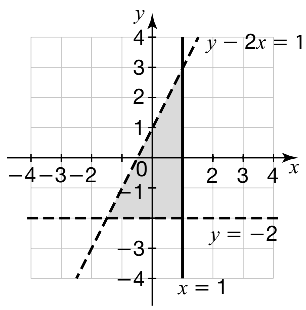
b $3x - 4 > 4x + 8$

$-x - 4 > 8$

$-x > 12$

$x < -12$

2 

3 **a** 

b Coordinates of points that lie in the shaded region or on the solid line:

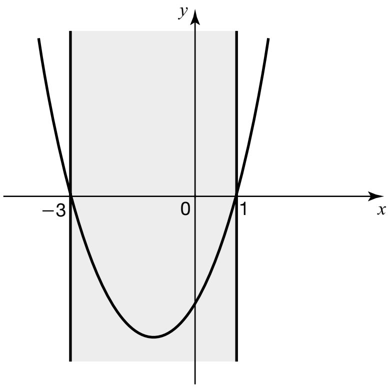
(1, 2), (1, 1), (1, 0), (1, −1), (0, 0), (0, −1), 4 $x^2 + 2x \le 3$

$x^2 + 2x - 3 \le 0$

Solving $x^2 + 2x - 3 = 0$:

$(x - 1)(x + 3) = 0$, giving $x = 1$ and $x = -3$

Sketch of the curve $y = x^2 + 2x - 3$:



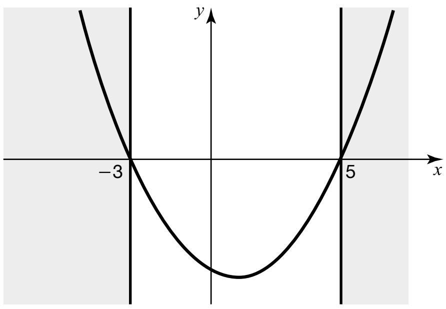
From graph, $x^2 + 2x - 3 \le 0$ when:

$-3 \le x \le 1$

5 Solving $x^2 - 2x - 15 = 0$:

$(x - 5)(x + 3) = 0$, giving $x = 5$ and $x = -3$

Sketch of the curve $y = x^2 - 2x - 15$:



From graph, $x^2 - 2x - 15 > 0$ when:

$x < -3$ and $5 < x$

Problem solving using algebra p.76

1 Let width = x

length = $x + 1$

perimeter = $x + x + 1 + x + x + 1 = 4x + 2$

perimeter = 26 so

$4x + 2 = 26$

$4x = 24$

$x = 6$

width = 6 m and length = 7 m

area = 6 × 7 = 42 m²

2 Let cost of adult ticket = x and cost of child ticket = y

$2x + 5y = 35$ (1)

$3x + 4y = 38.5$ (2)

Equation (1) × 3 and equation (2) × 2 gives:

$6x + 15y = 105$ (3)

$6x + 8y = 77$ (4)

Equation (3) – equation (4) gives:

$7y = 28$

$y = 4$

Substituting $y = 4$ into equation (1) gives:

$2x + 20 = 35$

$2x = 15$

$x = 7.5$

cost of adult ticket = £7.50

cost of child ticket = £4

3 **a** Let Rachel be x years and Hannah be y years.

$xy = 63$

$(x + 2)(y + 2) = 99$

$xy + 2x + 2y + 4 = 99$

Substitute for xy:

$63 + 2x + 2y + 4 = 99$

$2x + 2y = 32$

$x + y = 16$

The sum of their ages is 16 years.

b $y = x - 2$

$x + x - 2 = x + y$

$2x - 2 = 16$

$2x = 18$

$x = 9$

Rachel is 9 years old.

Use of functions p.77

1 **a** $f(3) = 5 \times 3 + 4 = 19$

b Set $f(x) = -1$

$5x + 4 = -1$

$5x = -5$

$x = -1$

2 **a** $fg(x) = f(g(x)) = f(x - 6) = (x - 6)^2$

b $gf(x) = g(f(x)) = g(x^2) = x^2 - 6$

3 **a** $f(5) = \sqrt{5 + 4} = \sqrt{9} = 3$ or -3

b $gf(x) = 2(\sqrt{x + 4})^2 - 3$

$= 2(x + 4) - 3$

$= 2x + 5$

4 Let $y = 5x^2 + 3$

$x = \sqrt{\frac{y - 3}{5}}$

$f^{-1}(x) = \sqrt{\frac{x - 3}{5}}$

Iterative methods p.78

1 Let $f(x) = 2x^3 - 2x + 1$

$f(-1) = 2(-1)^3 - 2(-1) + 1 = 1$

$f(-1.5) = 2(-1.5)^3 - 2(-1.5) + 1 = -2.75$

There is a sign change of $f(x)$, so there is a solution between $x = -1$ and $x = -1.5$.

2 $x_1 = (x_0)^3 + \frac{1}{9} = (0.1)^3 + \frac{1}{9} = 0\ 0.1121111111$

$x_2 = (x_1)^3 + \frac{1}{9} = (0.1121111111)^3 + \frac{1}{9} = 0.1125202246$

$x_3 = (x_2)^3 + \frac{1}{9} = (0.1125202246)^3 + \frac{1}{9} = 0.1125357073$

3 **a** $x_1 = 1.5182945$

$x_2 = 1.5209353$

$x_3 = 1.5213157$

$x_4 = 1.5213705 \approx 1.521$ (to 3 d.p.)

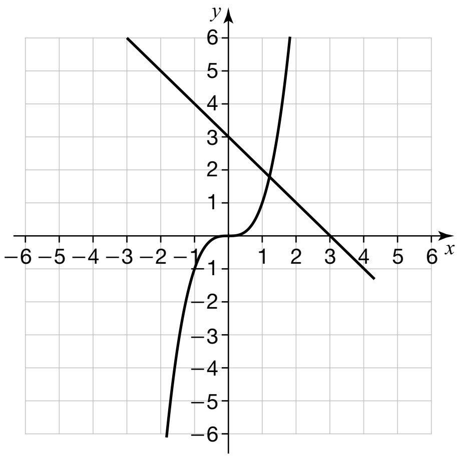
b Checking value of $x^3 - x - 2$ for $x = 1.5205, 1.5215$:

When $x = 1.5205$ f(1.5205) = −0.0052

$x = 1.5215$ f(1.5215) = 0.0007

Since there is a change of sign, the root is 1.521 correct to 3 decimal places.

4 a i



ii There is a real root of $x^3 + x - 3 = 0$ where the graphs of $y = x^3$ and $y = 3 - x$ intersect. The graphs intersect once so there is one real root of the equation $x^3 + x - 3 = 0$.

b $x_1 = 1.216440399$

$x_2 = 1.212725591$

$x_3 = 1.213566964$

$x_4 = 1.213376503$

$x_5 = 1.213419623$

$x_6 = 1.213409861 = 1.2134$ (to 4 d.p.)

Equation of a straight line p.80

1 Comparing the equation with the equation of a straight line, $y = mx + c$:

$y = -2x + 3$

gradient of line, $m = -2$ (line has a negative gradient).

intercept on the y-axis, $c = 3$

Correct line is A.

2 a gradient, $m = \frac{\text{change in } y\text{–values}}{\text{change in } x\text{–values}} = -\frac{4}{3}$

b gradient of CD, $m = \frac{1-5}{5-(-3)} = \frac{-4}{8} = -\frac{1}{2}$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 1 = -\frac{1}{2}(x - 5)$

$y = -\frac{1}{2}x + \frac{7}{2}$ or $x + 2y = 7$

c M is at $\left(\frac{-3+5}{2}, \frac{5+1}{2}\right) = (1, 3)$

gradient of perpendicular to $CD = \frac{-1}{-\frac{1}{2}} = 2$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 3 = 2(x - 1)$

$y = 2x + 1$

3 P is the point (x, y).

gradient of line $OP = \frac{y}{x} = 3$, so $y = 3x$

Using Pythagoras' theorem:

$OP^2 = x^2 + y^2$

$12^2 = x^2 + y^2$

$12^2 = x^2 + (3x)^2$

$144 = x^2 + 9x^2$

$144 = 10x^2$

$14.4 = x^2$

$x = 3.8$ (to 1 d.p.)

$y = 3x$

$= 3 \times 3.8$

$= 11.4$

P is the point (3.8, 11.4) (to 1 d.p.).

Quadratic graphs p.82

1 a $x^2 + 4x + 1 = 0$

$(x + 2)^2 - 4 + 1 = 0$

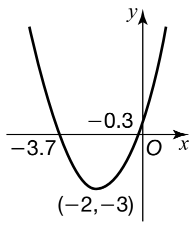
$(x + 2)^2 = 3$

$x + 2 = \pm\sqrt{3}$

$x = \sqrt{3} - 2$ or $-\sqrt{3} - 2$

$x = -0.3$ or -3.7 (to 1 d.p.)

b $x^2 + 4x + 1 = (x + 2)^2 - 3$, so turning point is at (−2, −3).



2 $5x^2 - 20x + 10 = 5[x^2 - 4x + 2]$

$= 5[(x - 2)^2 - 4 + 2]$

$= 5(x - 2)^2 - 10$

$a = 5$, $b = -2$ and $c = -10$

3 $2x^2 + 12x + 3 = 2[x^2 + 6x + \frac{3}{2}]$

$= 2[(x + 3)^2 - 9 + \frac{3}{2}]$

$= 2[(x + 3)^2 - \frac{15}{2}]$

$= 2(x + 3)^2 - 15$

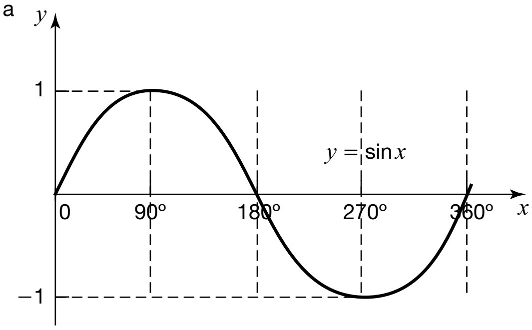
$a = 2$, $b = 3$ and $c = -15$

Recognising and sketching graphs of functions p.83

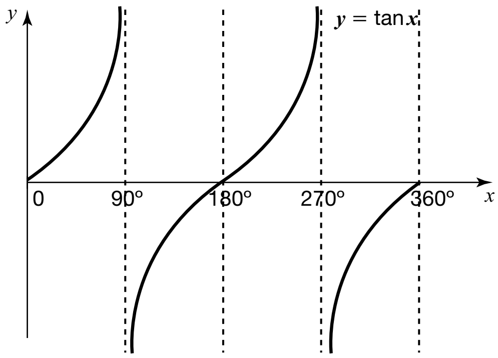
1

Equation	Graph
$y = x^2$	B
$y = 2^x$	D
$y = \sin x°$	E
$y = x^3$	C
$y = x^2 - 6x + 8$	A
$y = \cos x°$	F

2 a

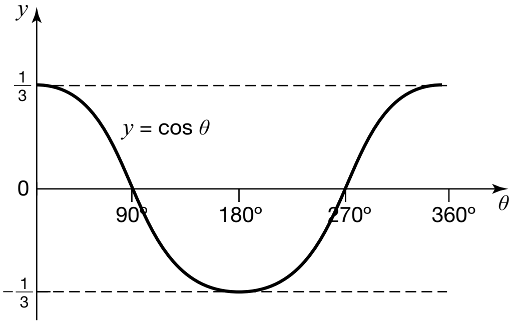


b



3 $3\cos\theta = 1$

$\cos\theta = \frac{1}{3}$



$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.5°$ (to 1 d.p.)

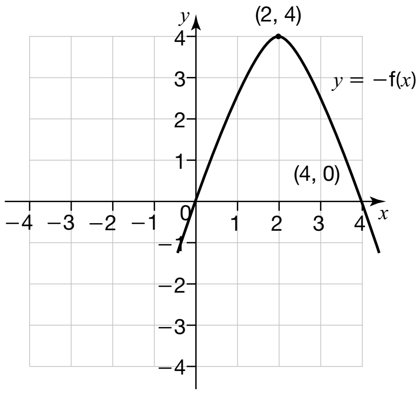
From the graph, $\cos\theta$ is also $\frac{1}{3}$ when $\theta = 360 - 70.5 = 289.5°$

$\theta = 70.5°$ or $289.5°$ (to 1 d.p.)

Translations and reflections of functions p.84

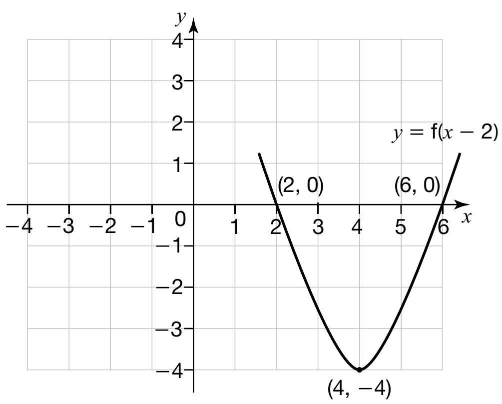
1 **a** $y = -\mathrm{f}(x)$ is a reflection in the x–axis of the graph $y = \mathrm{f}(x)$

The points on the x–axis stay in the same place and the turning point at (2, −4) is reflected to become a turning point at (2, 4).



b $y = \mathrm{f}(x - 2)$ is a translation by $\begin{pmatrix}2\\0\end{pmatrix}$ of the graph $y = \mathrm{f}(x)$.

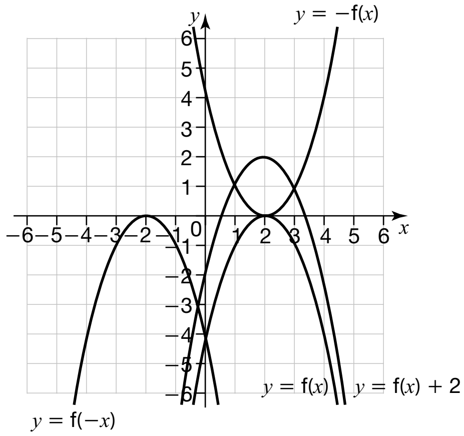
The y-coordinates stay the same but the x-coordinates are shifted to the right by 2 units.



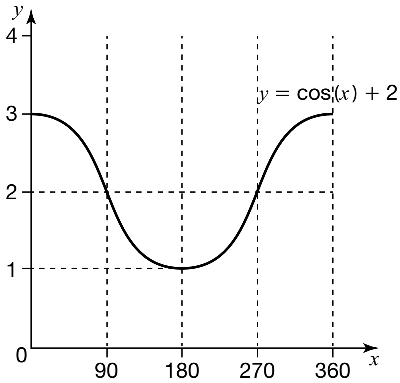
2 **a** $y = -\mathrm{f}(x)$: reflection in the x–axis.

b $y = \mathrm{f}(x) + 2$: translation of 2 units vertically upwards.

c $y = \mathrm{f}(-x)$: reflection in the y-axis.



3 The cosine graph is shifted two units in the vertical direction, i.e. a translation of $\begin{pmatrix}0\\2\end{pmatrix}$.



Equation of a circle and tangent to a circle p.85

1 **a** $x^2 + y^2 = 25$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{25} = 5$

b $x^2 + y^2 - 49 = 0$

$x^2 + y^2 = 49$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{49} = 7$

c $4x^2 + 4y^2 = 16$

$x^2 + y^2 = 4$

This equation is in the form $x^2 + y^2 = r^2$.

$r = \sqrt{4} = 2$

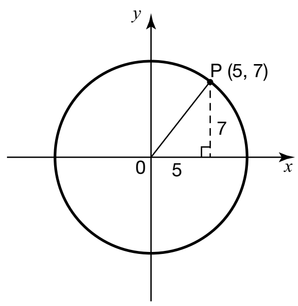
2 Radius of the circle $= \sqrt{21} = 4.58$

Distance of the point (4, 3) from the centre of the circle (0, 0)

$= \sqrt{16 + 9} = \sqrt{25} = 5$

This distance is greater than the radius of the circle, so the point lies outside the circle.

3 **a**



Using Pythagoras' theorem:

$OP^2 = 5^2 + 7^2$

$= 25 + 49$

$= 74$

OP = radius of the circle = $\sqrt{74}$

b equation of a circle, radius r, centre the origin: $x^2 + y^2 = r^2$

$x^2 + y^2 = 74$

c gradient of line $OP = \frac{7}{5}$

gradient of the tangent at $P = -\frac{5}{7}$

Substituting in $y - y_1 = m(x - x_1)$:

$y - 7 = -\frac{5}{7}(x - 5)$

$7y - 49 = -5x + 25$

$7y = -5x + 74$

$y = -\frac{5}{7}x + \frac{74}{7}$

Real-life graphs p.86

1 **a** acceleration = gradient of line = $\frac{10}{10}$ = 1 m/s²

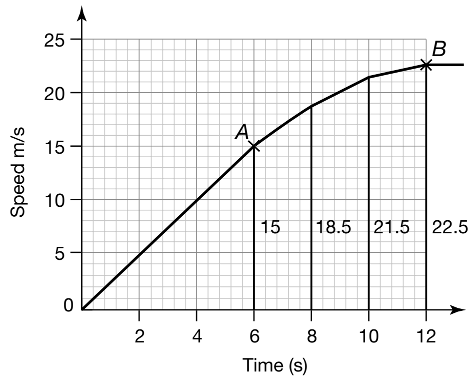
b total distance travelled = area under the velocity–time graph

$= \frac{1}{2}(30 + 15) \times 10 = 225$ m

2 **a** The graph is a straight line starting at the origin, so this represents constant acceleration from rest of $\frac{15}{6}$ = 2.5 m/s².

b The gradient decreases to zero, so the acceleration decreases to zero.

c area = area of 3 trapeziums



area $= \frac{1}{2}(15 + 18.5) \times 2 + \frac{1}{2}(18.5+21.5) \times 2 + \frac{1}{2}(21.5 + 22.5) \times 2$

$= 117.5$

distance travelled between A and B = 118 m (to nearest integer)

d It will be a slight underestimate, as the curve is always above the straight lines forming the tops of the trapeziums.

Generating sequences p.87

1 **a** **i** $\frac{1}{2}$ (term-to-term rule is divide by 2)

ii 243 (term-to-term rule is multiply by 3)

iii 21 (term-to-term rule is add 4)

b 4th term – 1st term = –12 – 27 = –39

common difference = –39 ÷ 3 = –13

missing terms are 14, 1

2 3rd term = 2 × 1 – 5 = –3

4th term = 2 × –3 – 5 = –11

3 **a** 25, 36 (square numbers)

b 15, 21 (triangular numbers)

c 8, 13 (Fibonacci numbers)

The *n*th term p.88

1 **a**

n	1		2		3		4
Term	2		6		10		14
Difference		4		4		4	

nth term starts with $4n$

$4n$	4	8	12	16
Term – $4n$	–2	–2	–2	–2

nth term = $4n - 2$

b nth term = $4n - 2 = 2(2n - 2)$

2 is a factor, so the nth term is divisible by 2 and therefore is even.

c Let nth term = 236

$4n - 2 = 236$

$4n = 238$

$n = 59.5$

n is not an integer, so 236 is not a term in the sequence.

2 **a** 2nd term = $9 - 2^2 = 5$

b 20th term = $9 - 20^2 = 9 - 400 = -391$

c n^2 is always positive, so the largest value value $9 - n^2$ can take is 8 when $n = 1$. All values of n above 1 will make $9 - n^2$ smaller than 8. So 10 cannot be a term.

3

Term	1		1		3		7		13
First difference		0		2		4		6	
Second difference			2		2		2		

The formula starts n^2.

n	1		2		3		4		
Term	1		1		3		7		13
n^2	1		4		9		16		25
Term – n^2	0		–3		–6		–9		–12
Difference		–3		–3		–3		–3	

The linear part of the sequence starts with $-3n$.

$-3n$	–3	–6	–9	–12	–15
Linear term – $(-3n)$	3	3	3	3	3

nth term = $n^2 - 3n + 3$

Checking:

When $n = 1$, term is $1^2 - 3 \times 1 + 3 = 1$

$n = 2$, term is $2^2 - 3 \times 2 + 3 = 1$

$n = 3$, term is $3^2 - 3 \times 3 + 3 = 3$

Arguments and proofs p.89

1 The only even prime number is 2.

Hence, statement is false because 2 is a prime number that is not odd.

2 **a** true: $n = 1$ is the smallest positive integer and this would give the smallest value of $2n + 1$, which is 3.

b true: 3 is a factor of $3(n + 1)$ so $3(n + 1)$ must be a multiple of 3.

c false: $2n$ is always even and subtracting 3 will give an odd number.

3 Let first number = x so next number = $x + 1$

Sum of consecutive integers = $x + x + 1 = 2x + 1$

Regardless of whether x is odd or even, $2x$ will always be even as it is divisible by 2.

Hence $2x + 1$ will always be odd.

4 $(2x - 1)^2 - (x - 2)^2 = 4x^2 - 4x + 1 - (x^2 - 4x + 4)$

$= 4x^2 - 4x + 1 - x^2 + 4x - 4$

$= 3x^2 - 3$

$= 3(x^2 - 1)$

The 3 outside the brackets shows that the result is a multiple of 3 for all integer values of x.

5 Let two consecutive odd numbers be $2n - 1$ and $2n + 1$.

$(2n + 1)^2 - (2n - 1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1)$

$= 8n$

Since 8 is a factor of $8n$, the difference between the squares of two consecutive odd numbers is always a multiple of 8.

(If you used $2n + 1$ and $2n + 3$ for the two consecutive odd numbers, difference of squares = $8n + 8 = 8(n + 1)$.)